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# SLOW RESONANT MHD WAVES IN ONE-DIMENSIONAL MAGNETIC PLASMAS WITH ANISOTROPIC VISCOSITY AND THERMAL CONDUCTIVITY

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## ABSTRACT

Slow resonant MHD waves are studied in a compressible plasma with strongly anisotropic viscosity and thermal conductivity. It is shown that anisotropic viscosity and/or thermal conductivity removes the slow singularity which is present in the linear ideal MHD equations. Simple analytical solutions to the linear dissipative MHD equations are obtained which are valid in the dissipative layer and in two overlap regions to the left and the right of the dissipative layer. Asymptotic analysis of the dissipative solutions enables us to obtain connection formulae specifying the variations or jumps of the different wave quantities across the dissipative layer. These connection formulae coincide with those obtained previously for plasmas with isotropic viscosity and finite electrical conductivity. The thickness of the dissipative layer is inversely proportional to the Reynolds number, in contrast to the case of isotropic dissipative coefficients, where it is inversely proportional to the cube root of the Reynolds number. The behavior of the perturbations in the dissipative layer is described in terms of elementary functions of complex argument.

*Subject heading:* conduction — MHD — plasmas — Sun: corona — waves

## 1. INTRODUCTION

Resonances are an intrinsic property of magnetohydrodynamic (MHD) waves propagating in inhomogeneous ideal plasmas. Dissipation (e.g., viscosity, finite electrical conductivity, and thermal conductivity) removes these resonances. However, in many solar and astrophysical situations dissipation is weak and the behavior of the resonant MHD waves deviates from that described by ideal MHD only in a narrow dissipative layer that embraces the ideal resonant position. In the dissipative layer the spatial gradients of the wave quantities are very large. This creates the possibility of strong damping of resonant MHD waves even in weakly dissipative plasmas. As a matter of fact, the damping rate of the resonant MHD waves is independent of the actual values of the dissipative coefficients in the limit of weak dissipation.

The property that resonant MHD waves are strongly damped in weakly dissipative plasmas has attracted a lot of attention in plasma physics and solar physics. Resonant MHD waves were first studied as a means for the supplementary heating of fusion plasmas (see, e.g., Uberoi 1972; Tataronis & Grossmann 1973; Grossmann & Tataronis 1973; Chen & Hasegawa 1974a; Hasegawa & Chen 1976). In the Earth's magnetosphere resonant MHD wave coupling is believed to establish low-frequency pulsations. In magnetospheric physics resonant MHD waves were considered by Lanzerotti et al. (1973), Southwood (1974), Chen & Hasegawa (1974b, 1974c), Southwood & Hughes (1983), Inhester (1986), Kivelson & Southwood (1986), Southwood & Kivelson (1986), and Smith, Goertz, & Grossmann (1986). Ionson (1978) proposed resonant MHD waves as a means to heat magnetic loops in the solar corona. After the original suggestion by Ionson, resonant MHD waves were extensively studied as a means to heat the solar corona (see, e.g., Kuperus, Ionson, & Spicer 1981; Ionson 1985; Davila 1987; Hollweg 1990, 1991; Goossens 1991). Recently, damping of resonant MHD waves was also suggested as a possible explanation of the observed loss of power of acoustic oscillations in the vicinity of sunspots (see, e.g., Hollweg 1988; Lou 1990; Sakurai, Goossens, & Hollweg 1991b; Goossens & Poedts 1992; Goossens & Hollweg 1993; Stenuit, Poedts, & Goossens 1993).

In the present paper we consider the driven problem for resonant MHD waves. This problem is concerned with an external source of energy that excites plasma oscillations. After a transitional time the system attains a steady state in which all perturbed quantities oscillate with the same frequency $\omega$. The driven problem for one-dimensional magnetic configurations was studied by Hollweg (1987), Davila (1987), and Hollweg & Yang (1988, hereafter HY) for planar geometry, and by Sakurai, Goossens, & Hollweg (1991a, hereafter SGH) and Goossens, Ruderman, & Hollweg (1995, hereafter GRH) for cylindrical geometry. The studies by Hollweg (1987) and HY were based on the ad hoc assumption that the Eulerian perturbation of total pressure does not change across the thin dissipative layer which is present near the ideal resonant position when the dissipative coefficients are small. SGH developed this idea and introduced the concept of connection formulae. The connection formulae specify the jumps in normal velocity and Eulerian perturbation of total pressure across the dissipative layer. They enable us to avoid solving the dissipative MHD equations when studying resonant waves. The dissipative layer can be considered as a surface of discontinuity. The ideal MHD equations are used to describe the waves to the left and the right of this surface of discontinuity. The connection formulae provide the boundary conditions necessary to connect the ideal solutions at the two sides of the discontinuity.

When deriving the connection formulae, SGH used a conservation law obtained in ideal MHD and assumed that this conservation law, which is essentially a generalization of Hollweg's constancy of the Eulerian perturbation of total pressure, remained valid in dissipative MHD. This conservation law was adopted by SGH ad hoc. GRH gave a

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rigorous mathematical derivation of the conservation law in dissipative MHD and obtained compact analytical solutions to the dissipative MHD equations that describe the waves in the dissipative layer.

Most papers on driven resonant MHD waves use isotropic viscosity and/or electrical resistivity. However, the solar corona is a well-known example of a plasma where viscosity is strongly anisotropic (see, e.g., Hollweg 1985). Hollweg (1987) studied resonant MHD waves in an incompressible plasma in the presence of anisotropic viscosity. He found that anisotropic viscosity only removes the singularity that is present in the ideal solution for resonant MHD waves that propagate along the equilibrium magnetic field. Subsequently, HY considered resonant MHD waves in a compressible plasma with anisotropic viscosity. They showed that anisotropic viscosity does not remove the Alfvén singularity. In addition, they found that anisotropic viscosity only slightly modifies the behavior of the resonant waves in the vicinity of the Alfvén resonant position for plasma parameters typical for the solar corona. Ofman, Davila, & Steinolfson (1994) and Erdélyi & Goossens (1995) numerically verified this result. HY pointed out that anisotropic viscosity might be important for slow resonant waves. However, the main emphasis of HY was on a cold plasma in which slow waves are absent. Therefore, they did not study the behavior of slow resonant waves in a plasma with anisotropic viscosity.

The aim of the present paper is to study the behavior of resonant MHD waves in the vicinity of the slow resonance in a compressible plasma with anisotropic viscosity. The most obvious and important question in this context is whether anisotropic viscosity removes the slow singularity or not. This is indeed an important question in the context of solar physics, since the first term in Braginskii's tensorial expression for viscosity is at least 5 orders of magnitude larger than the other terms for typical conditions in the solar corona. As a result, viscosity in the solar corona is strongly anisotropic. Estimations based on dimensional arguments lead us to expect that anisotropic thermal conductivity is of the same importance as anisotropic viscosity under conditions such as those in the solar corona, and that all other dissipative mechanisms can be neglected (see, e.g., discussion in Ruderman et al. 1996). Therefore, we also take anisotropic thermal conductivity into account.

The paper is organized as follows. In the next section we present the set of linear dissipative MHD equations and the equilibrium state. In § 3 we obtain the solution to the simplified dissipative MHD equations that describe the waves in the dissipative layer and in the two overlap regions to the left and the right of the dissipative layer. In § 4 we discuss our results.

## 2. GOVERNING EQUATIONS

We consider a collision-dominated infinitely conducting plasma. Having in mind applications to the solar corona, we assume that the plasma is strongly magnetized, so that $\omega_i \tau_i \gg 1$, where $\omega_i$ is the ion cyclotron frequency and $\tau_i$ is the mean free collisional time of the ions. Under this condition it is a good first approximation to retain only the first term in Braginskii's expression for the viscosity tensor $\hat{\pi}$ (see Braginskii 1965; Hollweg 1985). As a result we have the following linear expression for $\hat{\pi}$:

$$\hat{\pi} = \rho_0 \nu (\boldsymbol{b} \otimes \boldsymbol{b} - \tfrac{1}{3}\hat{I})[3\boldsymbol{b} \cdot \nabla(\boldsymbol{b} \cdot \boldsymbol{v}') - \nabla \cdot \boldsymbol{v}'] , \quad (1)$$

where $\rho$ is the density, $\boldsymbol{v} = (u, v, w)$ is the velocity, and $\boldsymbol{B} = (B_x, B_y, B_z)$ is the magnetic field; $\nu$ is the kinematic coefficient of viscosity, and $\boldsymbol{b} = \boldsymbol{B}_0/B_0$ is the unit vector in the direction of the equilibrium magnetic field. $\hat{I}$ is the unit tensor, and the symbol $\otimes$ denotes the tensorial product of vectors. The subscript zero refers to the equilibrium quantities, and a prime denotes an Eulerian perturbation.

In a strongly magnetized plasma thermal conductivity in the planes perpendicular to the magnetic field lines can be neglected in comparison with thermal conductivity along the magnetic field lines. As a result we arrive at the following linear expression for the heat flux $\boldsymbol{q}$ (see, e.g., Priest 1982):

$$\boldsymbol{q} = -\bar{\chi}\boldsymbol{b}(\boldsymbol{b} \cdot \nabla T') . \quad (2)$$

Here $T$ is the temperature and $\bar{\chi}$ is the coefficient of thermal conductivity.

From a physical point of view the viscosity tensor (eq. [1]) is characterized by the property that at any magnetic surface the viscous stresses are normal to the surface. The expression in equation (2) means that the heat flux is directed along the magnetic field.

In what follows we adopt the Cartesian coordinates $x$, $y$, and $z$. We consider a static one-dimensional equilibrium state and assume that the equilibrium quantities only depend on $x$ and that there is no equilibrium flow. For the sake of mathematical simplicity we consider a unidirectional equilibrium field $\boldsymbol{B}_0$ that has $y$ and $z$ components. In particular, $\boldsymbol{b}$ is constant. Note that $\nu$ and $\bar{\chi}$ can also depend on $x$. The equilibrium variables satisfy the condition of magnetostatics which requires total pressure to be constant,

$$p_0 + \frac{B_0^2}{2\mu} = \text{constant} , \quad (3)$$

where $p$ is the plasma pressure and $\mu$ is the magnetic permeability.

With the aid of equations (1) and (2), the linear equations of viscous thermal conductive MHD can be written as

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \boldsymbol{v}' + u\frac{d\rho_0}{dx} = 0 , \quad (4)$$

$$\rho_0 \frac{\partial \boldsymbol{v}'}{\partial t} = -\nabla P' + \frac{1}{\mu}(\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{B}' + \frac{B_x'}{\mu}\frac{d\boldsymbol{B}_0}{dx} + \left[\boldsymbol{b}(\boldsymbol{b} \cdot \nabla) - \frac{1}{3}\nabla\right]\{\rho_0 \nu[3\boldsymbol{b} \cdot \nabla(\boldsymbol{b} \cdot \boldsymbol{v}') - \nabla \cdot \boldsymbol{v}']\} , \quad (5)$$

$$\frac{\partial \boldsymbol{B}'}{\partial t} = (\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{v}' - u\frac{d\boldsymbol{B}_0}{dx} - \boldsymbol{B}_0 \nabla \cdot \boldsymbol{v}' , \quad (6)$$

$$\frac{\partial p'}{\partial t} + \gamma p_0 \nabla \cdot \boldsymbol{v}' + u\frac{dp_0}{dx} = (\gamma - 1)\bar{\chi}(\boldsymbol{b} \cdot \nabla)^2 T' , \quad (7)$$

$$\frac{p'}{p_0} = \frac{\rho'}{\rho_0} + \frac{T'}{T_0} . \quad (8)$$

Here $P = p + B^2/2\mu$ is the total (plasma plus magnetic) pressure, and $\gamma$ is the adiabatic index. As there is not any equilibrium velocity, we can omit primes when writing the components of the perturbed velocity. Equation (8) is the perturbed ideal gas law. The unperturbed ideal gas law takes the form

$$p_0 = \frac{2k_B}{m_p} \rho_0 T_0 , \quad (9)$$

where $k_B$ is the Boltzmann constant and $m_p$ is the proton mass.

The present paper focuses on the steady state of driven MHD waves. In this steady state all the linear wave variables are proportional to exp $(-i\omega t)$. As the equilibrium state only depends on $x$, we can Fourier-analyze equations (4)–(8), taking all perturbed variables proportional to exp $(ik_y y + ik_z z)$. This enables us to rewrite equations (4)–(8) as

$$\omega\rho' - \rho_0 \boldsymbol{k} \cdot \boldsymbol{v}_{\rm pl} + i \frac{d}{dx} (\rho_0 u) = 0 , \quad (10)$$

$$\omega\rho_0 u = -i \frac{d\tilde{P}'}{dx} - \frac{B_0(\boldsymbol{k} \cdot \boldsymbol{b})}{\mu} B'_x , \quad (11)$$

$$\omega\rho_0 \boldsymbol{v}_{\rm pl} = \boldsymbol{k}\tilde{P}' - \frac{B_0(\boldsymbol{k} \cdot \boldsymbol{b})}{\mu} \boldsymbol{B}'_{\rm pl} + \frac{i\boldsymbol{b}}{\mu}\frac{dB_0}{dx} B'_x - i\rho_0 \nu(\boldsymbol{k} \cdot \boldsymbol{b})\boldsymbol{b}Q , \quad (12)$$

$$\omega B'_x = -B_0(\boldsymbol{k} \cdot \boldsymbol{b})u , \quad (13)$$

$$\omega \boldsymbol{B}'_{\rm pl} = B_0[\boldsymbol{b}(\boldsymbol{k} \cdot \boldsymbol{v}_{\rm pl}) - (\boldsymbol{k} \cdot \boldsymbol{b})\boldsymbol{v}_{\rm pl}] - i\boldsymbol{b} \frac{d}{dx} (B_0 u) , \quad (14)$$

$$\omega p' - \gamma p_0 \left[ (\boldsymbol{k} \cdot \boldsymbol{v}_{\rm pl}) - i \frac{du}{dx} \right] + iu \frac{dp_0}{dx} = -i\chi(\boldsymbol{k} \cdot \boldsymbol{b})^2 S . \quad (15)$$

In these equations $\boldsymbol{k} = (0, k_y, k_z)$ is the wavevector, and we have introduced the components of the velocity $\boldsymbol{v}_{\rm pl} = (0, v, w)$ and the Eulerian perturbation of the magnetic field $\boldsymbol{B}'_{\rm pl} = (0, B'_y, B'_z)$ that are parallel to the $y$-$z$ plane. The quantities $Q$ and $S$, and the Eulerian perturbation of the total pressure modified by viscosity, $\tilde{P}'$, are determined by

$$Q = 3(\boldsymbol{k} \cdot \boldsymbol{b})(\boldsymbol{b} \cdot \boldsymbol{v}_{\rm pl}) - \boldsymbol{k} \cdot \boldsymbol{v}_{\rm pl} + i \frac{du}{dx} , \quad (16)$$

$$S = \frac{\gamma(\rho_0 p' - p_0 \rho')}{(\gamma - 1)\rho_0} , \quad (17)$$

$$\tilde{P}' = P' + \frac{i\rho_0 \nu}{3} Q . \quad (18)$$

The Eulerian perturbation of the total pressure, $P'$, is related to the Eulerian perturbation of pressure, $p'$, and to the Eulerian perturbation of magnetic field, $\boldsymbol{B}'$, by

$$P' = p' + \frac{B_0}{\mu} (\boldsymbol{b} \cdot \boldsymbol{B}'_{\rm pl}) . \quad (19)$$

The coefficient $\chi$ is given by

$$\chi = \frac{m_p(\gamma - 1)^2 \bar{\chi}}{2\gamma k_B \rho_0} . \quad (20)$$

When deriving equation (15) from equation (7), we have used equations (8) and (9). The set of equations (10)–(19) will be used in the next section to study the behavior of the driven slow resonant waves in the vicinity of the ideal resonant position.

## 3. SOLUTIONS IN THE DISSIPATIVE LAYER

The object of the paper is to determine the behavior of MHD waves that are driven at a frequency in the ideal slow continuum. In ideal MHD these driven waves are characterized by singular spatial solutions. The slow resonant position $x_c$ is determined by the condition

$$\omega^2 = \omega_c^2(x_c) . \quad (21)$$

The squares of the Alfvén and slow frequencies are given by

$$\omega_A^2 = \frac{(\boldsymbol{k} \cdot \boldsymbol{B}_0)^2}{\mu} , \quad \omega_c^2 = \frac{c_S^2}{c_S^2 + v_A^2} \omega_A^2 . \quad (22)$$

The squares of the Alfvén, sound, and cusp velocities are determined by

$$v_A^2 = \frac{B_0^2}{\mu\rho_0} , \quad c_S^2 = \frac{\gamma p_0}{\rho_0} , \quad c_T^2 = \frac{c_S^2 v_A^2}{c_S^2 + v_A^2} . \quad (23)$$

In ideal MHD the resonant slow waves are dominated by density and pressure perturbations, and by the dynamics in the magnetic surfaces parallel to the magnetic field lines. It is very likely that this behavior will persist in nonideal MHD when dissipation is weak. Results from ideal MHD provide us with a guideline for choosing the wave variables. It is convenient to introduce the components of $\boldsymbol{v}_{\rm pl}$ and $\boldsymbol{B}'_{\rm pl}$ that are parallel and perpendicular to the equilibrium magnetic field. We denote these components as $v_\parallel$, $B'_\parallel$, and $v_\perp$, $B'_\perp$, respectively. Indeed, in ideal MHD slow resonant waves are characterized by singular spatial solutions. The dominant singular behavior resides in $\rho'$, $p'$, $v_\parallel$, and $B'_\parallel$, which have a singularity of the form $(x - x_c)^{-1}$ at $x_c$ (see, e.g., SGH). The variables $u$ and $B'_x$ are also singular, but the singularity is of the form $\ln|x - x_c|$. The quantities $v_\perp$, $B'_\perp$, and the Eulerian perturbation of the total pressure $P'$ are continuous at $x_c$.

We eliminate all variables but $u$ and $\tilde{P}'$ from equations (10)–(15), to obtain a system of two differential equations,

$$\frac{d\tilde{P}'}{dx} = \frac{i\rho_0 A}{\omega} u , \quad (24)$$

$$C \frac{du}{dx} = \frac{i\omega D}{\rho_0(c_S^2 + v_A^2)A} \tilde{P}' + \nu\omega \frac{\omega^2 - 3c_S^2(\boldsymbol{k} \cdot \boldsymbol{b})^2}{3(c_S^2 + v_A^2)} Q - \frac{\chi\omega^2(\boldsymbol{k} \cdot \boldsymbol{b})^2}{\rho_0(c_S^2 + v_A^2)} S , \quad (25)$$

where

$$A = \omega^2 - \omega_A^2 , \quad C = \omega^2 - \omega_c^2, D = \omega^4 - (c_S^2 + v_A^2)k^2 C . \quad (26)$$

In what follows we also use an expression for $v_\parallel$, since this is one of the most singular variables in ideal MHD:

$$v_\parallel = \frac{\boldsymbol{k} \cdot \boldsymbol{b}}{\omega} \left( \frac{\omega^2 - v_A^2 k^2}{\rho_0 A} \tilde{P}' + \frac{iv_A^2}{\omega}\frac{du}{dx} - i\nu Q \right) . \quad (27)$$

The nonideal effects in the equations are characterized by the Reynolds number, $Re = \omega/\nu k^2$, and the Peclet number, $Pe = \omega/\chi k^2$. These two numbers measure the importance of viscosity and thermal conductivity, respectively. As we assume that both viscosity and thermal conductivity are small, we have $Re \gg 1$ and $Pe \gg 1$. Consequently, when calculating $Q$ and $S$, we can use results obtained on the basis of ideal MHD. As a result we arrive at the following approximate expressions:

$$Q = \frac{iA}{D}[\omega^2 - 3c_S^2(\boldsymbol{k}\cdot\boldsymbol{b})^2]\frac{du}{dx}, \tag{28}$$

$$S = -\frac{i\rho_0\omega c_S^2 A}{D}\frac{du}{dx} - \frac{i\rho_0 u}{\omega(\gamma-1)}\frac{dc_S^2}{dx}. \tag{29}$$

Substitution of equations (28) and (29) in equations (25) and (27) yields

$$\left[\!\left[(c_S^2 + v_A^2)C - \frac{i\omega A}{D} \times \left\{\frac{\nu}{3}[\omega^2 - 3c_S^2(\boldsymbol{k}\cdot\boldsymbol{b})^2]^2 + \chi\omega^2 c_S^2(\boldsymbol{k}\cdot\boldsymbol{b})^2\right\}\right]\!\right]\frac{du}{dx} = \frac{i\omega D}{\rho_0 A}\tilde{P}' + \frac{i\omega\chi(\boldsymbol{k}\cdot\boldsymbol{b})^2}{\gamma-1}\frac{dc_S^2}{dx}u, \tag{30}$$

$$v_\parallel = \frac{\boldsymbol{k}\cdot\boldsymbol{b}}{\omega}\left[\!\left[\frac{\omega^2 - v_A^2 k^2}{\rho_0 A}\tilde{P}' + \left\{\frac{iv_A^2}{\omega} + \frac{\nu A}{D}[\omega^2 - 3c_S^2(\boldsymbol{k}\cdot\boldsymbol{b})^2]\right\}\frac{du}{dx}\right]\!\right]. \tag{31}$$

In order to study the behavior of the waves in the vicinity of the ideal slow resonant point $x_c$, we follow GRH and introduce a new variable $s = x - x_c$. The analysis of equations (24), (30), and (31) is then restricted to the interval $[-s_c, s_c]$, where $s_c$ is determined by the condition that a linear Taylor polynomial is a good approximation to the function $\omega^2 - \omega_c^2(x)$. This leads to the restriction that $s_c$ is much smaller than the characteristic scale of the inhomogeneity. The coefficient functions in equations (24), (30), and (31) are expanded in Taylor series with respect to $s$, and only the first nonzero terms are retained in these expansions. As a result we obtain the simplified versions of equations (24), (30), and (31) that are valid in the vicinity of $x_c$:

$$\frac{d\tilde{P}'}{ds} = -\frac{i\omega\rho_0 v_A^2}{c_S^2}u, \tag{32}$$

$$\left(\Delta s + \frac{i\omega^3\lambda}{c_S^2 + v_A^2}\right)\frac{du}{ds} = -\frac{ic_S^2\omega^3}{\rho_0 v_A^2(c_S^2 + v_A^2)}\tilde{P}', \tag{33}$$

$$\left(\Delta s + \frac{i\omega^3\lambda}{c_S^2 + v_A^2}\right)v_\parallel = \frac{\omega c_S^2(\boldsymbol{b}\cdot\boldsymbol{k})}{\rho_0(c_S^2 + v_A^2)}\tilde{P}', \tag{34}$$

where

$$\Delta = \left.\frac{dC}{dx}\right|_{x=x_c}, \tag{35}$$

$$\lambda = \nu\frac{(2v_A^2 + 3c_S^2)^2}{3v_A^2 c_S^2} + \chi\frac{c_S^2 + v_A^2}{c_S^2}. \tag{36}$$

In equations (32)–(34) and (36) all equilibrium quantities are calculated at $x = x_c$. We have neglected the term proportional to $\chi(dc_S^2/dx)u$ in equation (33) because of the inequality $|(dc_S^2/dx)u| \ll |c_S^2(du/ds)|$. When deriving equation (34), we have used equations (32) and (33), and neglected terms proportional to $\nu$, $\chi$, and $s$ on the right-hand side. The quantity $\lambda$ is a dissipative coefficient that describes the combined effect of viscosity and thermal conductivity.

In ideal MHD ($\lambda = 0$) the system of equations (32) and (33) possesses a regular singular point at $s = 0$ ($x = x_c$) because the coefficient of $du/ds$ in equation (33) is equal to zero there. Dissipation ($\lambda \neq 0$) removes this singularity. Dissipation is only important in a thin dissipative layer which embraces the slow resonant position $x = x_c$, where the first and second terms in parentheses on the right-hand side of equation (33) are of the same order. This results in a dissipative layer with a thickness measured by the quantity $\delta_c$:

$$\delta_c = \frac{\omega^4}{k^2|\Delta|R(c_S^2 + v_A^2)}, \tag{37}$$

where the total Reynolds number $R$ is defined as

$$R = \frac{\omega}{\lambda k^2}. \tag{38}$$

The thickness of the dissipative layer is inversely proportional to $R$, in contrast to the case of isotropic viscosity, where the thickness of the dissipative layer is inversely proportional to the cube root of the Reynolds number (see, e.g., SGH and GRH).

As has already been stated in the Introduction, anisotropic viscosity does not remove the Alfvénic singularity. The Alfvénic singularity is only removed by the isotropic part of Braginskii's viscosity tensor (see, e.g., HY). Results by HY, Ofman et al. (1994), and Erdélyi & Goossens (1995) show that for typical conditions in the solar corona it is sufficient to retain the isotropic part of Braginskii's viscosity tensor when studying resonant Alfvén waves. For typical coronal conditions the Reynolds number calculated with the use of the coefficient of isotropic viscosity is of the order of $10^{14}$–$10^{16}$ (see, e.g., Ofman et al. 1994; Erdélyi & Goossens 1995). Since the ratio of the wavelength $L$ to the thickness of the Alfvénic dissipative layer $\delta_A$ is of the order of the cube root of the Reynolds number, we obtain $\delta_A/L \sim 2 \times 10^{-5}$–$5 \times 10^{-6}$.

In the case of slow resonant waves the ideal singularity is removed by viscosity described by the first term of Braginskii's tensorial expression, so that the Reynolds number has to be calculated with the use of Braginskii's first coefficient of viscosity, $\eta_0 = \rho_0\nu$. For typical coronal conditions the Reynolds number related to $\eta_0$ is of the order of $5 \times 10^2$–$10^4$ (see, e.g., Hollweg 1985; Ofman et al. 1994; Erdélyi & Goossens 1995), and the estimate of the thickness of the slow dissipative layer is $\delta_c/L \sim 2 \times 10^{-3}$–$10^{-4}$. Hence slow dissipative layers are much thicker than the Alfvénic dissipative layers ($\delta_c \gg \delta_A$).

In ideal MHD the most singular variables posses a singularity of the form $(x - x_c)^{-1}$ for slow resonant waves and of the form $(x - x_A)^{-1}$ for Alfvén resonant waves, where $x_A$ is an Alfvénic resonant position. Therefore, the ratio of the amplitude of the most singular variables in the dissipative layer to their amplitude far away from the dissipative layer is of the order of $L/\delta_c$ for slow resonant waves, and of the order of $L/\delta_A$ for Alfvén resonant waves. The direct consequence of the inequality $\delta_c \gg \delta_A$ is that the amplitudes of the most singular variables in a slow dissipative layer are much smaller than the amplitudes of the most singular variables in an Alfvénic dissipative layer when resonant slow and Alfvén waves have amplitudes of the same order far away from the dissipative layers.

The order of the system of ideal equations for $u$ and $P'$ is equal to 2. Let us recall the method for obtaining solutions to the dissipative MHD equations when the dissipative coefficients were isotropic. Dissipation with isotropic dissipative coefficients increases the order of the system of differential equations for $u$ and $P'$ from 2 to 6 (see, e.g., GRH). The determination of the solution to this system of dissipative equations for the resonant waves in the dissipative layer requires additional boundary conditions. The procedure for obtaining these boundary conditions is as follows: It is assumed that $\delta_c \ll s_c$, so that the simplified versions of ideal MHD equations are valid for $\delta_c \ll |s| \lesssim s_c$. This makes it possible to use the method of matched asymptotic expansions. In accordance with this method the ideal solution in the region $\delta_c \ll |s| \lesssim s_c$ has to be matched to the dissipative solution in the region $|s| \lesssim \delta_c$. The matching conditions require that the ideal and dissipative solutions coincide in the intermediate region $\delta_c \ll |s| \ll s_c$. They provide the additional boundary conditions for the dissipative system of equations for $u$ and $P'$.

In the case of anisotropic viscosity and thermal conductivity the mathematical analysis is much simpler. The system of dissipative equations (32) and (33) for $u$ and $\tilde{P}'$ has the same order as its ideal counterpart. Obtaining the solution to the dissipative system is not more difficult than solving the ideal system. In particular, no additional boundary conditions are needed in comparison with the ideal system, and there is no need to use the method of matched asymptotic expansions. Instead we simply look for the solution of the dissipative equations (32) and (33) in the whole region $|s| \lesssim s_c$. This enables us to reduce the restriction on $\delta_c$ in comparison with the case of isotropic dissipative coefficients and assume that $\delta_c \lesssim s_c$.

The system of equations (32) and (33) differs from its ideal counterpart only in that the singular position is shifted from $s = 0$ to $s = -i\delta_c \text{sign}(\Delta)$. Therefore, we can look for the solution in the form of a Frobenius series in the same way as in ideal MHD. The solution to equations (32) and (33) is obtained by simple substitution of $s + i\delta_c \text{sign}(\Delta)$ for $s$ in the ideal solution. The result is

$$\tilde{P}' = \text{constant} + O(|s + i\delta_c| \ln |s + i\delta_c|) , \quad (39)$$

$$u = -\frac{i\omega^3 c_S^2 \tilde{P}'}{\rho_0 v_A^2 (c_S^2 + v_A^2)\Delta} \ln [s + i\delta_c \text{sign}(\Delta)] + \text{constant} + O(|s + i\delta_c| \ln |s + i\delta_c|) . \quad (40)$$

The approximate constancy of the total pressure modified by viscosity, $\tilde{P}'$, in the dissipative layer parallels the approximate constancy of the total pressure, $P'$, found in dissipative MHD with isotropic dissipative coefficients (see, e.g., HY; SGH; GRH). Equation (34) shows that dissipation removes the singularity in $v_\parallel$ present in ideal MHD. However, $v_\parallel$ is very large for weak dissipation ($R \gg 1$). The perpendicular component of $\boldsymbol{v}_{\text{pl}}$, $v_\perp$, is nonsingular in ideal MHD. With the use of equations (10)–(15) we can express $v_\perp$ in terms of $\tilde{P}'$. With the use of this expression and equations (34) and (40), we obtain estimates valid in a slow dissipative layer:

$$\frac{v_\parallel}{u} \sim \frac{R}{\ln R} , \quad \frac{v_\parallel}{v_\perp} \sim R . \quad (41)$$

These estimates show that in the case of weak dissipation the dominant dynamics in a slow dissipative layer resides in motions that are in magnetic surfaces and parallel to the equilibrium magnetic field.

The quantity $\tilde{P}'$, which is constant in the dissipative layer, is determined by matching it to the ideal solution far away from the dissipative layer. In order to plot $u$ and $v_\parallel$ as functions of $\sigma$, we assume that $\tilde{P}'$ is real. The quantity $u$ is given up to an arbitrary additive constant, which is once again determined by matching conditions. Here we choose this constant so that $u = 0$ at $\sigma = 0$. The real and imaginary parts of $u/U$ and $v_\parallel/V$ versus $\sigma$ are shown in Figures 1 and 2, where the quantities $U$ and $V$ are given by

$$U = \frac{\omega^3 c_S^2 \tilde{P}'}{\rho_0 v_A^2 (c_S^2 + v_A^2) |\Delta|} , \quad V = \frac{\omega c_S^2 (\boldsymbol{b} \cdot \boldsymbol{k}) \tilde{P}'}{\rho_0 \delta_A (c_S^2 + v_A^2) \Delta} . \quad (42)$$

Note the similarity of the graphs shown in Figures 1 and 2 to the graphs of the real and imaginary parts of functions

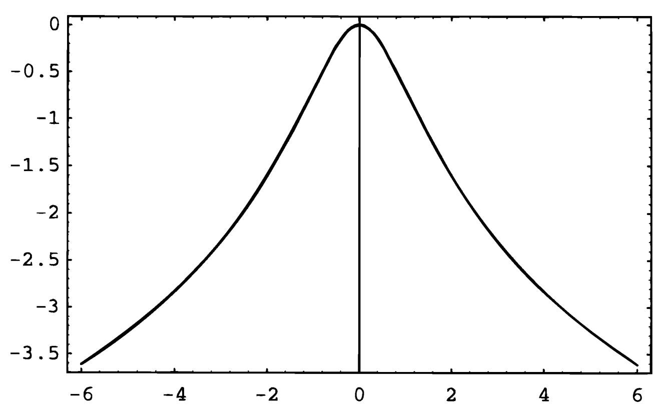

FIG. 1*a*

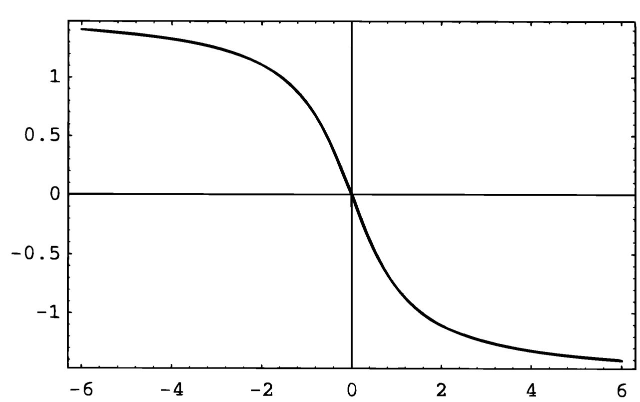

FIG. 1*b*

FIG. 1.—(*a*) Re $(u/U)$ and (*b*) sign $(\Delta)$ Im $(u/U)$ are shown

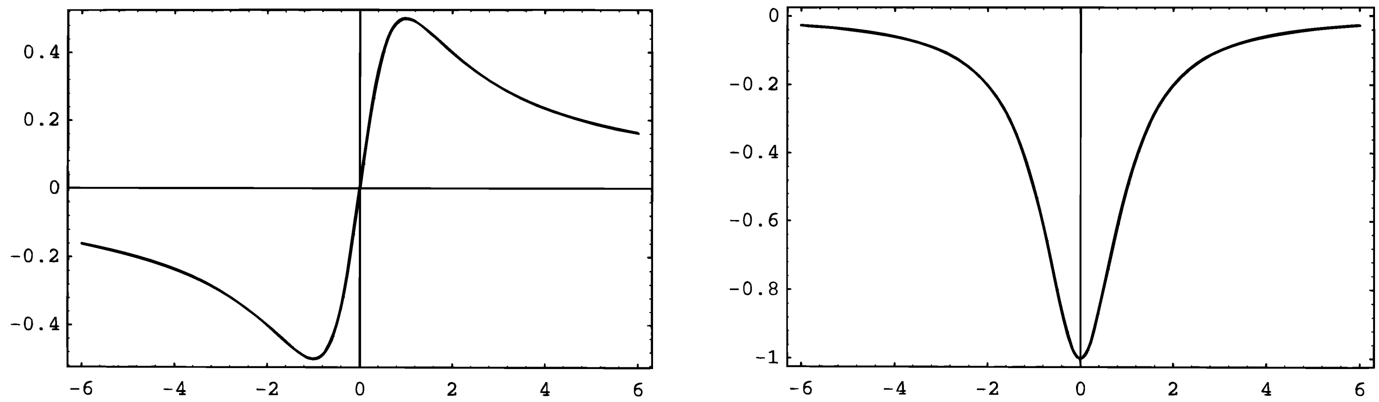


FIG. 2a FIG. 2b

FIG. 2.—(*a*) Re $(v_{\|}/V)$ and (*b*) sign(Δ) Im $(v_{\|}/V)$ are shown

$iG(\tau)$ and $-iF(\tau)$ that describe the behavior of $u$ and $v_{\|}$ in dissipative layers in plasmas with isotropic resistivity (see, e.g., GRH).

Let us now find the connection formulae that determine the jumps in $P'$ and $u$ across the dissipative layer. In order to do this, we assume that $\delta_c \ll s_c$ and introduce a new dimensionless variable $\sigma = s/\delta_c$, where $\sigma \sim 1$ in the dissipative layer, and $s \to \pm s_c$ correspond to $\sigma \to \pm\infty$. The jump in a function $f(\sigma)$ across the dissipative layer is defined as

$$[f] = \lim_{\sigma\to\infty} \{f(\sigma) - f(-\sigma)\} . \tag{43}$$

From equations (39) and (40) we obtain

$$[P'] = 0 , \tag{44}$$

$$[u] = -\frac{\pi\omega^3 c_S^2 \tilde{P}'}{\rho_0 v_A^2(c_S^2 + v_A^2)|\Delta|} . \tag{45}$$

The right-hand side of equation (45) is evaluated at $x = x_c$. When deriving equation (44), we use the fact that $\lim_{|\sigma|\to\infty} \tilde{P}' = P'$. The connection formula (44) coincides with that obtained in nonthermal conductive MHD with isotropic electrical conductivity, and the connection formula (45) only differs from its counterpart in that $P'$ is replaced by $\tilde{P}'$ (see, e.g., SGH). This supports the statement that the connection formulae are independent of the dissipative processes taken into account.

The connection formulae enable us to avoid solving the dissipative MHD equations when studying resonant MHD waves. We consider the dissipative layer as a surface of discontinuity. We use linear ideal MHD to describe the behavior of resonant MHD waves to the left and right of this surface of discontinuity. The connection formulae are used as boundary conditions at the surface of discontinuity.

Let us now clarify a result obtained by Hollweg (1987). This author showed that in an incompressible plasma anisotropic viscosity only removes the ideal singularity for waves that propagate along the equilibrium magnetic field. For waves that propagate obliquely to the equilibrium magnetic field, anisotropic viscosity removes the singularity in the component of the velocity parallel to the equilibrium magnetic field, $v_{\|}$, and does not remove the singularity in the component of the velocity perpendicular to both the equilibrium magnetic field and the direction of inhomogeneity, $v_{\perp}$. The singularity that is present in the linear ideal MHD equations for incompressible plasmas is often called the Alfvén singularity. However, this name disguises its physical nature. The approximation of an incompressible plasma corresponds to the mathematical limit of infinite sound speed. In this limit the cusp speed is equal to the Alfvén speed ($c_T = v_A$), and the slow and Alfvén singularities which are present in the linear ideal MHD equations for compressible plasmas are merged into one singularity in incompressible plasmas. For waves that propagate along the equilibrium magnetic field only the slow singularity is present in the linear ideal MHD equations for compressible plasmas. Therefore, in the case of a purely longitudinally propagating wave, the singularity in an incompressible plasma is the slow singularity. In the case of an obliquely propagating wave the singularity in an incompressible plasma is a mixture of the slow and Alfvén singularities. In a compressible plasma the dominant dynamics resides in $v_{\|}$ in the vicinity of the slow resonant point and in $v_{\perp}$ in the vicinity of the Alfvén resonant point. Hence, for an obliquely propagating wave in an incompressible plasma, $v_{\|}$ corresponds to the slow part of the singularity, while $v_{\perp}$ corresponds to the Alfvénic part of the singularity. We have shown in the present paper that anisotropic viscosity and/or thermal conductivity removes the slow singularity, and HY showed that anisotropic viscosity does not remove the Alfvén singularity. In view of these results, the results obtained by Hollweg (1987) are no longer surprising. Anisotropic viscosity in an incompressible plasma removes the slow part of the singularity, which resides in $v_{\|}$, and does not remove the Alfvén part of the singularity, which resides in $v_{\perp}$. In the case of longitudinally propagating waves the singularity is purely slow, and therefore it is completely removed by anisotropic viscosity.

## 4. CONCLUSIONS

We have studied the dissipation of driven slow resonant MHD waves in plasmas by strongly anisotropic viscosity and thermal conductivity in a planar one-dimensional magnetic configuration. We have shown that anisotropic viscosity and/or thermal conductivity removes the slow singularity from the linear MHD equations. The situation differs from that in the case of the driven Alfvén waves. As shown by HY, anisotropic viscosity does not remove the ideal Alfvén singularity.

We have obtained the complete solution to the dissipative MHD equations in the slow dissipative layer. With the use of asymptotic analysis of this solution, we have calculated the connection formulae that determine the jumps in the Eulerian perturbation of the total pressure and in the component of the velocity in the direction of the inhomogeneity. These formulae are the same as in the case of inviscid nonthermal conducting plasmas with isotropic finite electrical conductivity. This result supports the intuitive assumption that the connection formulae are independent of the type of dissipation as long as it removes the singularity. However, the behavior of the waves in the dissipative layer and the thickness of the dissipative layer depend on the nature of the dissipative mechanism. The mathematical description of the resonant waves in the dissipative layer is much simpler in the case of strongly anisotropic viscosity and thermal conductivity than in the case of isotropic viscosity and/or finite electrical conductivity. In particular, in the first case the behavior of the $x$ component of the velocity, $u$, is described in terms of the logarithmic function of a complex argument, while in the second case it is described in terms of an integral function related to the Airy function (see, e.g., GRH). The thickness of the dissipative layer is inversely proportional to the Reynolds number in the first case, and inversely proportional to the cube root of the Reynolds number in the second.

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