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# LIGHT CURVE OF A SOURCE ORBITING A BLACK HOLE: A FITTING FORMULA

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## ABSTRACT

A simple, analytic fitting formula for the photometric light curve of a source of light orbiting a black hole is presented. The formula is applicable for a source in a circular orbit with radius smaller than 45 gravitational radii from the black hole. This range of radii requires gravitational focusing of light rays and the Doppler effect to be taken into account with care. The fitting formula is therefore useful for modeling the X-ray variability of inner regions in active galactic nuclei.

*Subject headings:* accretion, accretion disks — black hole physics — galaxies: active — X-rays: galaxies

## 1. INTRODUCTION

In their seminal paper, Cunningham & Bardeen (1973) studied photometric light curves of a pointlike source of light orbiting in the equatorial plane of a rotating (Kerr) black hole. These authors adopted the approximation of geometric optics and presented a detailed discussion of periodic variations of the observed frequency-integrated flux. The origin of the variability is twofold: (1) the energy of an individual photon is affected by the Doppler effect (the effect of special relativity), and (2) photon trajectories are influenced by the gravitational field of the central black hole (a purely general relativistic effect). The motivation to study relativistic effects on light from a source near a black hole comes from astronomy; however, no direct observational comparisons were possible in the early 1970s, when the formalism was first established.

During the last decade, the interest of astronomers in modeling detailed features in the light curve of an orbiting source has revived. Satellite monitoring of active galactic nuclei (AGNs) has focused attention on X-ray variability (see Duschl, Wagner, & Camenzind 1991; Miller & Wiita 1991) as a means of investigating the central object, presumably a massive black hole surrounded by an accretion disk (Rees 1984). Variability timescales indicate that X-rays originate in the innermost region, of a few gravitational radii: $R \lesssim 25R_g$ (see, e.g., Heeschen et al. 1987). The power spectrum of AGNs has a particularly complex, featureless behavior at frequencies $\omega \gtrsim 10^{-5}$ Hz. The fluctuating signal can be represented, in the frequency domain, by a power law: $F_\omega \propto \omega^{-\alpha}$ with $1 \lesssim \alpha \lesssim 2$ (Lawrence et al. 1987). No strictly periodic AGNs have been discovered, but the power spectra of some objects (see Papadakis & Lawrence 1995) contain excessive power at frequencies $\omega \gtrsim 10^{-2}$ Hz, indicating quasi-periodic oscillations (QPOs) analogous to those associated with low-mass X-ray binaries (Lewin & Tanaka 1995).

In spite of the evident progress on the observational side, theoretical understanding of the short-term variability of AGNs remains rather insufficient. Several physical mechanisms have been proposed, referring to instabilities in accretion disks and associated jets. The possible existence and properties of vortices on accretion disks have been discussed by Abramowicz et al. (1992) and Adams & Watkins (1995). It is speculated that these vortices survive over several (perhaps many) local orbital periods. Radiation from separate vortices is modulated by their orbital motion around the black hole, and a large number of individual contributions result in the observed fluctuations. In a more phenomenological approach, one refers to spots (localized regions of enhanced or diminished emissivity) that reside on the surface of the disk (Abramowicz et al. 1991; Wiita et al. 1991; Zhang & Bao 1991). It is not crucial to the form of the power spectrum whether the spots are identified with vortices or whether they are of a different physical nature.

Our present contribution deals with the bright-spot model, but we would like to emphasize that other scenarios for the short-term variability of AGNs have been proposed. Krolik et al. (1991) explored ultraviolet continuum variability and pointed out that independent fluctuations originating at different radii in the source can produce power-law spectra. Mineshige, Ouchi, & Nishimori (1994) employed the theory of self-organized critical states that develop and persist in the disk material, also leading to the observed power spectra. Kanetake, Takeuti, & Fukue (1995) proposed that vertical oscillations of accretion disks are the source of QPOs and studied the characteristic periods of these oscillations. Ipser (1994) and Perez et al. (1996) associated QPO phenomena with frame-dragging effects that determine the behavior of relativistic disks and, in particular, with low-frequency modes in accretion disks near a Kerr black hole. Lamb et al. (1985), in the context of Galactic QPO sources, explored the interaction of a clumpy disk with the magnetosphere around a central object.

At present, it is probably fair to say that the relation between various possible instabilities and the resulting variable signal of AGNs is, at best, only vaguely understood. It is thus impossible to discriminate between intrinsically different models, even when they are testable in principle and the necessary information is contained in the observed signal. In other words, it is encouraging that some features of the power spectra can be explained by the bright-spot model, but in reality, this fact does not mean very much when the emission properties of individual spots are largely unknown. One could say (with Antoine de Saint-Exupéry) that "Truth is not that which is demonstrable, but that which is ineluctable."

Until the theory of (nonaxisymmetric) instabilities in acc-

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retion disks yields specific results about radiation outgoing from the disk surface, phenomenological models must take all possible values of their parameter space into account. This has not yet been possible with the bright-spot model, which has a number of degrees of freedom, such as the distribution of the spots across the disk, the intrinsic emissivity properties of individual spots, and the inclination of the observer. Several numerical codes to evaluate the light-curve profiles and corresponding power spectra have been developed (Asaoka 1989; Bao 1992; Karas, Vokrouhlický, & Polnarev 1992; Zakharov 1994), but models with a large number of different spots still remain too computationally expensive (some steps in the original derivation given by Cunningham & Bardeen 1973 need to be evaluated numerically). Authors (e.g., Abramowicz et al. 1991; Bao 1992) have had to impose additional assumptions on the model without a sound physical reason (a "canonical" number of 100 spots is just an example of such restriction). The main aim of the present contribution is to approximate the light curve of an individual orbiting source of light by a practical fitting formula. It is required that the formula, while reflecting relativistic effects in the light curve, can be implemented in a fast code and substitute the extensive ray tracing that is otherwise necessary. It will be specified later what can be considered "practical" and what is a suitable approximation for our purpose.

The fitting formula makes it possible to overcome the unnatural restrictions that are routinely imposed on the bright-spot model and to cover the parameter space of various models in a much more complete manner. The formula is presented in the next section.

## 2. THE FITTING FORMULA

### 2.1. *Assumptions*

It is assumed that the source of light is located in a circular orbit in the equatorial plane of a Kerr black hole (see Misner, Thorne, & Wheeler 1973). Prograde orbits of the source (corotating with the black hole) and direct-image trajectories of photons (not crossing the equatorial plane) are considered. The observer's position is specified, in Boyer-Lindquist coordinates, by $R \to \infty$, $\theta = \theta_0$. (The azimuthal coordinate is arbitrary, as a result of axial symmetry.) The observed radiation flux is determined by propagating photons along null geodesics from the source to the observer, in accordance with the approximation of geometric optics. It is thus straightforward to calculate the light curve of a pointlike source. The light curve of a finite-size source can be obtained by integrating over its surface. Calculation of the light curves of finite-size sources contributes significantly to the total computational cost of the bright-spot model, and we wish to make this step much faster. The technical details of our code to calculate light-curve profiles have been described by Karas et al. (1992).

Standard notation (see, e.g., Misner et al. 1973) will be adopted throughout this work, and geometrized units ($c = G = 1$) will be used; $M$ denotes the mass of the central black hole, and $a$ is its dimensionless angular momentum parameter. All lengths and times are made dimensionless by expressing them in units of $M$. The gravitational radius of the Kerr black hole is expressed in terms of $a$: $R_g = 1 + (1 - a^2)^{1/2}$, where $0 \le a \le 1$.

It turns out that the light-curve profile is very sensitive to the observer's inclination, $\theta_0$, and the radius of the orbit of the source, $R_s$. We will thus focus our attention on these two parameters. Rauch & Blandford (1994) studied the light curves of pointlike sources, which show extremely high peaks when the source crosses a caustic. The shape of the caustic depends on the angular momentum of the black hole, and the light curve is thus also sensitive to the value of $a$. The case is different when a finite-size source that covers the whole caustic is considered (or, alternatively, when the temporal resolution of the observation is lower than the caustic-crossing time); the caustic is then unresolved, and the high-magnification spikes are smoothed down. We will consider only a situation in which the source does not cross the caustic or in which its size, $d$, exceeds the cross-sectional size of the caustic, $s$. For $R_s \gg 1$, equation (6) of Rauch & Blandford (1994) yields an asymptotic formula: $s \approx 0.34 R_s^{-1} a^2 \sin^2 \theta_0$. In our calculations, it was assumed that each spot radiates isotropically in its local comoving frame and that the local emissivity decreases exponentially with the distance from the center of the spot; we checked that the computed profiles depend only weakly on these assumptions when the size of the spot satisfies the condition $s \ll d \ll R_s$.

It is only the variable component of the signal that is relevant for the source fluctuations. The zero level of our light curves was thus set at the minimum of the observed signal. The variable component of the radiation flux (counts per second) is given in arbitrary units. This arbitrary scaling of the profile contains sufficient information about the light curve when the radiating spots are all located at a constant radius. Additional information about the absolute value of the maximum count rate is necessary when a distribution of spots at different radii is considered. In this case, one has to prescribe the total flux from individual spots and their geometric shape as a function of radius. Since we pay little attention to the model-dependent quantities, the absolute scaling of the count rate is not discussed here (except for an illustrative example mentioned in § 2.2).

Analogously to the absolute value of the count rate, the phase of the light curve is also arbitrary. We scale the phase to the interval $0 \le \varphi \le 1$ and define the phase to be equal to 0.5 for the maximum magnification due to the focusing effect. This definition means that a phase of 0.5 corresponds to photons coming approximately (exactly, when $a = 0$) from the opposite side of the orbit, across the rotation axis, to the observer. The phase $\varphi$ increases linearly with time as measured by a distant observer. The relation between $\varphi$ and the orbital phase of the spot on the disk's surface is complicated because of time delay, but this has been taken into account automatically by integrating individual photon trajectories. Maximum Doppler enhancement of the observed radiation corresponds to a phase of $\approx 0.75$. Dimensionless phase can easily be converted to a time interval (as measured by a distant observer) when a rotation law for the spots is specified. For example, $\Delta t = R_s^{3/2} + a$ is the period of the Keplerian circular orbit (as measured by a distant observer), which is appropriate for a thin disk in the equatorial plane.

### 2.2. *Numerical Method and Results*

A series of light-curve profiles corresponding to a spot orbiting in the range of distances $R_s \le 45 R_g$ were computed. A model function $F(\varphi, R_s, \theta_0; p_k)$ was specified, depending nonlinearly on unknown parameters $\{p_k \,|\, k = 1, 2, \ldots, 8\}$. ($F$ is the measured frequency-integrated flux in arbitrary units.) The form of this function was chosen on the basis of asymptotic behavior and our experience with extensive calculations of the profiles (see Karas et al. 1992):

$$F = [p_3 \cos(\theta_0)/R_s + p_7(R_s - 1)^{2/5}] \times \{1 + \sin[2\pi(\varphi + p_4 R_s - \tfrac{3}{5}) + \tfrac{1}{2}\pi]\}^z \cos^{-2/3}\theta_0 + (p_1 + p_6 R_s^{1/3}) \exp(-p_2 |\varphi - \tfrac{1}{2}|^{9/5}) \cos^{-2}\theta_0 , \quad (1)$$

where

$$z = p_5 \cos^{1/2}\theta_0 + p_8 \cos^{3/2}\theta_0 .$$

The parameters $p_k$ were determined by the Levenberg-Marquardt method (nonlinear least-squares fitting; see Press et al. 1994). The range of fitting was restricted to

$$20° \le \theta_0 \le 80°, \quad 3R_g \le R_s \le 45R_g , \quad (2)$$

which is where relativistic effects on the light curve are most profound. Table 1 presents the results of the fitting for two values of the black hole's angular momentum: $a = 0$ (a nonrotating Schwarzschild black hole) and $a = 1$ (an extremely rotating Kerr black hole). The $a$-dependence is only weak, and it reflects a shift of relative phase of the two peaks in the light curve—the Doppler peak and the peak due to focusing (simultaneously visible only if $\theta_0 \gtrsim 70°$). The shift is caused by the frame-dragging effect that operates near rotating black holes. Figure 1 illustrates a typical form of the profiles, normalized to the maximum flux. The curves have been plotted according to equation (1), with parameters taken from Table 1.

Until now, only quantities that depend weakly on local properties of individual spots have been discussed. However, some applications are strongly model dependent, for example, when the radial distribution of spots or eclipses of spots by a thick disk are taken into account (Bao & Stuchlík 1992; Karas & Bao 1992; Mangalam & Wiita 1993). Vortices located across the surface of an accretion disk (as proposed by Abramowicz et al. 1992) can act as spots in our model; the maximum flux from spots at different radii must then be specified. Figure 2 illustrates the maximum flux as a function of radius for our simple model of isotropically radiating spots with constant size $d$. Relative fluxes from spots orbiting at different radii can be obtained by scaling the normalized light curves from Figure 1 by a corresponding value of the maximum flux. It is evident from Figure 2 that, at small radii, gravitational redshift decreases the total flux when $\theta_0 \lesssim 50°$ while the Doppler and lensing enhancements dominate for $\theta_0 \gtrsim 50°$. It is straightforward to obtain graphs analogous to Figure 2 for other models of the spots' local emissivity.

## 3. CONCLUSION

The variable signal from a source orbiting a black hole is approximated by a fitting formula (eq. [1]). This formula presents a simple *model* of the corresponding exact expression, which is too complex and inconvenient for numerous

TABLE 1

PARAMETERS OF THE FITTING FORMULA

| $a$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ |
|---|---|---|---|---|---|---|---|---|
| 0...... | 0.021696 | 190.7236 | 0.3476 | −0.0018 | 3.5106 | $-3.6 \times 10^{-5}$ | 0.0124 | −0.0231 |
| 1...... | 0.024258 | 181.8421 | 0.0958 | −0.0032 | 4.7862 | $-3.0 \times 10^{-5}$ | 0.0109 | −0.1527 |

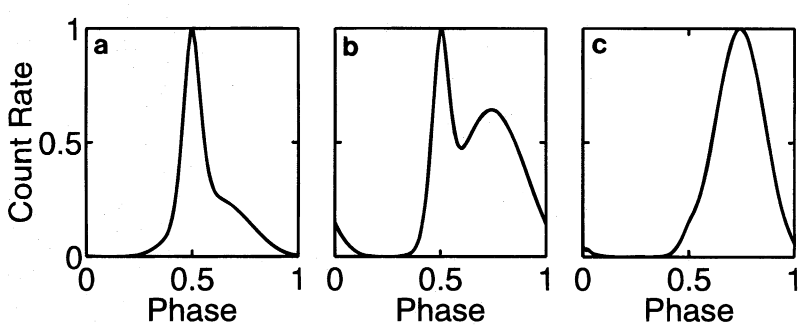


FIG. 1.—Illustration of the fitting formula. The light-curve profile normalized to the maximum flux (the count rate) is shown for three cases: $R_s = 3R_g$, $\theta_0 = 80°$ (*left*); $R_s = 44R_g$, $\theta_0 = 80°$ (*middle*); $R_s = 44R_g$, $\theta_0 = 20°$ (*right*). The shape of the normalized curve depends only weakly on $a$; here $a = 1$.

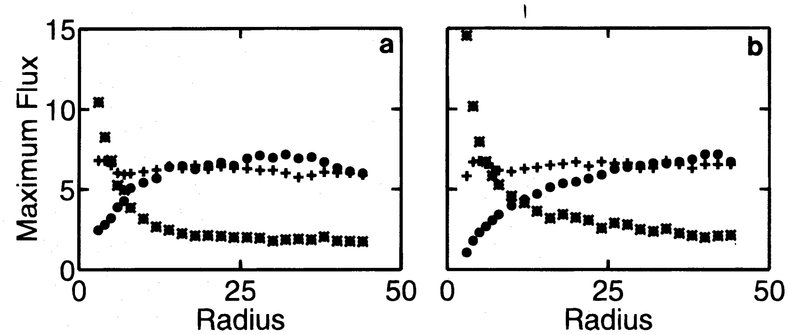


FIG. 2.—Maximum flux (in arbitrary units) from a spot as a function of the dimensionless radius $R_s/R_g$ of the orbit: $a = 0$ (*left*), $a = 1$ (*right*). The observer's inclination is indicated by symbols in each of the graphs: $\theta_0 = 20°$ (*circles*), $\theta_0 = 50°$ (*plus signs*), and $\theta_0 = 80°$ (*asterisks*).

applications. Within the range of validity of the formula, discussed above, the normalized light-curve profile is nearly independent of the source's shape. The approximations that have been adopted turn out to be adequate to many astrophysically relevant situations, e.g., exploring direct images of finite-size spots. This subject has been explored by numerous authors, but this fitting formula offers a much faster opportunity for studying light-curve profiles. We suggest that the formula is very practical for modeling short-term, featureless X-ray variability and quasi-periodic oscillations in active galactic nuclei within the framework of the bright-spot model (work in progress).

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