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THE MAXIMUM MASS OF A NEUTRON STAR

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ABSTRACT

Observational identification of black holes as members of binary systems requires the knowledge of the upper limit on the gravitational mass of a neutron star. We use modern equations of state for neutron star matter, fitted to experimental nucleon-nucleon scattering data and the properties of light nuclei, to calculate, within the framework of Rhoades & Ruffini (1974), the minimum upper limit on a neutron star mass. Regarding the equation of state as valid up to twice nuclear matter saturation density, ρ_{nm}, we obtain a secure upper bound on the neutron star mass equal to 2.9 $M_\odot$. We also find that in order to reach the lowest possible upper bound of 2.2 $M_\odot$, we need to understand the physical properties of neutron matter up to a density of $\sim 4\rho_{nm}$.
Subject heading: dense matter — stars: neutron

1. INTRODUCTION

The identification of a compact object as a black hole requires not only an accurate observational estimate of its mass but also knowledge of the maximum gravitational mass of a neutron star for stability against collapse into a black hole. A growing number of measured mass functions for X-ray binaries containing compact objects accreting mass from their companions has appeared in the literature in the last years. Knowledge of characteristics of the companion stars of these objects allows one to impose lower limits on their masses and to begin to identify black holes observationally (for recent reviews see Cowley 1994 and Tanaka & Lewin 1995). Determination of neutron star masses and the range they cover also provides significant information related to the mechanism responsible for neutron star formation and possible effects of the evolutionary history of their progenitors on their masses.

The calculation of an accurate maximum neutron star mass strongly depends on our knowledge of the equation of state of neutron matter up to very high densities, $\sim 5 \times 10^{15}$ g cm^{-3}. Modern equations of state are derived from accurate numerical solutions of the quantum mechanical nuclear many-body problem using two and three-body potentials fitted to experimental nucleon-nucleon (NN) scattering data as well the properties of light nuclei, which give information in the region of nuclear matter saturation density, $\rho_{nm} \sim 2.7 \times 10^{14}$ g cm^{-3}. See Wiringa, Fiks, & Fabrocini (1988, hereafter WFF88) for a description of such derivations and further references. The equations of state can be safely regarded as accurate up to $\sim 2\rho_{nm}$, and possibly extended up to $\sim 4\rho_{nm}$, while the study of neutron star structure demands an extrapolation toward much higher densities, where the properties of matter remain rather uncertain. In particular, less well-known intrinsic three-nucleon forces become more important with increasing density, as do further hadronic and eventually quark degrees of freedom (reviewed in Baym 1995).

A powerful approach to the problem is to derive least upper bounds to the maximum allowed gravitational mass of a neutron star by using the properties of neutron matter at density ranges where they can be accurately predicted, and imposing a minimum number of constraints at densities exceeding a higher fiducial density, ρ_0, e.g., subluminal sound velocity and thermodynamic stability. In this way, uncertainties related to the properties of high-density neutron matter can be circumvented in obtaining a secure upper limit to the maximum allowed mass of a neutron star. A number of authors have approached the problem at different levels of completeness and for different sets of fundamental assumptions. Hartle (1978) offers a thorough and detailed review of past work. Oppenheimer & Volkoff (1939) were the first to suggest that the mass of a stable neutron star becomes maximum for the stiffest possible equation of state that is consistent with fundamental physical constraints. Rhoades & Ruffini (1974), following this approach, were the first to prove this suggestion rigorously. Using a variational method in which they constrained the equation of state to have subluminal sound velocity and be stable against microscopic collapse, they proved that, in the regions where it is uncertain, the equation of state that produces the maximum neutron star mass is the one for which the sound speed is equal to the speed of light. As a result they found a maximum neutron star mass $\simeq 3.2$ $M_\odot$ assuming uncertainty in the equation of state above a fiducial density $\rho_0 = 4.6 \times 10^{14}$ g cm^{-3}.

In this paper we revisit the issue of the maximum neutron star mass, employing recent equations of state for neutron matter of WFF88 in the framework of the variational analysis of Rhoades & Ruffini (1974). In the next section we describe our calculation, the resulting neutron star models, and the maximum neutron star mass as a function of the fiducial density, ρ_0, up to which one believes the modern neutron star matter equations of state to be reliable. A principal result of this study is to show the dependence of the maximum neutron mass on current uncertainties in the equation of state. In § 3, we compare our results to those of earlier studies and discuss their implications regarding the observational identification of black holes in binary systems.

2. MAXIMUM NEUTRON STAR MASS

To calculate a least upper bound on the maximum mass of a neutron star stable against gravitational collapse we follow the initial argument of Rhoades & Ruffini (1974) and consider a neutron star to be divided in two parts: (i) an outer envelope of which the mass is calculated based on a specific neutron matter equation of state and (ii) an inner core whose mass we extremize. The interface between these two regions is at a specified fiducial density, ρ_0. The set of fundamental constraints, independent of the detailed physical properties of neutron matter, imposed on the equation of state of the inner core are the following: (i) the mass density, ρ, is non-negative, i.e., gravity is attractive; (ii) the pressure, P, at zero temperature is a function of ρ only, i.e., neutron matter is a fluid; (iii) $dP/d\rho \geq 0$, so that the zero-frequency sound speed of neutron matter $(dP/d\rho)^{1/2}$ is real and matter is stable against microscopic collapse; (iv) the sound speed does not exceed the speed of light, i.e., $dP/d\rho \leq c^2$, hence signals cannot be superluminal and causality is satisfied.[1] As Ruffini & Rhoades show, under these conditions the mass of the neutron star becomes maximum for the stiffest possible equation of state, one for which the sound speed, c_s, is equal to the speed of light:

$$c_s^2 = \frac{dP}{d\rho} = c^2 . \tag{1}$$

To evaluate the maximum mass of a neutron star, we adopt, at densities below ρ_0, the recent equations of state presented by WFF88, which represent the most complete microscopic calculations to now that use available low-energy nuclear data. WFF88 derive the equation of state for two different Hamiltonians, using the Argonne v_{14} (AV14) and the Urbana v_{14} (UV14) two-nucleon potentials. In contrast to mean field calculations fit to saturation properties, the two potentials are directly fitted with high accuracy to elastic nucleon-nucleon scattering data and the properties of the deuteron. The two-nucleon potentials are supplemented with the Urbana VII (UVII) three-nucleon potential constructed to provide the saturation of bulk nuclear matter at 16 MeV binding energy at ρ_{nm}, and to fit the properties of light nuclei. The results obtained by WFF88 represent the best microscopic equation of state for dense matter constrained by nucleon-nucleon scattering data. The two variations of the equation of state are tabulated in Table V of WFF88. For densities lower than 2.5×10^{14} g cm^{-3}, we use the equation of state given by Baym, Pethick, & Sutherland (1971) for convenience, since the details of the equation of state at these low densities do not affect our results. Above the fiducial density, ρ_0, we continue the WFF88 equations of state from their values at ρ_0 assuming $c_s = c$.

We note that for the two variations of the equation of state used above $\rho = 2.5 \times 10^{14}$ g cm^{-3}, the sound speed exceeds the speed of light at very high densities, $\rho \gtrsim 1.6 \times 10^{15}$ g cm^{-3}. This violation of causality arises from the form of the three-body forces used (WFF88). Essentially, a three-body force that produces an energy density $E_3 \sim n^3$, as in the simply UVII interaction, where n is the baryon density, leads to a contribution to the pressure $P_3 = 2E_3$, which tends to increase c_s above c at very high density. A more careful treatment of the momentum dependence of the three-body force is required to avoid this problem. Since subluminal sound velocity is one of the constraints imposed on the equation of state of the inner core, the results for fiducial densities exceeding 1.6×10^{15} g cm^{-3} become suspect; however, as we see below, the maximum mass as a function of ρ_0 reaches a plateau before this point.

Having specified the equation of state and the fiducial density, ρ_0, we then calculate a series of neutron star models for a range of central densities, ρ_c, by directly integrating the general relativistic equation of hydrostatic equilibrium, the Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{dP}{dr} = -\frac{G}{r^2}\left[\rho(r) + \frac{P(r)}{c^2}\right]\left[m(r) + 4\pi r^3 \frac{P(r)}{c^2}\right] \times \left[1 - 2G\frac{m(r)}{rc^2}\right]^{-1} , \tag{2}$$

out to the radius R of the neutron star, where $P(R)$ falls to zero; in equation (2)

$$m(r) = \int_0^r 4\pi r'^2 \rho(r')dr' . \tag{3}$$

The resulting neutron star masses as a function of central density, ρ_c, for different fiducial densities, ρ_0, are shown in Figure 1 for the AV14 plus UVII potential. As one would expect, as ρ_0 decreases and the stiffest part of the equation of state becomes more dominant, the neutron star mass at specified central density increases. Using these "maximal" models we calculate the maximum neutron star mass as a function of the fiducial density, ρ_0, shown in Figure 2 for the

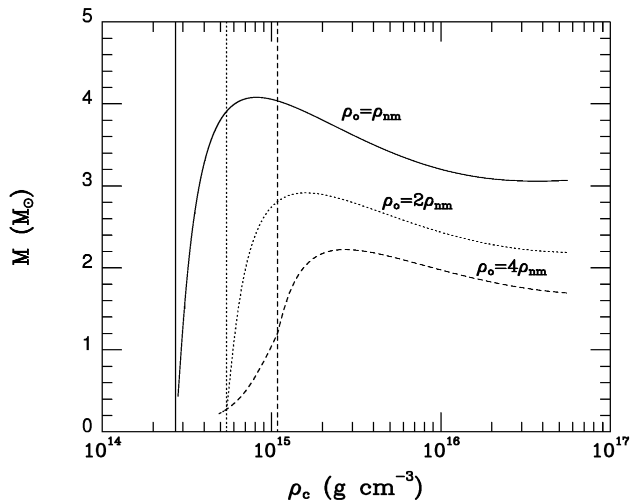

FIG. 1.—Neutron star mass, M, as a function of central density, ρ_c, for three different values of the fiducial density, ρ_0. At densities below ρ_0 the equation of state based on the AV14 plus UVII potential is used. Vertical lines lie at constant $\rho_0 = \rho_{nm}$, $2\rho_{nm}$, and $4\rho_{nm}$, where nuclear matter density $\rho_{nm} = 2.7 \times 10^{14}$ g cm^{-3}.

[1] The connection between the zero frequency sound velocity being greater than the speed of light and violation of causality, while physically plausible, is a tricky question, due to the presence of the frequency dependence of the sound velocity and sound wave damping. We are not aware of a general proof yet that the ground state of matter must obey $dP/d\rho \leq c^2$. See Hartle (1978) for further discussion of this issue.

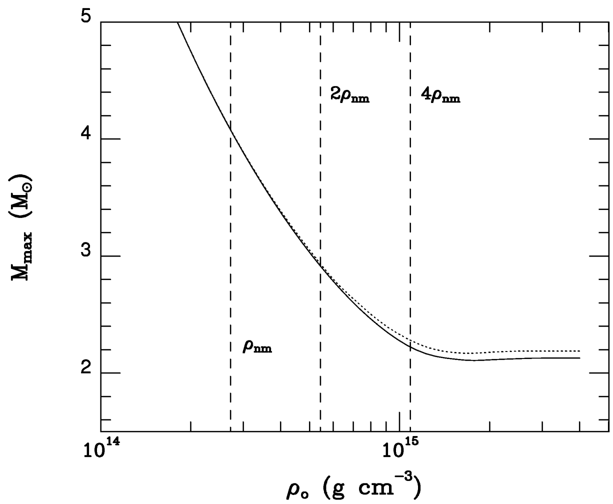

FIG. 2.—Maximum neutron star mass, $M_{\rm max}$, as a function of the fiducial density, ρ_0, for the two variations of the WFF88 equation of state: AV14 plus UVII potential (*solid line*) and UV14 plus UVII potential (*dotted line*). Vertical dashed lines lie at constant $\rho_0 = \rho_{\rm nm}$, $2\rho_{\rm nm}$, and $4\rho_{\rm nm}$.

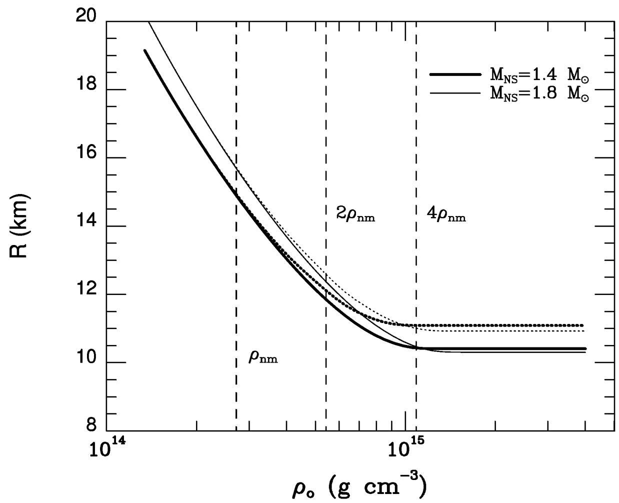

FIG. 3.—Neutron star radius, R, as a function of the fiducial density, ρ_o. Curves are plotted for two different neutron star masses, $M_{NS} = 1.4\, M_\odot$ (*thick lines*) and $M_{NS} = 1.8\, M_\odot$ (*thin lines*), and for the two variations of the WFF88 equation of state: AV14 plus UVII potential (*solid lines*) and UV14 plus UVII potential (*dotted lines*). Vertical dashed lines lie at constant $\rho_0 = \rho_{\rm nm}$, $2\rho_{\rm nm}$, and $4\rho_{\rm nm}$.

two variations of the equation of state used below ρ_0. At low densities the results are well approximated by

$$M_{\rm max} = 6.7\, M_\odot \left(\frac{\rho_0}{10^{14}\ {\rm g\ cm^3}}\right)^{-1/2} . \quad (4)$$

At densities higher than $\simeq 10^{15}$ g cm^{-3}, $M_{\rm max}$ approaches a limiting value $\simeq 2.2\, M_\odot$ independent of ρ_0. The reason is that at high fiducial densities the mass of the inner core becomes negligible compared to that of the outer envelope and consequently, the upper bound is governed by the maximum mass of the envelope, which is independent of ρ_0. Note that this limiting value is reached before the WFF88 equations of state become superluminal.

In Figure 3 we show the radii of the maximal neutron star models of mass 1.4 and 1.8 $M_\odot$, as a function of the fiducial density for the two equations of state of WFF88. Here the UV14 plus UVII equation of state (*dotted lines*) produces slightly larger stars at higher ρ_0 than the AV14 plus UVII equation of state (*solid lines*). The dependence of the neutron star radius on mass is weak for $\rho_0 \gtrsim 10^{15}$ g cm^{-3}. At low densities the radius of a 1.4 $M_\odot$ neutron star is well approximated by

$$R_{1.4} = 21.2\ {\rm km} \left(\frac{\rho_0}{10^{14}\ {\rm g\ cm^3}}\right)^{-0.35} . \quad (5)$$

3. DISCUSSION

We have taken into account recent neutron star matter equations of state to determine a new upper bound on the mass of a neutron star stable against gravitational collapse. Our results agree with the qualitative behavior of $M_{\rm max}$ as a function of the fiducial density, ρ_0, found in earlier studies (see Hartle 1978). The normalization of our empirical relation at low densities is also consistent with earlier results. At higher densities $M_{\rm max}$ becomes independent of ρ_0, and approaches a plateau value $\simeq 2.2\, M_\odot$, higher than previously found. Effects of rotation become important only if the neutron stars rotate close to breakup. Friedman & Ipser (1987) have studied the effect of uniform rotation (differential rotation is efficiently damped; Hegyi 1977) and found that maximum rotation can cause an increase of ~25% on the maximum neutron star mass.

The lowest value of $M_{\rm max}$ is determined by the maximum value of ρ_0 up to which one can be confident of the WFF88 equation of state. Their results can be regarded as valid up to $\rho_0 = 2\rho_{\rm nm}$, at which density $M_{\rm max} = 2.9\, M_\odot$. We consider this value to be our safest estimate. If we take their results as valid up to $\rho_0 \sim 4\rho_{\rm nm}$, $M_{\rm max}$ is further reduced to $\simeq 2.2\, M_\odot$. Further increase of ρ_0 does not affect the value of $M_{\rm max}$, which remains constant at $\simeq 2.2\, M_\odot$. The appearance of this plateau is significant because it indicates that in order to have an accurate estimate of the lowest possible upper bound on the neutron star mass, we need only understand the detailed properties of neutron matter up to a density $\sim 4\rho_{\rm nm}$, which is a feasible extrapolation from low energy nuclear data.

The updated upper bound of 2.9 $M_\odot$ on the neutron star mass can be used to discriminate between neutron stars and black holes based on measurements of the mass function of the companion to the compact object. The mass function depends on the inclination, i, i.e., the angle between the line of sight and the normal to the orbital plane, and the mass of the companion. Although the inclination of the plane of the orbit is generally not known, its maximum value ($i = 90°$) can be used along with a lower limit to the mass of the companion to set a lower limit on the mass of the compact object. If this limit exceeds the value of $M_{\rm max}$, one has strong evidence that the compact object is a black hole. In this way, nine black holes candidates have been identified so far. The lower limits of their masses lie in the range from 3.1 to 6 $M_\odot$. Since the value of $M_{\rm max} = 2.9\, M_\odot$ we report here is substantially lower than the one used to date (3.2 $M_\odot$), it is quite probable that more compact objects can be identified as black holes.

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