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A REANALYSIS OF MULTIWAVELENGTH OBSERVATIONS OF THE PULSATING DA WHITE DWARF G117–B15A

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ABSTRACT

In the light of recent developments in the study of the atmospheric properties of DA white dwarfs, we present a reanalysis of the multiwavelength observations of the ZZ Ceti star G117–B15A obtained by Robinson et al. Following a procedure similar to that used by these authors, we compare the observed amplitudes of the dominant pulsation mode of that star in six different bandpasses with theoretical amplitudes computed from model atmospheres in order to constrain the pulsation index l of the mode and the atmospheric parameters of G117–B15A. Our approach, however, allows for the sensitivity of the predicted pulsation amplitudes to the convective efficiency used in the model atmosphere calculations. Thus, we use three grids of model atmospheres based on the ML2, ML2/$\alpha = 0.6$, and ML1 parameterization of the mixing-length theory (MLT). We find, as did Robinson et al., that the observations are best explained if $l = 1$. However, for each version of the MLT, the best-fitting models form a different family of solutions in the $T_{\rm eff}$–log g plane. Since we cannot discriminate fully between the various versions of the MLT on the basis of the pulsation data alone, we invoke constraints derived in an independent analysis of the time-averaged optical and ultraviolet spectra of G117–B15A. These constraints indicate that the pulsation results are incompatible with the time-averaged spectrum of that star if model atmospheres based on ML2 and ML1 convection are used in the analysis. We find, however, that a model of G117–B15A with $T_{\rm eff} \sim 11{,}500$ K, log $g \sim 8.0$, and ML2/$\alpha = 0.6$ convection is consistent with both the time-averaged spectroscopic observations and the observed pulsation amplitudes.

Subject headings: stars: atmospheres — stars: oscillations — white dwarfs

1. INTRODUCTION

In a recent paper, Robinson et al. (1995) have presented an analysis of the variable star G117–B15A using photometric observations in six different bandpasses covering the effective wavelength range from 1570 to 6730 Å. G117–B15A is a pulsating DA white dwarf (a ZZ Ceti star) whose luminosity variations were reported first by McGraw & Robinson (1976) and studied in more detail by Kepler et al. (1982). As in other ZZ Ceti stars, the light variations are caused by the simultaneous presence of excited pulsation modes of the gravity (g) type. More recent studies, based on observations obtained at higher sensitivity, have revealed that the light curve of G117–B15A contains at least 11 harmonic oscillations but can be understood in terms of only three independent pulsation modes and their associated low-amplitude harmonics and cross frequencies (Brassard et al. 1993; Fontaine & Brassard 1994). The dominant pulsation mode in G117–B15A has a period of 215.2 s and a (white-light) amplitude of ~2.1% of the mean intensity of the star. The two other modes have periods of 270.5 s and 304.1 s and mean white-light amplitudes of ~0.6% and ~0.8%, respectively. The analyses of Brassard et al. (1993) and Fontaine & Brassard (1994) show that, in order to account for the low-amplitude harmonics and cross frequencies observed in the light curve of G117–B15A, the three independent pulsation modes *must* have the same value of the index l *and* the same value of the index m of the spherical harmonic function Y_l^m, which specifies the angular geometry of the temperature wave associated with each mode. The most likely value of l is 1, while m *must* be equal to 0.

The observations of Robinson et al. (1995) are particularly noteworthy in this regard, since they provide an independent means to infer the pulsation index l, at least for the dominant mode in G117–B15A. The method of analysis used by these authors exploits the wavelength dependence of the amplitude of a given pulsation mode. From the work of Robinson, Kepler, & Nather (1982), it can be shown (see Robinson et al. 1995; Brassard, Fontaine, & Wesemael 1995) that for white dwarfs undergoing g-mode pulsations, the ratio of the amplitude of a mode at two different wavelengths bears the signatures of the l index as well as the atmospheric parameters of the star. In a first approximation, that ratio is independent of the intrinsic amplitude of the mode and of the viewing aspect (specified by the indices l and m and the angle of inclination of the symmetry axis of the pulsation with respect to the line of sight). These two parameters are, of course, unknown a priori. Thus, in principle, multiwavelength photometry can be used to infer the pulsation index l of a mode and the atmospheric parameters of a pulsator. By using such an approach in conjunction with model atmospheres constructed on the basis of a version close to the standard (ML1) parameterization of the mixing-length theory (MLT), Robinson et al. (1995) have determined that the dominant (215.2 s) pulsation mode in G117–B15A has $l = 1$. Furthermore, they found a family of solutions which is correlated tightly in the $T_{\rm eff}$–log g plane for model atmospheres that fit their data. Adopting log $g = 7.97$ from the analysis of the time-averaged optical spectrum of G117–B15A (Daou et al. 1990), they inferred further $T_{\rm eff} = 12{,}375 \pm 125$ K for that star.

We must, however, take exception to their statement that "either the convective efficiency used in [their] model atmospheres is correct or [their] method is insensitive to convective efficiency." On the first account, we note that the formal effective temperature derived by Koester, Allard, & Vauclair (1994) by fitting the time-averaged (*Hubble Space Telescope*) [*HST*]) ultraviolet spectrum of G117−B15A using the *same* ~ML1 model atmospheres is only $T_{\rm eff}$ = 11,577 K (with a very small formal uncertainty). More importantly, however, Koester et al. (1994) mention that this particular formal fit is not acceptable and that a higher convective efficiency than that provided in the ~ML1 version must be used to explain the observed spectrum. On the second account, Bergeron, Wesemael, & Fontaine (1992) first demonstrated the high sensitivity of the emergent Eddington flux of DA white dwarf atmospheres on the assumed convective efficiency (most notably in the ZZ Ceti range of effective temperature). Since the method used by Robinson et al. (1995) relies on the temperature derivative of the emergent specific intensity (see below), it is clear that it must be sensitive also to convective efficiency, as discussed in Brassard et al. (1995).

In a still more recent analysis, Bergeron et al. (1995) have derived fundamental constraints on the convective efficiency to be used in modeling the atmospheres of DA white dwarfs in general, and of ZZ Ceti stars in particular. They demonstrated first that fits to the time-averaged optical spectra of ZZ Ceti stars lead to a mass distribution for these stars that bears the signature of the convective efficiency. It is important to realize here that the quality of the optical fits itself is *not* sensitive to the convective efficiency. Indeed, comparable quality fits are obtained, for example, to the optical spectrum of G117−B15A by using either the ML1 ($T_{\rm eff}$ = 11,000 K; log g = 8.04), ML2 ($T_{\rm eff}$ = 12,620 K; log g = 7.84), or ML3 ($T_{\rm eff}$ = 14,140 K; log g = 7.70) version of the MLT. However, taken as a group, ZZ Ceti stars have a mean mass (0.58 $M_\odot$) in excellent agreement with that of hotter DA stars (0.59 $M_\odot$), whose atmospheres are completely radiative, if the ML2 parameterization is used. ML1 and ML3 models, on the other hand, yield masses that are, respectively, too high (0.70 $M_\odot$) or too low (0.51 $M_\odot$).

Bergeron et al. (1995) also considered time-averaged *International Ultraviolet Explorer* (*IUE*) and *HST* spectroscopic observations of ZZ Ceti stars. They showed that a unique solution for $T_{\rm eff}$ and log g cannot be achieved on the basis of ultraviolet spectroscopy alone, and that one of these parameters needs to be constrained independently. When log g values from the optical analysis are adopted, the analysis of the ultraviolet data requires a parameterization less efficient than ML2. Models calculated with ML2/α = 0.6 (intermediate in efficiency between ML1 and ML2; ML2 is characterized by α = 1.0) provide an excellent internal consistency between ultraviolet and optical temperatures. Moreover, the atmospheric parameters obtained with these models are consistent with the observed photometry, the trigonometric parallax measurements, and the gravitational redshift masses. However, the mean mass of the sample of ZZ Ceti stars increases to a value ~0.06 $M_\odot$ larger than that of the hotter DA stars, but this result could be explained if the red edge of the instability strip has a weaker mass dependence than the blue edge (see Bergeron et al. 1995). The ML2/α = 0.6 solution for G117−B15A is $T_{\rm eff}$ = 11,620 ± 350 K, log g = 7.97 ± 0.05.

Given these recent developments, we have felt that it would be most worthwhile to broaden the analysis of the multiwavelength observations of Robinson et al. (1995). On the one hand, these data are unique (since the ultraviolet observations were gathered with the now decommissioned fast photometer on board *HST*), and, on the other hand, as mentioned above, they provide independent constraints on the index l of the dominant pulsation mode as well as on the atmospheric parameters of G117−B15A. It seems particularly important to test the sensitivity of the central result of Robinson et al. (1995)—namely, that the dominant pulsation mode in G117−B15A has $l = 1$—on the choice of the convective efficiency used in the model atmospheres. Furthermore, it would be of considerable interest to find a model of G117−B15A that is consistent with both the time-averaged (ultraviolet and optical) spectroscopic observations and the observed pulsation amplitudes in different bandpasses. We point out, at the outset, that such a consistency is not guaranteed automatically because the analysis of the former uses directly the emergent Eddington flux, while that of the latter uses the temperature derivative of the local emergent specific intensity (see eq. [3] below), a more subtle characteristic of a model atmosphere.

In the following, we reanalyze the data of Robinson et al. (1995) on G117−B15A with the help of three grids of model atmospheres based on the ML1, ML2/α = 0.6, and ML2 parameterizations of the MLT. Our method and results are presented and discussed in the next sections.

2. METHOD

The theoretical spectral variations of low-amplitude g-mode pulsations in DA white dwarfs have been discussed first by Robinson et al. (1982). Their approach has been extended and generalized by Brassard, Wesemael, & Fontaine (1987) and, more recently, by Robinson et al. (1995) and Brassard et al. (1995). We follow the notation of the latter authors. To first order in the temperature perturbation, the amplitude of a g-mode in the Fourier spectrum (expressed as a percentage of the mean brightness of the star in filter x) is given by

$$a_1^x = A_l^x \epsilon_T T_0 \bar{Y}_l^m(i) , \qquad (1)$$

where ϵ_T is the dimensionless amplitude of the temperature perturbation, T_0 is the unperturbed effective temperature, and $\bar{Y}_l^m(i)$ is an associated Legendre function corresponding to the viewing aspect of the mode of angular indices l and m as seen along a line-of-sight making an angle i with the pulsation axis. The quantity A_l^x is a frequency-integrated quantity given by

$$A_l^x \equiv \frac{\int_0^\infty W_\nu^x A_{l\nu}\, d\nu/\nu}{\int_0^\infty W_\nu^x H_{\nu,0}\, d\nu/\nu} \times 100 , \qquad (2)$$

where W_ν^x is the transmission function for filter x, $H_{\nu,0}$ is the unperturbed emergent Eddington flux, and

$$A_{l\nu} \equiv \frac{1}{2}\int_0^1 \left.\frac{\partial I_\nu}{\partial T}\right|_{T_0} P_l(\mu)\mu\, d\mu , \qquad (3)$$

where I_ν is the local emergent specific intensity, and $P_l(\mu)$ is a Legendre polynomial of index l. The latter quantity corresponds to an integral over the visible disk of a star as it would be seen pole-on.

From equation (1), we find that the ratio of the observed amplitude of a pulsation mode in two different bandpasses, say x and y, is given by

$$\frac{a_1^x}{a_1^y} = \frac{A_l^x}{A_l^y} . \qquad (4)$$

This ratio depends on the index l, on the atmospheric parameters of a star, and on the assumed convective efficiency, but *not* on the unknown intrinsic amplitude of the mode and on the viewing aspect.

We have computed the quantity A_l^x for three grids which make use of the ML1, ML2/$\alpha = 0.6$, and ML2 parameterizations of the MLT. Each grid consists of 91×91 points distributed equally in the $T_{\rm eff}$–log g plane. These points were obtained by interpolation from an original coarser grid of 45 pure hydrogen model atmospheres covering the range of effective temperature 10,000 K $\leq T_{\rm eff} \leq$ 14,500 K in steps of 500 K, and the range of surface gravity $7.50 \leq \log g \leq 8.50$ in steps of 0.25. The physics included in these models is identical to that described in Bergeron et al. (1995). The values of l considered are 1 through 3, but smaller grids were also obtained for values of l up to 8. We used the transmission functions provided by Robinson et al. (1995) in their Figure 1. These consist of two ultraviolet bandpasses, M and W, with effective wavelengths 1570 and 1920 Å, respectively, and four optical bandpasses similar, but not identical, to the standard Johnson and Cousins filters U, B, V, and R with effective wavelengths 3480, 4410, 5450, and 6730 Å, respectively.

Figure 1 shows the behavior of some of the coefficients A_l^x as a function of the unperturbed effective temperature for model atmospheres with log $g = 8.0$ and two different convective efficiencies. Insight into the behavior of the coefficient A_l^x in terms of atmospheric parameters can be obtained from the detailed discussion presented in Brassard et al. (1995), to which the reader is referred. For our present needs, we want to emphasize mostly the dependence on the convective efficiency. At both extremes of the range of unperturbed effective temperature shown here, the values of A_l^x tend to be the same for the two parameterizations of the MLT. This arises because at high effective temperatures, there is little flux carried by convection while, at lower temperatures, the stratification tends to become adiabatic. In both cases, this leads to an atmospheric structure that is only weakly dependent on the details of the MLT parameterization. In between, however, where the ZZ Ceti instability strip is found, Figure 1 shows clearly that the pulsation amplitude is a strong function of the assumed convective efficiency.

Figure 2 shows the predicted amplitudes (normalized arbitrarily to that of the V band) in the six bandpasses of interest for ML1 model atmospheres with log $g = 8.0$ and with three different but typical values of the unperturbed effective temperature. The figure shows explicitly the distinct signature of the l index, from 1 through 6. Our calculations, which actually extend to $l = 8$, show that starting with $l = 3$, odd-valued l modes tend to have large relative amplitudes in the ultraviolet, while even-valued l modes tend to have small amplitudes in that wavelength range. This tendency is reversed in the red region of the spectrum. This behavior is explained readily in terms of the steeper limb-darkening law obtained generally in the ultraviolet (as compared to a relatively flat law obtained in the red) in conjunction with the weight function $P_l(\mu)\mu$ (see eq. [3]), which alternatively favors the rim or the central regions of the visible disk. In view of these results, we fail to understand why the predicted relative amplitudes associated with the $l = 3$ and $l = 4$ cases are so similar in Figures 6 and 7 of Robinson et al. (1995).

In the rest of this paper, we focus on a comparison of the predicted pulsation amplitudes coming out of our model atmosphere grids with the observational data of Robinson et al. (1995), as given in the second column of their Table 4. We quantify this comparison through the evaluation of the

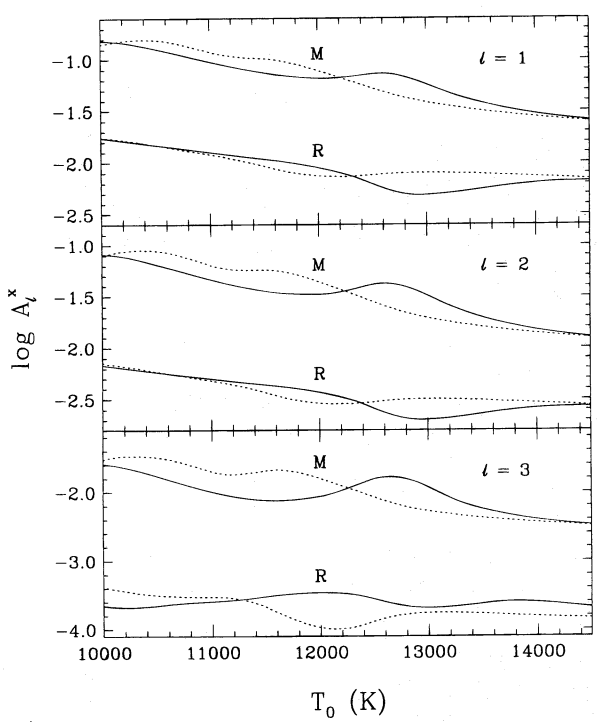

FIG. 1.—Behavior of the coefficient A_l^x for the M and R bandpasses in terms of unperturbed effective temperature at fixed surface gravity (log $g = 8.0$) for ML1 (*dashed curves*) and ML2 (*solid curves*) models. Each panel refers to a specific value of the pulsation index l, from 1 through 3.

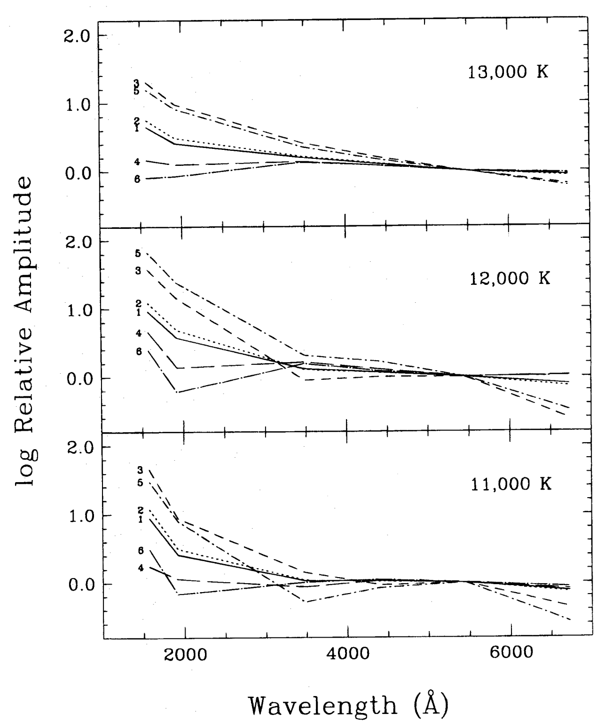

FIG. 2.—Normalized pulsation amplitudes for the six bandpasses of interest computed from ML1 model atmospheres with log $g = 8.0$ and with three different effective temperatures. The solid (dotted, dashed, long-dashed, dot-dashed, dot-long-dashed) curve corresponds to a g-mode with $l = 1$ (2, 3, 4, 5, 6).

quantity

$$\chi^2(l) = \sum_{i=1}^{6} \left(\frac{fA_l^i - O^i}{\sigma^i} \right)^2 , \quad (5)$$

where the summation is taken over the six bandpasses of interest. In this equation, O^i is the observed amplitude in bandpass i and σ^i is its related uncertainty as given by Robinson et al. (1995). The quantity f is a scale factor that acts as a normalization constant. At each point of one of our 91 × 91 grids in the $T_{\rm eff}$–log g plane, we determined the value of f that minimizes $\chi^2(l)$ with the help of a steepest descent technique (Press et al. 1986). We note that one of the advantages of this method is that it does not rely on the arbitrary choice of a specific bandpass for normalization purposes. This is in contrast, for example, to the approach used by Robinson et al., in which both the observed and theoretical pulsation amplitudes are normalized to those of the V band. However, as demonstrated by these authors, the latter approach is still useful, since it reveals immediately that modes with $l \geq 3$ can be ruled out for the main (215.2 s) observed pulsation mode in G117−B15A. Indeed, a comparison of the observational data found in column (3) of Table 4 in Robinson et al. (1995) with, for example, the theoretical values shown in Figure 2 (both normalized to the V band) indicates that the expected normalized amplitudes at both extremes of the spectrum are much larger than the observed normalized amplitudes for modes with $l \geq 3$. Therefore, we focus, in what follows, on the $l = 1$ and $l = 2$ alternatives.

In ending this section, we must raise a technical point, first discussed by Brassard et al. (1995) in their Appendix B. Robinson et al. (1995) compute their theoretical pulsation amplitudes on the basis of their equation (4), which neglects implicitly the temperature dependence of the limb-darkening law. This is usually *not* a good approximation in ZZ Ceti model atmospheres. In contrast, our approach does include this dependence (see eq. [3] and the discussion in Brassard et al. 1995), and our derived theoretical pulsation amplitudes must be considered more accurate.

3. RESULTS

Our results can be studied first qualitatively by examining the behavior of the quantity $\chi^2(l)$ considered as a surface above the $T_{\rm eff}$–log g plane. For a given parameterization of the MLT and for a given value of l (either 1 or 2), such a surface is generated by computing the 8281 values of $\chi^2(l)$ covering our grids of model atmospheres (91 unperturbed effective temperatures and 91 surface gravities).

Figures 3*a*–3*c* illustrate the surface $\chi^2(l = 1)$ for the ML2, ML2/α = 0.6, and ML1 parameterizations of the MLT, respectively. The viewing angle is the same for the three figures and is chosen so that the reader can see the bottoms of two "valleys" in which, locally, the predicted pulsation amplitudes best fit the observational data. Such a double-valued solution is a characteristic of DA white dwarf atmospheres in the ZZ Ceti range of effective temperature (see Bergeron et al. 1995; Brassard et al. 1995 for a discussion of this point). For the ML2 models (Fig. 3*a*), the "cold" valley is significantly deeper than the "hot" valley, and it is therefore in the former that the optimal models are found. The pattern reverses itself with decreasing convective efficiency. Thus, for ML1 convection (Fig. 3*c*), the optimal models are actually found in the hot valley. The case of ML2/α = 0.6 convection (Fig. 3*b*) is intermediate between the two previous cases; the optimal solutions are found in either the hot or cold valley, depending on the value of the surface gravity. We can point out from these figures that, on a relative scale, the ML2 parameterization provides the best fits to the observational data, while the overall goodness of fit (as measured by the depths of the valleys in which the optimal models are found) tends to decrease with decreasing convective efficiency (from ML2 to ML2/α = 0.6 and, then, to ML1).

A similar situation is encountered for the $\chi^2(l = 2)$ surfaces (not shown), which have morphologies similar to those illustrated in Figures 3*a*–3*c*. In general, the $\chi^2(l = 2)$ valleys nearly overlap with the $\chi^2(l = 1)$ valleys, but the former surfaces are always located higher above the $T_{\rm eff}$–log g plane than the latter surfaces. In particular, *whatever the chosen parameterization of the MLT*, the smallest values of $\chi^2(l = 2)$ remain systematically larger than their $\chi^2(l = 1)$ counterparts. This suggests that the $l = 1$ identification is to be preferred for the 215.2 s pulsation mode in G117−B15A, in agreement with the central conclusion of Robinson et al. (1995).

This result can be put on a more quantitative basis. To this end, we follow Press et al. (1986) and compute the probability, Q, that a given set of atmospheric parameters ($T_{\rm eff}$, log g) provides an acceptable fit to the observed pulsation amplitudes in the six bandpasses of interest. In the present case, we have one free parameter (the scale factor f) in our calculations of $\chi^2(l)$, so the number of degrees of freedom, ν, reduces to 5. The quantity Q [function of $\chi^2(l)$ and ν] is estimated through the use of a truncated gamma function (see Press et al. 1986). For ML2 convection, the optimal fits at the bottom of the deeper valleys have, on the average, the values $\chi^2(l = 1) \simeq 4.8$ and $\chi^2(l = 2) \simeq 13.8$. This corresponds to a ratio $Q(l = 1)/Q(l = 2) \simeq 26$. For ML2/$\alpha$ = 0.6 and ML1 convection, this ratio climbs to 1964 and 4140, respectively. Thus, for all three parameterizations of the MLT considered here, the $l = 1$ solutions are favored heavily over the $l = 2$ possibilities.

We can use again the formalism of the probability Q to determine quantitatively which of the models of our grids provide acceptable fits. This time, contrary to the previous paragraph, we consider Q in an *absolute* sense. Following Press et al. (1986), we estimate conservatively that fits with $Q \geq 10^{-3}$ reproduce the observational data in an acceptable manner. In the present context, this criterion corresponds to fits with $\chi^2(l) \leq 20$. We find that such fits are those in which simultaneously in the six bandpasses, the predicted pulsation amplitudes fall within 3 σ of the observed amplitudes. The σ values here are the experimental uncertainties quoted by Robinson et al. (1995) in their Table 4.

In all cases of interest, the optimal models are found at the bottom of a relatively flat valley, which means that these models all provide fits to the observational data of equivalent quality. Thus, we generally find a family of solutions (for each parameterization of the MLT) such that a higher gravity is compensated for by a higher effective temperature. This is consistent with the independent results of Robinson et al. (1995), who also found such a family of solutions. Hence, it is not possible, on the basis of the multiwavelength photometry of Robinson et al. (1995) *alone*, to constrain independently the atmospheric parameters of G117−B15A. Independent constraints must be used.

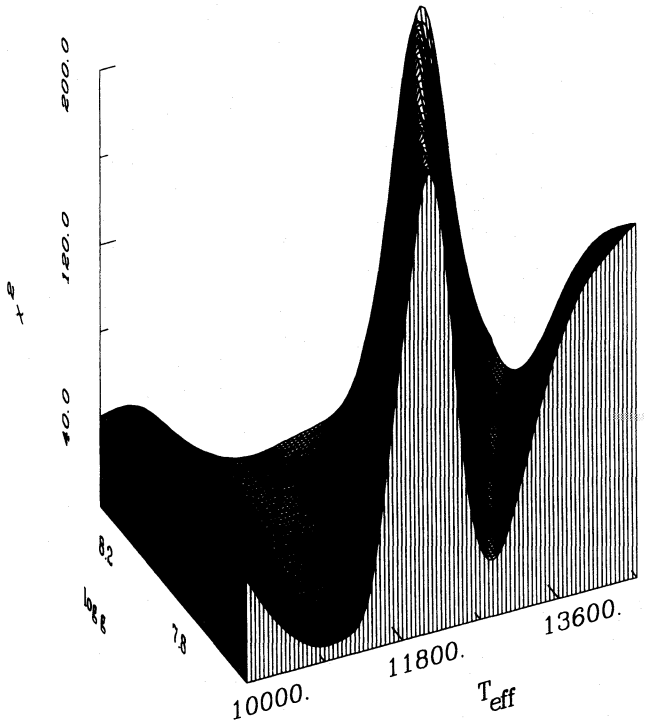

FIG. 3a

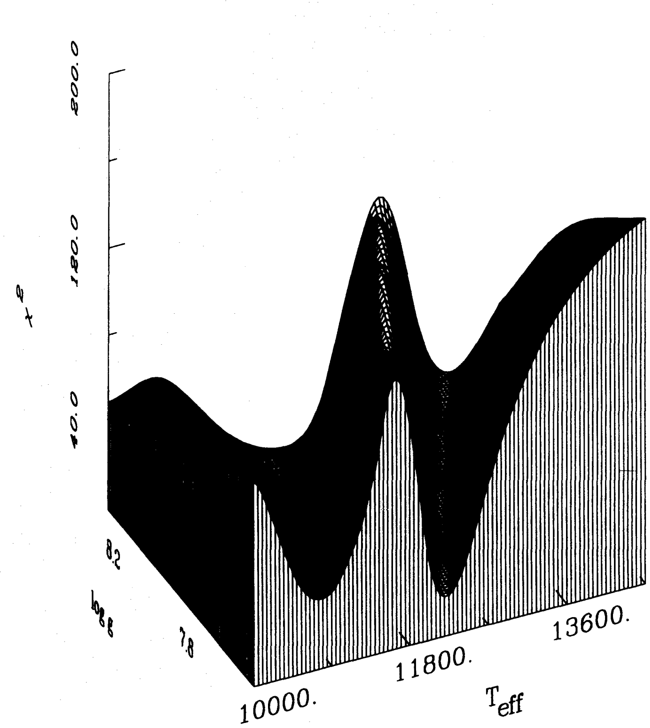

FIG. 3b

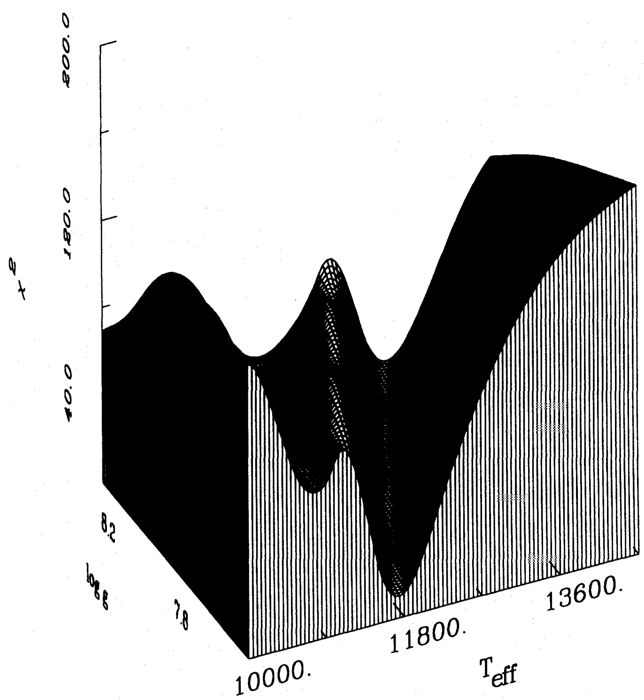

FIG. 3c

FIG. 3.—(*a*) Three-dimensional representation of the surface $\chi^2(l = 1)$ over the $T_{\rm eff}$–log g plane for the ML2 model atmospheres. (*b*) Similar to (*a*), but for ML2/$\alpha = 0.6$ models. (*c*) Similar to (*a*), but for ML1 models.

Table 1 summarizes the properties of the families of models which best fit the observed pulsation amplitudes, assuming that the pulsation index of the dominant mode in G117−B15A is either $l = 1$ or $l = 2$. For each version of the MLT (col. [1]) and each gravity in our original grid of model atmospheres (col. [2]), we list the effective temperature (col. [3]) of the optimal model for $l = 1$. A quantitative estimate of the goodness of fit is next provided in column (4), which gives the associated value of $\chi^2(l = 1)$ for a given model atmosphere. In column (5), a more qualitative indicator of the quality of the fit is presented: we list the filters for which the predicted pulsation amplitudes are *more* than 2 σ (indicated by a letter) or 3 σ (indicated by a letter with an asterisk) away from the observed amplitudes. Finally, in the last three columns, we give the corresponding data for the $l = 2$ case. Note that the optimal models for

TABLE 1

Optimal Solutions for $l = 1$ and $l = 2$

MLT (1)	log g (2)	$T_{\rm eff}$ ($l = 1$) (3)	χ^2 ($l = 1$) (4)	Filters[a] ($l = 1$) (5)	$T_{\rm eff}$ ($l = 2$) (6)	χ^2 ($l = 2$) (7)	Filters[a] ($l = 2$) (8)
ML2	7.50	10,950	6.937	...	10,950	12,21	*W*
	7.75	11,200	4.298	...	11,150	14.53	*W*
	8.00	11,450	3.890	...	11,350	14.68	*W*, *B*
	8.25	11,500	4.263	...	11,400	14.10	*B*
	8.50	11,750	4.533	...	11,700	13.62	*B*
ML2/$\alpha = 0.6$	7.50	12,200	18.05	*W*, *V*	12,400	33.20	*W**, *V**
	7.75	11,100	19.06	*U*, *V*	12,800	31.23	*W**, *V**
	8.00	11,350	14.61	*U*	13,250	31.37	*W**, *U*, *V**
	8.25	11,650	9.228	...	11,550	30.12	*M**, *W**, *B*
	8.50	12,050	7.256	...	12,000	29.19	*M*, *W**, *B*
ML1	7.50	11,750	9.919	*V*	11,950	33.64	*W**, *U*, *V**
	7.75	12,100	11.66	*V*	12,300	32.21	*W**, *U*, *V**
	8.00	12,500	14.83	*U*, *V*	12,700	33.17	*W*, *U**, *V**
	8.25	12,850	16.92	*W*, *U*, *V*	13,100	34.15	*W*, *U**, *V**
	8.50	13,300	20.35	*U*, *V*	13,550	35.14	*W*, *U**, *V**

[a] Letter alone indicates predicted pulsation amplitudes of more than 2 σ; letter with asterisk indicates predicted pulsation amplitudes of more than 3 σ.

ML2/$\alpha = 0.6$ convection do not form a single family in the $T_{\rm eff}$–log g plane; they belong to either the cold or the hot valley, depending on the value of the surface gravity.

To complement Table 1, we compare in Figures 4*a*–4*c* (referring to ML2, ML2/$\alpha = 0.6$, and ML1 convection, respectively) the observed pulsation amplitudes of Robinson et al. with the predicted amplitudes computed on the basis of the optimal models assuming that either $l = 1$ or $l = 2$. The models chosen have log g = 7.5, 8.0, and 8.5 and span the range of surface gravity considered in this investigation. From the figures (and this is true in general for all the models listed in Table 1), it is again clear that the $l = 1$ solution is to be preferred to the $l = 2$ possibility. And indeed, the values of $\chi^2(l = 1)$ listed in Table 1 are less than the values of $\chi^2(l = 2)$ by, typically, a factor of 2 or more. Furthermore, according to the Q criterion discussed above, all the $l = 2$ solutions for ML2/$\alpha = 0.6$ and ML1 convection should be rejected because their associated values of $\chi^2(l = 2)$ are larger than 20. This confirms further the conclusion of Robinson et al. (1995) that the dominant pulsation in G117–B15A is a mode with $l = 1$. We concentrate on that result in what follows.

Table 1 and Figures 4*a*–4*c* reveal, once again, that the optimal solutions (for $l = 1$) are sensitive to the assumed parameterization of the MLT. Furthermore, the overall goodness of fit is also sensitive to that parameterization. The ML2 models provide the best fits to the observational data. For all surface gravities of interest, those models lead to predicted pulsation amplitudes for the six bandpasses that are *all* within 2 σ of the observed amplitudes (see Table 1 and Fig. 4*a*). Models computed with ML1 convection, on the other hand, lead to comparatively worse fits. For instance, the predicted amplitude in the V bandpass never falls within 2 σ of the observed amplitude for the optimal ML1 models (see Table 1). Nevertheless, the ML1 models still provide quite *acceptable* solutions, as can be appreciated from Figure 4*c*. In fact, those models satisfy the Q criterion, and the predicted amplitudes in all six filters are at least within 3 σ of the observed amplitudes. Thus, it is not possible to rule out ML1 convection on the basis of the observations of Robinson et al. (1995). Finally, ML2/$\alpha = 0.6$ models are characterized by a convective efficiency intermediate between ML2 and ML1. Not surprisingly, and as a consequence, the resulting best fits to the observational data are also of intermediate quality. Note that, on an absolute scale, these fits are again quite good, as can be observed in Figure 4*b*. They all satisfy the Q criterion. Moreover, the models with log $g \geq 8.25$ lead to predicted amplitudes that are within 2 σ of the observed pulsation amplitudes in all six bandpasses (see Table 1).

4. DISCUSSION

Our results reinforce the conclusion of Robinson et al. (1995) that the main pulsation mode in G117–B15A has a pulsation index $l = 1$. However, contrary to the assertion of these authors, the models that can explain the observations have parameters that *are* sensitive to the convective efficiency assumed in the computations. Furthermore, the pulsation observations cannot be used to discriminate fully between the various versions of the MLT. Independent constraints must be sought to derive a unique model for G117–B15A. We use the recent results of Bergeron et al. (1995) to derive such constraints in what follows.

Fits to the time-averaged optical spectra of 22 ZZ Ceti stars have been presented by Bergeron et al. (1995) on the basis of ML2 model atmospheres (see their Fig. 2). Although the formal fitting errors for each individual spectrum are quite small, the typical (external) uncertainties are $\Delta T_{\rm eff}$ = 350 K and Δ log g = 0.05. Their ML2 solution for G117–B15A is $T_{\rm eff}$ = 12,620 K and log g = 7.84. This is completely inconsistent with our present results: for a gravity log g = 7.84, our optimal ML2 models suggest a much smaller effective temperature, $T_{\rm eff} \simeq$ 11,250 K, well outside the uncertainties associated with the optical analysis. Moreover, the fit to the pulsation data obtained with the atmospheric parameters derived by Bergeron et al. (1995) is very poor. It corresponds to $\chi^2(l = 1) \simeq 205$, a value which must be formally rejected according to the Q criterion discussed above. Because the model atmospheres used in the present study are *identical* to those used in Bergeron et al. (1995), this discrepancy must be considered significant. It is possible that the present discrepancy between the results of the optical spectroscopic analyses of Bergeron et al. (1995) and the results of our analysis of the

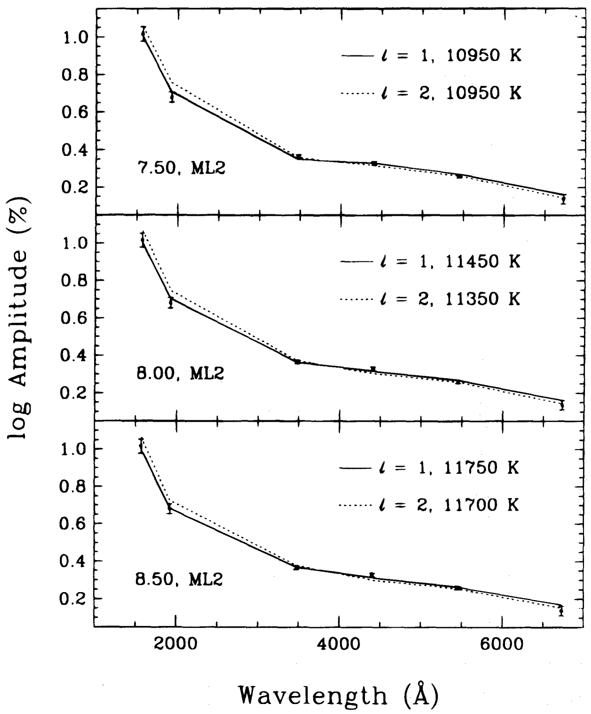

FIG. 4*a*

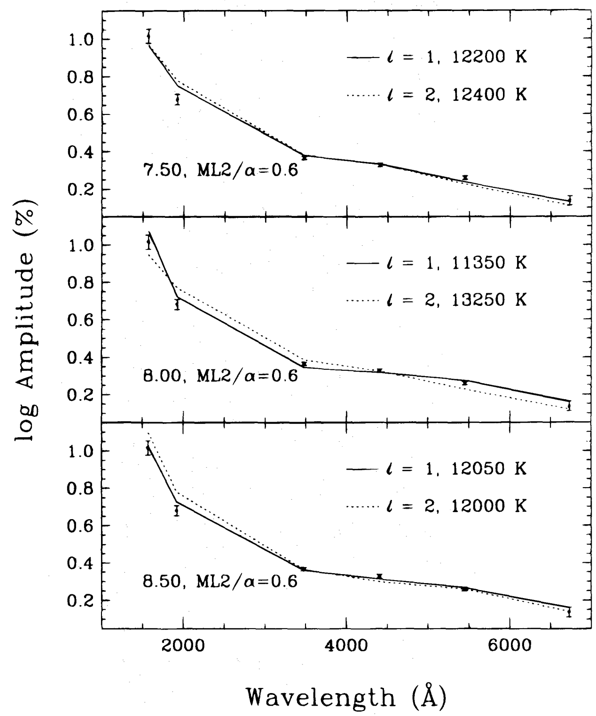

FIG. 4*b*

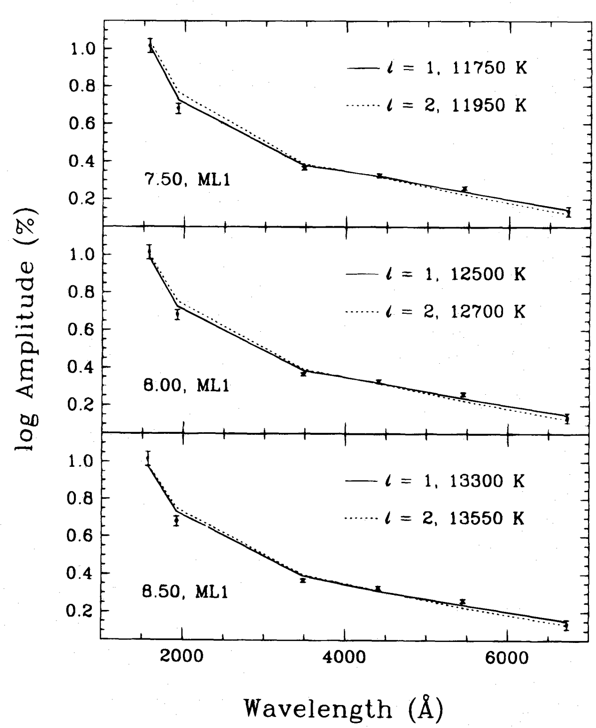

FIG. 4*c*

FIG. 4.—(*a*) Comparison of the observed pulsation amplitudes (*points with* 1 σ *error bars*) with the predicted amplitudes (*solid curve*) in the six bandpasses of interest for three models (log $g = 7.5$, 8.0, 8.5) belonging to the family of best-fitting ML2 models under the assumption that $l = 1$. A similar comparison with models belonging to the family of optimal ML2 models under the assumption that $l = 2$ is provided also by the dotted curves. (*b*) Similar to (*a*), but for ML2/$\alpha = 0.6$ models. (*c*) Similar to (*a*), but for ML1 models.

pulsation data of G117−B15A reflects the inability of ML2 convection to describe adequately the atmospheres of DA white dwarfs. This would be in line with Bergeron et al. (1995), who were forced to reject the ML2 parameterization for ZZ Ceti stars because the ultraviolet spectra could not be fitted simultaneously with the optical spectroscopy (see their Fig. 12). Alternatively, whatever the parameterization of the MLT, the current model atmospheres may not be able to account simultaneously for the spectroscopic and pulsation data because, as noted above, these observations are not related to the same properties of the models. Pending further investigations (but see below), we favor the first hypothesis, and therefore we reject the ML2 solutions for G117–B15A, even though they provide the best overall fit to the observed pulsation amplitudes.

We reject also the ML1 solutions because our optimal models for the pulsation data are inconsistent with the optical analysis of Bergeron et al. (1995), which suggests $T_{\rm eff} = 11{,}000$ K and $\log g = 8.04$ for G117–B15A. This time, following a behavior opposite to that observed for the ML2 models, our results suggest a significantly *higher* effective temperature for a given surface gravity than derived by the optical analysis. For a value $\log g = 8.04$, Table 1 indicates an optimal value $T_{\rm eff} \simeq 12{,}550$ K.

The models computed with ML2/$\alpha = 0.6$ convection fare better. The analysis of Bergeron et al. (1995) on the basis of such model atmospheres indicates that G117–B15A should have $T_{\rm eff} = 11{,}620$ K and $\log g = 7.97$. Note that this locates G117–B15A in the middle, and not at the blue edge of the ZZ Ceti instability strip, in contrast to many previous estimates. This solution is consistent, within the uncertainties quoted above, with the family of optimal models for ML2/$\alpha = 0.6$ convection. As *representative* of that family, it is useful to converge on the model with $T_{\rm eff} =$ 11,350 K and $\log g = 8.0$, since we already summarized its properties in Table 1 and in Figure 4*b* (*middle panel*). We reiterate here the fact that this model leads to predicted pulsation amplitudes that provide a very good fit to the observational data (see Fig. 4*b*). The theoretical amplitudes are all within 2 σ of the observed amplitudes, except for the *U* filter, but the criterion is missed only barely in that case. The quality of the fit shown in Figure 4*b* is comparable to the best fit obtained by Robinson et al. (1995) in their analysis. Given the fact that the observational data considered in this paper probe different properties of the model atmospheres than the spectroscopic, photometric, and trigonometric data analyzed by Bergeron et al. (1995), we find it gratifying that a common set of atmospheric parameters (within reasonable uncertainties) can be invoked to account for the different observed properties of G117–B15A.

In this connection, it will be interesting to verify if the *white-light optical data* that we have gathered at the Canada-France-Hawaii Telescope for G117–B15A can also be explained with the same set of atmospheric parameters ($T_{\rm eff} \sim 11{,}500$ K, $\log g \sim 8.0$, ML2/$\alpha = 0.6$ convection). This high-quality data set has been analyzed first in a preliminary fashion by Brassard et al. (1993), who *assumed* the atmospheric parameters derived from a ML2 analysis of the time-averaged optical spectrum of G117–B15A. A more detailed study was presented by Fontaine & Brassard (1994), who actually *inferred* the effective temperature and surface gravity from the pulsation data using ML2 model atmospheres. They also found two possible solutions from the pulsation data and actually retained the "hot" solution. In both cases, the model atmospheres were of earlier generations and did not incorporate, in particular, the latest developments discussed briefly by Bergeron et al. (1995) and related to the important opacity source associated with the H_2^+ and H_2 quasi-molecular absorption features (see Allard et al. 1994). In the light of these developments, the results of Bergeron et al. (1995), and our present results, it is indeed worthwhile to reanalyze the white-light photometry of G117–B15A, and this is under investigation. We know already, however, that the main conclusions of Brassard et al. (1993) and Fontaine & Brassard (1994) are insensitive to changes in atmospheric parameters, and therefore we expect that these conclusions will remain unchanged: the three observed pulsation modes in G117–B15A have the same value of l (probably $l = 1$) and the same value of m ($m = 0$).

In ending this discussion, we remind the reader that, while the ML2/$\alpha = 0.6$ parameterization of the MLT proposed by Bergeron et al. (1995) appears to give a reasonable description of convection in the atmospheric layers of DA white dwarfs in which the observable flux originates, that parameterization does not necessarily extend to the deeper regions and in the envelope. Wesemael et al. (1991) were the first to point out that there exists an inconsistency between the MLT parameterization used in the model atmosphere analyses of the ZZ Ceti stars and that needed to explain the blue edge of the instability strip in the few nonadiabatic pulsation calculations currently available. The analyses of flux measurements are sensitive to the treatment of convection in the *outer atmospheric layers*, while the theoretical blue edge temperature is sensitive to convection mostly through the location of the base of the convection zone in the *envelope*. The existing calculations show, for a given version of the MLT, that the theoretical blue edge predicted through nonadiabatic pulsation calculations is cooler than the effective temperatures inferred from model atmosphere analyses (see, e.g., Fig. 14 of Bergeron et al. 1995). This led Wesemael et al. (1991) to suggest that the convective efficiency may increase with depth in a ZZ Ceti star. The exploratory hydrodynamical calculations of Ludwig, Jordan, & Steffen (1994) show that the temperature structure of such a star cannot be calculated rigorously within the MLT, but they also indicate that the convective efficiency (as measured in a MLT context) does indeed increase with depth. In practical terms, this means that adiabatic and nonadiabatic pulsation calculations of models of G117–B15A may require a MLT efficiency larger than the ML2/$\alpha = 0.6$ version which has been tested successfully in this paper.

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