

ON THE LIGHT CURVE–LUMINOSITY RELATION OF RR LYRAE STARS

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ABSTRACT

We use a large sample of RRab stars in globular clusters and in the Sculptor dwarf galaxy to decipher the relation between the Fourier decomposition and the luminosity. For fixing the zero point of the relation, we use the latest Baade-Wesselink (BW) results. It is shown that the most plausible representation of the absolute brightness (in V color) consists of a linear expression of the period and of the Fourier parameters A_1 and φ_{31} (computed also from the V light curve). We derive an average $M_V = \alpha[\text{Fe}/\text{H}] + \beta$ relation from all the available Fourier data. Our results exclude any values of α larger than 0.19, in agreement with most of the BW and evolutionary studies. We give age and reddened distance modulus estimations for the clusters entering in our analysis.

Subject headings: globular clusters: general — stars: abundances — stars: fundamental parameters — stars: horizontal-branch — stars: oscillations — stars: variables: other (RR Lyrae)

1. INTRODUCTION

Determination of the luminosity of RR Lyrae stars is a high-priority task in the study of these stars because they play a key role as distance and age indicators in our Galaxy and in its close neighborhood. With the rapid progress of massive photometry, the number of RR Lyrae stars with accurate light curves will soon be increased by 1 or 2 orders of magnitude. To utilize these data in mapping the not too distant parts of the universe, we need accurate relations between the physical and the light-curve parameters. Several previous attempts tried to tackle this question. In the simplest case, a linear relation is sought for M_V either with $[\text{Fe}/\text{H}]$ (Clementini et al. 1995, and references therein) or with $[\text{Fe}/\text{H}]$ and the period P (see, e.g., Nemeč, Linnell Nemeč, & Lutz 1994). From the theoretical side, we mention Simon & Clement (1993) who used nonlinear pulsation models of RRc stars to derive relations between the Fourier parameters and the luminosity. It is important to note that the often quoted dependence of M_V on $[\text{Fe}/\text{H}]$ is suggested by the evolutionary theories and is obtained through an *ensemble average* of many evolutionary tracks. The several empirical variants of this relation suffer from the ambiguity due to the limited number of stars used. In addition, because of the recently discovered tight dependence of $[\text{Fe}/\text{H}]$ on P and φ_{31} (Jurcsik & Kovács 1996, hereafter JK), the single-parameter dependence of M_V can clearly no longer be held.

In this Letter, we follow JK and derive a linear relation for the luminosity of the RRab stars. Up to the zero point, the result is *free* of any assumptions and relies solely on the empirical relation between *directly observed* quantities.

2. THE DATABASE

For the purpose of the empirical determination of the relation between the luminosity and the shape of the light curve, we use the cluster data as compiled by JK. Here we omit the LMC clusters NGC 1835 and 2257 mostly because of the low number of the stars but also because of their lower quality. Additional data come from the very recent Optical Gravitational Lensing Experiment (OGLE) observations of the Sculptor dwarf spheroidal galaxy and those of the Galactic globular

clusters Ruprecht 106 (Kaluzny et al. 1995a; Kaluzny, Krzem-
iński, & Mazur 1995b) and M5 (Reid 1996). We use (as in JK) Johnson V color throughout this Letter. For the absolute calibration, we apply the compilation of the Baade-Wesselink (BW) luminosities as given by Clementini et al. (1995) (their Table 21, col. [5]). All these data sets are listed in Table 1. Since we use only the best quality light curves without any peculiarity or sign of Blazhko behavior, the numbers shown in the table are lower than the ones given for the RRab stars in the respective publications. In the Fourier analysis, we follow the same method as outlined in JK. Reddening corrections are applied only in the case of M4 because of its considerable differential extinction (Cacciari 1979; Liu & Janes 1990).

3. THE $(P, A_1, \varphi_{31}) \rightarrow M_V$ RELATION

First, we use only the globular cluster and the Sculptor data and do not rely on the BW luminosities. In finding the relation between the Fourier parameters and the luminosity, we follow essentially the same method as in JK.

Here, we utilize the fact that the distance moduli are the same for all stars in each cluster. In addition, it is assumed that the same thing is true also for the reddenings. Since the luminosity should follow a general relation, independently of the cluster considered, for a consistent representation of the apparent magnitudes, we fit a certain number of Fourier parameters and $n - 1$ constants, where n is the number of clusters. Including the period and all the Fourier amplitudes and phases up to order 6, we search for the best linear relation representing the observed (*intensity-averaged*) brightness. In this way, we get an optimum set of *reddened relative distance moduli* and a formula representing the relation between the light curve and the absolute magnitude. Stars with discrepant luminosities are left out of the fit. We do not discuss these stars in detail; we only remark that there might be several reasons for their peculiarity, e.g., crowding effects, inhomogeneous extinction, Blazhko behavior, etc. Although the fairly high noise level introduces some ambiguity into the selection process, we try to avoid it by keeping as many stars as possible and reaching a situation when both the fitting accuracy and the regression coefficients seem to settle on a stationary value. We

TABLE 1
DATA SETS AND THEIR SOURCES

Name	N	Source
Galactic clusters.....	71	JK, Kaluzny et al. 1995b, Reid 1996
LMC.....	25	JK
Sculptor.....	93	Kaluzny et al. 1995a
BW.....	31	Clementini et al. 1995

think that we reach this state by omitting five stars from the globular clusters and seven stars from Sculptor. Finally, we end up with 177 stars. Since the fitting accuracy shows a leveling off when using more than three Fourier parameters (see Table 3), we settle on the following formula that fits the data with an accuracy of 0.047 mag:

$$M_V = \text{const} - 1.398P - 0.471A_1 + 0.104\varphi_{31}. \quad (1)$$

To obtain an absolute calibration, we repeat the above process by including also the BW stars. We emphasize that we use the BW stars merely to *fix the zero point*. We find that the BW luminosities of the following stars do not conform to the basic trend of the clusters: WY Ant, UU Cet, DX Del, SS Leo, AV Peg, BB Pup, W Tuc, M5 V28, M92 V1, and M92 V3. After leaving out these stars, we get a set of 198 stars that we refer hereafter as the *calibrating data set*. This yields the following expression:

$$M_V = 1.221 - 1.396P - 0.477A_1 + 0.103\varphi_{31}, \quad (2)$$

with the corresponding error formula

$$\sigma_{M_V}^2 = 0.2275\sigma_{A_1}^2 + 0.0106\sigma_{\varphi_{31}}^2 + \sum_{i,j=1}^4 K_{ij}p_i p_j, \quad (3)$$

where $p_1 = 1$, $p_2 = P$, $p_3 = A_1$, $p_4 = \varphi_{31}$, and the correlation coefficients K_{ij} are given in Table 2. In this formula, we have omitted the completely negligible error of the period. We recall, that just as in JK, the Fourier parameters refer to a *sine* decomposition and that the phase should be chosen as the closest value to 5.1.

In Figure 1 we show the calculated versus the observed absolute magnitudes for the best single and the above three-parameter fits. The “observed” absolute magnitudes are calculated by the use of the apparent brightnesses and the computed reddened distance moduli. It is clear that the inclusion of the Fourier parameters caused a visible improvement in the representation of the data compared with that of the traditional period-luminosity relation.

It is important to address the question of the significance of the three-parameter relation over the two-parameter one. In order to do so, we generate artificial data with the following formula:

$$M_V(i) = 1.736 - 1.242P(i) - 0.659A_1(i) + \xi(i), \quad (4)$$

TABLE 2
CORRELATION COEFFICIENTS K_{ij} ($= K_{ji}$) IN EQUATION (3)

i	j	K_{ij}	i	j	K_{ij}
1	1	0.0101309	2	3	0.0020546
1	2	-0.0018854	2	4	-0.0001767
1	3	-0.0056066	3	3	0.0048972
1	4	-0.0014288	3	4	0.0005610
2	2	0.0036403	4	4	0.0002674

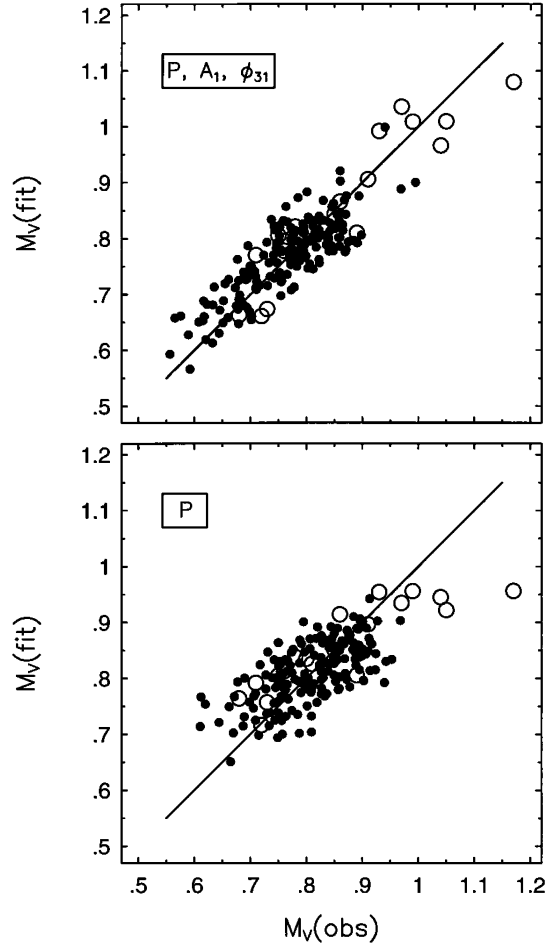


FIG. 1.—Calculated vs. observed M_V relations. *Top*: three-parameter fit (eq. [2]); *bottom*: single-parameter fit. The calibrating data set is plotted. Open circles denote the BW stars. The 45° lines are shown for guidance.

where $\xi(i)$ is a Gaussian random number with $\sigma_\xi = 0.05$. The regular part of equation (4) corresponds to the best two-parameter fit to the calibrating data set, and the indices run through this set. For each realization of $\{\xi(i)\}$ we find the best n -parameter fit just as in the case of real data. Denoting the *unbiased* estimation of the standard deviation of the best n -parameter fit by σ_n , we ask what is the probability that the relative reduction of σ_n (i.e., $1 - \sigma_n/\sigma_{n-1} \equiv \rho_n$) exceeds a certain limit when the three- and two-parameter fits are compared. After a large number of simulations, the distribution function shown in Figure 2 is obtained. This can be used to estimate the significance of ρ_3 obtained in the case of the observed data of the calibrating set (Table 3). We see that ρ_3 is about 3 times greater than the maximum reduction of the dispersion found in the random simulation. Therefore, we conclude that the three-parameter formula (2) is statistically *highly significant* in the representation of the luminosity. In addition, as can be seen in Table 3, the fitting accuracy clearly levels off for the higher parameter regressions. This shows that the three-parameter description is not only necessary but is also sufficient for the representation of the observations.

Finally, we mention that because of the close interrelations among the amplitudes and phases (see JK), there exist compatible formulae that contain other Fourier components. Our equation (2) not only has the highest fitting accuracy but also

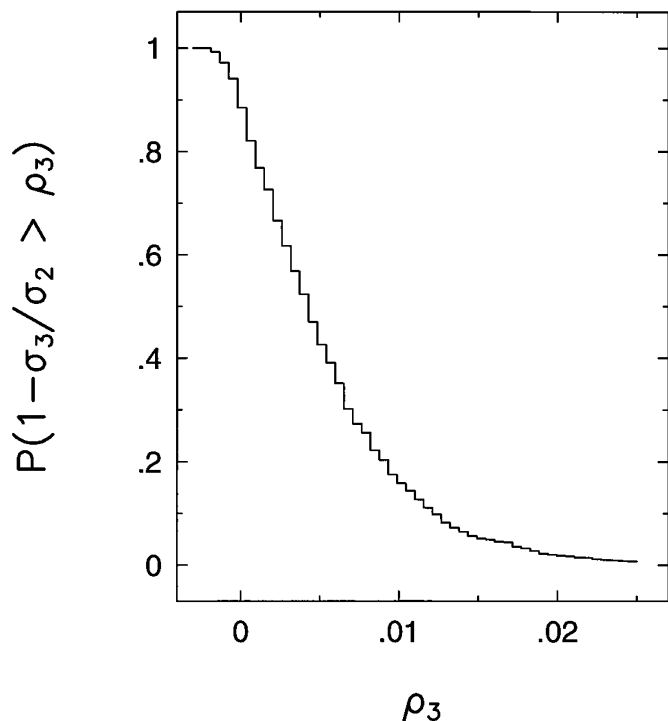


FIG. 2.—Probability distribution of the relative reduction of the standard deviation for artificially generated data (eq. [4]; see text for details). The three-parameter models are tested against a two-parameter one.

contains low-order Fourier components, which helps to diminish observational errors.

4. THE $[\text{Fe}/\text{H}] \rightarrow M_V$ RELATION

In deriving an average $[\text{Fe}/\text{H}] \rightarrow M_V$ relation, we utilize the fact that with the aid of the formula of JK and equation (2), we are able to estimate both $[\text{Fe}/\text{H}]$ and M_V for any RRab star with reliable Fourier decomposition. Using all data of this Letter and those of JK, we get the result shown in Figure 3. It is seen that the $[\text{Fe}/\text{H}] \rightarrow M_V$ relation suffers from a considerable *intrinsic scatter* because of the extra dependence on the Fourier parameters. The straight line is a least-squares fit and corresponds to the following expression:

$$M_V = 0.19[\text{Fe}/\text{H}] + 1.04. \quad (5)$$

This formula is in nice agreement with the recent BW results as summarized by Clementini et al. (1995) and also with the evolutionary calculations (see, e.g., Lee 1990). Since equation

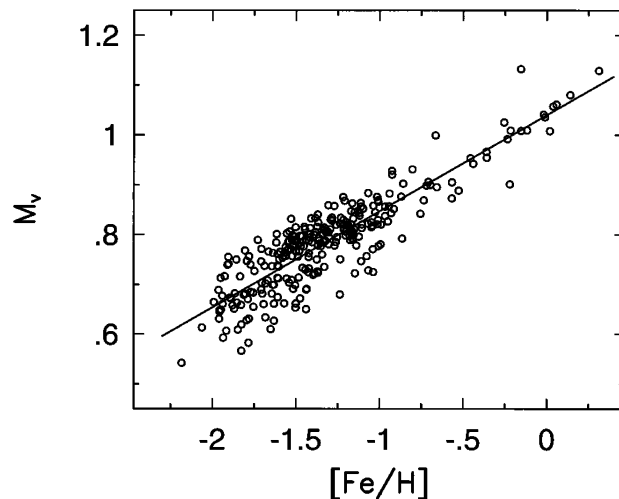


FIG. 3.—Relation between $[\text{Fe}/\text{H}]$ and M_V for 278 stars with reliable Fourier decompositions. $[\text{Fe}/\text{H}]$ and M_V are computed from eq. (3) of JK and eq. (2) of this Letter. The linear regression (eq. [5]) is shown as a solid line.

(2) depends somewhat on the sample of stars used in its derivation, this dependence translates to equation (5). Our experiences show that for all reasonable samples, the coefficients of equation (5) are always in the ranges of 0.19–0.16 and 1.04–0.99, respectively. Furthermore, changing our greater values of $[\text{Fe}/\text{H}]$ to the lower spectroscopic ones for M68, M92, and NGC 1841, we get only a slight decrease in the coefficients; namely, they become 0.18 and 1.03, respectively.

Therefore, our results *undoubtedly exclude* large $[\text{Fe}/\text{H}]$ coefficients as sometimes quoted in the literature (Buonanno et al. 1990; Longmore et al. 1990; Sandage 1993).

5. DISTANCE MODULI AND AGES

With equation (2) it is easy to get a reddened distance modulus that can be directly (i.e., *without* reddening correction) applied to estimate the age of a cluster if the apparent turnoff luminosity $V(\text{TO})$ is known. The results are shown in Table 4. The distance moduli and the apparent turnoff luminosities are *reddened*, except for M4, where reddening was taken into account as mentioned in § 2. Errors of the distance moduli correspond to the standard deviations of the distance moduli obtained for the stars in each cluster. Superscripts refer to the sources of $V(\text{TO})$. Abundances are calculated according to JK. To estimate the ages, we apply the formula of Straniero & Chieffi (1991, their eq. [4]). We recall that using the reddened distance moduli and the observed (also reddened) $V(\text{TO})$, the absolute magnitude of the turnoff, $M_V(\text{TO})$, and, therefore, the age can be directly estimated without resorting to an additional observable (i.e., the mean brightness difference between the horizontal branch and the turnoff point, the estimation of which introduces further ambiguities—see Caputo & Degl’Innocenti 1995). The formal errors of the ages are between 1 and 2 Gyr. We remark that according to our formula, M5 and Rup 106 exhibit a relatively large spread in the metallicity with $\sigma_{[\text{Fe}/\text{H}]} \approx 0.2$.

Because it is not the subject of this Letter to discuss the ages of globular clusters, we just mention that there is an *age spread* among them, as has also been stated by other studies applying low steepness in the $[\text{Fe}/\text{H}] \rightarrow M_V$ relation (see, e.g., Walker 1992b).

TABLE 3

VARIATION OF THE STANDARD DEVIATION σ_n AND ITS RELATIVE REDUCTION ρ_n AS A FUNCTION OF THE NUMBER OF PARAMETERS FITTED TO THE CALIBRATING DATA SET

n	Parameters	σ_n	ρ_n
1	P	0.0597	0.245
2	P, A_1	0.0499	0.164
3	P, A_1, φ_{31}	0.0471	0.056
4	$P, A_1, \varphi_{31}, \varphi_{51}$	0.0470	0.002
5	$P, A_1, \varphi_{31}, \dots$	0.0465	0.011
6	$P, A_1, \varphi_{31}, \dots$	0.0465	0.000
7	$P, A_1, \varphi_{31}, \dots$	0.0466	-0.002
8	$P, A_1, \varphi_{31}, \dots$	0.0467	-0.002

TABLE 4
 REDDENED DISTANCE MODULI AND AGES (in Gyr) OF SOME GLOBULAR
 CLUSTERS AND THE DWARF GALAXY SCULPTOR

Cluster	Distance Modulus	$V(TO)$	$M_V(TO)$	[Fe/H]	Age
M4 ^a	11.17 ± 0.02	15.60 ^b	4.43	-1.03	18.8
M5	14.24 ± 0.05	18.60 ^c	4.36	-1.14	18.1
M68	14.91 ± 0.04	19.05 ^d	4.14	-1.76	17.6
M92	14.44 ± 0.04	18.70 ^e	4.26	-1.98	21.2
M107	14.77 ± 0.06	19.20 ^e	4.43	-0.75	17.0
N3201	13.95 ± 0.05	18.25 ^f	4.30	-1.33	18.1
Ru 106	16.99 ± 0.03	21.05 ^g	4.06	-1.51	15.0
N1466	18.57 ± 0.03	-1.54	...
N1841	18.62 ± 0.03	22.60 ^h	3.98	-1.73	14.9
Reticulum	18.28 ± 0.03	22.50 ⁱ	4.22	-1.45	17.3
Sculptor	19.36 ± 0.05	-1.49	...

^a Dereddened distance modulus and $V(TO)$.

^b Kanatas et al. 1995.

^c Buonanno et al. 1989.

^d Walker 1994.

^e Ferraro & Piotto 1992.

^f Brewer et al. 1993.

^g Buonanno et al. 1993.

^h Walker 1990.

ⁱ Walker 1992a.

6. CONCLUSIONS

On the basis of the latest observational material, we have shown that the luminosity of RRab stars depends on three observables, namely on the period and on the Fourier param-

eters A_1 and φ_{31} (eq. [2]). The relative accuracy of the estimated absolute V magnitudes is better than 0.05 mag, the standard deviation of the residuals of the calibrating data set. This is comparable to the corresponding value of the $P \rightarrow M_K$ regressions obtained from infrared photometry (Longmore et al. 1990; Jones et al. 1992). We think that a considerable part of the scatter comes from the inhomogeneous reddening inside the clusters. In the absolute calibration, there is still an error probably smaller than $\approx 0.1-0.15$ mag both in M_V and in M_K because they all rely on the Baade-Wesselink results. Our average $[Fe/H] \rightarrow M_V$ relation (eq. [5]) yields a similar dependence on $[Fe/H]$ to those given by the recent Baade-Wesselink and evolutionary studies. We think that future, more accurate CCD measurements in clusters or galaxies with large $[Fe/H]$ spread will drastically improve the correlation between the luminosity and the Fourier parameters. This will enable us to determine the luminosity of *any* RRab star, provided that we can measure its light curve with a reliable accuracy.

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