The Astrophysical Journal, 468:L127–L130, 1996 September 10

NEW STELLAR REACTION RATE FOR $^{12}C(\alpha, \gamma)^{16}O$

L. Buchmann
TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada
Received 1996 May 8; accepted 1996 July 2

ABSTRACT

The $^{12}C(\alpha, \gamma)^{16}O$ reaction rate has been reevaluated using recently published cross section estimates. The results suggest that the temperature dependence of the reaction rate as given by Caughlan & Fowler should be revised, even though the absolute cross section of $^{12}C(\alpha, \gamma)^{16}O$ still needs a better determination at low energies. Values for the stellar reaction rate of $^{12}C(\alpha, \gamma)^{16}O$ have been calculated for a recommended cross section as well as for the experimental upper and lower limits of the cross section. In addition, analytic expressions for these reaction rates are given for $0.03 \leq T_9 \leq 2$.

Subject headings: Galaxy: abundances — stars: abundances — stars: evolution — supernovae: general

1. INTRODUCTION

The triple-α reaction and the $^{12}C(\alpha, \gamma)^{16}O$ reaction compete with each other during core helium burning in massive stars. Energy released from these two reactions drives the hydrodynamics of the convective core. Additional α-captures by the ^{16}O nuclei thus produced are strongly supressed because of the nuclear state structure of ^{20}Ne. While the stellar reaction rate for the triple-α process is well determined (Rolfs & Rodney 1988), the reaction rate for $^{12}C(\alpha, \gamma)^{16}O$ is not. However, the ratio of the two rates determines directly the ratio of ^{12}C and ^{16}O after helium burning, as well as the abundances of elements produced in the later evolution of massive stars (Weaver & Woosley 1993; Hashimoto 1995). Since the dispersion into the galaxy of these medium-mass elements is among the most important effects of Type II supernova explosions, galactic chemical evolution is determined directly by the $^{12}C(\alpha, \gamma)^{16}O$ reaction rate (Timmes et al. 1995a, 1995b; Woosley & Weaver 1995). In addition, the later structural evolution of massive stars is influenced by the $^{12}C/^{16}O$ ratio after helium burning. The $^{12}C(\alpha, \gamma)^{16}O$ reaction rate will therefore influence the value of the initial stellar mass for which the supernova remnant will be either a neutron star or a black hole (Hashimoto 1995).

2. THE $^{12}C(\alpha, \gamma)^{16}O$ CROSS SECTION: THE PROBLEM

The cross section for radiative α-capture by ^{12}C into the ground state of ^{16}O is largely the result of the resonance capture into the tails of two states below the $^{12}C + \alpha$ threshold (subthreshold states) in the compound nucleus ^{16}O. One is at the nuclear excitation energy[1] $E_x = 7.117$ MeV, with spin and parity $J^\pi = 1^-$ ($E = -45$ keV), and the other one is at $E_x = 6.917$ MeV ($E = -245$ keV) with $J^\pi = 2^+$. Typically, stellar reaction energies are around $E = 300$ keV for core helium burning. In contrast, direct measurements of the radiative α-capture have been published for energies down to $E = 0.94$ MeV only. However, with the present precision and energy range of the measurements of the radiative α-capture on ^{12}C, the influence of the resonance tails associated with the subthreshold states is found to be too small to lead to unambiguous extrapolations into the low-energy regions of the cross section (Buchmann et al. 1996). The required extrapolation of the cross section to 300 keV is complicated by the interference of components to the cross section above the α-threshold of ^{16}O for both the $l = 1$ and $l = 2$ partial waves. For the $l = 1$ component, an interference of the subthreshold state with the broad $J^\pi = 1^-$ state at 9.6 MeV in ^{16}O occurs, while for the $l = 2$ component, a similar interference of the subthreshold state with the nonresonant (direct) electric quadrupole capture (DC) is presumed. Besides the radiative α-capture into the ground state of ^{16}O, direct radiative α-capture into bound excited states of ^{16}O is also feasible (cascade transitions). However, these captures into excited states of ^{16}O are relatively weak and are estimated to contribute only about 10%–20% to the total cross section.

Because the α-spectrum resulting from the β-decay of ^{16}N is sensitive to the α-strength of the $J^\pi = 1^-$, $E_x = 7.117$ MeV state in ^{16}O, measurements of the β-delayed α-spectrum of ^{16}N by Azuma et al. (1994) have been used to yield improved estimates of the $^{12}C(\alpha, \gamma)^{16}O$ cross section. Thus, from this measurement, the $E1(l = 1)$ component of the radiative α-capture on ^{16}O is known to within about 25%, with a cross section factor[2] S at $E = 300$ keV of $S_{E1}(300) = 80 \pm 20$ keV barns. However, the situation is different for the $E2(l = 2)$ part of the reaction, where no model-independent measurement of the reduced α-width of the $E_x = 6.917$ MeV state is available. In Buchmann et al. (1996), estimates of the $E2$-transition cross section are given, using R- and K-matrix parameterizations, to fit the available data sets.[3] The final result is $S_{E2}(300) \leq 140$ keV barns, and for the total cross section factor, 62 keV barns $\leq S(300) \leq 270$ keV barns.

From the analysis presented in Buchmann et al. (1996), it is difficult and somewhat arbitrary to establish a recommended value of the $^{12}C(\alpha, \gamma)^{16}O$ cross section in view of the uncertainty of the $E2$ component. From estimates based on α-transfer reactions and theoretical models, a value of $S_{E2}(300) = 50$ keV barns seems at least acceptable. The sum

[1] E_x denotes nuclear excitation energies, while E represents energies in the $^{12}C + \alpha$ center-of-mass system.

[2] The magnitude of the $^{12}C(\alpha, \gamma)^{16}O$ cross section is expressed in terms of the cross section factor S, which is $S(E) = E\sigma(E) \exp(2\pi\eta)$ (Rolfs & Rodney 1988), where η is the Sommerfeld parameter $\eta \propto E^{-1/2}$. Because a center-of-mass energy of 300 keV corresponds to a stellar temperature of $T_8 = 2$ in $^{12}C(\alpha, \gamma)^{16}O$, the cross section factor at 300 keV, $S(300)$, has been discussed, in most instances, in the literature dealing with the $^{12}C(\alpha, \gamma)^{16}O$ problem.

[3] References to the data and a description of the R- and K-matrix formalism are given in Buchmann et al. (1996).

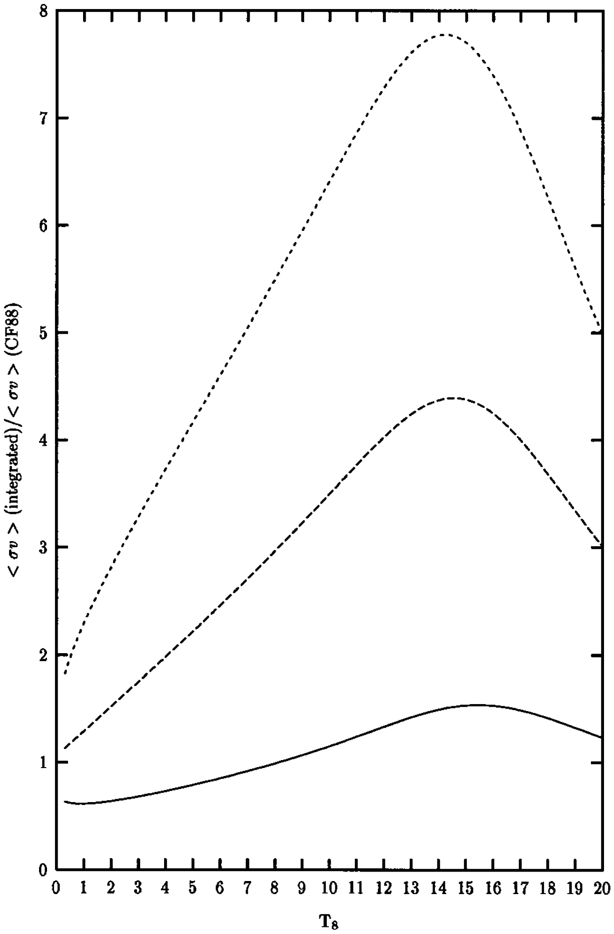

FIG. 1.—The ratio of the numerical integrated stellar reaction rate for $^{12}C(\alpha, \gamma)^{16}O$ ("integrated") and that of CF88 for cross section factors $S(300)$ of 62 (*solid line*), 146 (*long-dashed line*), and 270 keV barns (*short-dashed line*) from $T_8 = 0.3$–20.

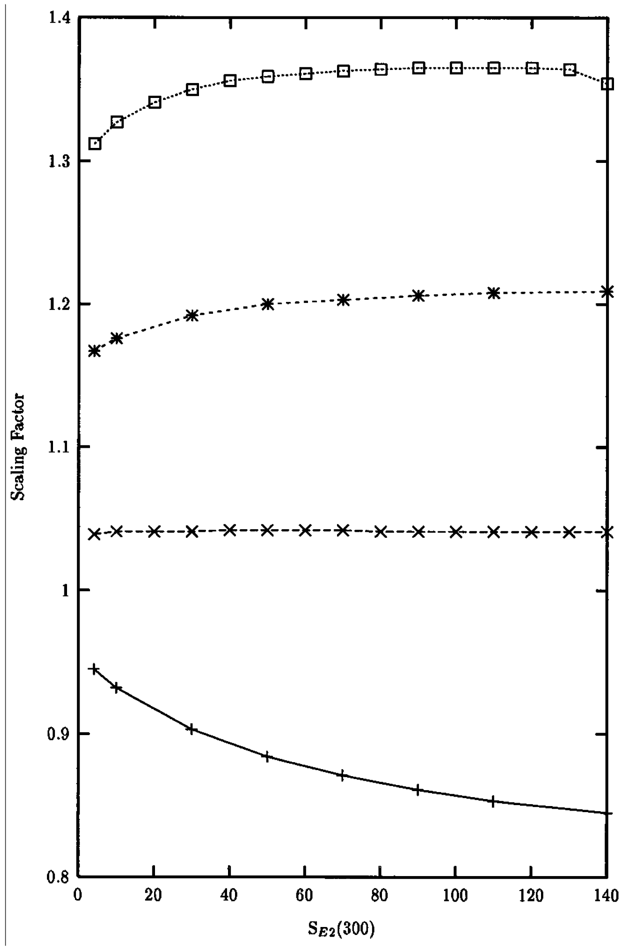

FIG. 2.—The scaling factor for the $S(300)$ normalized reaction rate of CF88 of $^{12}C(\alpha, \gamma)^{16}O$ as a function of $S_{E2}(300)$ for the recommended values of $S_{E1}(300)$ and $S\Sigma_{casc}(300)$. The scaling factor is given for temperatures of $T_8 = 1$ (*plus signs*), 2 (*crosses*), 3 (*asterisks*), and 4 (*squares*). For additional details, see text.

of the best estimates for the cascade transitions is found to be 16 keV barns (Buchmann et al. 1996), bringing $S(300)$ to 146 keV barns, which is taken here as the recommended cross section value.

3. RESULTS OF THE $^{12}C(\alpha, \gamma)^{16}O$ CROSS SECTION INTEGRATION

Stellar reaction rates are derived by folding nuclear cross sections with the Maxwell-Boltzmann distribution (Rolfs & Rodney 1988)

$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu}\right)^{1/2} \left(\frac{1}{kT}\right)^{3/2} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \frac{b}{E^{1/2}}\right) dE, \quad (1)$$

where μ is the reduced mass of the two-particle system, k the Boltzmann constant, and $b = 0.989 Z_1 Z_2 \mu^{1/2}$ the Gamow factor ($Z_{1,2}$ the nuclear charges). With N_A, the Avogadro number, $N_A\langle\sigma v\rangle$ is then either tabulated or given as an analytic expression in compilations of stellar reaction rates (e.g., Caughlan & Fowler 1988, hereafter CF88). For most cases, the integral cannot be solved analytically; however, some approximations to it have been found and used in the past.

In the compilations of stellar reaction rates initiated by W. A. Fowler (Fowler et al. 1967, 1975; Harris et al. 1983; Caughlan et al. 1985; CF88), the analytic expression for the rate of $^{12}C(\alpha, \gamma)^{16}O$ consists of the sum of two to four terms (rational functions combined with exponentials, see, e.g., eq. [2]), depending on the publication. The first term in CF88 results from the tail of the $J^\pi = 1^-$, $E_x = 7.117$ MeV state, already present in Fowler et al. (1967). In Fowler et al. (1975), it is reported that this term has been matched by numerical integration to the cross section extrapolation as performed in Dyer & Barnes (1974). The second term of CF88, only introduced in Harris et al. (1983), is a term resulting from the $J^\pi = 2^+$, $E_x = 6.917$ MeV state. The third term is a narrow resonance expression for the $E_x = 9.6$ MeV state in ^{16}O, present since Fowler et al. (1967). The fourth term was introduced by Fowler et al. (1975) and has never been changed. The other terms have been altered in subsequent compilations according to newly available information. No explanation for this fourth term is obvious nor has one been found by the author. However, this term and the narrow resonance term are only important for temperatures[4] above $T_9 = 1.5$. Below this temperature, the first two terms (subthreshold state terms) in CF88 are the only ones of significance. In many cases (e.g., Weaver & Woosley 1993), the rate

[4] As usual, temperatures are given as $T_8 = T/10^8$ K and $T_9 = T/10^9$ K.

TABLE 1

PARAMETERS p_i OF THE FITS TO THE NUMERICAL INTEGRATED REACTION RATES OF $^{12}C(\alpha, \gamma)^{16}O$[a]

Parameter	$N_A\langle\sigma v\rangle_{62}$[b]	$N_A\langle\sigma v\rangle_{146}$[b]	$N_A\langle\sigma v\rangle_{270}$[b]
p_1	-5.1029×10^8	-3.5738×10^7	1.7399×10^8
p_2	16.686	0.65711	13.608
p_3	29.877	30.834	40.171
p_4	86.063	0.40104	−11.013
p_5	3.4183×10^8	5.0464×10^8	7.6099×10^8
p_6	1.4064	0.66456	0.47121
p_7	30.955	31.332	31.666
p_8	−0.71494	−1.6054	−0.78718
p_9	15.514	16.272	14.781
p_{10}	1.7916×10^5	85028	56546
p_{11}	18617	8844.5	5881.1
p_{12}	36.057	33.033	31.322

[a] According to eq. (2) for the three cases of $S(300)$ = 62, 146, and 270 keV barns.

[b] Reaction rate in units cm^3 s^{-1} mole^{-1}.

of CF88 has been scaled linearly with respect to $S(300)$, in order to incorporate different values of the $^{12}C(\alpha, \gamma)^{16}O$ cross section.

The stellar rate expressions used in CF88 are only based on an approximation of the real cross section (§ 2). For higher temperatures, the peak of the integrand (Gamow Peak)[5] of equation (1) moves into the interference region for the two partial waves, an effect which was not taken into account by CF88. In the present work, stellar reaction rates have been derived by numerically integrating three different sets of R-matrix cross section factors $S(E)$ from Buchmann et al. (1996). In the first of these calculations, the best fit to the combined experimental data sets of Buchmann et al. (1996) was chosen and was used with the recommended value for $S(300)$. The two other sets of cross section factors $S_i(300)$ (i = 1, 2) used were those corresponding to the upper and lower limits found in Buchmann et al. (1996). The S-factor for cascade transitions was treated as constant, which is justified by the fits shown in Redder et al. (1987) and Barker & Kajino (1991). Because the high-energy $J^\pi = 4^+$ resonance at E_x = 10.35 MeV in ^{16}O was not included in the cross section, the temperature range is limited to only $T_9 = 2$, starting at $T_9 = 0.03$. The results of the calculations are shown in Figure 1, normalized to the reaction rate of CF88. Obviously, the temperature dependence of the stellar reaction rate found in these calculations is steeper than the one in CF88. For $T_8 > 1.8$, the reaction rate [for $S(300)$ = 100 keV barns] is larger than the one in CF88, while below $T_8 = 1.8$, it is smaller.

The dependence of the stellar reaction rate of $^{12}C(\alpha, \gamma)^{16}O$ on the $l = 1$ and $l = 2$ partial waves has also been investigated. In particular, the dependence of $\langle\sigma v\rangle$ on the largely unrestricted value of $S_{E2}(300)$ has been explored for the recommended case of $S_{E1}(300)$ = 80 keV barns and $S\sum_{\rm casc}(300)$ = 16 keV barns. The stellar reaction rate was calculated as a function of $S_{E2}(300)$ for the four temperatures of T_8 = 1, 2, 3, and 4 (core helium burning). These rates, normalized to $S(300)$ = 100 keV barns (CF88) and the total cross section factor $S(300)$, are shown in Figure 2 and may be used as scaling factors to correct the rates of CF88 for any desired value of $S_{E2}(300)$ within the range of the graph. It can

[5] For T_8 = 1, 2, 4, 7, 10, and 20, the peak of the integrand in eq. (1) is at approximately 0.19, 0.30, 0.48, 0.69, 0.80, and 1.40 MeV, respectively.

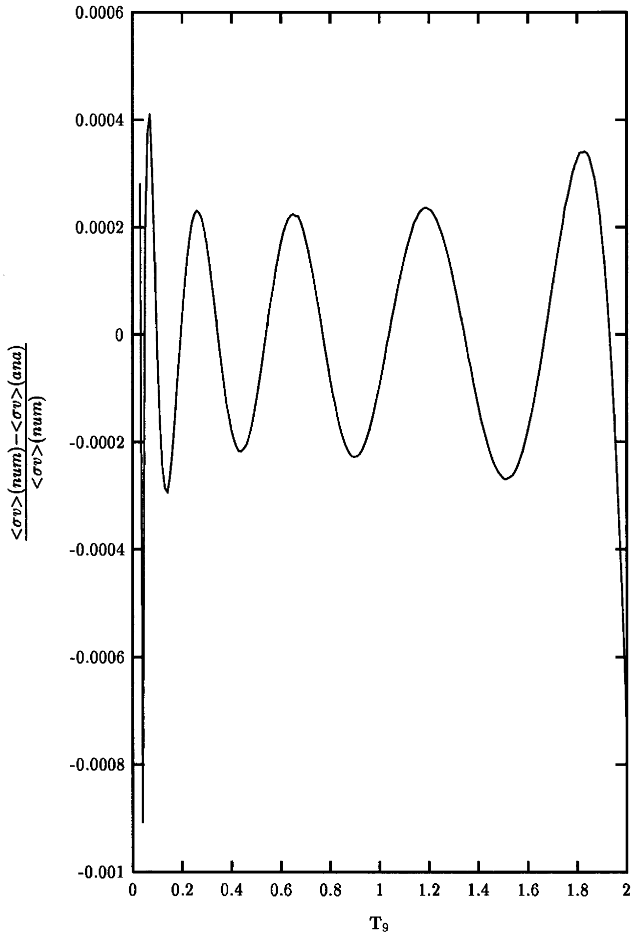

FIG. 3.—The difference between the numerically integrated reaction rate of $^{12}C(\alpha, \gamma)^{16}O$ and the analytic expression of eq. (2) normalized to the numerically integrated rate for the recommended stellar reaction rate of $S(300)$ = 146 keV barns; "num" labels the numerical integrated rate, "ana" the rate calculated from the analytical expression.

be seen that the reaction rate scales similar to that in CF88 for $T_8 = 2$, most likely because of the matching numerical results reported in Fowler et al. (1975). For lower (higher) temperatures than $T_8 = 2$, the stellar reaction rate drops (increases) relative to the value from CF88 with increasing $S_{E2}(300)$. Therefore, for these temperatures, the stellar reaction rate for $^{12}C(\alpha, \gamma)^{16}O$ is dependent at the 10% level on the details of the composition of the cross section, a dependence which strongly increases for higher or lower temperatures.

4. ANALYTIC EXPRESSION FOR THE STELLAR RATE OF $^{12}C(\alpha, \gamma)^{16}O$

For computational reasons, it is practical to express stellar reaction rates in terms of analytic forms, as was done previously in the compilations of stellar reaction rates (e.g., CF88). While these analytic expressions resulted often from approximations of equation (1), the choice of such approximations is not obvious for expressions based on numerical integration. On the other hand, for computational purposes, there is no particular need that the analytic expressions be correlated to the underlying physics. An expression similar to those in CF88 has therefore been employed here and is shown in equation (2):

$$N_A \langle \sigma v \rangle = \frac{p_1}{T_9^2 (1 + p_2/T_9^{2/3})} \exp\left(\frac{-p_3}{T_9^{1/3}} - \frac{T_9^2}{p_4}\right) + \frac{p_5}{T_9^2 (1 + p_6/T_9^{2/3})} \exp\left(\frac{-p_7}{T_9^{1/3}}\right) + \frac{p_8}{T_9^{3/2}} \exp\left(\frac{-p_9}{T_9}\right) + \frac{p_{10}}{T_9^{2/3}} (1 + p_{11} T_9^{1/3}) \exp\left(\frac{-p_{12}}{T_9^{1/3}}\right), \quad (2)$$

where the parameters p_i (i = 1–12) are the free parameters of the fit. For the temperature range of T_9 = 0.03–2, these parameters have been varied to minimize the difference with respect to the reaction rates resulting from the numerical integration of equation (1) for the lower limit rate [$S(300)$ = 62 keV barns], the recommended rate [$S(300)$ = 146 keV barns], and the upper limit rate [$S(300)$ = 270 keV barns]. Equal fitting weight has been employed over the entire temperature range. The parameters p_i of these fits are listed in Table 1. Note that any extrapolation of the analytic reaction rate to outside the fitted temperature range will lead to unreliable results. In Figure 3, the relative deviation between the reaction rate from numerical integration and the reaction rate from the analytic one is shown for the $S(300)$ = 146 keV barns case. The figure shows an oscillatory structure between the two rates, with a maximum deviation of 0.09% at the extremes but being around 0.02% for most temperatures. For the rates at the limit [$S(300)$ = 62, 270 keV barns], the deviations between the integral and the analytic expression are somewhat larger, reaching 1.2% for T_9 = 0.04 in the $S(300)$ = 62 keV barns case.

5. CONCLUSIONS

By using the cross sections for $^{12}C(\alpha, \gamma)^{16}O$ derived in Buchmann et al. (1996), which employed R-matrix parametrizations, it is found that the temperature dependence of the stellar reaction rate given in CF88 has to be revised. The degree of the revision depends on the value of the absolute cross section factor $S(300)$. It also has been shown that this correction depends somewhat on the detailed composition by partial waves of the $^{12}C(\alpha, \gamma)^{16}O$ cross section. For core helium burning of massive stars, believed to occur between T_8 = 0.15 and 3, this correction spans a range of about 30%, relative to the rate of CF88 when normalized to the cross section factor S at 300 keV. However, for higher and lower temperatures, larger deviations are found. Contrary to intuition, it is also found that the uncertainties in the stellar reaction rate of $^{12}C(\alpha, \gamma)^{16}O$ are larger at higher temperatures up to T_9 = 1.5 than at T_9 = 0.2. To make it possible to include the changes of the reaction rate derived here into future stellar model calculations, a revised analytic expression for the stellar reaction rate of $^{12}C(\alpha, \gamma)^{16}O$ has been calculated covering the temperature range of T_9 = 0.03–2.

I would like to thank Stan Woosley of the University of California, Santa Cruz, for comments on the newly derived reaction rate of $^{12}C(\alpha, \gamma)^{16}O$. I would also like to express my gratitude to Richard E. Azuma of the University of Toronto for extensive corrections to the manuscript.

REFERENCES

Azuma, R. E., et al. 1994, Phys. Rev. C, 50, 1194
Barker, F. C., & Kajino, T. 1991, Australian J. Phys., 44, 369
Buchmann, L., Azuma, R. E., Barnes, C. A., Humblet, J., & Langanke, K. 1996, Phys. Rev. C, in press
Caughlan, G. R., & Fowler, W. A. 1988, At. Data Nucl. Data Tables, 40, 283 (CF88)
Caughlan, G. R., Fowler, W. A., Harris, M. J., & Zimmerman, B. A. 1985, At. Data Nucl. Data Tables, 32, 197
Dyer, P., & Barnes, C. A. 1974, Nucl. Phys. A, 233, 495
Fowler, W. A., Caughlan, G. R., & Zimmerman, B. A. 1967, ARA&A, 5, 525
———. 1975, ARA&A, 13, 69
Harris, M. J., Fowler, W. A., Caughlan, G. R., & Zimmerman, B. A. 1983, ARA&A, 21, 1965
Hashimoto, M. 1995, Prog. Theor. Phys., 94, 663
Redder, A., Becker, H. W., Rolfs, C., Trautvetter, H. P., Donoghue, T. R., Rinkel, T. C., Hammer, J. W., & Langanke, K. 1987, Nucl. Phys. A, 462, 385
Rolfs, C. E., & Rodney, W. S. 1988, Cauldrons in the Cosmos (Chicago: Univ. Chicago Press)
Timmes, F. X., Woosley, S. E., Hartman, D. H., Hoffman, R. D., Weaver, T. A., & Matteuci, F. 1995a, ApJ, 449, 204
Timmes, F. X., Woosley, S. E., & Weaver, T. A. 1995b, ApJ, 449, 204
Weaver, T. A., & Woosley, S. E. 1993, Phys. Rep., 227, 65
Woosley, S. E., & Weaver, T. A. 1995, ApJS, 101, 181