THE PULSATING PRE–WHITE DWARF STAR PG 0122+200

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ABSTRACT

We present an analysis of single-site time-series photometry of the pulsating pre–white dwarf PG 0122+200. We show the pulsations are consistent with a pattern of modes equally spaced in period; both the observed period range and spacing confirm that PG 0122 is a g-mode pulsator. PG 0122 shows a pattern similar to that seen in multisite observations of PG 2131+066 and PG 1159−035. The measured period spacing, combined with the spectroscopic temperature, constrain the stellar mass much more precisely than the published measurement of its surface gravity. Based on stellar models, the mass of PG 0122 falls in the range 0.66–0.72 $M_\odot$. Fine structure in the power spectrum indicates that PG 0122 rotates once every 1.6 days. Future multisite observation (e.g., using the Whole Earth Telescope) should increase the precision of these results and reveal detailed information on the internal structure of this variable pre–white dwarf star.

Subject headings: stars: individual (PG 0122+200) — stars: oscillations — stars: white dwarfs

1. INTRODUCTION

The PG 1159 stars represent the ephemeral penultimate stage in the life of low- and intermediate-mass stars. Gaining knowledge of their properties provides insight into their predecessors—central stars of planetary nebulae and AGB stars—and their descendants—the white dwarfs. The pulsations exhibited by some PG 1159 stars help us in this endeavor. With the tools of asteroseismology in hand, the observed frequencies can be used to determine the total stellar mass and surface layer mass of the compositionally stratified pre–white dwarfs. This, then, constrains their genealogy and structure.

The variable PG 1159 stars (GW Vir stars) are nonradial g-mode pulsators. Theory suggests—and observations show—that for such high surface gravity objects ($\log g \sim 7$), the power spectra should be rich but essentially well ordered: we expect to see patterns of modes equally spaced in period. High radial overtone ($n \gg 1$) modes of low spherical harmonic index l can show multiplet structure; rotation can split each one into $2l + 1$ peaks in the power spectrum. Other effects, such as a stellar magnetic field, cause frequency splitting about the $m = 0$ mode in a given multiplet. This splitting can be asymmetric depending on the field geometry.

Despite the potential complexity, only modes with low l have been identified in GW Vir stars. Thus, the power spectrum is not necessarily complex beyond comprehension. Kawaler & Bradley (1994) showed that the average period spacing between modes of the same l and consecutive n depends primarily on the total stellar mass, with lesser dependence on luminosity and only a very slight dependence on composition. Periodic deviations from mean period spacing can reveal the existence of a composition interface (Kawaler 1988; see Kawaler & Bradley 1994). Determination of the values of n, l, and the azimuthal quantum number m, therefore, reveals a wealth of information about the structure of the star (see Winget et al. 1991 and Bradley 1994).

The task of decoding the power spectrum of a real star is complicated because not all possible modes are necessarily present. Also, the periodic intervention of the Earth between the telescope and the star introduces aliases into the frequency spectrum that confuse mode identification. The aliasing problem currently is addressed using the Whole Earth Telescope (WET; Nather et al. 1990). A major success of WET came in 1989, when observers at telescopes around the globe obtained 2 weeks of almost continuous data on the class prototype, PG 1159−035 (hereafter PG 1159). This minimized the aliasing and allowed Winget et al. (1991) to conclusively identify pulsation modes. They found an unbroken sequence of almost 20 multiplets—each corresponding to $l = 1$ and consecutive n—along with several $l = 2$ modes. The total stellar mass, the rotation rate, and the envelope mass were determined with unprecedented precision, showing that asteroseismological models could be used with confidence to study the PG 1159 pulsation phenomenon. Recently, Kawaler et al. (1995) analyzed a second PG 1159 star, PG 2131+066, again using WET data. PG 2131 displayed a much simpler frequency spectrum: only a few consecutive $l = 1$ triplets, and no $l = 2$ modes, were identified. The data from WET observations of other PG 1159 stars are currently in analysis.

The star PG 0122+200 (BB Psc, $m_b = 16.13$) was identified as a member of the PG 1159 spectral class by Wesemael, Green, & Liebert (1985). Dreizler et al. (1995) report a surface gravity of $\log g = 7.5 \pm 0.5$, and an effective temperature of $75,500 \pm 5000$ K, placing PG 0122 among the coolest PG 1159 stars. Bond & Grauer (1987) discovered variability in PG 0122, reporting a power spectrum dominated by variations at 402.3 and 443.7 s. Hill, Winget, & Nather (1987, hereafter HWN) observed the star in white light on four consecutive nights in late 1986 with the 2.1 m reflector at McDonald Observatory. They tentatively identified eight pulsation modes between 300 s and 700 s and suggested a mean period spacing of 16.4 s. This implied a mass of ~ 0.7 $M_\odot$, based on models developed by Kawaler (1987).

At the time HWN published their observations, rotational splitting had not been resolved in the power spectrum of any of the hot degenerate stars, so that identification of the value of l and n for individual modes in these stars remained

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uncertain. WET observations have since led to measurement of $l = 1$ rotational splitting for PG 1159 ($\delta v = 4.22$ μHz) and PG 2131 ($\delta v = 27.4$ μHz), implying rotation rates (see eq. [3] below) of 1.38 days and 5.07 hr, respectively. All of the high-amplitude variation in both stars is attributed to $l = 1$ triplets, with a series of low-amplitude $l = 2$ modes discovered in PG 1159.

Armed with insight gained from these observations, we reanalyzed the HWN data. We hypothesized that the power spectrum of PG 0122 was comprised of $l = 1$ rotationally split triplets. If PG 0122 had the complex mode structure of PG 1159, this analysis—based on single-site data—would have failed. Luckily, PG 0122 is a simple star that is remarkably similar to PG 2131 in the sparseness of its frequency spectrum. This allowed us to test our hypothesis without multisite observations. This paper describes our reanalysis of the HWN data. In the next section, we describe the observations and their reduction. Section 3 outlines our analysis, including our efforts to separate peaks from aliases and the evidence for equal period spacing in the power spectrum. In § 4, we use the periods to constrain the physical properties of the star, and we conclude with § 5.

2. LIGHT CURVES AND POWER SPECTRUM

HWN obtained a total of 4609 5 second and 6137 10 second integrations over four consecutive nights (see Table 1). We reduced these data following the procedures in Nather et al. (1990). Figure 1 presents the data after summing to produce 40 s integrations. Subsequent analysis

TABLE 1
OBSERVING LOG FOR THE ARCHIVAL McDONALD 2.1 m
OBSERVATIONS OF PG 0122

Run Name	Start Date (UT)	Start Time (UT)	Duration (hr : minutes : s)
RUN22......	1986 Nov 28	01:47:00	5:58:25
RUN24......	1986 Nov 29	01:33:50	2:20:40
RUN25......	1986 Nov 29	03:54:43	3:44:50
RUN26......	1986 Nov 30	01:19:00	2:47:40
RUN27......	1986 Nov 30	04:08:10	3:14:50
RUN29......	1986 Dec 01	02:49:30	5:22:30

was performed on the unsummed data. Note the change in amplitude in the light curve from night to night, which was mentioned by HWN as possibly deriving from fine structure in the frequency spectrum of PG 0122 (i.e., beating).

Figure 2 shows the power spectrum of the entire data set out to the highest frequency showing significant power above noise. The power spectrum was obtained by squaring the modulus of the Fourier transform (FT) of the light curve, resulting in a plot of modulation power (mp) versus frequency, where $mp = (\Delta I/I)^2$, as discussed in Winget et al. (1994). The low-frequency power (below about 1000 μHz) is due primarily to extinction and sky brightness variations in the atmosphere. The decrease of power at zero frequency is a consequence of the data reduction procedure.

The dashed lines in Figure 2 indicate the power P corresponding to different false alarm probabilities $FAP(P)$, calculated according to

$$FAP(P) = 1 - (1 - e^{-P/\langle P \rangle})^N \qquad (1)$$

where N is the number of data points and $\langle P \rangle$ is the average power, calculated over the $N/2$ independent frequencies f_i that characterize the data:

$$f_i = i/T , \quad i = 1, 2, 3, \ldots, N/2 , \qquad (2)$$

where T is the total time spanned by the data set. $FAP(P)$ is the probability that noise will generate a peak of a given power at least once among the frequencies in equation (2). Though power spectra are usually presented over-resolved for clarity, calculating the FT at more than $N/2$ frequencies adds no information about the data since the "extra" frequencies are not independent of those in equation (2) (see Horne & Baliunas 1986). Unlike a statistical analysis, which evaluates confidence by considering only how many standard deviations a peak stands above the local mean, the false alarm probability also accounts for the increasing probability that a peak of specific size will occur by chance as the number of independent data points increases. For example, when an FT includes 25,000 independent frequencies, the chance that at least one noise peak will have 3 times the mean power is $\sim 99\%$. The false alarm probability provides a much more conservative and realistic criterion
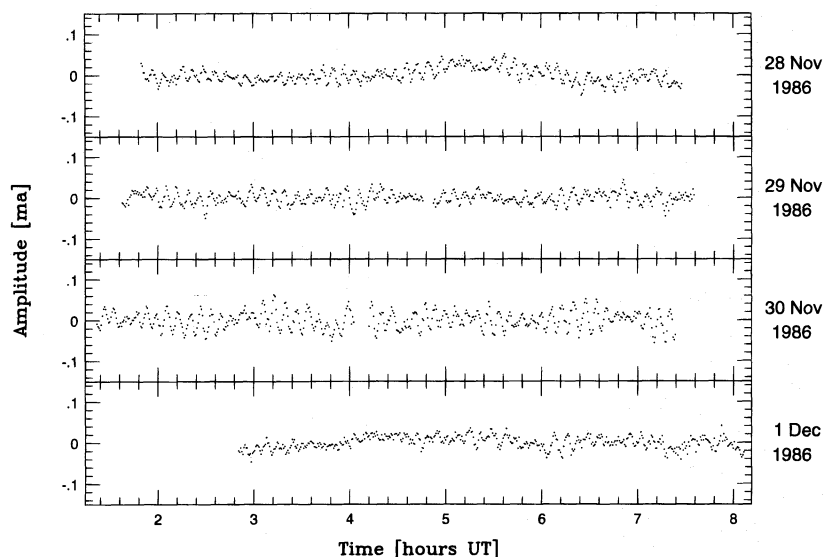

FIG. 1.—Light curve of PG 0122. The vertical axis has the units of modulation amplitude, $ma = \Delta I/I$. Each panel represents a single night's data.
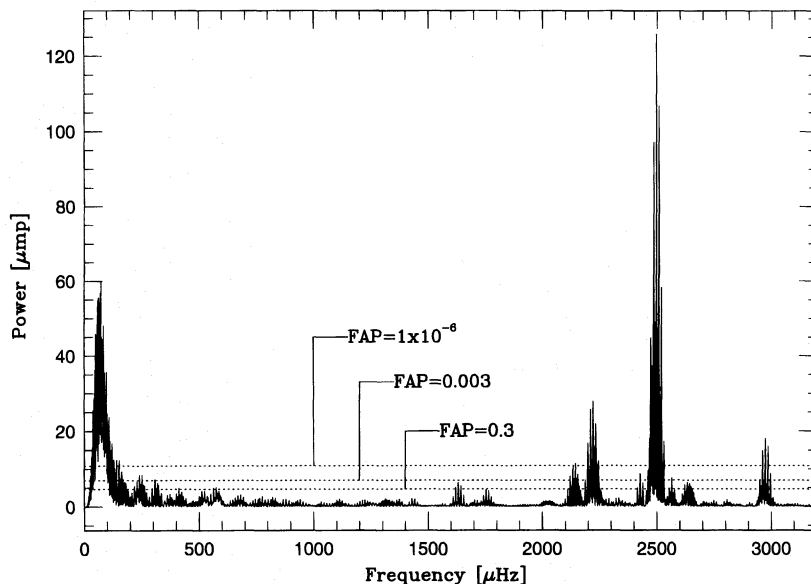
FIG. 2.—Power spectrum of the PG 0122 data, out to the highest frequency showing significant power above noise. The power is given in units of $\mu mp = mp/10^6$, where $mp = (\Delta I/I)^2$. The dashed lines correspond to false alarm probabilities of 30%, 0.3%, and 0.0001%, indicating the chances that random noise will generate a peak at these power levels at least once in the frequency range 0–18,000 μHz.

for judging the significance of a peak in the power spectrum.

We reiterate that we calculate $\langle P \rangle$ over *all* the frequencies in equation (2), from $f = 1/T$ to the Nyquist frequency $N/2T$. Thus, even though *FAP* analysis assumes normally distributed noise, we chose not to filter the data, either to remove the low-frequency "atmospheric noise" (which is certainly *not* normally distributed) evident in Figure 2 or to remove the real stellar variations. As a result, our FAP levels are overly conservative for the frequency range we analyze, but we prefer this to the alternative, which requires subjective evaluation of atmospheric noise. Consequently, our *FAP*s should not be regarded as an absolute measure of peak significance but as an objective,

conservative guide that the reader can apply to the power spectrum of PG 0122.

Figure 3 shows the frequency region of primary power (excluding low-frequency atmospheric variations) in the spectrum of PG 0122. For comparison, the spectral window (on the lower right of Fig. 3) depicts the transform of a single noise-free sinusoid, sampled at the same times as the actual data. This power spectrum shows seven groups of peaks, six of which correspond to periods identified in 1987. We were unable to confirm the existence of significant variation at 435 and 364 s (2300 and 2750 μHz) reported by HWN. There is evidence for slight excess power at these periods, but so close to noise that we did not include them
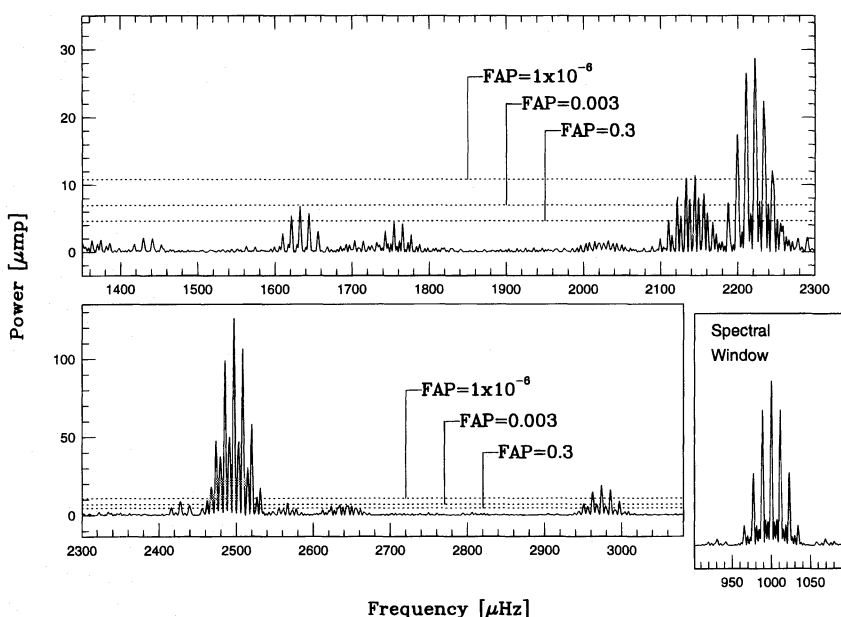

FIG. 3.—Region of the power spectrum analyzed in this paper. Note that the vertical scale is different in each panel. The false alarm probabilities for three different power levels are again shown as in Fig. 2. For comparison, the spectral window shows the power spectrum of a single noise-free sinusoid sampled at the same times as the data.

in our analysis. One additional peak, not listed in the 1987 paper, is clearly discernible in the transform, at 570 s (1755 μHz).

3. DATA ANALYSIS

3.1. *Frequency Identification*

Comparison to the spectral window shows that most frequency groups are more complex than the pattern formed by a single periodicity, indicating the possible presence of a companion. Only the group near 1630 μHz (and perhaps the one at 1755 μHz) apparently represents the spectrum of a single sinusoid. The other groups, and especially the region around 2640 μHz, are more complex than the window pattern. In particular, the group at 2220 μHz seems at first glance to represent the combination of two window-like peak groups, one of which is smaller in amplitude and offset from the other by a few μHz in frequency.

To separate closely spaced frequencies, we consecutively calculated linear least-squares fits for all eight identified frequency groups. The data were then "prewhitened" at the largest peak in a region to look for lower amplitude variations. This was accomplished by subtracting from the light curve, point by point, a sine wave of the frequency, amplitude, and phase determined by the least-squares analysis of a given peak. Smaller peaks identified in this way were then fitted with sine waves simultaneously with the larger peaks in the original data set. The precision of frequency determinations for the previously known peaks was also greatly improved by this process. This procedure has proven highly successful in recovery of low-amplitude variations from WET data, where even the residual alias can swamp small peaks. We describe below the details of this method as we applied it to the archival PG 0122 data. Table 2 summarizes the frequencies, amplitudes, and mode identifications for the periodicities found in the light curve of PG 0122.

In the process of prewhitening we encountered the inevitable problem of distinguishing real power from sampling aliases. In every case, we applied the simple algorithm of choosing the largest (and therefore most statistically likely) peak as representing the real power. Thus, although our identification of peaks is not the only one consistent with the data, it is the most likely solution. In the next section we show that the rules we followed lead to a set of observed peaks with frequencies that are entirely consistent with rotationally split $l = 1$ triplets equally spaced in period.

Starting with the region of greatest power (the group at 2500 μHz), we assumed that the largest peak represented a true pulsation mode and we removed it. Figure 4 shows the result. The remaining power spectrum, depicted in panel Figure 4b, illustrates how prewhitening works. We could in principle defend the choice of any of the three largest remaining peaks as representing real power, but we again chose the largest and most probable peak and removed it. The remaining power, shown in Figure 4c, is also consistent with a single periodicity. When it too is removed, no power remains above noise, as shown in panel Figure 4d. Figure 5 further demonstrates that our solution is consistent with the original data. The top panel shows the power spectrum of PG 0122, while the lower panel depicts the power spectrum of a noise-free time series, sampled at the same times as the data, constructed using three sine waves of the frequency, amplitude, and phase determined by a simultaneous least-squares solution. Choosing different aliases at each stage of prewhitening might lead to equally good reconstructions but would require that we choose less probable peaks, leading in some cases to the further necessity of invoking a greater number of peaks to explain the same region of power.

To within the formal error from the least-squares fit, the splitting is uniform, with $\langle \delta v \rangle = 3.5 \pm 0.3$ μHz. This group is thus entirely consistent with our hypothesis that the power in PG 0122 consists primarily of $l = 1$ triplet modes.

The next largest groups are those at 2220 and 2970 μHz. Successive prewhitening, using the same prescription as above, revealed that each consists of two peaks separated by roughly twice δv found in the previously identified triplet: $\delta v = 7.9 \pm 0.2$ and 7.4 ± 0.2 for the 2220 and 2970 μHz groups, respectively. Again, in selecting peaks we always chose the most probable peak remaining after prewhitening to be "real." This choice is supported by identification of frequency splittings consistent with those in the largest group. These two groups can be identified as $l = 1$ if we allow that in each case a central ($m = 0$) mode has not been detected. This is highly plausible in light of the results of the study of PG 1159 and PG 2131; in both stars the

TABLE 2
PERIODICITIES OF PG 0122

Period (s)	Frequency (μHz)	σ_f (μHz)	Amplitude (ma)	σ_A (ma)	l	m	δv (μHz)	$\sigma_{\delta v}$ (μHz)
612.4......	1632.8	0.2	2.6	0.3	?	?	...	...
570.0......	1754.4	0.2	2.5	0.3	?	?	...	...
466.4......	2144.2	0.2	3.3	0.4	2	?	5.4	0.3
465.2......	2149.6	0.2	2.3	0.4	2	?		
451.9......	2213.1	0.2	4.8	0.4	1	+1	7.9 (2 × 3.95)	0.2
450.2......	2221.0	0.1	6.3	0.4	1	−1		
401.6......	2490.0	0.1	7.3	0.4	1	+1	3.6	0.2
401.0......	2493.6	0.2	3.0	0.4	1	0		
400.5......	2496.7	0.1	12.3	0.4	1	−1	3.3	0.2
380.1......	2631.0	0.2	1.9	0.4	1	+1, 0	3.3	0.3
379.6......	2634.3	0.2	2.1	0.4	1	0, −1		
337.1......	2966.2	0.2	3.1	0.4	1	+1	7.4 (2 × 3.70)	0.2
336.3......	2973.6	0.1	5.1	0.4	1	−1		

NOTES.—Numbers in parentheses show the mean consecutive m splitting if the observed doublet is assumed to represent $m = \pm 1$, $l = 1$. Commas separate possible m identifications when constraint to a single value was not possible. Frequency and amplitude errors derive from a formal least-squares analysis of the data. The amplitude units are modulation amplitude, $ma = \Delta I/I$.
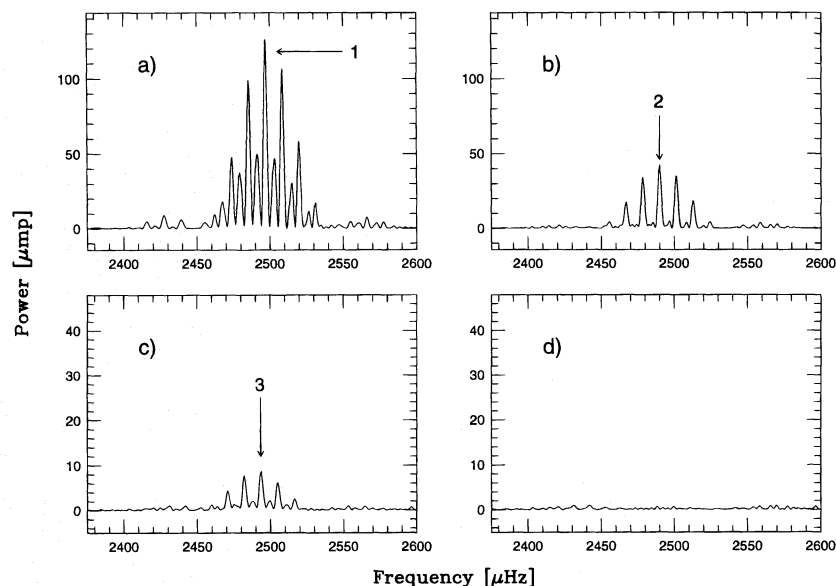
Fig. 4.—Prewhitening sequence for the region of largest power in the spectrum; (a) shows the original spectrum; (b), (c), and (d) show the effects of subtraction in the time domain of one, two, and three sine waves (with periods corresponding to the peaks labeled 1–3), respectively, from the light curve. Note that the vertical scale is smaller in (c) and (d) than in the first two panels. Frequencies, amplitudes, and phases for each subtracted mode were obtained by simultaneous least-squares fitting to the original light curve.

ratio of the amplitudes of modes of different m within a multiplet can approach an order of magnitude. This degree of amplitude asymmetry could easily place one or more peaks in a given multiplet below the level of noise.

There are two more groups that show apparent multiplet structure, at 2640 μHz and at 2140 μHz. The first of these was found to be a doublet with frequency splitting $\delta\nu = 3.3 \pm 0.3$ μHz. Here, too, our simple algorithm of
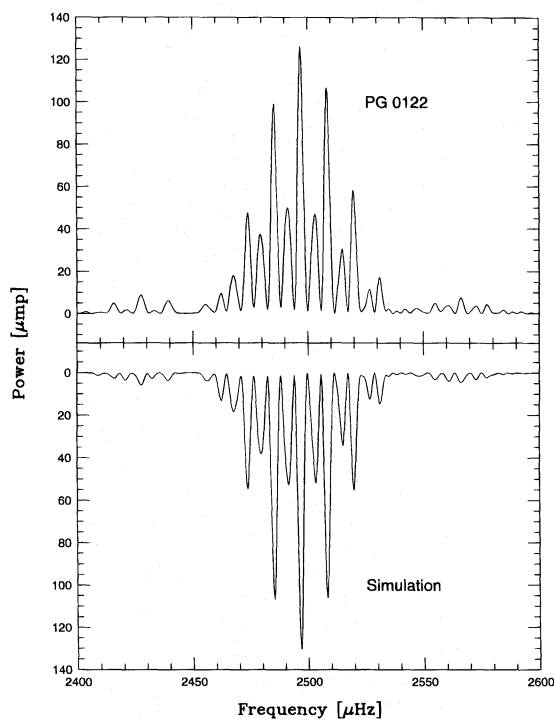

Fig. 5.—Comparison of a portion of the power spectrum of PG 0122 to the spectrum of a (noise-free) light curve constructed using the three peaks identified by prewhitening. The transform of the simulated light curve has been inverted in order to aid comparison of the two spectra by eye.

selecting the most likely peaks as real leads to the identification of a splitting consistent with every other group analyzed thus far. This leaves only the group at 2140 μHz, which we now discuss at some length, since it does not fit the established pattern.

The prewhitening procedure shows that the 2140 μHz group is a doublet with a splitting of 5.4 $\pm$ 0.3 μHz, a number inconsistent with identification of this mode with the same value of l as the other peaks in the power spectrum. However, the ratio of the splitting in the $l = 1$ multiplets to that of the 2140 μHz doublet is 0.67 $\pm$ 0.04, which is approximately the asymptotic ratio of 0.6 between $l = 1$ and $l = 2$ rotational splittings (see below). Identifying this group as $l = 2$ explains its failure to conform to the splitting found in every other multiplet.

In the final stages of writing this paper, we received a preprint of a paper by Vauclair et al. (1995) reporting on analysis of photometry of PG 0122 in 1990 October. One of the principal problems they discuss is the identification of l for the 2140 μHz multiplet. In our data two consecutive-m modes are present, allowing us to positively identify the $l = 2$ frequency splitting of 5.4 μHz. The two peaks present in their data are not consecutive in m. This is not unusual behavior for the GW Vir stars; they frequently show season-to-season amplitude variations. Thus, the $l = 2$ splitting was not apparent in 1990, and Vauclair et al. decided that this mode was probably a mixture of $l = 1$ and $l = 2$. When our peak list for this multiplet is combined with the one from Vauclair et al., the result is a single group of three peaks with splittings of $\delta\nu = 5.4 \pm 0.3$ μHz and $\delta\nu = 13.9$ μHz $= 3 \times 4.6$ μHz. The combined list represents three of five possible $l = 2$ components with a splitting of ≈ 5 μHz.

In an important respect, Vauclair et al.'s observations also reinforce our idea of the 2970 μHz multiplet as an $l = 1$ mode. In our data, we see a doublet ($f = 2966.2$ and 2973.6 μHz) with the central ($m = 0$) mode missing. Vauclair et al. see a mode at 2970.2 μHz, halfway between our two modes,

providing further confirmation of the identification of the $l = 1$ splitting of 3.60 ± 0.2 μHz.

At this point we have no doubt this analysis, based at every stage on the simplest and most likely deconvolution of individual regions of power, has borne out the hypothesis that the power spectrum of PG 0122 is dominated by $l = 1$ modes. A total of 13 independent periods were identified in the light curve (see Table 2); all but two are members of multiplets in the power spectrum. The average frequency splitting in these multiplets can now be used to determine the rotation rate of PG 0122.

The frequency splitting for peaks of consecutive m in each mode is set by the stellar rotation period Π_{rot} according to

$$\delta v_{n,l} = \Pi_{rot}^{-1}(1 - C_{n,l}) \qquad (3)$$

where $C_{n,l} \approx [l(l + 1)]^{-1}$. This splitting is not uniform among the $l = 1$ modes we identified. The minimum range, given the uncertainty in the frequency determinations, is 7% from the smallest splitting to the largest. This result is not unusual, however; the range in δv among the most certain modes identified in PG 1159 is close to 10% (Winget et al. 1991). The average value found in the four $l = 1$ modes in the power spectrum of PG 0122 is 3.60 ± 0.08 μHz. This implies that PG 0122 rotates about its axis once every 1.61 ± 0.04 days. Note that the identification of the peaks near 2140 μHz as $l = 2$ allows a determination of the rotation rate from that mode alone of 1.79 ± 0.10 days; this is approximately the same rotation rate derived using the triplet modes.

3.2. Period Spacings

The most striking feature of the frequency pattern is its simplicity—very few modes account completely for a complex light curve. With the exception of the $l = 2$ doublet at 465 s (2140 μHz), every multiplet is consistent with $l = 1$. We show below that, as expected from theory and previous experience, the $l = 1$ modes are each separated from the others by multiples of a fundamental period spacing.

The evidence for equal period spacing in the power spectrum of PG 0122 is summarized in Table 3. This simple model, with $\Delta\Pi = 21.2$ s, accounts for every $l = 1$ mode identified in the star. This period spacing, for $l = 1$, implies a mass of ~ 0.6 $M_\odot$. In addition, the two singlet modes at 570 s (1754 μHz) and 612 s (1632 μHz) fit the $l = 1$ period spacing quite well, though of course the value of l for these two peaks cannot be constrained from frequency splitting. Finally, the singlet just above the surrounding power at

1430 μHz fits well into the pattern. Though these singlet peaks cannot stand alone in support of the proposed pattern, they are accounted for by it.

The proximity of the 465 s mode to the one at 450 s implies that one of them has a different value of l than the other modes. The 465 s mode fits the period spacing pattern far better than does the 450 s mode, which tends to support the identification of the 465 s mode as $l = 1$ and the 450 s mode as $l > 1$. Unfortunately, as discussed in the previous section, the frequency splittings in each mode support the opposite conclusion: $l = 2$ for the 465 s mode, and $l = 1$ for the 450 s mode. However, we prefer the identifications based on frequency splitting, since mode trapping can cause the period spacing to vary from mode to mode over a much larger range than will the frequency splittings ($\Delta\Pi_{n,n+1}$ varies by about 20% of $\langle\Delta\Pi\rangle$ in PG 1159, for instance).

The pulsational data are thus consistent with a pattern of g-mode periods seen independently in previous observations of PG 1159 and PG 2131. Our model accounts for every peak, and there is none left over. In the only case where the spectrum does not conform to both equal period spacing and uniform frequency splitting—the 465 s doublet—theory accounts readily for the anomaly. The power spectrum of PG 0122 is a set of rotationally split $l = 1$ multiplets equally spaced in period, with one mode identified as $l = 2$. These results are in accord with those of the other PG 1159 stars previously analyzed.

These results do not agree with the conclusions reached by Vauclair et al. (1995). Unfortunately, the absence of some modes robbed them of the ability to detect the 21.2 s period spacing. They find 10 periods in their data. Seven of their 10 peaks are at essentially the same frequencies in the HWN data we analyzed, but the amplitudes in the Vauclair et al. data averaged 3.6 times smaller. In particular, the triplet at 379.6 s was absent in their data. As mentioned before, this behavior is not atypical of the GW Vir stars.

The primary problem this created was a misidentification of the 465 s multiplet as a mixture of $l = 1$ and $l = 2$. This caused Vauclair et al. to conclude incorrectly that the period spacing was 16.1 s. Discarding the 465 s group from measurements of $l = 1$ period spacing makes the observations of Vauclair et al. inadequate to determine $\Delta\Pi$ without ambiguity; they only have three $l = 1$ modes, none consecutive in n. We are fortunate to have two consecutive $l = 1$ modes (379.6 s and 401.0 s) in our data. The spacing between them (21.4 s) pointed us toward a pattern with an average period spacing of 21.2 s. We are confident that $l = 1$ modes found in future observations of PG 0122 will conform to this pattern and not to one based on 16.1 s.

4. THE MASS OF PG 0122

The period spacing between $l = 1$ modes constrains the global properties of PG 0122. To determine these properties, we computed evolutionary stellar models with masses between 0.56 and 0.66 $M_\odot$, over a range of effective temperatures from 140,000 K down to 70,000 K, using the stellar evolution code ISUEVO. Details concerning this code can be found in Dehner (1995) and Dehner & Kawaler (1995). The helium layer thickness in all the models was the same as that of the best model found by Kawaler & Bradley (1994) for PG 1159. Since there are not enough consecutive modes in PG 0122 to attempt a separate determination of the helium layer thickness, and since this parameter has only a very slight effect on the period spacing in the models

TABLE 3

COMPARISON OF THE PERIOD SPECTRUM ($l = 1$, $m = 0$) WITH A STRICT 21.2 s EQUAL SPACING MODEL

$\Pi_{observed}$ ($m = 0$) (s)	$\Pi_{predicted}$ ($\Delta\Pi = 21.2$ s) (s)	Δn	$\Pi_{observed} - \Pi_{predicted}$ (s)
612.4 ($^{+1.4}_{-1.5}$)	613.0	+10	−0.6
570.0 ($^{+1.2}_{-1.3}$)	570.6	+8	−0.6
451.0*	443.4	+2	+7.6
401.0	401.0	0	...
379.6, 380.1	379.8	−1	+0.2, −0.3
336.7*	337.4	−3	−0.7

NOTES.—An asterisk (*) indicates that the period represents the calculated center of a doublet splitting. Numbers in parentheses show the effects of assuming a value of m other than $m = 0$ for the identified peak.

anyway (Kawaler & Bradley 1994), an order-of-magnitude estimate is sufficient for the present analysis.

Using the same techniques as Kawaler & Bradley (1994) and Kawaler et al. (1995), we directly compared the periods of each model to those found in the star. The best-fit model (in a least-squares sense) has a mass of 0.66 $M_{\odot}$, but a systematic residual exists in the fit. Therefore, we caution against taking this value too seriously until more careful modeling can be done. The model with the best-fitting periods has an average period spacing of 21.6 s.

By varying both the model mass and temperature, many models are found that do match the period spacing of PG 0122. Figure 6 shows the region of the log (g)–log ($T_{\rm eff}$) plane occupied by the models. A linear fit to the $m = 0$ periods in PG 0122 yields a final spacing $\Delta\Pi = 21.21^{+0.32}_{-0.35}$ s. The solid line in the figure indicates the interpolated position of models that satisfy this condition. We would like of course to reduce the range of possibilities from a line to a point. This suggests a two-parameter fit to the periods (of both the star and the models) of the form

$$\Pi_n = \Delta\Pi(n + \epsilon) , \qquad (4)$$

where both $\Delta\Pi$ and ϵ are free parameters. This is simply the asymptotic period equation derived using a WKB analysis, familiar from standard stellar pulsation theory (see Unno et al. 1989). Though ϵ does not represent any obvious physical quantity, its value is set by the input physics and by the equilibrium stellar structure. This is apparent since we found that the fit of the periods to equation (4) for every model, over the entire range of masses and temperatures mentioned above, gives essentially the *same* value of ϵ, ($\sim$2.2). This method of model comparison has the advantage that both $\Delta\Pi$ and ϵ are evaluated as an average over several modes, and therefore are affected only slightly by changes in the helium layer thickness, even when only a few modes are used (as long as the periods span several trapping cycles). We find with this analysis that none of the models adequately reproduce ϵ as measured in the stars themselves with sufficient accuracy to distinguish them from one another, and therefore all models with the correct period spacing—regardless of the periods themselves—represent equivalent fits to the power spectrum of PG 0122. Put another way, it was not possible to find a satisfactory and unique model that reproduced the periods and the mean spacing simultaneously.

We are reduced to using the spectroscopic temperature to eliminate some models from consideration. The area of the H-R diagram where the correct range of period spacing intersects the allowed temperature region is indicated by the parallelogram in Figure 6. Since this area falls outside the range of available models, the final mass was found by extrapolating both the 21.21 s line and the model masses. The mass of PG 0122 thus falls in the range 0.66–0.72 $M_{\odot}$. Until models with the same period spacing can be distinguished from one another (using ϵ or some other parameter), no independent estimate of the effective temperature (and therefore of the luminosity and distance) can be given. We are preparing a paper that describes the details of this method of model comparison and its application to other PG 1159 stars.

One final point should be made concerning the power spectrum of PG 0122. This is the third pre–white dwarf star, out of three studied in detail so far, with a period spacing near 21.5 s. Why the mass and temperature should conspire, over a large range of both quantities, to give the same period spacing in all three stars is a mystery. Perhaps this value for $\Delta\Pi$ is also a condition for pulsation? We intend to explore the implications of these results for the instability strip occupied by the PG 1159 stars.

5. SUMMARY AND CONCLUSIONS

Reanalysis of 1986 time-series photometry of the pulsating PG 1159 star PG 0122 revealed a wealth of new information about this pre–white dwarf star. PG 0122 is a nonradial g-mode pulsator with several $l = 1$ modes and one $l = 2$ mode present. Fine structure in the power spectrum indicates that it rotates once every 1.6 days. Comparison of the calculated period spacing of PG 0122 with that of stellar models implies a mass of 0.66–0.72 $M_{\odot}$, given the spectroscopic constraint of its effective temperature.
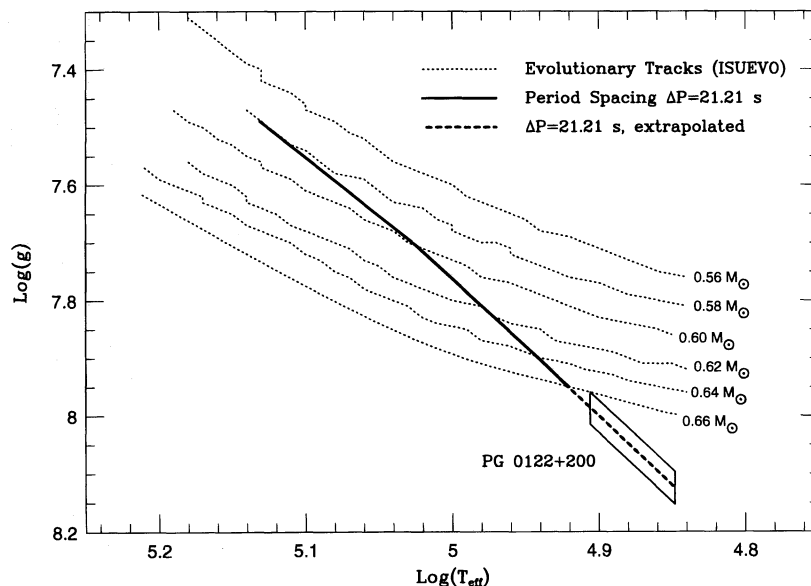

Fig. 6.—Logarithmic plot of surface gravity vs. effective temperature showing evolutionary tracks based on ISUEVO. The position occupied by PG 0122 in this diagram is constrained by the spectroscopically determined temperature and by the measured period spacing.

PG 0122 is similar to PG 1159 in its pulsational structure and rotation rate and closely resembles PG 2131 in both mass and in the quantity of modes observed. We found an insufficient number of consecutive-n modes with which to analyze possible mode-trapping in PG 0122. Future multisite observation could provide a means to study such structure and to thereby measure the surface helium layer mass. This measurement is crucial, since it is the surface layer thickness that primarily governs the subsequent evolution of these stars as they become white dwarfs. However, with global parameters of three pulsating pre–white dwarfs now asteroseismologically constrained, we have begun to establish and to understand the general characteristics of this important evolutionary link between the AGB stars and the white dwarfs.

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