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HOT BOTTOM BURNING IN ASYMPTOTIC GIANT BRANCH STARS AND THE TURBULENT CONVECTION MODEL

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ABSTRACT

We investigate the effect of two different local turbulent convection models on the structure of intermediate-mass stars (IMSs, $3.5\ M_\odot \leq M \leq 7\ M_\odot$) in the asymptotic giant branch (AGB) phase where, according to observations, they should experience hot bottom burning (HBB). Evolutionary models adopting either the mixing length theory (MLT) or the Canuto & Mazzitelli (CM) description of stellar convection are discussed.

It is found that, while the MLT structures require some degree of tuning to achieve, at the bottom of the convective envelope, the large temperatures required for HBB, the CM structures spontaneously achieve these conditions. Since the observational evidence for HBB (existence of a class of very luminous, lithium-rich AGB stars in the Magellanic Clouds showing low $^{12}C/^{13}C$ ratios) is quite compelling, the above result provides a further, successful test for the CM convective model, in stellar conditions far from solar.

With the aid of the CM model, we then explore a number of problems related to the late evolution of this class of objects, and give first results for (1) the luminosity evolution of IMSs in the AGB phase (core mass–luminosity relation and luminosity range in which HBB occurs) for Population I and Population II structures, (2) the minimum core mass for semidegenerate carbon ignition ($\sim 1.05\ M_\odot$), (3) the relation between initial mass and final white dwarf (WD) mass (also based on some observational evidences about the upper AGB stars), and (4) the expected mass function of massive WDs.

Confirmation of the theoretical framework could arise from an observational test: the luminosity function of AGB stars is expected to show a gap at $M_{\rm bol} \sim -6$, which would distinguish between the low-luminosity regime, in which AGBs become carbon stars, and the upper luminosities, at which they undergo HBB, have very low $^{12}C/^{13}C$ ratios, and become lithium rich.

Subject headings: convection — stars: AGB and post-AGB — stars: evolution — stars: interiors

1. INTRODUCTION

There are a number of relevant, still open astrophysical problems about the advanced evolutionary phases of low- and intermediate-mass stars (IMSs). Among these are a quantification according to first principles of the various mechanisms that lead nuclearly processed matter to the surface; the occurrences driving mass loss, and the relation between initial stellar mass ($M_{\rm in}$) and final white dwarf mass ($M_{\rm WD}$); the value of the minimum mass for intermediately degenerate carbon ignition ($M_{\rm up}$); and the origin of neon- and magnesium-rich WDs. Solution of these problems is required as a whole, and as an input for Galactic chemical evolution, population synthesis, interpretation of colors of distant galaxies, and so on.

All the above problems deal with evolutionary phases following central helium exhaustion, from the base of the asymptotic giant branch (AGB) to the final ejection of planetary nebula, after which the blueward excursion leading to WD begins. This field has been explored in many important theoretical investigations in the last quarter century, starting with the earlier work of Paczyński (1970a) and Iben (1975) and following papers (see, e.g., the review in Iben 1984), up to Schönberner (1979), Wood & Zarro (1981), and ending with Lattanzio (1989), just to list a few of the more relevant contributions. Unfortunately, the two basic ingredients (mass loss and turbulent convection) that are the key to really understanding what happens in stars during these phases are still treated, in present-day modeling, according to simplified schematizations. In our opinion, one of the two above ingredients (mass loss) will be subject to semi-empirical parameterization (see, e.g., Vassiliadis & Wood 1993; Boothroyd & Sackmann 1992) for some time to come, while recent advancements in the application to stars of modern treatments of turbulent convection (and others still to follow in the near future) can make this aspect of the problem more manageable. We will then concentrate our efforts on the problems of convection in AGB giants.

This paper will first discuss the consequences when some of the more recent updates in microphysical inputs (opacities, thermodynamics, etc.) are included in the evolutionary models, still in a mixing length theory (MLT) framework, and ignoring mass loss. Then we will show what happens when convection is described according to the Canuto & Mazzitelli (1991, hereafter CM) model. In fact, the first tests provided interesting enough results to warrant their publication, before undertaking the much more complete and cumbersome study necessary to gain a wide, consistent picture of the AGB phase. The check of the two convection models will be the modalities of occurrence of "hot bottom burning" (HBB). Static envelope models (Sackmann, Smith, & Despain 1974; Scalo, Despain, & Ulrich 1975) showed that, at large luminosity ($L \geq 2 \times 10^4\ L_\odot$), the base of the convective envelope could reach temperatures ($T \gtrsim 5 \times 10^7$K) allowing nuclear burning, whose effects could be seen at the surface. In these models, carbon is converted into nitrogen, and the ratio $^{12}C/^{13}C$ decreases. The theoretical predictions qualitatively agree with observations, since a drastic decrease of the $^{12}C/^{13}C$ ratio in the spectra of luminous AGBs is a well-known

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feature.

It has also been suggested that ^{3}He burning at the bottom of the convective envelope is connected to the presence of lithium in the late AGB stars (via the "fast ^{7}Be transport" mechanism; Cameron & Fowler 1971) in the Large and Small Magellanic Clouds (Smith & Lambert 1989, 1990; Plez, Smith, & Lambert 1993). HBB is the best framework in which this mechanism seems to work (Sackmann & Boothroyd 1992).

Most previous computations with updated physical and numerical inputs, in which the mixing length theory was adopted to describe turbulent convection, showed that the occurrence of HBB requires large values of the ratio mixing length to pressure scale height ($\alpha = l/H_p \geq 2$). In § 2 we confirm this requirement. We then test (§ 3) the Canuto & Mazzitelli (1991) model for overadiabatic convection and find that it quite naturally provides HBB.

This last successful application of the CM model encouraged us to extend the computations to metallicity ranges for which there is still insufficient observational evidence of HBB to tune MLT models. We suggest that the lithium production phenomenon will in the future likely be described well enough theoretically to be used as a signature of the absolute luminosity of AGB stars and, as such, as a distance indicator, since, at variance with MLT, the CM model need not be tuned. In § 4 we describe the results for our Population I models and for the test models of lower metallicity.

Understanding of the general AGB evolutionary framework requires computations much more extensive than the present ones; computations in which the influence of other physical and chemical inputs (e.g., low-T opacities, helium content, detailed nucleosynthesis of s-elements, etc.) is explored. Nevertheless, these first results allow us to briefly comment on some of their consequences. Section 5 shows the $M_{\rm in}$-$M_{\rm WD}$ relation predicted, § 6 deals with the minimum mass for semidegenerate carbon ignition and the maximum AGB luminosity, and § 7 shows the mass function for WDs resulting from our $M_{\rm in}$-$M_{\rm WD}$ relation. Conclusions and plans for future work are given in § 8.

2. MODEL INPUTS AND MLT MODELS

The main features of the stellar evolution program are described in Mazzitelli, D'Antona, & Caloi (1995) and references therein. We recall the main updates in microphysical inputs:

1. New radiative opacities both for stellar interiors (from the astrophysical database of Rogers & Iglesias 1992, 1993, according to the solar Z distribution by Grevesse & Noels 1993) and for the low-temperature external layers (Alexander & Ferguson 1994);
2. Neutrino losses from Itoh et al. (1992) and references therein;
3. Electron conduction from Itoh & Kohyama (1993) and references therein;
4. Equation of state from Rogers, Swenson, & Iglesias (1995) for $T \gtrsim 5000$ K, and from Däppen et al. (1988) for lower temperatures.

Since there is still a poor understanding of what happens at the boundaries of a convective region (Xiong 1985; Canuto 1992), we decided to simply stick to the Schwarzschild criterion to fix the lower convective boundaries. Other choices are possible (e.g., the more sophisticated algorithm used by Sackmann's group), so that the temperature at the bottom of our convective envelopes may be slightly underestimated in our computations.

The formulation of the MLT has been taken from Cox & Giuli (1968). Note that there are different possible choices for some parameters appearing in the theory (i.e., the shape factor a_0 can be put to 9/4, or to 1, etc.), and that the α value required to fit the Sun depends on these choices.

With the above inputs, the fit of the present solar luminosity at an age 4.6 ± 0.1 Gyr with $Z = 0.0175$ (Grevesse & Noels 1993) requires $Y = 0.271$, and this same composition is adopted to represent Population I chemistry in the present IMS models. In the MLT case, the fit to the observed solar $T_{\rm eff}$ is obtained with $\alpha = 1.55$. However, since the solar metal abundance chosen is a lower limit, we decided to also test the upper limit, $Z = 0.02$ (Grevesse 1984). In this case, the He abundance has to be increased up to $Y = 0.282$, and the MLT α value required increases up to 1.58. This effect (increase of α with Z) is discussed by Sackmann, Boothroyd, & Fowler (1990). The global effect on α is very small: the increase in Z, which would give lower $T_{\rm eff}$ and thus require a larger α, is in fact partially compensated by the necessity of a larger Y to fit the present solar luminosity.

In Figure 1 we show the H-R diagram relative to the evolution of a 5 $M_\odot$ model, from the homogeneous main sequence to an advanced AGB phase, obtained in the MLT framework with $\alpha = 1.5$, 2.0, and 2.5. The latter track has been computed starting from one of the previous sequences, so only the AGB evolution is shown.

Figure 2 shows the temperature at the bottom of the convective envelope ($T_{\rm bce}$) as a function of the H-exhausted core mass $M_{\rm core}$ in AGB, for the three MLT evolutionary tracks above. After the first six pulses, the MLT solar-tuned evolution is still far from HBB conditions, and, even hypothesizing that $T_{\rm bce}$ will go on increasing pulse by pulse with the same slope (an unlikely situation, since after the first few pulses saturation conditions are quickly achieved;

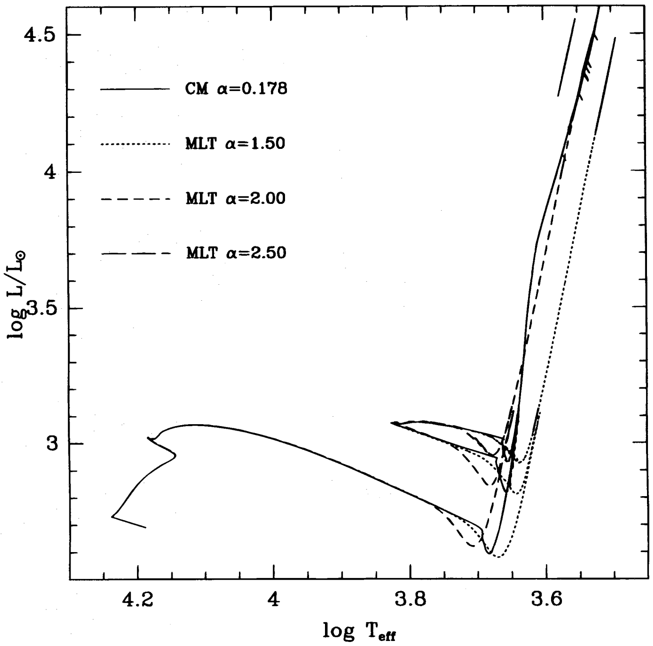

FIG. 1.—H-R diagram for 5 $M_\odot$ with MLT and CM convection

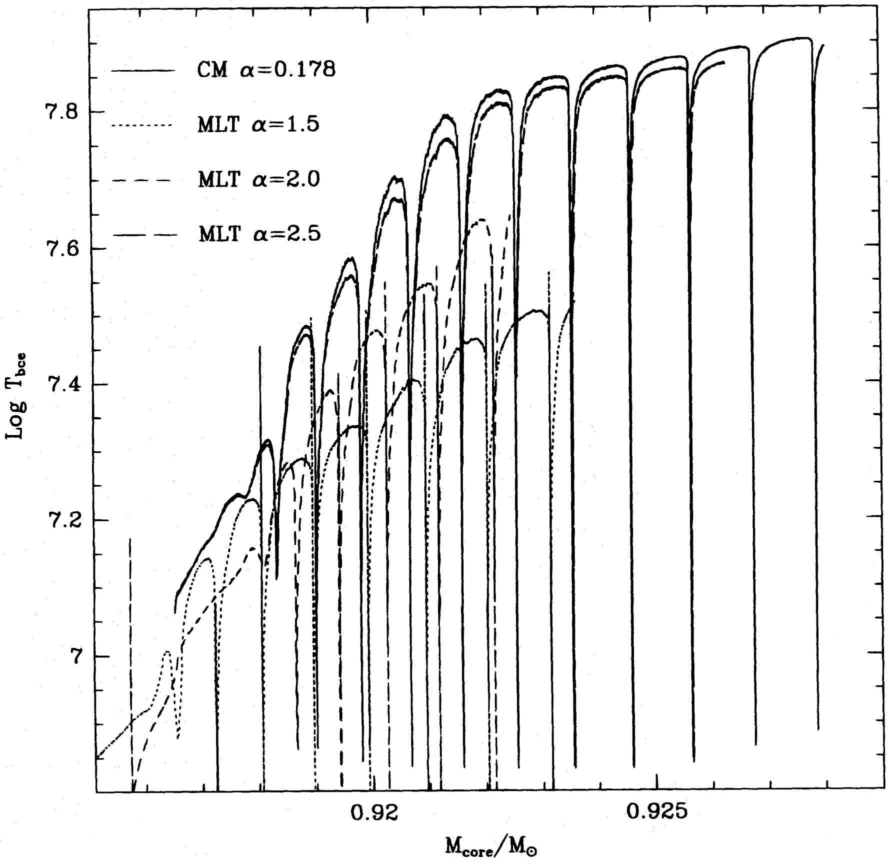

FIG. 2.—Run of temperature at the bottom of the convective envelope for the 5 $M_\odot$ models with different convection treatments

e.g., Mazzitelli 1987), it would reach HBB conditions only after ~15 pulses. In the $\alpha = 2$ case, the situation is better, but immediate HBB is reached only with $\alpha = 2.5$. Just for reference, let us recall that, for a solar model, such an α value would lead to a radius approximately 12% smaller than the true solar radius. Sackmann & Boothroyd (1992) obtained HBB with $\alpha = 2.1$, but they used different radiative opacities, so a direct comparison to our results is not allowed.

In Figure 3 the surface luminosity evolution as a function of $M_{\rm core}$ is depicted, for the three MLT cases. The sequence attaining HBB also shows a fast luminosity increase at the beginning of the AGB, consistent with the findings by Blöcker & Schönberner (1991). In fact, while $T_{\rm bce}$ rises from 10 to 60 × 10^6 K and HBB sets in, the luminosity increases by a factor ~2 while the core mass increases only by ~0.04 $M_\odot$. As was first shown by Blöcker & Schönberner (1991), HBB and this fast luminosity increase are related to each other. AGB stars conform to an almost linear core mass–luminosity relation (Paczyński 1970a) only as long as there is a relatively large radiative buffer between the H-exhausted core and the convective envelope, allowing thermal "decoupling" between core and surface. In HBB stars this condition is no longer met, since the inner convective boundary approaches so close to the extremely thin burning shell that it causes it to leak.

Figure 4 shows the surface ^{3}He abundance for the same tracks and on the same abscissa. Only the MLT track with $\alpha = 2.5$ burns ^{3}He at the bottom of the envelope shortly after the beginning of the thermal pulses (TPs), and along the same track the surface ratio ^{12}C/^{13}C (not shown) is reduced from ~20 to ~3. Since ^{3}He depletion is in any case likely to be a prerequisite for lithium production, these models should correspond to the lithium-enriched AGB stars.

Let us now recall an argument that seems to have been overlooked until recently. The MLT tuning of α is meant to get the correct value of $T_{\rm eff}$; that is, we play with *one* free parameter to fit *one* observed quantity. Only recently, and only for the Sun, have we met a case in which the tuning of α should provide not only the $T_{\rm eff}$, but also the observed spectrum of oscillations, that is, *more than one* observed quantity. With a parametric convection theory, this procedure could prove successful only by chance, and, in fact, the MLT performs quite poorly when fitting both $T_{\rm eff}$ and high-frequency oscillations (Baturin & Miranova 1995).

For the AGB stars, we have a second case in which a stellar convection theory must fulfill *two* independent constraints, that is, the observed $T_{\rm eff}$, and the existence of HBB. It is perhaps too early to draw definitive conclusions, since not only the theoretical stellar models, but also the transformations of the observational parameters to the theoretical plane (mainly for AGBs) are still affected by uncertainties; nevertheless, there is at least a hint suggesting that the MLT will prove only marginally able—if at all—to pass the test. The hint comes from two facts: (1) we computed MLT models of several masses (see below), finding that an early onset of HBB always requires $\alpha \geq 2.5$ *independent of the mass*, and (2) Stothers & Chin (1995) have recently shown that, to fit the observed values of $T_{\rm eff}$ for red giants, the MLT value of α *must change with the mass*, being $\alpha = 2.8$ for 3 $M_\odot$, and $\alpha = 2.1$ for 5 $M_\odot$ (whereas the CM convection model fits the observations without any tuning). It is not clear from their paper how much consideration of

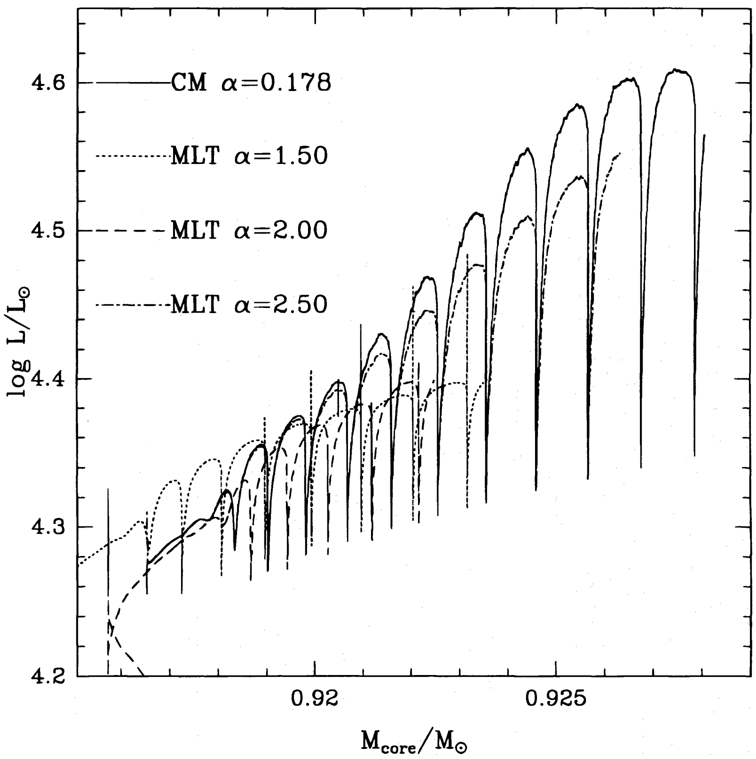

FIG. 3.—Luminosity evolution of the 5 $M_\odot$ models. Only the CM model and, though in a different way, the $l/H_p = 2.5$ model present an early sharp increase in luminosity following the first TPs. At this point, T_{bce} reaches the temperature necessary for the fusion of ^{3}He.

metallicities differing from cluster to cluster might affect this result, but they have recently shown that α apparently does not depend on the metallicity of the stars in consideration (Stothers & Chin 1996). In principle, one can object that Stothers & Chin considered only red giant stars, while we are discussing AGB stars. Actually, from Figure 1 (and from any theoretical model) one can see that, for stars having large convective envelopes (1 $M_\odot$ or more), the location of both red giants and AGBs is almost coincident with the Hayashi track, so that the conclusions reached for the $T_{\rm eff}$'s of red giants are also valid for the AGB. We regard the above result as another indication that the MLT is not so satisfactory when sound theoretical quantitative predictions are needed, since it not only requires tuning, but the results

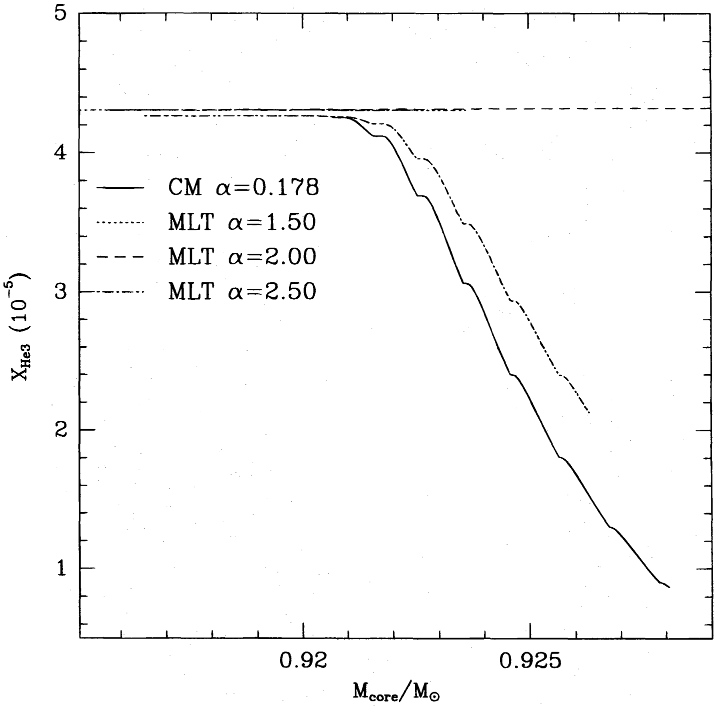

FIG. 4.—Burning of ^{3}He in the CM models of 5 $M_\odot$. In the MLT sequences of $l/H_p = 1.5$ and 2, the ^{3}He remains constant, while it is burned in the sequence with $l/H_p = 2.5$. It is possible that the other MLT models present HBB after many TPs.

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are, in the best of the cases, only marginally consistent with the observations. It is then necessary to examine the performances of the more modern and updated CM model.

3. THE CM MODEL AND THE 5 $M_\odot$ EVOLUTION

We now describe the results obtained when applying the CM model to the evolution of a 5 $M_\odot$ star. We see from Figure 1 that the CM track (*solid line*) has a different AGB location than does the MLT track (*dotted line*) with the solar-tuned value $\alpha = 1.5$, a better fit being obtained with $\alpha = 2.0$. The value $\alpha = 2.5$, required for an early onset of HBB, produces a hotter track ($\delta T_{\mathrm{eff}} \sim +7\%$–$8\%$) than the CM one. Stothers & Chin (1995), working with microphysical inputs close to the present ones, find the fit of the observed T_{eff} for the 5 $M_\odot$ red giant with $\alpha = 2.1$, and with the CM model without any tuning. Our results are then largely consistent with those of Stothers & Chin.

Figure 2 shows that, after the first TPs, the CM structures very quickly reach high temperatures at the bottom of convection, even larger than in the $\alpha = 2.5$ MLT structures. Also, ^{3}He depletion is similar along these last two sequences (Fig. 4). Figure 3 shows that the CM track presents, at the beginning of the AGB phase, an even sharper luminosity increase than the MLT, $\alpha = 2.5$ track, corresponding to the sudden shrinking of the radiative buffer between the H-burning shell and the convective envelope, and to the onset of HBB.

Contrary to the MLT, then, the CM model might be able to provide both the observed Hayashi track location in AGB (we extrapolate to larger luminosities the results obtained by Stothers & Chin for red giants—a safe procedure for stars with large convective envelopes) and HBB without tuning. Of course, the already quoted uncertainties, mainly in the correlations between theoretical and observational quantities, are not yet negligible; the differences in T_{eff} between the MLT models fitting the observations of red giants and the MLT models undergoing HBB are then such that one cannot definitely dismiss the MLT on these grounds only. Nevertheless, we regard this result as another significant indication that the CM model is to be preferred to the MLT model for the description of convection in *any* kind of star, and in the following we will discuss only results obtained with the CM model.

4. HOT BOTTOM BURNING AND THE SECOND DREDGE-UP

For Population I composition, we computed the evolutionary sequences of stars of masses 4, 4.5, 5, 5.5, and 6 $M_\odot$ from the main sequence through the first few TPs. A model of 7 $M_\odot$ was also evolved until it ignited carbon. The evolution of luminosity versus time for these sequences in AGB is shown in Figure 5 (*lower panel*). The age has been arbitrarily normalized to allow for an easier comparison among the various tracks. Models of $M \geq 5$ $M_\odot$ show the initial, fast luminosity increase linked to the onset of HBB, whose surface chemical signature (depletion of ^{3}He) is shown in the upper panel.

Figure 6 shows the relation between the H-exhausted core mass (M_{core}) and the stellar luminosity for the Population I evolutionary sequences. Also shown is the core mass–luminosity relation by Paczyński (1970a) and by Wood & Zarro (1981). The "second dredge-up," leading to a reduction of the hydrogen-exhausted mass (Becker & Iben 1979) is clearly present only for masses $M \geq 5$ $M_\odot$. For the

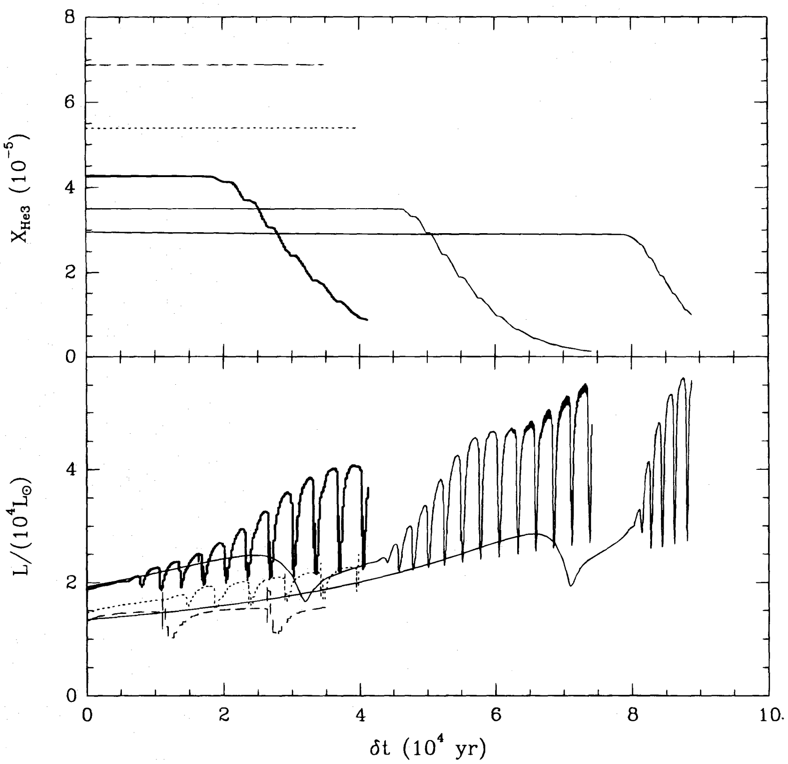

FIG. 5.—Temporal evolution of $L/L_\odot$ for the Population I stellar models (*bottom part*). The time is normalized so that all the evolutions fit in the same scale. Starting from lower luminosities, the models refer, respectively, to 4 (*dashed line*), 4.5 (*dotted line*), 5 (*thick line*), 5.5 and 6 $M_\odot$ (continuous lines) evolution. Notice how the time between two consecutive TPs decreases when increasing the mass, and the large increase in L during the first TPs in the 5 $M_\odot$ evolution. The upper part shows the ^{3}He mass fraction in the stellar envelope, whose decrease is a sign of the occurrence of HBB in the 5, 5.5, and 6 $M_\odot$ models. Notice how the rapid increase of stellar luminosity corresponds to the ^{3}He burning.

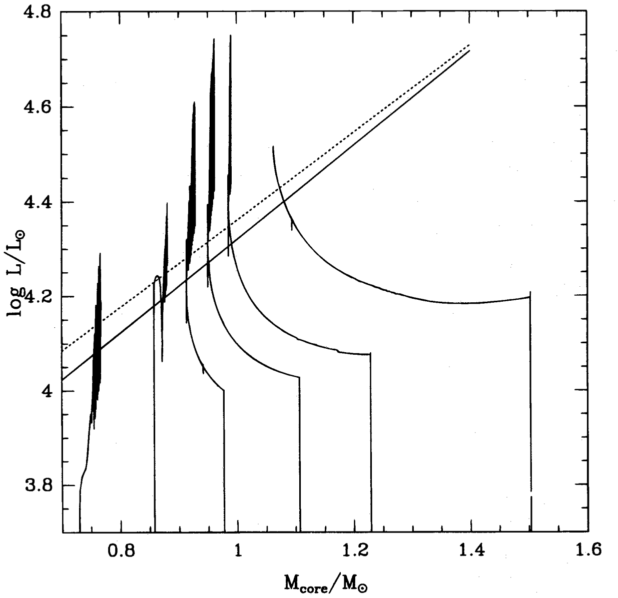

FIG. 6.—Evolution of stars of masses 4, 4.5, 5, 5.5, 6, and 7 $M_\odot$ in the core mass ($M_{\rm core}$)–luminosity plane. Stars up to 4.5 $M_\odot$ do not suffer the second dredge-up, which is present for $M \geq 5\ M_\odot$. These latter stars evolve through a rapid rise of stellar luminosity through the first thermal pulses. The 7 $M_\odot$ track finishes with C-ignition, while the second dredge-up was being completed. The two $M_{\rm core}$-luminosity relations by Paczyński (1970a) (*solid line*) and Wood & Zarro (1981) (*dotted line*) are also shown.

5 $M_\odot$ model, the core mass is reduced from $\sim$0.98 $M_\odot$ at the end of the core He burning to $\sim$0.91 $M_\odot$ after the dredge-up and prior to the beginning of the TP phase. The 7 $M_\odot$ ignites C burning just around the maximum deepening of convection during the second dredge-up, when its H-exhausted core is reduced to $\sim$1.06 $M_\odot$. The 7 $M_\odot$ is then very close to the lower mass limit for semidegenerate C ignition.

We also computed some Population II evolutionary sequences, finding that, for these stars, the onset of HBB occurs at lower luminosity and mass. Figure 7 presents the evolution of luminosity and surface ^{3}He versus time for the tracks of 3.5 and 4 $M_\odot$ with $Y = 0.24$ and $Z = 10^{-3}$. While the 3.5 $M_\odot$ undergoes neither the second dredge-up nor the HBB, the 4 $M_\odot$ shows both features. A last sequence of 3.5 $M_\odot$, with $Y = 0.23$ and $Z = 10^{-4}$, has also been computed. Neither second dredge-up nor HBB have been found (the second dredge-up is however found by I. J. Sackmann 1996, private communication) in $Z = 0.001$ models of $M > 2.5$).

From the above framework, some conclusions can be drawn.

1. Only Population I stars of mass $\gtrsim$4.5 $M_\odot$ and Population II stars of mass greater than 3.5 $M_\odot$ undergo the second dredge-up.
2. Only the masses undergoing the second dredge-up achieve HBB. In our models this holds both for Population I and Population II.
3. Stars undergoing HBB quickly evolve at *larger L* than would be expected on the basis of a linear core mass–luminosity relation (which, in turn, is a linear core mass–$M_{\rm in}$ relation). This result is in agreement with Blöcker & Schönberner (1991), Lattanzio (1991), and Boothroyd & Sackmann (1992).

This last feature clearly shows up both from Figure 6 and when we plot the relation between $M_{\rm in}$ and the $M_{\rm bol}$ achieved at the beginning of the TP phase (Fig. 8), where there is a sharp step in luminosity ($M_{\rm bol}$ rises from ~ -6 to ~ -6.7) in the range 4.5 $M_\odot \leq M_{\rm in} \leq 5\ M_\odot$. Figure 8 also shows that the minimum luminosity for HBB increases with metallicity (see also Sackmann & Boothroyd 1992).

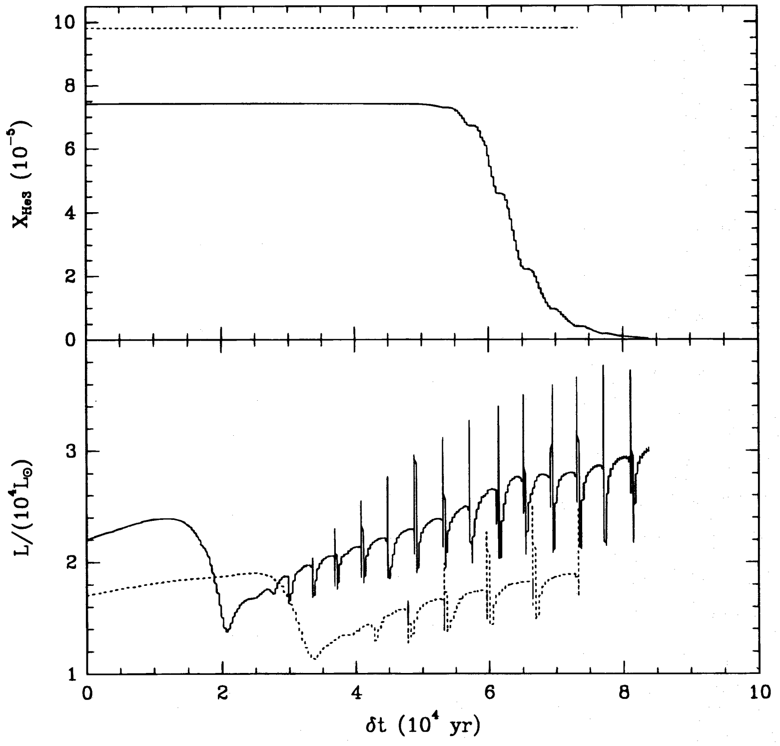

FIG. 7.—For the Population II stellar models of 3.5 and 4 $M_\odot$ we show the temporal evolution of $L/L_\odot$ (*lower panel*) and of the ^{3}He abundance (*upper panel*).

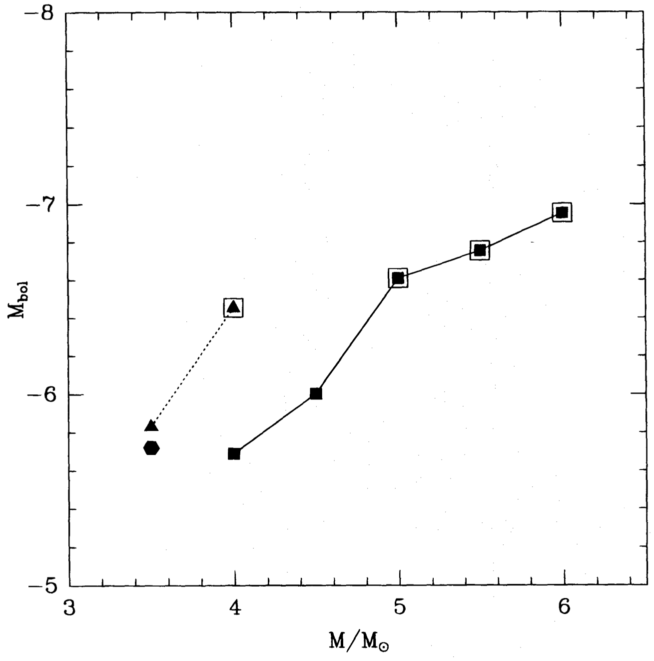

FIG. 8.—Points corresponding to the early AGB evolution of the considered models in the M_{in}-M_{bol} plane (or log $L/L_{\odot}$). Filled squares represent the Population I models, filled triangles the $Z = 10^{-3}$ models, and the filled hexagon the $Z = 10^{-4}$ model. The open squares contain the models that suffer hot bottom burning. Notice the good correspondence between M_{bol} for these models and the M_{bol} of lithium-rich AGBs in the Magellanic Clouds ($M_{bol} = -6.3$ to -7).

In principle, the sharp rise in luminosity at the onset of HBB, predicted by theoretical evolutions, should find an observational counterpart. Actually, the minimum $M_{bol} \sim -6.4$ for HBB at $Z = 10^{-3}$ corresponds quite well with the minimum M_{bol} at which the Li-rich AGBs appear in the Small Magellanic Cloud, while this class of stars is a bit more luminous in the LMC, which is also more metal rich (Smith & Lambert 1990), consistent with the present results. If further statistical studies of the luminosity distribution of AGBs in the LMC and SMC will also show a "gap" in the luminosity function around $M_{bol} \sim -6$, above which the stars with a low $^{12}C/^{13}C$ ratio appear, the whole theoretical framework will be confirmed.

5. THE M_{in}-M_{WD} RELATION

As already noted, quantification of mass loss from first principles is still far from complete, at least for the present generation of stellar modeling. If our goal is to derive a realistic M_{in}-M_{WD} relation, we must seek information on the termination of the AGB phase based on observational, rather than theoretical, evidence.

In this framework, a key role can be played by the so-called "carbon star mystery" (Iben 1981). Detailed study of the LMC and SMC luminous red giants showed that C-rich AGB stars are found at luminosities much lower than predicted by the canonical theory of the "third dredge-up" during the thermal pulses, and that they do not last until luminosities as large as would be expected. While the first problem is still with us, and is probably related to the

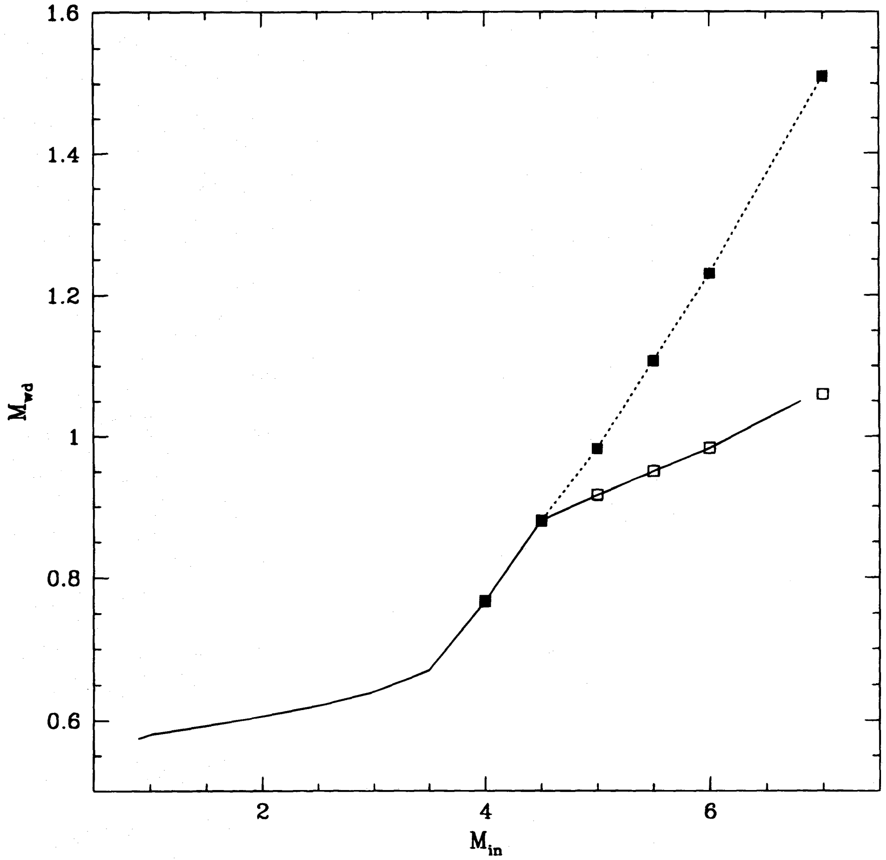

FIG. 9.—Relation between the initial stellar mass (*abscissa*) and the mass of the He-exhausted core (*dotted line*) or the mass after the second dredge-up (*full line*). The dots represent the computed models. Below 4 $M_{\odot}$ the behavior is extrapolated on the basis of the semiempirical M_{in}-M_{WD} relation in order to compute the predicted mass function of white dwarfs (Fig. 10).

"local" treatment of convection (which ignores partial mixing outside the formally convective boundaries), the lack of luminous carbon stars is likely to be explained in terms of large mass-loss rates when climbing the AGB. In fact, AGBs coming from IMS progenitors are present in an amount smaller than expected. Wood, Bessell, & Fox (1983) showed, for instance, that in the MCs there are AGB stars more luminous than $M_{\rm bol} \gtrsim -6$, which are not carbon stars. Since these stars also show signs of HBB, carbon processing through CN cycle at the base of the convective envelope has been invoked to explain the absence of C stars. This picture can be supported by theoretical models (e.g., Boothroyd & Sackmann 1993), but the scarcity of these M-type AGB very luminous stars (e.g., in the young clusters of the Magellanic Clouds; Frogel, Mould, & Blanco 1990) makes another hypothesis more appealing: the AGB phase is soon terminated by some mechanism, not yet well understood, of enhanced mass loss at large luminosities (e.g., envelope ejection via radiation pressure; see Wood & Faulkner 1986). If this is true, we must regard the observed AGB more as a "locus" than as a "sequence."

The latter suggestion receives support from the observational results by Smith & Lambert (1989), who showed that *all* the upper AGB stars discovered by Wood, Bessell, & Fox in the MCs were lithium rich. If we believe (Sackmann et al. 1974; Sackmann & Boothroyd 1992) that lithium survival in the convective envelope cannot last longer than a few times 10^4 yr, we may infer that (1) the stars become Li rich as soon as they begin their AGB life (as also confirmed by the present theoretical models), and (2) the AGB evolution is terminated no longer than a few times 10^4 yr after the star becomes Li rich.

At present, the above conclusions seem to be the only ones fully consistent with both observations and theory, even if several points (mainly a more realistic quantification of Li-survival times in a nonlocal convective framework) still need further clarification. If we provisionally accept them, we can raise two main consequences. First, the IMSs eject in the interstellar medium almost all the HBB processed envelope, so this feature becomes highly relevant for the Galactic chemical evolution, in particular of lithium (D'Antona & Matteucci 1991; Matteucci, D'Antona, & Timmes 1995). Second, the white dwarf remnant mass is nearly equal to the core mass when the AGB evolution begins, within a very few hundredths of $M_\odot$, that is, only slightly larger than the H-exhausted core mass after the thick He-shell burning phase (at the end of the "second dredge-up," if present).

Figure 9 shows the relation between the initial mass and the core mass before and after the second dredge-up from our models. This last must be considered close to the relation between the initial stellar mass and final mass of the remnant WD.

6. CARBON IGNITION, THE MAXIMUM AGB LUMINOSITY, AND THE PROGENITOR MASS OF SIRIUS B

As we anticipated in § 4, our 7 $M_\odot$ model ignites carbon off-center when the H-exhausted core is reduced to ~ 1.06 $M_\odot$, while the He-exhausted core is ~ 1.03 $M_\odot$. The thinness of the He intershell suggests that, by now, the "second dredge-up" phase is almost over—which is confirmed by our models. Complete references to previous work and a detailed discussion of the boundary in mass between degenerate and nondegenerate carbon ignition can be found in Becker & Iben (1980). With the present input physics, then, the minimum core mass for partly degenerate carbon ignition should be ~ 1.05 $M_\odot$. We followed C burning as long as the hydrostatic approximation was adequate to describe the structures, that is, as long as the term $\rho\ddot{r}$ was less than a few percent of the term $GM\rho/r^2$. By that time, two off-center flashes had occurred, the second one being more internal (and more degenerate) than the first. During each of the flashes, convection from the peak of C burning almost reached the He/C interface, and most of the carbon present in the convective region was burnt into Ne and Mg. As long as hydrostatic models can then make sense, and if further C-flashes eventually reaching the center will behave as the first two (Dominguez, Tornambè, & Isern 1993), at the end of C burning we would be left with an O-Ne-Mg structure devoid of carbon and unable to ignite oxygen because of the low temperature. This degenerate core would give rise to a single O-Ne-Mg WD if the envelope is blown away in the meantime. Otherwise, evolution should proceed until *e*-capture and collapse of the core (Nomoto 1982). Formation of such O-Ne-Mg WDs was predicted to occur only in close binaries (Iben & Tutukov 1985) if mass transfer is able to reduce the total stellar mass below 1.1 $M_\odot$ before C-ignition; the present framework suggests the possibility of forming *single* O-Ne-Mg WDs, although the computations are still much too preliminary to draw firm conclusions.

Interestingly enough, the observed luminosity limit of the AGB $M_{\rm bol} \sim -7$ is generally referred to $M_{\rm core} = 1.4$ $M_\odot$—according to the core mass–luminosity relation by Paczyński (1970a)—at which carbon ignites in conditions of large degeneracy (Paczyński 1970b), giving rise to Type Ia supernovae. Actually, both Blöcker & Schönberner (1991) and the present results show that the above core mass–luminosity relation is not valid in HBB conditions. The core mass at $M_{\rm bol} = -7$ is instead only ~ 1.05 $M_\odot$. This is just the limit for *semidegenerate* carbon ignition, which, in any case, seems to terminate the AGB. It is then possible that the more massive single WDs have an internal O-Ne-Mg composition, even if only the repeated nova outbursts in binaries can be able to lead these elements to the surface.

It is immediately recognized that the mass of the WD Sirius B ($M = 1.05 \pm 0.028$ $M_\odot$; Gatewood & Gatewood 1978) is just around the minimum mass for C ignition. According to our models, then, Sirius B's progenitor mass must have been ~ 7 $M_\odot$, for which the maximum pre-WD radius consistent with no–Roche lobe overflow and common envelope evolution to Sirius A is ~ 470 $R_\odot$ (D'Antona 1982). The 7 $M_\odot$ at 1.06 $M_\odot$ core mass has a radius of 494 $R_\odot$. This figure is perhaps marginally consistent with evolution without Roche lobe overflow, if we account for possible uncertainties in the theory; in any case, future models for the evolution of Sirius B must take into account that its mass is close to the that needed for off-center carbon ignition. Is Sirius B an O-Ne-Mg WD?

7. THE MASS FUNCTION OF MASSIVE WHITE DWARFS

From the $M_{\rm in}$-$M_{\rm WD}$ relation of Figure 9, extrapolated at $M < 4$ $M_\odot$ on the basis of known results of stellar evolution at smaller $M_{\rm in}$ (see Weidemann 1990 and Mazzitelli 1988), we can derive the mass function of white dwarfs in a coeval stellar population. In this case we have

$$\frac{dN}{dM_{\rm WD}} = \frac{dN}{dM_{\rm in}} \frac{dM_{\rm in}}{dM_{\rm WD}} . \quad (1)$$

Adopting an initial IMF of the type

$$\frac{dN}{dM_{\rm in}} = kM^{-(1+x)} , \qquad (2)$$

we can predict the form of the white dwarf mass function. This is shown in Figure 10 for $x = 0.7$, 1.0, and 1.3, where we adopted a $M_{\rm in}$-$M_{\rm WD}$ relation for masses smaller than 4 $M_\odot$ according to the observational and theoretical indications that this relation sharply flattens between 1 and 3 $M_\odot$. As we can see, the theory predicts a small secondary peak in the relation at $M_{\rm WD} \sim 0.9$ $M_\odot$. Observationally, a secondary peak is suggested by Bergeron, Saffer, & Liebert (1992) at $M_{\rm WD} \sim 0.8$, but the statistics are still too poor for any firm conclusion.

8. CONCLUSIONS

In this paper we have shown that the Canuto & Mazzitelli (1991) model for turbulent convection is successful in describing the properties of the thermally pulsating AGB stars of intermediate mass. In particular, it correctly predicts hot bottom burning. The same results in terms of HBB and fast luminosity evolution can be obtained by tuning of the α parameter in MLT structures, but in this case, one cannot reproduce also the $T_{\rm eff}$ predicted by the CM model, which seems instead more adequate to fit observations. It seems then worth exploring the parameter space of AGB evolution (especially chemistry) by adopting the CM treatment of convection, as this choice allows us to make sound theoretical predictions without the need to tune the models "a posteriori" on the grounds of observations.

Following work will be devoted to this purpose. In particular, we intend to investigate the dependence of the luminosity at which HBB occurs on the physical and chemical inputs, to understand whether the lithium-rich AGB stars and/or the AGBs having low ^{12}C/^{13}C ratios can be successfully used as a stellar distance indicator. Detailed nucleosynthesis of light elements is then a relevant input in this framework.

A fast luminosity increase at the beginning of the HBB phase, predicted by Blöcker & Schönberner (1991), is also found in our models. Theoretical population synthesis will show whether a gap in the luminosity functions of AGB stars is predicted, which should occur between the carbon stars and the Li-rich AGBs. Observational confirmation of this feature can be looked for in the Magellanic Clouds, and, if found, it would establish an important confirmation of theoretical evolution.

Our input physics predicts that, for solar chemistry, the minimum mass for semidegenerate carbon ignition is $\sim$7 $M_\odot$, at core mass $\sim$1.06 $M_\odot$. It is possible that repeated C flashes would lead to carbon exhaustion, and to an O-Ne-Mg WD core. This evolution cannot be followed by a long AGB phase, which would occur at $M_{\rm bol} < -7$; in fact, observationally, the AGB is not brighter than $M_{\rm bol} \sim -7$ (e.g., Smith & Lambert 1989, for the Magellanic Clouds stars). From the results of present models, then, Sirius B has a mass at the borderline of carbon ignition.

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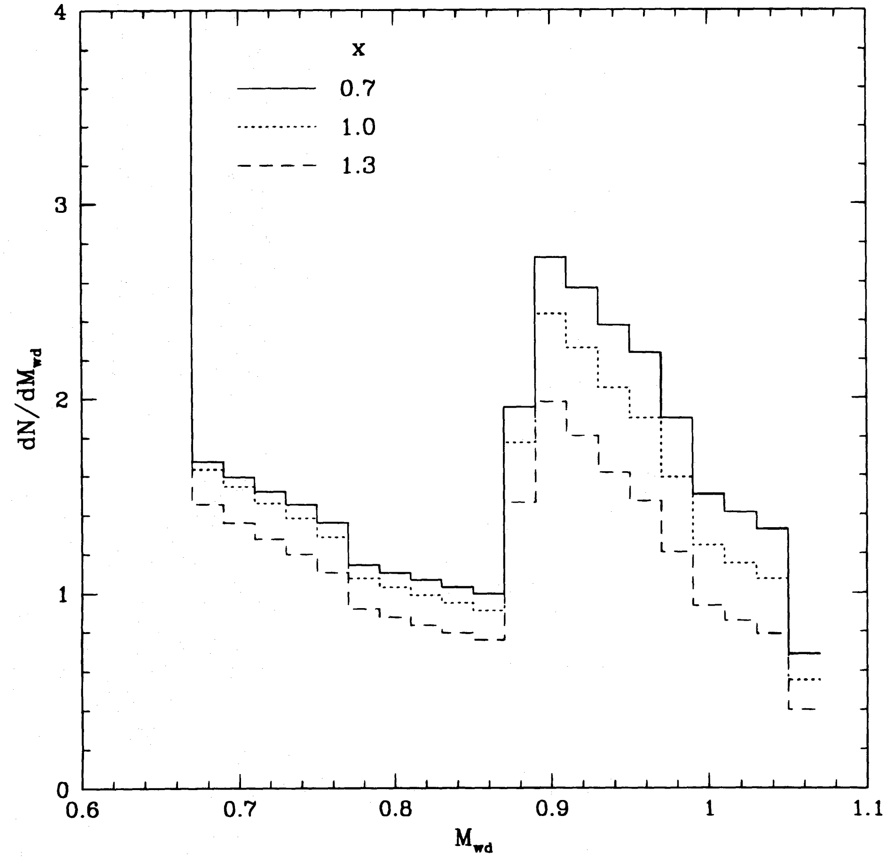

FIG. 10.—Mass function for white dwarfs in a coeval system, based on the $M_{\rm in}$-$M_{\rm WD}$ relation of Fig. 9 and on power-law initial mass functions (eq. [2]), for the three labeled values of the exponent x.

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