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POSTCOLLAPSE EVOLUTION OF GLOBULAR CLUSTERS

JUNICHIRO MAKINO
Department of Graphics and Information Science, College of Arts and Sciences,
University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo 153, Japan
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ABSTRACT

A number of globular clusters appear to have undergone core collapse, in the sense that their predicted collapse times are much shorter than their current ages. Simulations with gas models and the Fokker-Planck approximation have shown that the central density of a globular cluster after the collapse undergoes nonlinear oscillation with a large amplitude (gravothermal oscillation). However, the question whether such an oscillation actually takes place in real N-body systems has remained unsolved because an N-body simulation with a sufficiently high resolution would have required computing resources of the order of several GFLOPS-yr. In the present paper, we report the results of such a simulation performed on a dedicated special-purpose computer, GRAPE-4. We have simulated the evolution of isolated point-mass systems with up to 32,768 particles. The largest number of particles reported previously is 10,000. We confirm that gravothermal oscillation takes place in an N-body system. The expansion phase shows all the signatures that are considered to be evidence of the gravothermal nature of the oscillation. At the maximum expansion, the core radius is $\sim 1\%$ of the half-mass radius for the run with 32,768 particles. The maximum core size, r_c, depends on N as $\langle r_c \rangle \propto N^{-1/3}$.

Subject headings: globular clusters: general — methods: numerical — stars: kinematics

1. INTRODUCTION

A number of globular clusters appear to have undergone core collapse (Cohn & Hut 1984; Hut & Djorgovski 1992). Djorgovski & King (1986) have shown that 15% of Galactic globular clusters have unresolved density cusps. Recent observations with the *Hubble Space Telescope* (*HST*) have demonstrated that some of these clusters have core sizes smaller than 0".03, which is as far as one can go with present techniques (see, e.g., Sosin & King 1995; Yanny et al. 1994).

The theoretical study of the evolution after core collapse was pioneered by Henon (1975), who incorporated the energy production by binaries into his Monte Carlo calculations. He found that the whole cluster expanded homologously within the thermal timescale. Similar results were obtained with gas models and Fokker-Planck calculations.

Sugimoto & Bettwieser (1983; see also Bettwieser & Sugimoto 1984) found that the postcollapse expansion is unstable if the energy production is inefficient. In the unstable case, the central density showed an oscillation with a large amplitude. They called this *gravothermal oscillation.* They modeled the postcollapse evolution of globular clusters by using a conducting gas sphere with artificial energy production. The efficiency of the energy production is related to the total number of particles N, and for large N efficiency is small. Thus their result implies that an N-body system with a number of particles larger than some critical number should exhibit gravothermal oscillation.

Since several other calculations based on similar models failed to reproduce the result of Sugimoto & Bettwieser (1983), the validity of their result was controversial for a few years after their first paper. However, calculations with improved accuracy have confirmed that this oscillation takes place in both gas models and Fokker-Planck (FP) calculations (Goodman 1987; Heggie & Ramamani 1989; Cohn, Hut, & Wise 1989).

What has been found in gas models and FP calculations can be summarized as follows: There is a homologously expanding solution for the postcollapse evolution of the cluster (Goodman 1984). In this solution, the size of the core is proportional to $N^{-2/3}$. Thus, the core is smaller for larger numbers of particles. If the core is too small, this homologous expansion becomes unstable in a way similar that in which an isothermal sphere is unstable, and the core starts to show an oscillatory behavior. The critical number of particles is ~ 7000 (Goodman 1987). If the total number of particles is slightly larger than this critical value, the oscillation is regular. However, at $N \simeq 10{,}000$, "period doubling" takes place, and the density shows oscillation with double peaks. For $N > 50{,}000$, the oscillation becomes apparently chaotic (Heggie & Ramamani 1989; Breeden, Cohn, & Hut 1994).

Both the contraction and the expansion are driven by the gravothermal instability, and the energy production by binaries acts as the trigger to start the expansion. During expansion, thermal energy flows in from the halo to the core, and this heat flux is supported by so-called temperature inversion. This temperature inversion itself is formed through the expansion of the core driven by the binary heating in the following way.

Binaries deposit energy to the core via "indirect heating" (Hut 1985). When a binary hardens through interaction with a third star, that star is likely to be kicked out of the core, because the recoil velocity is much larger than the velocity dispersion of the core. As a result, the binding energy of the core becomes smaller. The core expands to reestablish dynamical equilibrium. This expansion causes the velocity dispersion of the core to decrease. Thus the central temperature becomes lower than the temperature of the region just outside the core.

Once this happens, heat starts to flow inward. This inward heat flux lets the core expand further, and the temperature of the core continues to decrease. This self-supported expansion can continue as long as the temperature gradient outside the core is small. In practice,

however, the temperature gradient becomes larger as the core size becomes larger, and the temperature inversion vanishes at a certain core size. After that, the core starts to recollapse.

The critical number of particles for multicomponent systems is larger than that for single-component systems, and for a "realistic" mass function, the critical number of particles can be as large as 3×10^5 (Murphy, Cohn, & Hut 1990).

The question whether such gravothermal oscillation would take place in real globular clusters or in N-body systems has not yet been settled. Gas models and FP models have many simplifying assumptions that might make the evolution completely different from that of a real cluster. For example, both assume spherical symmetry while, in N-body simulations, cores are known to wander around (Makino & Sugimoto 1987; Heggie, Inagaki, & McMillan 1994), and both assume that the energy production by binaries is smooth while, in N-body systems, binaries are formed stochastically and emit energy intermittently.

An N-body simulation with a sufficiently large number of particles has been impossible simply because the requirement for computational power has been excessive. Makino, Tanekusa, & Sugimoto (1986) performed 100-body simulations of an equal-mass cluster, using both softened and unsoftened potentials. They used a softened potential in order to reduce the energy production from binaries. They found oscillatory behavior in both cases, but with larger amplitude for the softened potential. They argued that the fact that the amplitude of oscillation was larger for reduced energy production implied that the observed oscillation was driven by the gravothermal instability and not purely by the heating by binaries. McMillan & Lightman (1984; see also McMillan 1986a) developed a hybrid calculation code in which the central region of a cluster was treated as an N-body system while the outer region was treated in FP formalism. They observed oscillatory behavior but concluded that it was not gravothermal. Inagaki (1986) performed 1000- and 3000-body simulations but found no sign of oscillation. He used the standard Salpeter mass function as the distribution of the masses of particles. Makino (1989) performed simulations of a 3000-body equal-mass system and saw some signs of oscillation, including temperature inversion. Spurzem & Aarseth (1996) performed a 10,000-body simulation and saw oscillatory behavior, but they did not see any clear evidence of the gravothermal oscillation. N-body simulations in the last decade can be summarized as follows: The central density has shown some oscillatory behavior, and its amplitude is seemingly larger for larger numbers of particles. However, each expansion phase is associated with large energy input from one or a few binaries. As a result, whether the expansion is driven by instability or by energy input has been unclear. In addition, in all but one simulation, the number of particles was smaller than the critical value of 7000. The 10,000-body simulation by Spurzem & Aarseth (1996) was not long enough to draw a clear conclusion. Therefore it was not clear whether oscillatory behavior is caused by gravothermal instability or not.

Heggie (1989) and Heggie et al. (1994) tried to see whether gravothermal expansion would take place if one constructs an N-body system with an initial temperature inversion. They constructed an N-body system that mimicked the density and temperature structures of the expansion phase of the gas-model calculation by Heggie & Ramamani (1989) and followed the evolution of the N-body system. They found that the core expanded in the thermal timescale. Thus they at least proved that the core of an N-body system can expand gravothermally.

In the present paper, we describe the results of N-body simulations with 2048–32,768 particles. This is the first calculation with a number of particles well beyond the critical number of 7000 that covers a sufficiently long time after the collapse to determine the nature of the postcollapse evolution. All simulations were performed on GRAPE-4 (Taiji et al. 1996), a special-purpose computer for collisional N-body simulations. Our main results are the following: First, for large N, the core density and core mass exhibit oscillations with large amplitude. The core mass at the maximum expansion is in good agreement with FP and gas-model results (1% of the total mass for 16K [$=16 \times 2^{10}$] and 32K particle runs). Second, we confirm that the observed oscillation is driven by gravothermal instability. Several long expansion periods without any energy input were observed. The temperature inversion was visible in these phases. The behavior of the core density is strikingly similar to the result of FP calculations with a stochastic heat source (Takahashi & Inagaki 1991), which strongly suggests that the mechanisms are the same. The trajectory in the central density–central temperature plane clearly indicates that the inward heat flux supports the expansion.

The structure of this paper is as follows: In § 2, we describe the initial model, the numerical method, and the computer used. In § 3, we present the results. Section 4 is for discussion.

2. MODEL AND NUMERICAL METHOD

2.1. *Initial Models and the System of Units*

We followed the evolution of isolated systems of point-mass particles. For all calculations, we used random realizations of the Plummer model as the initial condition. We adopted standard (Heggie) units (Heggie & Mathieu 1986), in which $G = 1$, $M = 1$, and $E = -\frac{1}{4}$, where G is the gravitational constant, M is the total mass of the cluster, and E is the total energy of the cluster. All particles have the same mass, $m = 1/N$, where N is the total number of particles. In these units, the half-mass crossing time, $t_{\rm hc}$, is $2\sqrt{2}$.

2.2. *Numerical Method*

For all calculations, we used NBODY4 (Aarseth 1985, 1996), modified for GRAPE-4 (Taiji et al. 1996). The numerical integration scheme adopted in NBODY4 is the fourth-order Hermite scheme (Makino 1991a; Makino & Aarseth 1992) with the individual (hierarchical) time step algorithm (McMillan 1986b; Makino 1991b), to use the GRAPE hardware or parallel/vector computers efficiently. Close two-body encounters and stable binaries are handled by KS regularization (see Kustaanheimo & Stiefel 1965). Special treatment for compact few-body subsystems is also possible.

The fourth-order Hermite scheme has several advantages over traditional Adams-Bashforth-Moulton (ABM) type schemes. It is a self-starting scheme and therefore much easier to implement than the multistep ABM scheme of the same order. Its local truncation error has a much smaller coefficient, which allows a larger time step for the same

accuracy. On general-purpose computers, this advantage is partly canceled by the additional CPU time to calculate the first time derivative, but on GRAPE-type machines the advantage is significant (Makino & Aarseth 1992; Makino, Kokubo, & Taiji 1993).

The outputs are taken at intervals of a fixed time, which is a certain fraction of the crossing time. We recorded the central density, core radius, number of particles in the core, radii of Lagrangian shells, velocity dispersion within Lagrangian shells, and binaries and their binding energies. The core parameters are calculated following Casertano & Hut (1985), with modifications described in McMillan, Hut, & Makino (1990).

The accuracy parameter of the time integration is adjusted so that the energy error between two outputs is smaller than a certain prescribed value. The value we used was $\sim(1 \times 10^{-5})$–(1×10^{-6}), depending on N. We required higher accuracy for larger N since the duration of the simulation is longer. When the energy error is very large, the program automatically reads the output at the previous checkpoint and restarts with a reduced accuracy parameter.

The N-body simulation of point-mass systems with a large number of particles poses several technical problems besides the necessary computational power. The difficulty comes from the wide range of the timescale. The critical separation of two particles at which we apply the KS regularization is $\sim 1/N$. The time step for particles involved is of the order of $1/(100N^{1.5})$. On the other hand, the timescale of the evolution of the system is of the order of N. Thus, the time steps of particles that are integrated in the KS formalism are $1/(100N^{2.5})$ of the system time, which, for $N \sim 10^5$, is about as small as the limit of double-precision numbers. (Note that this estimate is valid for a soft binary. A hard binary would require time steps smaller by several orders of magnitude than described above.) Of course, a clever treatment can overcome this kind of difficulty, but the amount of programming work and the complexity of the resulting code would be considerable. The simplest solution would be to use a longer number format (e.g., quadruple precision). This would have been a viable solution in 1970s, when mainframe machines could handle quadruple-precision arithmetic reasonably fast. Unfortunately, most present reduced instruction set (RISC) computers are extremely slow at quadruple-precision arithmetic.

2.3. *Hardware and Calculation Cost*

For all calculations, we used GRAPE-4 (Taiji et al. 1996), a special-purpose computer designed to accelerate N-body simulations using the Hermite integrator and hierarchical time step algorithm. The total system consists of 1692 pipeline processor chips and has a theoretical peak speed of 1.08 TFLOPS. The simulations reported in the present paper were performed while the assembly and testing of the GRAPE-4 system were underway. Thus the number of processors varied during the calculation. For most of the 32K particle run, we used one-quarter of the system, with 423 processor chips, which has a peak speed of 270 GFLOPS. Calculations with smaller N were performed when the available number of processors was smaller.

The actual performance depends on many factors, but most strongly on the number of particles. With the present host computer (DEC 3000/900), the average speed we obtained for the 32K particle run was ~ 50 GFLOPS. This run took ~ 3 CPU-months. In comparison, the 10K particle run reported by Spurzem & Aarseth (1996) took 2 CPU-months on a Cray Y-MP. Since the calculation cost of globular cluster simulations is proportional to $N^{3.3}$, one-quarter of GRAPE-4 is effectively 50 times faster than a Cray Y-MP.

At present, a 32K particle system is about the largest we can try for the equal-mass case. Since the calculation speed of GRAPE-4 is still limited by the speed of the host computer, the actual calculation speed is roughly proportional to N for $N < 2 \times 10^5$. Thus the CPU time for globular cluster simulation on GRAPE-4 is proportional to $N^{2.3}$. A 64K particle run would take ~ 1 yr.

The above estimate is for an equal-mass isolated cluster of point-mass particles. The computational cost of more realistic calculations depends on many factors. Among them, the mass spectrum and the presence of primordial binaries are most important, and have adverse effects.

Systems with a mass spectrum evolve faster than equal-mass systems (see, e.g., Inagaki & Wiyanto 1984; Murphy & Cohn 1988). Therefore it is possible to handle a larger system. For example, if the evolution of the system is p times faster, GRAPE-4 can handle a number of particles $\sqrt{p}$ times larger. Thus, if the collapse is 4 times faster, GRAPE-4 can finish a 32K particle run in 3 weeks or a 64K particle run in 3 months.

The presence of primordial binaries would increase the CPU time. At present, however, actual cost is difficult to estimate because the calculation cost depends strongly on the core size, of which we have little knowledge (see § 4). It should be noted that the number of floating-point operations used to follow the life of a binary depends only weakly on the total number of particles in the system (Makino & Hut 1990). Therefore, while the CPU time for the orbit integration of single stars would increase as $N^{2.3}$, the cost of binaries would increase only as N. It is likely that for large N the cost of binaries is relatively small, even for simulations with primordial binaries.

3. RESULTS

3.1. *Core Oscillation*

Figure 1 shows the temporal evolution of the central density for all runs. The time is scaled so that thermal timescale is the same for all runs. The scaling factor is $t_r(1000)/t_r(N) = 212.75 \log (0.11N)/N$ (Giersz & Heggie 1994). The core density shows oscillations with a large amplitude in calculations with large N (>16 K). No matter what the real nature of this oscillation, it is at least clear that the core density of the N-body system shows oscillation with an amplitude comparable to that observed in gas models or FP calculations.

For runs with small N (2K and 4K), there are some oscillation-like features, but they are hardly distinguishable from fluctuations. This result is similar to those of previous studies (e.g., Makino 1989). For large values of N, however, the oscillation with large amplitude is clearly visible.

Note that in Figure 1 there is no clear transition from stable expansion to oscillation or from regular oscillation to chaotic oscillation, which were observed in gas and FP models (Heggie & Ramamani 1989; Breeden et al. 1994). The reason is that binaries emit energy intermittently (Makino & Sugimoto 1987). Takahashi & Inagaki (1991) incorporated this stochastic nature of the energy source into their FP model and found that the core density showed

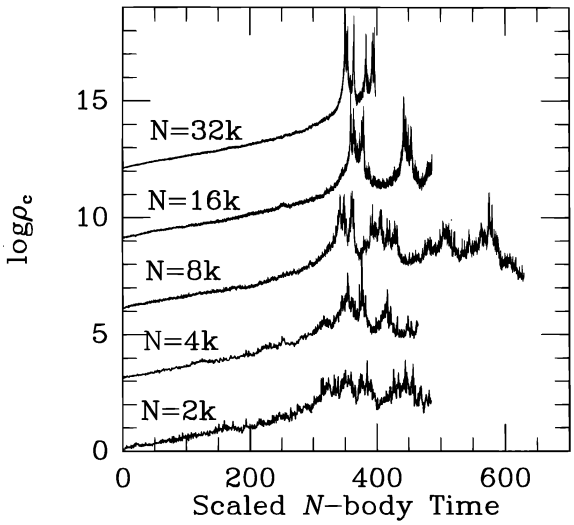

FIG. 1.—Logarithm of the central density plotted as a function of the scaled N-body time. Curves for different values of N are vertically shifted by 3 units.

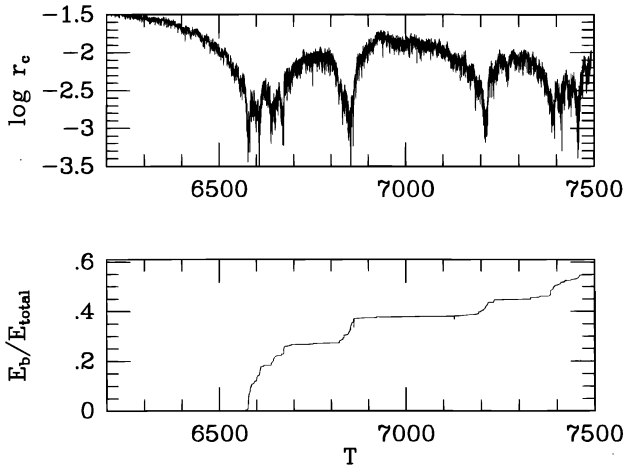

FIG. 2.—Core radius (*top*) and binding energy of binaries (*bottom*) as a function of time for the 32K particle run.

chaotic oscillatory behavior even if the energy production rate was larger than the critical value for transition between stable expansion and regular oscillation. They also found that the amplitude of the oscillation was smaller for larger energy input (smaller N), which is consistent with the present result. In fact, it would be difficult to distinguish our N-body calculation results from their stochastic FP results. The only visible difference is that their result is smoother while the central density is low. This is because the FP result is perfectly smooth as long as there is no binary heating.

3.2. *Analysis of the 32K Run*

In the previous section, we summarized the postcollapse evolution of N-body point-mass systems with 2K–32K particles. In this section, we take a closer look at the 32K particle simulation to see whether we can find direct signatures of gravothermal expansion. It is generally believed that a long expansion phase without significant energy input and a temperature inversion during this phase are the most direct signatures of gravothermal expansion (Bettwieser & Sugimoto 1984; McMillan & Engle 1996). In this section, we investigate both.

Figure 2 shows an enlarged view of the time variation of the core radius as compared with the sum of the binding energies of all binaries. It is clear that most of the energy is generated when the core is very small.

Continued expansion without energy input from binaries has been considered to be the most direct signature of gravothermal oscillation. In Figure 2, one can clearly see three such expansions, at $t = 6700$–6740, $t = 6860$–6920, and $t = 7220$–7280. All these expansions continue for more than 10 half-mass crossing times, which is hundreds of the core relaxation time. These expansions cannot be driven simply by binary heating. If the expansion were driven only by binary heating, it could not continue without energy input for a timescale more than 100 times longer than the core relaxation time.

Figure 3 shows temperature profiles for the contracting and expanding phases. Near the end of the expanding phase, a temperature inversion of the order of 5% is clearly visible. Since these profiles are time-averaged over 10 time units (80 snapshots), the actual inversion might be somewhat stronger. Note that the temperature inversion is also visible only near the end of long expansion phases in gas-model and FP calculations (Bettwieser & Sugimoto 1984; Cohn et al. 1989).

Figure 4 shows the relation between the central density and the central velocity dispersion. The trajectory bears a striking resemblance to that obtained by gas-model calculations (Goodman 1987). The fact that the trajectory shows clockwise rotations means that this is a refrigeration cycle, in which the central region absorbs heat when the temperature is low and releases heat when the temperature is high (Bettwieser & Sugimoto 1984; Bettwieser 1985). In particular, the later phase of the large expansions (indicated by the arrow marked "B") is nearly isothermal. Since the binding energy of binaries is unchanged during this phase, this nearly isothermal expansion is driven by heat supplied from outside the core. In other words, the expansion is gravothermal.

3.3. *Core Size—Results and Interpretation*

Figure 5 shows the evolution of the number of particles in the core. For all runs, the number of particles at the maximum contraction is of the order of 10 while that at the maximum expansion is $\sim 1\%$ of the total number of particles. This result is again in good agreement with gas models and FP calculations (e.g., Heggie & Ramamani 1989; Breeden et al. 1994).

Figure 6 shows the fraction of time for which the number of particles in the core is smaller than the value N_c as a function of N_c/N, for the postcollapse phase. In other words, this figure shows the cumulative distribution of the core mass. For $N > 8192$, the median core mass is $\sim 0.5\%$–0.6%. This corresponds to $r_c/r_h \sim 0.01$. For $N = 32{,}768$, the core mass is somewhat smaller than that for 16K or 8K runs.

Whether the core size in the gravothermal oscillation phase depends on the total number of particles is not well understood. Bettwieser & Sugimoto (1984) argued that clusters spend most of their time in the most expanded state and

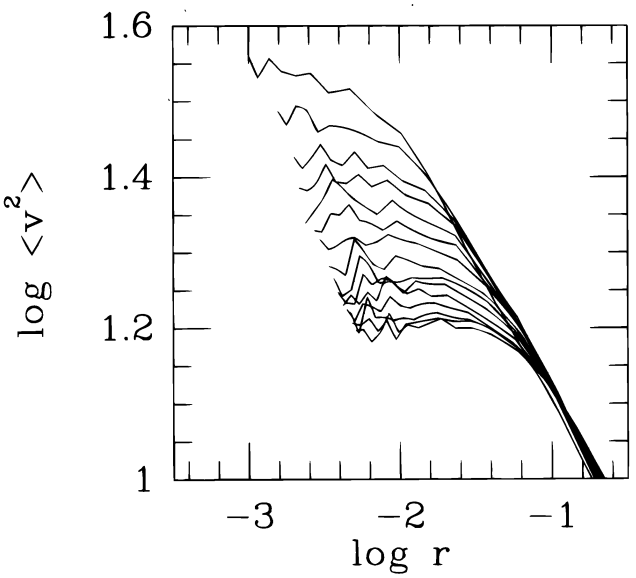

FIG. 3a

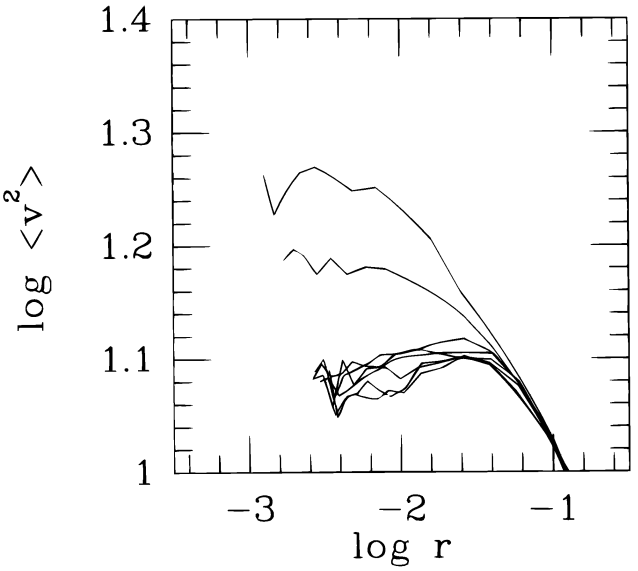

FIG. 3b

FIG. 3.—Velocity dispersion profiles for (*a*) contracting and (*b*) expanding phases. Each profile was obtained by time-averaging over 80 snapshots (10 time units). The interval between curves is 5 time units.

that the structure of the cluster at the maximum expansion does not depend on the total number of particles. However, it is clear from Figure 6 that the cumulative time is roughly proportional to the core radius and that the coefficient is larger for larger N. In other words, the size of the core depends on N.

Simple theoretical argument suggests that the core size must depend on N. The time-averaged energy production in the core would be the same as that for the stable expansion (Goodman 1987; Heggie & Ramamani 1989). This requirement imposes a constraint on the time-averaged core size.

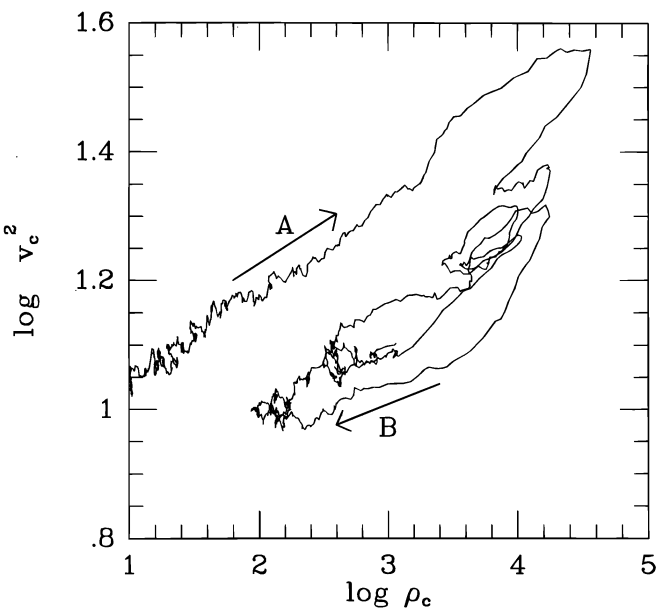

FIG. 4.—Change of the central density and the central velocity dispersion. Each data point is a time-averaged value over 80 snapshots. Arrows indicate the direction of evolution.

The energy production rate is expressed as

$$dE/dt \propto M_c \rho_c^2 , \tag{1}$$

where M_c and ρ_c are the core mass and core density. Here we have ignored the dependence on the velocity dispersion since the inner part is almost isothermal. For M_c and the core radius, r_c, we have the relation $M_c \propto r_c$, again because the inner part of the cluster is almost isothermal. Thus

$$dE/dt \propto M_c^{-3} . \tag{2}$$

The time-averaged energy production rate must be equal to that of the homologous expansion solution obtained by Goodman (1984). The core size for the homologous expansion is $N^{-2/3}$ (Goodman 1984; Giersz & Heggie 1994). Thus we obtain

$$\langle M_c^{-3} \rangle^{-1/3} \propto N^{-2/3} , \tag{3}$$

where angle brackets denote a time average.

Figure 7 shows the arithmetic mean and the maximum of the core mass after the collapse as functions of N. We also plotted the quantity $M_e = \langle M_c^{-3} \rangle^{-1/3}$ since we have a theoretical prediction only for this quantity. For maximum core mass, the FP results of Breeden et al. (1994) are plotted for $N = 5 \times 10^4$ and 10^6 (these values were read by eye from their Fig. 5). The quantity M_e indeed shows reasonable agreement with the theoretical prediction of equation (3).

However, the average and maximum core radii show a dependence noticeably weaker than the theoretical prediction. It should be noted that our N-body results for the maximum core mass and the FP results of Breeden et al. (1994) are on one line expressed as $M_c \propto N^{-1/3}$. From Figure 7, the core mass at the maximum expansion is approximated as

$$M_{c,\max} \sim 0.006(N/10^5)^{-1/3} M . \tag{4}$$

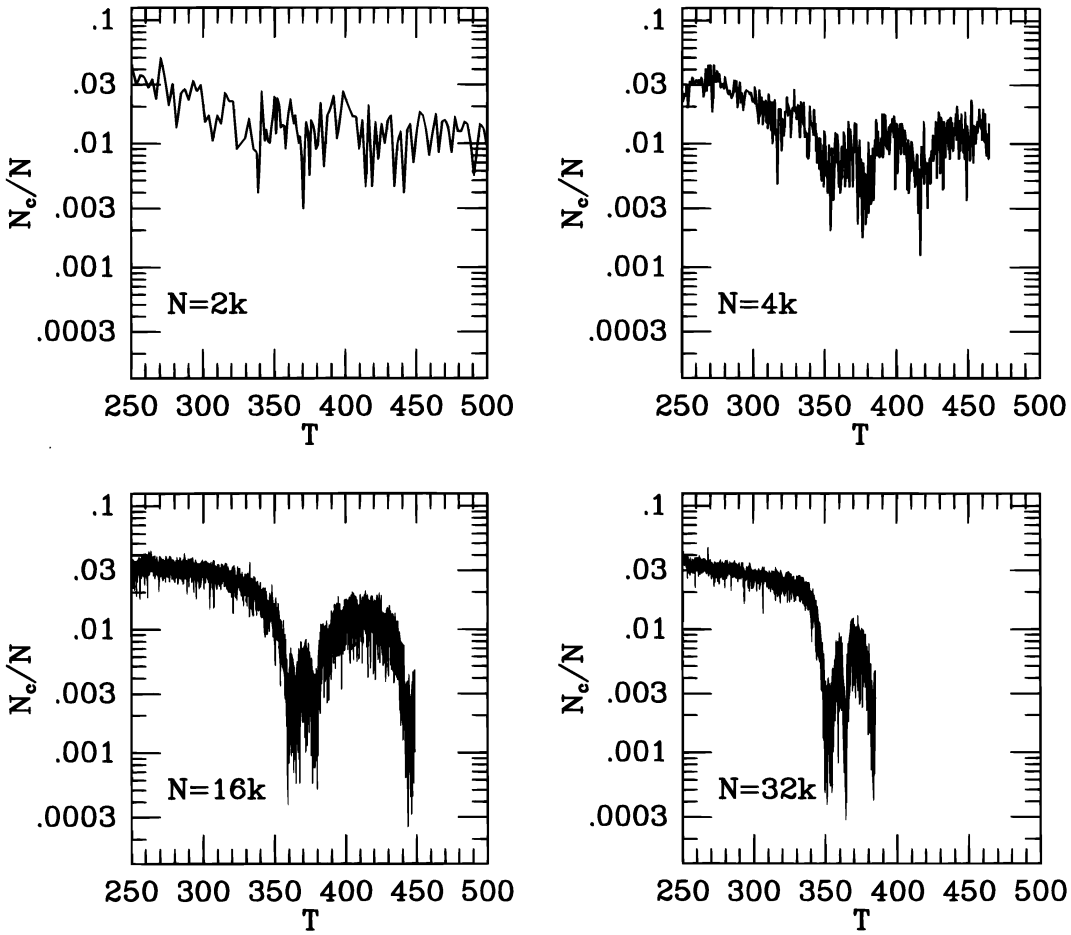

FIG. 5.—Number of particles in the core as a function of the scaled time, for simulations with 2K, 4K, 16K, and 32K particles

The slope of the average core size seems to be somewhat steeper.

Qualitatively, it is natural that the ratio between the maximum core mass and M_e is larger for larger N, since the ratio between the minimum core mass and M_e is larger for larger N. The minimum core mass is $O(1/N)$ since the number of particles in the core at the core bounce is always $\sim$10–30. In other words, the energy production rate at the

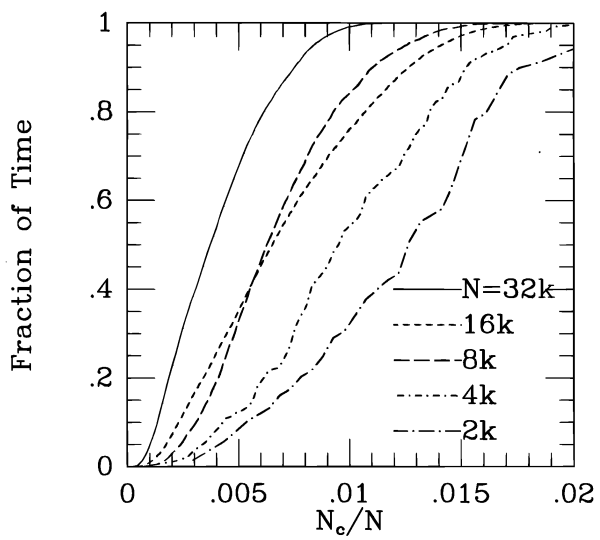

FIG. 6.—Fraction of time for which the number of particles in the core is smaller than N_c, as a function of N_c/N.

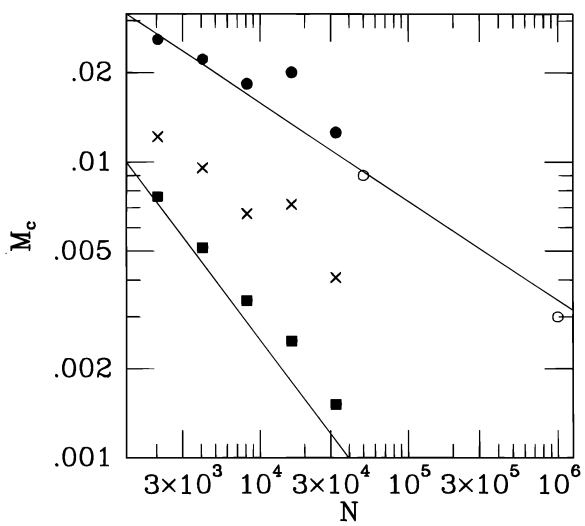

FIG. 7.—Core mass plotted as a function of the total number of particles N. Squares are M_e (see § 3.3). Crosses are arithmetic means. Filled circles are maximum values from present N-body simulations. Open circles are maximum values from FP calculations of Breeden et al. (1994). Straight lines indicate $M_c \propto N^{-1/3}$ (*top*) and $M_c \propto N^{-2/3}$ (*bottom*).

core bounce is larger than the time-averaged rate by a factor proportional to N. Thus, roughly speaking, the core mass has to stay at a value larger than the value at the core bounce for most of the time. The typical relaxation timescale of the core not at the maximum contraction is, therefore, N times longer than that at the maximum contraction. Since the relaxation time is proportional to M_c^2, this implies that the typical core mass is $N^{-1/2}$. This is in fact a reasonable fit for the average core mass in Figure 7.

One interesting question is whether it is possible to distinguish the core in the gravothermal oscillation and the core dominated by primordial binaries. The theoretical prediction of Goodman & Hut (1989) yields $r_c/r_h \sim 0.02$. The Fokker-Planck calculation by Gao et al. (1991) yielded a similar result. On the other hand, N-body simulations by McMillan et al. (1990) yielded $r_c/r_h \sim 0.2$. McMillan et al. (1990) used 1136 particles. If their N-body results can be extrapolated to larger values of N, cores with primordial binaries and cores in gravothermal oscillation would be clearly distinguishable. On the other hand, the FP result of Gao et al. (1991) indicates that the typical core size at the "primordial binary burning" phase and that at the gravothermal oscillation phase would not be much different. To obtain a definitive answer, we need to perform simulations of clusters with primordial binaries using a number of particles larger than that employed by McMillan et al. (1990).

3.4. *Core Size—Comparison with Observations*

Djorgovski & King (1986) showed that 15% of Galactic globular clusters have unresolved density cusps. They classified these clusters as "post–core collapse" (PCC). Recent ground-based observations (Lugger, Cohn, & Grindlay 1995) and *HST* results (Sosin & King 1995; Guhathakurta et al. 1996; Yanny et al. 1994) have demonstrated that some of the PCC clusters have cusps that are unresolved even with *HST* (NGC 6624, M15). For these clusters, the surface density profile obeys a power law down to 0".03. For other clusters (e.g., M30), although the slope seems to level off toward the center, it turned out to be difficult to determine the slope accurately because there are too few stars. In the case of M30, the core radius can be anywhere smaller than 1".5.

Many researchers have claimed that there is a good agreement between the theory and observation of the core radius. For example, Djorgovski & Meylan (1994) found that no observed cluster has a core radius smaller than 1% of the half-light radius, using the data compiled by Trager, Djorgovski, & King (1993), and argued this to be in good agreement with the model that assumes a binary-dominated core for PCC clusters (Vesperini & Chernoff 1994).

This apparently good agreement between the "observation" and "theory" is actually due to the fact that both of them overestimated the core size. On the observational side, high-resolution observations have shown that the real core size of the PCC clusters is significantly smaller than what was assumed in Djorgovski & Meylan (1994). Trager et al. (1993) adopted the half-width at half-maximum (HWHM) as the core size of PCC clusters. Since the HWHM has no relation to the real core radius of the PCC clusters, it is quite natural that observations with higher resolution obtained smaller core radii. On the theoretical side, our present result suggests that the typical size of the core in the gravothermal oscillation phase is significantly smaller than the previous claims, for two reasons. First, the time-averaged core size is about half the maximum size. Second, the core size depends on N as $N^{-1/\alpha}$, with $\alpha \sim 2$–3.

For NGC 6624 and M15, core sizes seem to be too small for a stably expanding cluster, no matter what the heat source. So it is highly likely that they are undergoing gravothermal oscillation.

4. DISCUSSION

We have performed direct N-body simulations of the postcollapse evolution of globular clusters. We confirm that gravothermal oscillation actually takes place in a point-mass N-body system.

Whether real globular clusters undergo gravothermal oscillation is a question that requires further research. If clusters contain many primordial binaries, then even after core collapse they might still be burning the primordial binaries. In addition, the effect of two-body binaries on the evolution of the cluster is still unclear. In fact, the cross section for binary formation by tidal capture is not fully understood yet (Mardling 1995a, 1995b).

The most straightforward way to study the effect of primordial binaries or two-body capture is by direct N-body simulation. In principle, we can put primordial binaries and their evolution into FP calculations, as Gao et al. (1991) did. However, the standard one-dimensional FP calculation, which assumes an isotropic distribution, is not appropriate to follow the binary population since most binaries are formed in the core and their orbits are nearly radial. Thus we have to solve the FP equation in at least three dimensions (E, J, and the binary binding energy E_b). This would require prohibitively large computational power. In addition, reliable two-dimensional FP calculation has become possible only very recently (Takahashi 1995). One could also use the Monte Carlo approach, but its results still must be compared with N-body simulations.

We have demonstrated that N-body simulation with a number of particles close to real globular clusters is now possible, thanks to the extremely powerful special-purpose computer GRAPE-4 and its full-time availability. We are now able to use direct N-body simulations to study various aspects of the evolution of globular clusters.

Our 32K particle calculation took $\sim$3 months of CPU time on one-quarter of GRAPE-4. If we tried to perform a similar calculation on a Cray T90 vector supercomputer, it would have taken several years of CPU time. It is simply impossible to do with present-day supercomputers. If we want to finish the calculation in, say, 1 CPU-month, we need a computer 50–100 times faster than a Cray T90, which will be available some 10 years from now.

The CPU time of 3 months is still very long. However, for many simulations, we do not need as many as 32K particles. A 16K particle calculation was finished in 2–3 weeks on one-eighth of GRAPE-4. Thus to run many simulations of 16K particle systems is now practical.

If we can continue the development of this special-purpose computer, we will have a system 100–1000 times faster than the present GRAPE-4 in the next 5 years. Such a system will make it possible to run 50K–100K particle simulations routinely, while 500K–1000K particle simulations will still take months.

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