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THE POWER SPECTRUM IN A STRONGLY INHOMOGENOUS UNIVERSE

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ABSTRACT

The standard picture of the matter clustering, the cold dark matter model (and its variants), assumes that, on scales smaller than a certain "flattening scale" λ_f, the power spectrum increases with the scale, while on much larger scales it decreases so as to match the tiny fluctuations observed in the microwave background. However, there is no consensus on whether the turnaround λ_f has been detected or not, and on the actual importance of the luminosity segregation effect. We show that, due to the finiteness of the sample, the standard analysis applied to fractal distributions yields a turnaround for scales close to the survey scale, and a systematic amplitude shift with the survey scale. We point out that both features, bending and scaling, are in agreement with recent determinations of the power spectrum, in particular with the CfA2 spectrum in redshift space. Therefore, our conclusion is that the CfA2 sample is consistent with a fractal clustering.

Subject headings: galaxies: clusters: general — large-scale structure of universe

Identifying the scale at which our universe becomes homogeneous, if any, is a crucial task of contemporary cosmology. There is no doubt that the assumption of homogeneity worked pretty well so far, and explains many cosmological observations. However, when we come to consider the homogeneity of the luminous matter at the present, the situation becomes much less clear (see for instance the general review in Baryshev et al. 1994, hereafter BSLMP94). Essentially all the currently elaborated models of galaxy formation (see, e.g., Peebles 1993) *assume large-scale homogeneity* and predict that the galaxy power spectrum (PS), i.e., the power spectrum of the density contrast, decreases both toward small scales and toward large scales, with a turnaround somewhere in the middle, at a scale λ_f that can be taken as separating "small" from "large" scales. Because of the assumption of homogeneity, the power spectrum amplitude should be independent of the survey scale, any residual variation being attributed to luminosity bias (or to the fact that the survey scale has not yet reached the homogeneity scale). However, the crucial clue to this picture, the firm determination of the scale λ_f, is still missing, although some surveys do indeed produce a turnaround scale around 100 h^{-1} Mpc (Baugh & Efstathiou 1994; Feldman, Kaiser, & Peacock 1994). Recently, the CfA2 survey analyzed by Park et al. (1994, hereafter PVGH) (and confirmed by SSRS2—da Costa et al. 1994, hereafter DVGHP), showed a $n = -2$ slope up to $\sim 30h^{-1}$ Mpc, a milder $n \approx -1$ slope up to 200 h^{-1} Mpc, and some tentative indication of flattening on even larger scales. PVGH also find that deeper subsamples have higher power amplitude, i.e., that the amplitude scales with the sample depth.

We argue here that both features, bending and scaling, are a manifestation of the finiteness of the survey volume, and that they might not be due to the convergence to homogeneity nor to a power spectrum flattening. The systematic effect of the survey finite size is in fact to suppress power at large scale, mimicking a real flattening. Clearly, this effect occurs whenever galaxies have a large correlation scale, with respect to the survey, and it has often been studied in the context of standard scenarios (Itoh, Suginohara, & Suto 1992; Colombi, Bouchet, & Schaeffer 1994). We push this argument further, by showing that even a fractal distribution of matter, which never reaches homogeneity, shows a sharp flattening. Such a flattening is partially corrected, but not quite eliminated, when the correction proposed by Peacock & Nicholson (1991) is applied to the data. We show also how the amplitude of the power spectrum depends on the survey size as long as the system shows long-range correlations.

The standard power spectrum (SPS) measures directly the contributions of different scales to the galaxy density contrast $\delta\rho/\rho$. It is clear that the density contrast, and all the quantities based on it, is meaningful only when one can define a constant density, i.e., reliably identify the sample density with the average density of all the universe. When this is not true, and we argue that is indeed a questionable assumption in all the cases investigated so far, a false interpretation of the results may occur since both the shape and the amplitude of the power spectrum (or correlation function) depend on the survey size, as Coleman & Pietronero (1992, hereafter CP92) have shown for the CfA1 redshift survey. We stress that a sample that contains a portion of a fractal distribution is a *statistically fair sample*, even being nonhomogeneous.

Let us recall the basic notation of the power spectrum analysis. Following Peebles (1980) we imagine that the universe is periodic in a volume V_u, with V_u much larger than the (presumed) maximum correlation length. The survey volume $V \in V_u$ contains N galaxies at positions $\boldsymbol{r}_i$, and the galaxy density contrast is $\delta(\boldsymbol{r}) = [n(\boldsymbol{r})/\hat{n}] - 1$, where it is assumed that there exists a well-defined constant density $\hat{n}$, obtained averaging over a sufficiently large scale. Expanding the density contrast in its Fourier components we have $\delta_{\boldsymbol{k}} = N^{-1}\sum_{j\epsilon V} e^{i\boldsymbol{k}\boldsymbol{r}_j} - W(\boldsymbol{k})$, where $W(\boldsymbol{k}) = V^{-1}\int d\boldsymbol{r}W(\boldsymbol{r})e^{i\boldsymbol{k}\boldsymbol{r}}$ is the Fourier transform of the survey window $W(\boldsymbol{r})$, defined

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to be unity inside the survey region and zero outside. If $\xi(\boldsymbol{r})$ is the correlation function of the galaxies ($\xi(\boldsymbol{r}) = \langle n(\boldsymbol{r})n(0)\rangle/\hat{n}^2 - 1$), the true PS $P(\boldsymbol{k})$ is defined as the Fourier conjugate of the correlation function $\xi(r)$. Because of isotropy the PS can be simplified to $P(k) = 4\pi\int\xi(r)\,[\sin(kr)/kr]r^2\,dr$. The variance of δ_k is (Peebles 1980) $\langle|\delta_k|^2\rangle = N^{-1} + V^{-1}\tilde{P}(\boldsymbol{k})$. The first term is the usual additional shot noise term, while the second is the true PS convolved with a window function that describe the geometry of the sample (PVGH):

$$\tilde{P}(\boldsymbol{k}) = \frac{V}{(2\pi)^3}\int\langle|\delta_{\boldsymbol{k}'}|^2\rangle|W(\boldsymbol{k}-\boldsymbol{k}')|^2\,d^3\boldsymbol{k}'\,. \qquad (1)$$

We apply now this standard analysis to a fractal distribution. In a self-similar system the number of points inside a certain radius r scales according to the mass-length relation (Mandelbrot 1982) $N(r) = Br^D$, with $D < 3$ ($D = 3$ in the homogenous case) and the constant B is related to the lower cutoffs, i.e., to the minimum length at which we have fractal correlations (CP92). The average density for a spherical sample of radius R_s is therefore $\hat{n} = N(R_s)/V(R_s) = (3/4\pi)\,BR_s^{-(3-D)}$. It is simple to calculate the expression of the $\xi(r)$ in this case (CP92): $\xi(r) = [(3-\gamma)/3]\,(r/R_s)^{-\gamma} - 1$, where $\gamma = 3 - D$. Notice that the volume integral of $\xi(r)$ over the sphere is bound to vanish, since $\hat{n}$ is calculated from the data themselves. This implies "anticorrelation" for r close to R_s. A key point of our discussion is that on scales larger that R_s the $\xi(r)$ cannot be calculated without making assumptions on the distribution outside the sampling volume. In fact, any shell larger than R_s is incomplete, and thus any evaluation of $\xi(r > R_s)$ is an arbitrary reconstruction of what *cannot be known from the sample itself*. For instance, a disklike survey of galaxies of radius R and thickness $h \ll R$ would be judged to be completely homogeneous up to the scale $2R$, if analyzed the standard way, even if outside the survey there are in fact no galaxies at all, that is, even if the homogeneity is in reality only up to the scale h! The arbitrary reconstruction introduces (unless the distribution is really homogeneous) spurious, scale-dependent effects, as we will explicitly show below in the case of a pure fractal. By the same reasons, one should avoid in the evaluation of $\xi(r)$ *at all scales* shells not completely included within the sample's boundary. When the survey volume is not spherical, the scale R_s is of the order of the largest sphere completely contained inside the survey. Let us remind the reader that in a fractal the so-called correlation length r_0 [defined as $\xi(r_0) = 1$] is a linear function of the (spherical) sample size R_s: $r_0 = [(3-\gamma)/6]^{1/\gamma}R_s$. A linear dependence of r_0 on the sample size R_s has indeed been found in the whole CfA1 sample (Coleman, Pietronero, & Sanders 1988). Davis et al. (1988) also found that r_0 increases with the depth. However, the slope they find is smaller than unity. This discrepancy may be due to their way of averaging over incomplete shells, i.e., on the implicit boundary conditions, which, as already explained, introduce spurious homogenization, and thus a systematic decrease of the r_0 scaling. Notice also that $\xi(r)$ is a power law only for $[(3-\gamma)/3]\,(r/R_s)^{-\gamma} \gg 1$, hence, for $r \le r_0$; for larger distances there is a clear deviation from a power-law behavior. Both the amplitude and the shape of $\xi(r)$ are therefore scale dependent in the case of a fractal distribution. It is clear that the same kind of finite size effects are also present when computing the SPS.

The SPS for a fractal distribution inside a sphere of radius R_s is

$$P(k) = \int_0^{R_s} 4\pi\,\frac{\sin(kr)}{kr}\left[\frac{3-\gamma}{3}\left(\frac{r}{R_s}\right)^{-\gamma} - 1\right]r^2\,dr$$

$$= \frac{a(k, R_s)\,R_s^{3-D}}{k^D} - \frac{b(k, R_s)}{k^3}\,. \qquad (2)$$

Notice that the integral has to be evaluated inside R_s because we want to compare $P(k)$ with its *estimation* in a finite size spherical survey of scale R_s. Equation (2) shows the two scale-dependent features of the PS. First, the amplitude of the PS depends on the sample depth. Secondly, the shape of the PS is characterized by two scaling regimes: the first one, at high wavenumbers, is related to the fractal dimension of the distribution in real space, while the second one arises only because of the finiteness of the sample. In the case $D = 2$, one has in equation (2): $a = (4\pi/3)\,[2 + \cos(kR_s)]$ and $b = 4\pi\sin(kR_s)$. In practice, the rapidly varying functions a, b are averaged out on k shells. The PS is then a power law with exponent -2 at high wavenumbers, it flattens at low wavenumbers and reaches a maximum at $k \approx 4.3/R_s$, i.e., at a scale $\lambda \approx 1.45R_s$. The scale at which the transition occurs is thus related to the sample depth. In a real survey, things are complicated by the window function, so that the flattening (and the turnaround) scale can only be determined numerically (see below).

As we have seen, the analysis of a distribution on scales at which homogeneity is not reached must avoid the normalization through the mean density if the goal is to produce results that are not related to the sample size and thus misleading. To this aim, now we consider the scale-independent PS (SIPS) of the density $\rho(\boldsymbol{r})$, a quantity that gives an unambiguous information of the statistical properties of the system. We first introduce the density correlation function $G(\boldsymbol{r}) = \langle\rho(\boldsymbol{x}+\boldsymbol{r})\rho(\boldsymbol{x})\rangle = Ar^{-(3-D)}$, where the last equality holds in the case of a fractal distribution with dimension D, and where A is a constant determined by the lower cutoffs of the distribution (CP92). Defining the SIPS as the Fourier conjugate of the correlation function $G(r)$, one obtains that in a finite spherical volume $\Pi(k) \sim A'k^{-D}$, (where $A' = 4\pi[1 - \cos(kR_s)]$ if $D = 2$) so that the SIPS is a single power law extending all over the system size, without amplitude scaling with the sample size (except for $kR_s < 2\pi$). In analogy to the procedure above, we consider the Fourier transform of the density $\rho_{\boldsymbol{k}} = V^{-1}\sum_{j\in V}e^{-i\boldsymbol{k}\boldsymbol{x}_j}$ and its variance $\langle|\rho_{\boldsymbol{k}}|^2\rangle = V^{-1}\tilde{\Pi}(\boldsymbol{k}) + N^{-1}$, where $\tilde{\Pi}(\boldsymbol{k})$ is the same as in equation (1), with $\langle|\rho_{\boldsymbol{k}'}|^2\rangle$ instead of $\langle|\delta_{\boldsymbol{k}'}|^2\rangle$.

To study in detail the finite size effects in the determinations of the PS we have performed some tests on artificial distributions with a priori assigned properties. We distribute the sample in a cubic volume V_u. We determine $P(k)$ [$\Pi(k)$] defined as the directionally averaged $P(\boldsymbol{k})$ [$\Pi(\boldsymbol{k})$]. Following PVGH, the estimate of the noise-subtracted PS given in equation (1) for a strongly peaked window function is

$$P(\boldsymbol{k}) = \left(\langle|\delta_{\boldsymbol{k}}|^2\rangle - \frac{1}{N}\right)\left(\sum_{\boldsymbol{k}}|W_{\boldsymbol{k}}|^2\right)^{-1}(1 - |W_{\boldsymbol{k}}|^2)^{-1} \qquad (3)$$

(in this expression the volume V_u inside which the Fourier transform is evaluated is assumed conventionally to be $1\,h^{-1}\,\mathrm{Mpc}^3$, following the normalization of PVGH). For the lowest wavenumbers the power spectrum estimation (3) is not

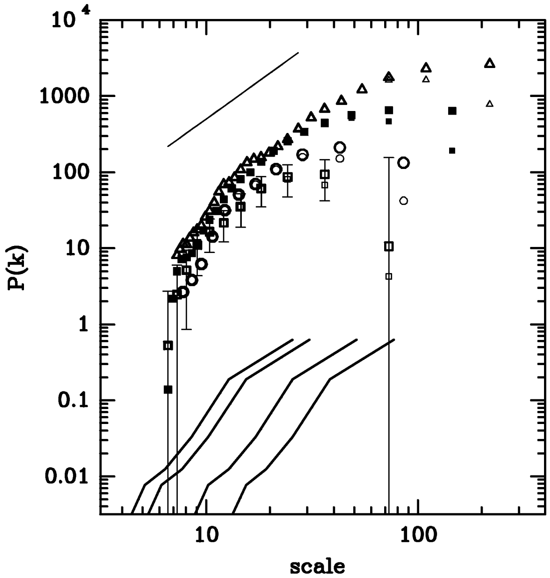

FIG. 1.—Symbols represent the power spectra for a fractal distribution with dimension $D \approx 2$ vs. the scale $2\pi/k$ without the correction factor (*smaller symbols*) and with the correction factor (*larger symbols*). The error bars, shown for clarity only on one sample, represent the scatter among the different observers of the same fractal. The four set of symbols refer, from top to bottom, to artificial fractal samples of 75 (*open triangles*), 50 (*filled squares*), 32.5 (*crosses*), and 25 (*open squares*) h^{-1} Mpc. The straight line shows the slope $D = 2$. The flattening and the turnaround at high wavenumbers is spurious. The continuous lines at the bottom show, respectively, the four window functions.

acceptable, because then the window filter flattens sensibly (see, e.g., PVGH). The factor $(1 - |W_k|^2)^{-1}$ has been introduced by Peacock & Nicholson (1991) as an analytical correction to the erroneous identification of the sample density with the population density. However, the correction itself rests on the assumption that the power spectrum is flat on very large scales, which is just the feature we are testing for. PVGH actually correct their results by comparing them to the power spectra of N-body simulations; their conclusion is that the power spectrum correction is a procedure reliable for wavelengths smaller than $\sim 200\ h^{-1}$ Mpc.

We have generated $D \approx 2$ fractal distributions with the random β model algorithm (Benzi et al. 1984). Then we have constructed artificial volume-limited catalogs with roughly the same geometry of the CfA2 survey (our model contains no dynamics, so that we can think of our fractal set as lying in redshift space, as the CfA2 galaxies). We have computed the quantity $P(k)$ from equation (3) averaging over 50 random observers (located on one of the particles) for each realization. In Figure 1 it is shown the $P(k)$ versus the scale $2\pi/k$ with and without the correction factor, for some different survey scales R_s, together with the angle-averaged window power spectrum $|W_k|^2$. The slope of the PS at high wavenumbers is $\approx -D$ in agreement with equation (2). As anticipated, the flattening at low wavenumbers is here completely spurious, i.e., it is due to the finite volume effects on the statistical analysis performed. In fact, comparing the PS at the various sample scales, one can see that the turnaround of the PS occurs always near the scale $\lambda = 1.5R_s$ as predicted previously. Notice that the PS starts flattening before the window spectrum flattens; as in PVGH, the change in slope occurs for $|W_k|^2 < 0.2$, value that they assumed as preliminary condition for the estimation (3) to be valid. The amplitude of the power spectrum scales according

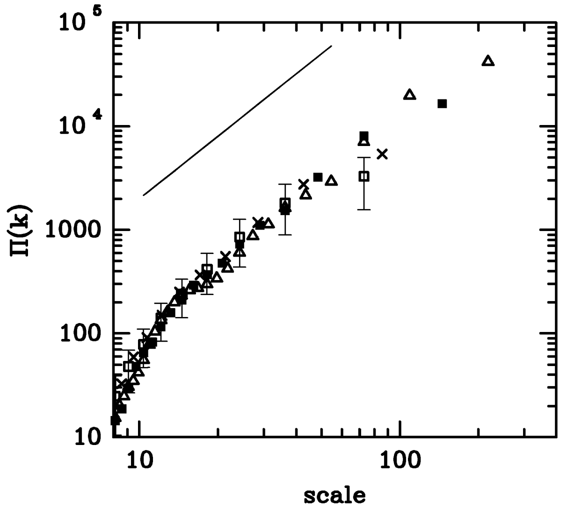

FIG. 2.—Scale-independent power spectrum for the same fractal and the same four scales as in Fig. 1. Now the spectrum is a single power law, up to the very few largest scales as expected. The reference line has a slope $D = 2$. The amplitude of the spectrum is now constant inside the errors.

to equation (2). In Figure 2 it is shown the behavior of $\Pi(k)$ computed from equation (3) with $\langle|\rho_k|^2\rangle$ in place of $\langle|\delta_k|^2\rangle$ and without the Peacock-Nicholson correction: as predicted, its amplitude does not scale with the sample size and it is characterized by a single power-law behavior, up to very large scales. Finally, in Figure 3 we compare directly the PS of our artificial catalogs with the PS of the CfA2 subsamples obtained, generating two volume-limited subsamples at 130 and 101 h^{-1} Mpc (PVGH). The physical scale for the artificial fractals has been computed matching the CfA2-101 galaxy average density. Both the shapes and the amplitudes are compatible with a fractal distribution: for CfA2-

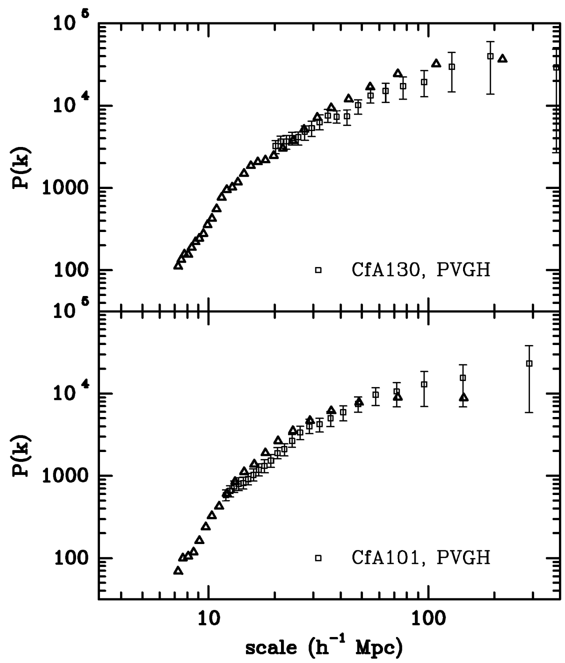

FIG. 3.—Comparison of power spectra of fractal distribution (*triangles*) with the CfA2 survey (*squares*). In the top panel, we plot the PS of the subsample CfA2-130 (PVGH) along with the PS of our artificial fractal distribution (without the error bars for clarity). In the bottom panel, we plot CfA2-101 (PVGH) and a subsample of the same fractal as above, with a correspondingly scaled depth.

101 the agreement is excellent; for CfA2-130 the two curves are compatible inside the errors.

As we have shown, the standard analysis on a pure fractal would lead to the conclusions that the fractal distribution has a spectrum approximated by two power laws, that a turnaround occurs on the largest scales, and that the amplitude scales with the sample depth; these conclusions are clearly dependent on the size of the sample and have nothing to do with the real distribution. The fact that recent evaluations of the galaxy PS showed just these features motivated us to consider whether or not a pure fractal can explain observational data. Let us remark that the scaling effect is particularly important in evaluating the matter/galaxy bias factor. Keeping the matter power spectrum fixed, the galaxy PS scaling as $\sim R_s^{3-D}$ implies indeed a scaling of the bias factor $b \sim R_s^{(3-D)/2}$.

Let us summarize the results of PVGH, by confronting them with the analysis of the PS for a fractal distribution: (1) for $k \geq 0.25$ ($\lambda \leq 25\ h^{-1}$ Mpc) the PS is very close to a power law with slope $n = -2.1$. In our view, this is the behavior at high wavenumbers connected with the real fractal dimension. (2) For $0.05 \leq k \leq 0.2$ ($120\ h^{-1}$ Mpc $> \lambda > 30\ h^{-1}$ Mpc) the spectrum is less steep, with a slope about -1.1. This bending is, in our view, solely due to the finite size of the sample. (3) The amplitude of the volume-limited (VL) subsample CfA2-130 PS is $\sim$40% larger than for CfA2-101. This linear scaling of the amplitude can be understood again considering that the sample is fractal with $D = 2$. It is worthwhile to notice that this trend is qualitatively confirmed by the results of Peacock & Nicholson (1991), who find a higher PS amplitude for a deep radio galaxy survey. On the other hand, PVGH explain this fact considering the dependence of galaxy clustering on luminosity: brighter galaxies correlate more than fainter ones. They support this interpretation observing that brighter galaxies tend to avoid underdense regions; they also analyze separately two subsets of the same VL sample of CfA2, one brighter than the other, and find in some cases a luminosity-amplitude correlation. It is certainly possible that both mechanisms, the luminosity segregation and the intrinsic self-similarity, are correct, and each one explains part of the scaling. However, PVGH do *not* detect such a luminosity segregation for the two largest subsamples, CfA2-101 and CfA2-130, which we are comparing here. It seems therefore that the amplitude scaling at these scales can be entirely attributed to the fractal scaling. We also mention the possibility that deeper samples may have larger redshift distortions due to the possible presence of richer clusters; this would induce a spurious increase in D and consequently an increase in clustering at scales smaller than $\sim 20\ h^{-1}$ Mpc (we thank the referee for this comment).

Another piece of evidence against the dominant role of the luminosity segregation has been put forward in Baryshev et al. (1994). In this paper two VL subsamples with the same absolute magnitude limit, $M \geq -20$, but with different depth, CfA1-80 (limited at $d \leq 80\ h^{-1}$ Mpc) and CfA2-130, have been compared. If the hypothesis of luminosity segregation holds then one should find that there is no difference between the amplitude of the correlation function (and of the PS) computed in these subsamples since they contain galaxies with the same average absolute magnitude. On the contrary one finds that the amplitude scales as predicted by the fractal behavior. In fact, for CfA1-80 $r_0 = 7\ h^{-1}$ Mpc, while for CfA2-130 $r_0 = 11\ h^{-1}$ Mpc (PVGH). Our conclusion is then that the fractal nature of the galaxy distribution can explain, to the scales surveyed in the CfA2 survey, the shift of the amplitude with sample depth of the PS and of $\xi(r)$.

However, several authors find different results from ours. For example, Strauss et al. (1992) and Fisher et al. (1993) find that in the *IRAS* samples the value of r_0 is $4\ h^{-1}$ Mpc, and it is independent of the sample depth. This result is quite surprising because the visual impression (Strauss et al. 1992) is that the *IRAS* galaxies belong to the same structures of the optical ones and do not fill the voids. We have done a detailed analysis of the properties of *IRAS* versus optical galaxies (Sylos Labini et al. 1996), and we have found that the difference among them is due to a systematic effect related to the poor sampling of the *IRAS* surveys. The point we have made in Sylos Labini et al. (1996) is that a sparse fractal set *will look like a (noisy) homogeneous sample*. A similar effect can also explain the scaling with depth of the amplitude of the angular correlation function. In fact in this case, even if there is a large number of points in the angular surveys, one cannot make an average over observers, so that the spurious effects related to finite sampling dominate the behavior of such a quantity (this holds also for the galaxy number counts).

Finally, we observe that the fractal dimension of the galaxy clustering in *redshift space* rises from $D \sim 1.5$ for CfA1 to $D \sim 2$ for CfA2, in agreement with the results of other independent surveys: Perseus-Pisces (Guzzo et al. 1991; Sylos Labini et al. 1996), and ESP (BSLMP94, Sylos Labini et al. 1996; Di Nella et al. 1996). This is an indication that a large sample is needed to estimate accurately the fractal dimension. Another explanation for this change in D may be that, in the fractal view, $\xi(r)$ *is a power law only for* $r \ll r_0$, after which it curves downward to negative values. Such a behavior induces a systematic underestimation of the fractal dimension obtained fitting $\xi(r)$ (and, similarly, fitting the angular correlation).

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Note added in proof.—A power spectrum scaling analogous to the one reported in CfA2 has been found recently also in the Las Campanas survey (Lin, H., et al., ApJ, 456, L1 [1996]).