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OCCLUSION EFFECTS AND THE DISTRIBUTION OF INTERSTELLAR CLOUD SIZES AND MASSES

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ABSTRACT

The frequency distributions of sizes of "clouds" and "clumps" within clouds are significantly flatter for extinction surveys than for CO spectral line surveys, even for comparable size ranges. A possible explanation is the blocking of extinction clouds by larger foreground clouds (occlusion), which should not affect spectral line surveys much because clouds are resolved in velocity space along a given line of sight. We present a simple derivation of the relation between the true and occluded size distributions, assuming that clouds are uniformly distributed in space or are all at about the same distance. Because the occlusion is dominated by the largest clouds, we find that occlusion does not affect the measured size distribution, except for sizes comparable to the largest size, implying that occlusion is not responsible for the discrepancy if the range in sizes of the samples is large. However, we find that the range in sizes for many of the published observed samples is actually quite small, which suggests that occlusion does affect the extinction sample and/or that the discrepancy could arise from the different operational definitions and selection effects involved in the two samples. We suggest that studies of isolated small dark clouds and of *IRAS* cloud cores support the former interpretation.

Subject headings: ISM: clouds — ISM: structure — radio lines: ISM

1. INTRODUCTION

The mass spectrum of density fluctuations, defined in various operational ways as "clouds," is an important function that must be related in some way to the processes by which clouds form and evolve, as well as to the mass spectrum of stars that form within these clouds. A fairly large and growing number of studies of (mostly) molecular clouds yield a differential mass spectrum, which, if fitted by a power law, has a form $f(m) \sim m^{\gamma}$, with $\gamma \sim -1.5 \pm 0.2$. These studies are primarily based on masses derived from column densities inferred from ^{12}CO and ^{13}CO spectral line observations and linear size. Some of the results are for surveys that cover a significant area of the Galactic disk: e.g., the second quadrant survey of Casoli, Combes, & Gerin (1984), who find $\gamma = -1.4$ to -1.6 in both the Perseus and Orion arms; the ^{12}CO first quadrant surveys by Sanders, Scoville, & Solomon (1985), Solomon et al. (1987), and Solomon & Rivolo (1989), who find $\gamma \sim -1.5$ or -1.6 using virial masses; and the recent comparison of 204 inner and outer Galaxy molecular clouds by Brand & Wouterloot (1995), who find $\gamma = -1.6$ for outer Galaxy clouds and $\gamma = -1.8$ for all 204 clouds. These surveys together cover a mass range from about 1 $M_{\odot}$ to over 10^6 $M_{\odot}$, although each individual study generally covers a much smaller mass range over which a power law is an adequate fit, usually a range of about a factor of 20. Other work has concentrated on the mass spectrum of "clumps" within individual clouds complexes, and these studies find similar mass spectra in regions as different in star formation properties as the following: the Maddalena-Thaddeus cloud (Williams, de Geus, & Blitz 1994), which shows no evidence for star formation; the ρ Oph core region (Loren 1989, as revised by Blitz 1993), which is forming low- to intermediate-mass stars; the Rosette Molecular Cloud (Williams et al. 1994; Williams, Blitz, & Stark 1995); the M17SW cloud (Stutzki & Güsten 1990); and the Orion region (Lada, Bally, & Stark 1991; Tatematsu et al. 1993). Each of these regions is actively forming stars up to large masses and even lower mass clumps in MBM 12, a molecular cloud that is not gravitationally bound (Pound 1994). All these studies give $\gamma \sim -1.5 \pm 0.3$ (the flattest being the Williams et al. result for the Rosette cloud, with $\gamma \sim -1.3$).[2]

However, there is a probable discrepancy when these results are compared with studies of the mass spectra of clouds that are derived using extinction surveys, which are also based on masses from sizes and column densities. If the distribution of sizes is given by $f(r) \sim r^{\alpha}$ and the cloud internal density n is related to size by $n \propto r^p$, then $f(m) = f(r)\, dr/dm$ with $dm = r^{p+2}\, dr$, and the mass spectrum is $f(m) \sim \mathrm{m}^{\gamma}$ with $\gamma = (\alpha - p - 2)/(3 + p)$. Estimates of p are uncertain and vary from (at least) $p \sim -1.2$ (see Scalo 1985, 1987) to $p = 0$ (sizes or masses uncorrelated with density; e.g., Casoli et al. 1984; Williams et al. 1995); alternatively, the correlation may be, at least in part, an artifact caused by selection effects (Scalo 1990). The spectral line studies mentioned earlier give values of α around -2 to -2.5, based either on the published size data, when available, or on the above transformation between mass and size spectra.

Scalo (1985) presented the frequency distribution of angular surface areas of dark clouds from the catalogs of Lynds (1962) and Khavtassi (1960). The resulting size spectrum, if fitted by a power law, has $\alpha \sim -1.4 \pm 0.2$, which is much flatter than the size distributions inferred from the spectral line surveys. The implied mass spectra ($\gamma \sim -1.2 \pm 0.3$) seem significantly flatter than the spectral line mass spectrum, but, because of the above relationship between size and mass spectra (which gives $\Delta\alpha = (2\text{–}3)\Delta\gamma$ for $p = -1$ to 0) and because size is a directly measured quantity in both types of studies, the discrepancy is more clearly seen in the size spectrum. Feitzinger & Stüwe (1986)

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[2] Based on the simulation studies of Williams et al. (1994) and the number of clouds used for fitting power laws in the surveys, it appears that the uncertainty in γ for any individual survey must be at least 0.2–0.4.

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studied the statistics of the combined sample of Lynds clouds and their own Southern dark cloud survey, and they found a distribution of areas proportional to $(\text{area})^{-1}$. The corresponding size spectrum has $\alpha = -1$. This gives a mass spectrum index of -1 for any p. Thus, the discrepancy with the molecular line survey size or mass spectra is even larger. Other published studies of mass spectra based on extinction are not so clear, but they point in the same direction, especially for lower mass clumps (e.g., Bhatt, Rowse, & Williams 1984) for Lynds clouds in Orion, ρ Oph, and Taurus. Drapatz & Zinnecker (1984) give size and mass spectra for several samples, based on both extinction and CO.

In the present paper, we examine the possibility that this flatter size spectrum seen in extinction is caused by the effects of occlusion (smaller clouds being hidden behind large clouds) on the extinction studies; this effect would not affect the spectral line studies nearly as much because, in that case, two clouds along the same line of sight can be distinguished in velocity space. (Of course, occlusion in velocity space can also occur, and we discuss this briefly in § 3, below.) We derive an expression for the real size distribution of clouds in terms of the measured distribution that is affected by binary occlusion, and we derive the range of parameters over which the difference in size spectra between the two approaches can be reconciled.

A relation between the distribution of physical sizes of clouds and their angular sizes is established in § 2, while a relation between the actual distribution of angular sizes and the distribution measured in the presence of occlusion is presented in § 3.

2. "APPARENT" SIZES OF CLOUDS

Consider that $N_1(l)$ is the "real" size distribution and $N_2(\theta)$ is the "apparent" angular size distribution, without accounting for occlusion. In this section, we define the relation between $N_1(l)$ and $N_2(\theta)$. This problem is similar to the one discussed in Feitzinger & Stüwe (1986). Because of the geometry of diverging lines of sight, clouds with the same physical size but at different distances from the observer will fall into different ranges of apparent angular sizes. As a first approximation, assume the "true" properties of clouds to be independent of the distance from the observer. This is probably reasonable for observations in the Galactic plane and of nearby individual cloud complexes (e.g., Taurus, Oph, Chameleon, Orion, etc.)

In our model, the distribution of clouds at distance r is given by the product $\varrho(D)N_1(l)$, where $\varrho(D)$ is the total density of clouds at distance D, and we take $N_1(\ell)$ normalized to unity. Then the number of clouds within the distance interval D, $D + dD$ is $\varrho(D)\omega D^2\, dD$, where ω is the solid angle. Within this volume, the clouds with sizes from $D\theta$ to $(\theta + d\theta)D$, where θ is the angular size of clouds, will contribute to the apparent angular cloud distribution $N_2(\theta)$. The total number of "projections" with angular sizes $(\theta, \theta + d\theta)$ within the solid angle ω can be found by integrating $\varrho(D)\omega D^2 N_1(D\theta)D\, d\theta\, dD$ over the line of sight. Therefore,

$$N_2(\theta)\omega\, d\theta = \omega \int_{D_{\min}}^{D_{\max}} D^3 \varrho(D) N_1(D\theta) dD\, d\theta\,. \tag{1}$$

A change of variables $D\theta = x$ results in

$$N_2(\theta) = -\frac{1}{\theta^4}\int_{\theta D_{\min}}^{\theta D_{\max}} x^3 \varrho\left(\frac{x}{\theta}\right) N_1(x) dx\,. \tag{2}$$

Assuming $\varrho(x) =$ constant, differentiation gives

$$\frac{1}{D_{\max}}\,[N_2(\theta)\theta^4]' = \theta^3 D_{\max}^3 \varrho N_1(\theta D_{\max}) - \theta^3 D_{\min}^3 \varrho N_1(\theta D_{\min}) \tag{3}$$

For power law $N_1(\ell) = N_1(\ell_{\min})(\ell/\ell_{\min})^{-\alpha}$, the first term is the most important if $\alpha < 3$, whereas if $\alpha > 3$, the second term dominates. The cases of greatest interest here have $\alpha < 3$. Whenever the second term is negligible and $N_1(\ell)$ is a power-law distribution, $N_2(\theta)$ is also power law with the same index.

Similarly, for $D_{\min} = 0$,

$$N_1(\theta D_{\max}) = \frac{1}{\theta^3 D_{\max}^4 \varrho}\,[N_2(\theta)\theta^4]'\,, \tag{4}$$

and for any other $D_{\min}$, the power-law distribution $N_1(l)$ entails a power-law distribution $N_2(\theta)$ with equal slope. Therefore, the index of the size distribution is not affected by the differing distances of the clouds in the sample, and the index of the angular size distribution is the same as the index of the linear size distribution. An exception occurs for a delta function linear size distribution, i.e., when all clouds have the same size. In that case, the apparent angular size distribution varies as θ^{-4} (see Bhatt et al. 1984). In what follows, we therefore identify $N_2(\theta)$ with the "real" distribution of sizes, with the understanding that clustering of clouds and gradients in the number density of clouds with distance could alter this identification.

3. OCCLUSION EFFECT

In this section, we estimate the effect of occlusion on the apparent size distribution. By "occlusion" we mean cases in which a small cloud is hidden from view by the presence of a larger foreground cloud along the same line of sight. In reality, extinction clouds are not perfectly opaque, and so it is possible to detect some small clouds through larger clouds, depending on the relative column densities and the radial distance between the clouds. For example, a small cloud with $A_V = 6$ might appear as a condensation within a larger foreground cloud with $A_V = 2$ if the small cloud is not located too far behind the larger cloud. However, here we ignore this effect and treat the clouds as if they were completely opaque. In this sense, we overestimate the severity of occlusion.

Another effect we ignore occurs when two clouds of roughly similar angular sizes lie near enough to each other on the plane of the sky that their envelopes overlap, giving the appearance of two clouds within a larger (e.g., peanut-shaped, if the clouds were spherical) cloud. This effect is sometimes referred to as "blending." The larger "parent" cloud is an artifact, and so the effect tends to flatten the apparent size distribution. However, it is easy to see that the number of such partially merged projected clouds is much smaller than the number of occluded clouds, since the blending effect depends on the circumference of the larger cloud, while occlusion depends on the area. The effect of blending is only comparable to that of occlusion when both clouds have comparable sizes.

However, our treatment may be generalized to include these effects and other types of selection functions peculiar to particular survey techniques and cloud identification criteria. The prices to be paid for such a generalization would

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be a much more complicated analytic solution and a severe model dependency, and, thus, we postpone any discussion of these effects.

The quantity $N_2(\theta)$ is the projected apparent angular size distribution of clouds when occlusion is ignored; i.e., it is the angular size distribution corresponding to the "real" linear size distribution. If occlusion is "switched on," some of smaller clouds are hidden behind (or in front of) bigger ones. Let $N_3(\theta)$ be the distribution of projections in the presence of occlusion. Then

$$\pi N_3(\theta)\theta^2 \omega \, d\theta \tag{5}$$

is the angular area covered by cloud projections with sizes within the range θ, $\theta + d\theta$. The part of the sky not covered by cloud projections with angular sizes greater than θ is

$$1 - \frac{\pi}{A}\int_{\theta}^{\theta_u} N_3(x)x^2 \, dx \,, \tag{6}$$

where A is the angular area covered by the survey and θ_u is the upper size limit for the sample. Therefore, the number of cloud projections of angular size θ that are *not* occluded by larger clouds is

$$N_2(\theta)\omega \, d\theta\left[1 - \frac{\pi}{A}\int_{\theta}^{\theta_u} N_3(x)x^2 \, dx\right] . \tag{7}$$

Since this is the number of clouds that is seen, equating it to $N_3(\theta)\omega \, d\theta$ gives

$$N_3(\theta) = N_2(\theta)\left[1 - \frac{\pi}{A}\int_{\theta}^{\theta_u} N_3(x)x^2 \, dx\right] . \tag{8}$$

The real size distribution $N_2(\theta)$ can, therefore, be derived from the apparent (occluded) size distribution $N_3(\theta)$ as

$$N_2(\theta) = N_3(\theta)\bigg/\left[1 - \frac{\pi}{A}\int_{\theta}^{\theta_u} N_3(x)x^2 \, dx\right] . \tag{9}$$

The second term in the denominator is just the fraction of the survey area A covered by clouds with sizes greater that θ. The largest value this fraction can have occurs at $\theta = \theta_\ell$, the minimum size detected in the survey, for which the second term is the total area filling factor of clouds detected in the survey (<1).

It is also possible to derive the observed distribution $N_3(\theta)$, which would result from a given real distribution $N_2(\theta)$, as shown in the Appendix. However, that formulation is not as useful for the purposes of the present paper because the solution involves the unknown properties of the real distribution.

To illustrate the properties of the N_2–N_3 relation, assume that the observed occluded distribution is a power law, $N_3(\theta) = c_3\theta^{-\alpha_3}$. Then

$$N_2(\theta) = (c_3\theta^{-\alpha_3})\bigg/\left[1 - \frac{\pi}{A}\frac{c_3}{(3-\alpha_3)}\right](\theta_u^{-r_3+3} - \theta^{-\alpha_3+3}) . \tag{10}$$

The total areal filling fraction is

$$f_{3,\mathrm{tot}} = \frac{1}{A}\int_{\theta_\ell}^{\theta_u} N_3(\theta)\pi\theta^2 \, d\theta = \frac{\pi c_3}{A(3-\alpha_3)}(\theta_u^{-\alpha_3+3} - \theta_\ell^{-\alpha_3+3}) . \tag{11}$$

The second term is negligible for $\theta_\ell \ll \theta_u$ and $\alpha_3 < 3$. Therefore, from equations (10) and (11) we see that for θ significantly smaller than θ_u, $N_2(\theta) = N_3(\theta)/(1 - f_{3,\mathrm{tot}})$; i.e., for small clouds the real number of clouds is larger than the observed number by a factor $(1 - f_{3,\mathrm{tot}})$, but the power-law index is unaffected. The probability of a small cloud to be hidden by a large cloud is independent of its size if its size is much smaller than θ_u, since the areal filling is dominated by the largest clouds (if $\alpha_3 < 3$).

To see this more clearly, consider the local logarithmic slope of the real distribution at size θ (i.e., the exponent of a local power-law fit at that size). From equation (10), we obtain

$$\begin{aligned}\alpha_2(\theta) &= \frac{d \ln N_2(\theta)}{d \ln = n\theta} \\ &= \alpha_3 + \frac{\pi c_3}{A}(\theta^{-\alpha_3+3})\bigg/\left[1 - \frac{\pi c_3}{A(3-\alpha_3)}(\theta_u^{-\alpha_3+3} - \theta^{-\alpha_3+3})\right] \\ &\equiv \alpha_3 + \Delta\alpha(\theta)\end{aligned} \tag{12}$$

The maximum value of the change in exponent $\Delta\alpha(\theta)$ occurs for θ near θ_u, at which size $\Delta\alpha(\theta) = \pi c_3 \theta_u^{-\alpha_3+3}/A \approx (3 - \alpha_3) f_{3,\mathrm{tot}}$ (for $\theta_\ell \ll \theta_u$ and $\alpha_3 < 3$). If $f_{3,\mathrm{tot}} \approx 0.5$, as is typical for dark cloud surveys (not selected according to opacity class or size), then the dark cloud power law, $\alpha_3 \sim 1.4$, gives $\Delta\alpha \approx 1.6 f_{3,\mathrm{tot}} \sim 0.8$. While this is about the value needed to reconcile the extinction size distribution with the spectral line size distribution, it only occurs very close to θ_u. At smaller θ, for instance $x\theta_u (x < 1)$, $\Delta\alpha$ is reduced by a factor of $x^{-\alpha_3+3} \sim x^{1.6}$ for the parameters chosen. Thus, even for clouds half or a third of the size of the largest clouds, $\Delta\alpha$ is too small to account for the discrepancy, and for $x = 0.1$, $\Delta\alpha$ is essentially negligible.

The same considerations hold even if the observed distribution $N_3(\theta)$ is not a power law, as long as it is not locally too steep ($\alpha > 3$): the real size distribution tracks the observed distribution (although at larger amplitude), except for sizes close to θ_u, at which sizes the real distribution is steeper than the observed distribution.

We would be tempted to conclude that occlusion cannot account for the discrepancy, except for the fact that the range in sizes in the observed surveys is actually quite small. For both types of surveys, the cloud masses are proportional to the square of some characteristic size times a column density; so the range in sizes, which is a directly observed datum, only corresponds to the square root of a given range in the masses (which is what is usually displayed). Since, for the published spectral line surveys of clumps within cloud complexes, power laws are only good fits over a limited mass range (limited by small numbers at the largest masses and resolution, incompleteness, and other effects at small masses), usually a factor of 10–100, the range in sizes is not very large. The range in *sizes* for a few early surveys is listed in Drapatz & Zinnecker (1984). The range in sizes for the line surveys of Stutzki & Güsten (1990), Lada et al. (1991), Tatematsu et al. (1993), Williams et al. (1984), and Williams et al. (1995) is less than a factor of 10, although the range in *masses* used to derive the mass spectra is larger in some of the surveys. This suggests that the mass distributions derived from spectral line surveys will be very sensitive to the definition of, and systematic uncertainties in the measurement of, cloud sizes. For the extinction sample, the range of sizes over which the power-law size spectrum is applicable is less than a factor of about 10 in all cases, even for the full sample of the Lynds and Khavtassi surveys, and

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various selection effects come into play at smaller and larger masses (see, for a discussion, Scalo 1985, § III.B.2).

Thus, we conclude that the discrepancy between size distributions derived from extinction surveys and spectral line surveys *may* be due to occlusion effects in the extinction surveys, because the minimum size is not much smaller than the maximum size in both types of surveys, *or* because of different operational definitions of size in the two types of surveys. Actually, these two possibilities are not independent because the size range is related to how clouds are defined, as well as by the noise level. In some of the surveys, the noise level is large enough so that the surveys may only be observing, in effect, the "tips of the mountain range" if the column density map is thought of as a two-dimensional surface with height equal to column density.

We know of two studies whose results are relevant for resolving this ambiguity.

Clemens & Barvainis (1988) compiled a catalog of isolated small dark cloud ("globules") identified on POSS plates, and they compiled properties based on their CO observations. For clouds with mean size larger than 3'.5 (smaller size clouds are probably affected by incompleteness), we can fit the frequency distribution of angular sizes and, hence, linear sizes if the clouds are uniformly distributed: by $f(r) \sim r^{-2}$, which gives a power-law mass spectrum with $\gamma = -1.5$ to -1.7 for $p = -1$ or 0. These clouds were selected to be small and isolated, in such a way that occlusion should not be important. Since this result agrees with the molecular line surveys, it suggests that the flatter size and mass spectra derived from general extinction surveys are products of occlusion effects.

The study of 255 *IRAS* cloud cores by Wood, Myers, & Daugherty (1994) derives sizes and masses, based on *IRAS* 100 μm optical depth, for "cores" with visual extinction greater than about 4 mag. Wood et al. find power-law fits to the frequency distribution of sizes and masses with indices -2.1 and -1.5, respectively. (The indices quoted in the Wood et al. paper are for the cumulative distributions; we thank B. Elmegreen for pointing this out to us.) The sizes and masses for the fits have ranges of well over a factor of 1000. The cores are optically thin at 100 μm, so occlusion should not be important; a small core behind a larger core would be seen as a column density enhancement of about a factor of 2 because all the cores in the sample have about the same column density. The situation is not so clear, however, because these "cores" were defined by contours at a single column density in such a way that a small background cloud with large column density might not be counted as a separate entity, an effect that might flatten the apparent size distribution. In any case, the fact that the Wood et al. result agrees with the molecular line surveys, despite the very different techniques used to identify clouds, again suggests that the flatter extinction survey size and mass spectra are due to occlusion.

The same argument used above for purely spatial occlusion can be somewhat extended to include the effects of blending in velocity space for spectral line surveys. In this illustrative example, we assume that each identified "cloud" or "clump" (for convenience we use the latter term in what follows) has an internal velocity dispersion $\Delta v(\theta)$ that is strictly correlated with the size of the clump, as found in several surveys, at least for clumps in which self-gravity is important. In that case, the fraction of the total survey volume of the data cube AV (A = area of the survey in the plane of the sky, V = radial velocity extent of the survey) occupied by occluded clumps of size θ is

$$f_v(\theta) = \frac{1}{AV} \int_\theta^{\theta_u} \pi\theta^2 N_v(\theta) \Delta v(\theta) d\theta ,$$

where $N_v(\theta)$ is the size distribution found in the (blended) survey. The real size distribution is, therefore,

$$N_2(\theta) = \frac{N_v(\theta)}{1 - f_v(\theta)} .$$

The maximum value of $f_v(\theta)$ occurs at θ_ℓ and is the total volume filling factor of observed clumps in the data cube. Since this number is small for the surveys that we are aware of (see Fig. 7 in Williams et al. 1995), the effect of this type of occlusion (owing to finite internal velocity dispersion of the clumps) on the derived size distribution must be negligible, at least for velocity resolutions much smaller than the minimum Δv. However, this analysis does not account for the fact that clumps with similar *centroid* velocities may lie along the same line of sight. Taking this effect (which probably dominates the blending in velocity space) into account would involve calculating the probability that, for a prescribed centroid velocity distribution, two clouds along a given line of sight have a centroid velocity difference smaller than the sum of the line widths of the two clouds (which is a function of θ), a calculation that we postpone to a later publication.

4. CONCLUSIONS

Our study has examined the effect of occlusion on extinction surveys. The predicted change in the shape of the frequency distribution of cloud sizes for extinction surveys compared to spectral line surveys is small, except very near the maximum cloud size. Thus, the discrepancy between the empirical results for the two types of surveys probably cannot be attributed to occlusion in the extinction survey if the size range of both types of survey is large. However, an examination of the literature shows that many of the observed surveys employ a very limited range of sizes. In these cases, the discrepancy might still be due to occlusion. The isolated small dark cloud survey of Clemens & Barvainis (1988) and the *IRAS* cloud core survey of Wood et al. (1994) both give size spectra similar to the results of molecular line surveys, suggesting that occlusion is indeed responsible for the flatter distributions found in general extinction studies.

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APPENDIX

Rather than solve for the real distribution function in terms of the observed (occluded) distribution, it is possible to derive the observed distribution $N_3(\theta)$ that would result from a given real distribution $N_2(\theta)$. Differentiating equation (9) with respect

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to θ gives

$$N_3'(\theta) = N_3(\theta)\left[\frac{\pi\theta^2}{A} + \frac{N_2'(\theta)}{N_2^2(\theta)}\right]N_2(\theta)\,. \tag{A1}$$

Integrating this equation with a lower integration limit, θ_ℓ, gives

$$\frac{N_3(\theta)}{N_3(\theta_\ell)} = \frac{N_2(\theta)}{N_2(\theta_\ell)} \exp\left[\frac{\pi}{A}\int_{\theta_\ell}^{\theta} N_2(x)x^2\,dx\right]. \tag{A2}$$

We can obtain $N_3(\theta_\ell)/N_2(\theta_\ell)$ by imposing the condition that the largest cloud in the sample cannot suffer any occlusion, i.e., by substituting $N_3(\theta_u) = N_2(\theta_u)$ at $\theta = \theta_u$ in equation (A2). This condition results in

$$\frac{N_3(\theta_\ell)}{N_2(\theta_\ell)} = \exp\left[\frac{\pi}{A}\int_{\theta_\ell}^{\theta_u} N_2(x)x^2\,dx\right] = \exp\left(-A_{\rm tot}/A\right)\,, \tag{A3}$$

where $A_{\rm tot}$ is now the total area covered by all clouds in the unoccluded (real) distribution and may be greater than the survey area A. Dividing the integral from θ_ℓ to θ_u into parts from θ_ℓ to θ and from θ to θ_u, and substituting into equation (A2), gives

$$\begin{aligned} N_3(\theta) &= N_2(\theta)\exp\left[-\frac{\pi}{A}\int_{\theta}^{\theta_u} N_2(x)x^2\,dx\right] \\ &= N_2(\theta)\exp\left[-A(>\theta)/A\right]\,, \end{aligned} \tag{A4}$$

where $A(>\theta)$ is the area covered by clouds with sizes greater than θ in the unoccluded distribution and $A(>\theta)$ may be greater than unity. For a power law $N_2(\theta) = c_2\theta^{-\alpha_2}$, we find

$$N_3(\theta) = N_2(\theta)\exp\left[\frac{\pi c_2}{(3-\alpha_2)A}\left(\theta_u^{-\alpha_2+3} - \theta^{-\alpha_2+3}\right)\right]. \tag{A5}$$

The local logarithmic slope of the predicted occluded distribution is then (assuming $\theta_\ell \ll \theta_u$ and $\alpha_2 < 3$)

$$\alpha_3(\theta) = \frac{d\ln N_3(\theta)}{d\ln\theta} = \alpha_2 - \frac{A_{\rm tot}}{A}\left(\frac{\theta}{\theta_u}\right)^{-\alpha_2+3}\,. \tag{A6}$$

Once again we see that, although the change in local logarithm slope may be large near θ_u, the effect becomes increasingly negligible for $\theta \ll \theta_u$.

However, this formulation is not as useful as that given in the main text [which expressed $N_2(\theta)$ in terms of $N_3(\theta)$] because the total covering fraction of the real distribution is unknown, although it can be evaluated for a model that specifies the total number of clouds in the distribution (again unknown from observations).

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