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A MODEL FOR THE GALACTIC POPULATION OF BINARY SUPERSOFT X-RAY SOURCES

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ABSTRACT

A population synthesis model for the Galactic binary supersoft X-ray sources with white dwarf accretors is derived. It is shown that three major subpopulations may exist: with low-mass main-sequence donors (~500–850 objects), with low-mass subgiant donors (~460 objects), and with (super)giant donors (~500–600 objects). The orbital period ranges of the three groups of sources are 80 minutes–12 hr, 10 hr–20 days, and 10 days–10 yr, respectively. Each of these families contains both permanent and recurrent sources, but in different proportions. The former are steady hydrogen-burning white dwarfs, while the latter are white dwarfs in a post–nova explosion state, burning the remainders of hydrogen after a thermonuclear runaway. The estimated number of recurrent sources in the "on" state is a sensitive function of the assumptions on the duration of the supersoft emission stage.

Less populated families include planetary nebula nuclei, double degenerate systems and "helium Algols." For all the major groups of sources, we derive distributions over potentially observable parameters: orbital periods, masses of components, and "on" times. Intrinsic absorption of supersoft X-rays by circumbinary matter and the low transparency of the interstellar medium reduce the number of "detectable" galactic sources to ~20–30, a considerable proportion of which have probably already been detected.

It is estimated that the total rate at which accreting white dwarfs in Galactic binaries reach the Chandrasekhar mass is $\sim 3 \times 10^{-5}$ yr^{-1}. The only type of systems in which the accumulation of a He shell which is able to detonate is possible, are the systems with subgiant donors, in which such events occur at the rate of $\sim 3 \times 10^{-4}$ yr^{-1}.

Subject headings: accretion, accretion disks — binaries: general — white dwarfs — X-rays: stars

1. INTRODUCTION

Supersoft X-ray sources (hereafter SSSs) are a class of luminous objects emitting mainly below 0.4 keV, with characteristic radiation temperatures of 30–60 eV. The luminosity of these sources is estimated to be of order of 10^{37}–10^{38} ergs s^{-1}. It is clear that despite the similarity in their X-ray properties, the family of SSSs is highly inhomogeneous. Galactic sources include postnovae, symbiotic nova systems, and permanent sources. Among the extragalactic sources, there is also a suspected planetary nebula nucleus (PNN). Some of the sources are variable. Known orbital periods of SSSs range from 0.059 to 554 days. The history of the discovery of SSSs, and a discussion of their main properties may be found, e.g., in Hasinger (1994) and Kahabka & Trümper (1995).

Altogether, seven SSSs are now identified in the Galaxy, 11 in the Magellanic Clouds, 15 in M31 (plus a comparable number of suspected sources), and candidates exist in M101, NGC 253, and M33 (see reviews by Hasinger 1994; Kahabka & Trümper 1995; Cowley, Schmidtke, & Crampton 1995a). The discovery of most of the SSSs in the Galaxy is certainly prevented by very high absorption of their emission by the interstellar medium (ISM). An estimate of the expected number of Galactic SSSs may be obtained, for example, as suggested by Di Stefano & Rappaport (1994), by scaling their numbers in external galaxies that are at high Galactic latitude, because then the column density of neutral hydrogen through the Galaxy is less than $N_{\rm H} \sim 10^{21}$ cm^{-2}. For binary SSSs, most of which are probably "relatives" of cataclysmic variables (CVs), and hence not a very young population, the appropriate scaling factor may be the ratio of the H luminosities, like for novae (Della Valle & Livio 1994). If, for a crude estimate, we neglect internal absorption within the source itself, in the parent galaxy, and within the Milky Way, then scaling, e.g., with the ratio of $L_{\rm H}$ for the Milky Way and M31 ($S_{\rm H} \approx 0.5$), gives $N_{\rm SSS} \sim 50$. On the other hand, if the SSSs contain predominantly massive white dwarfs, and hence descend from relatively massive stars, i.e., a younger population, then scaling with the ratio of the luminosities in the blue may be more appropriate. Scaling with $S_B \approx 0.29$ (Di Stefano & Rappaport 1994) gives $N_{\rm SSS} \sim 100$. The actual number of sources in M31 may be ~30–200 times higher than detected because of interstellar absorption (Di Stefano & Rappaport 1994), with a corresponding increase in the Galactic incidence. An analysis of the positions of the Galactic SSSs that have already been detected suggests that the number of SSSs with a luminosity $L \gtrsim 10^{38}$ ergs s^{-1} is certainly higher than 20 (Motch, Hasinger, & Pietch 1994).

In fact, there presently exists only one almost universally accepted model of SSSs, originally suggested by van den Heuvel et al. (1992) and later elaborated on by Rappaport, Di Stefano, & Smith (1994b, hereafter RDS) and Di Stefano & Rappaport (1994, 1995). On the basis of the estimates for the observed luminosities of SSSs, the persistence of the LMC sources CAL 83 and CAL 87, and their orbital periods (1.04 days and 10.6 hr, respectively), van den Heuvel et al. suggested that SSSs may be relatively massive (~1 $M_\odot$) white dwarfs (WDs), which steadily burn at their surface hydrogen accreted from a Roche lobe overfilling

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companion, which is either a main-sequence star or a slightly evolved subgiant. The necessary accretion rate for steady burning is $\dot{M} \sim (1\text{–}4) \times 10^{-7}\ M_\odot\ \text{yr}^{-1}$ (Paczyński & Żytkow 1978). To sustain such a high $\dot{M}$, it was necessary to assume that the donors lose a substantial fraction of their mass on a thermal timescale. RDS performed a population synthesis study for the model of steady-burning sources. They considered as SSSs steady-burning WDs in systems with initial mass ratios of the components $M_{2,0}/M_{1,0} > 1$, in which the constant accretion rate was assumed to be equal to $\dot{M} = 0.5(M_{2,0} - M_{1,0})/\tau_{2,\text{th}}$, where $M_{2,0}$ and $M_{1,0}$ are the masses of the donor and the accretor immediately following the common envelope phase in which the WD was produced, and $\tau_{2,\text{th}}$ is the thermal timescale of the donor. For different assumptions on the initial distributions of binaries over the mass ratios of the components and the efficiency of expulsion of the common envelope, they derived the number of such objects in the Galaxy ($\sim$100–1800, depending on model assumptions) and estimated, among other things, the possible range of their orbital periods ($\sim$7.5 hr to $\sim$3 days).

Kahabka (1995) derived an approximate analytical model for transient SSSs, considering them as WDs accreting at rates slightly below the steady hydrogen-burning limit, and hence, experiencing thermonuclear runaways. This model relates the masses of accretors, the constant mass accretion rate, the recurrence period, and the postoutburst decay time. As expected, if the variability of several SSSs on a timescale of $\lesssim$10 yr is due to thermonuclear runaways, then Kahabka's model favors very high-mass WDs and high mass exchange rates for SSSs (see, however, Southwell et al. 1996).

For certain sets of model assumptions, RDS have shown that steady-burning SSSs may grow to the Chandrasekhar mass M_{Ch} with a rate as high as $\sim$0.006 yr^{-1}, close to the upper limit for the observationally inferred Galactic Type Ia supernova (SN Ia) rate $\sim$0.003–0.007 yr^{-1} (Wheeler & Harkness 1990; van den Bergh & Tammann 1991; see, however, Branch et al. 1995).

The properties of the nebulae, which may be associated with steady-burning SSSs were studied by Rappaport et al. (1994a), Remillard, Rappaport, & Macri (1995), and Di Stefano, Paerels, & Rappaport (1995).

A variation of the model described above, in which the white dwarf steadily burns helium accreted from a Roche lobe filling companion with a carbon-oxygen core and a thick helium mantle ("helium Algol") was suggested by Iben & Tutukov (1994). Based on simple considerations of the possible range of progenitor masses, the mass ratios, orbital separations, and the mass of helium available for burning, these authors roughly estimated the number of helium Algols in the Galaxy to be $\sim$100. The orbital periods of these systems, as estimated more accurately in the present paper, are between 1 and 20 hr.

Yet another variant on the SSSs model, involving stable accretion of helium, was suggested by Yungelson & Tutukov (1995) and Tutukov & Yungelson (1995a). This model consists of a semidetached binary system with a carbon-oxygen white dwarf accretor and a degenerate low-mass white dwarf helium donor. Such systems may have a short phase of evolution immediately after Roche lobe overflow (RLOF), in which the mass transfer rate is sufficiently high to sustain stable He burning and even enable the temperature in the accretion disk to reach values in excess of 3×10^5 K, thus transforming the system into an SSS. Population synthesis models similar to the model presented in the present paper, yielded a number of such systems of $\sim$100, and orbital periods of the order of 2–4 minutes. In fact, a serious difficulty for all the models involving He burning is the apparent presence of hydrogen features in the spectra of all the optically identified SSSs (M. Pakull 1995, private communication).

It was noticed that accreting white dwarfs in symbiotic binaries may also manifest themselves as supersoft X-ray sources (e.g., Sion & Starrfield 1994; RDS; Yungelson et al. 1995), but this model was never explored in detail, despite the identification of two symbiotic stars with SSSs.

In the present paper we model, by means of a population synthesis code, possible formation paths for the entire population of SSSs with white dwarf accretors and derive numbers and potentially observable properties of the SSS population. The model follows both mass exchange stages on a thermal timescale and the subsequent stages controlled by nuclear evolution and/or angular momentum losses. It includes both steady hydrogen-burning sources and *recurrent* sources, assuming that the latter are in a post–nova explosion state. We consider both systems in which accretion occurs from Roche lobe filling main-sequence or subgiant donors and detached systems with stellar wind accretion. We discuss the implications of different sets of model assumptions for the model and give a detailed comparison with the RDS model of SSSs, which represents a subset of the present model. We also consider, by means of population synthesis, the contribution to the population of SSSs by binary systems containing steady helium-burning accretors, white dwarf systems with hot accretion disks, and hot planetary nebula nuclei (both single and binary). As a part of this comprehensive investigation, we explain the existence of systems in a broad range of orbital periods. We also briefly discuss the selection effects that are involved in the detection of the observed systems and their optical identification. We discuss briefly the potential contribution of SSSs to the Galactic SN Ia's via channels involving WDs increasing in mass to M_{Ch} and off-center helium ignition.

A description of the model is given in § 2, the results are presented in § 3, and a summary and conclusions follow in § 4.

2. THE MODEL

Our model follows a pattern that involves as its basic element accretion of hydrogen or helium onto a white dwarf, as suggested by van den Heuvel et al. (1992) and Iben & Tutukov (1994). The modeling involves three major steps.

In the first step, we model a population of systems of different types, in which a white dwarf is accompanied by a mass-losing star. These systems include semidetached binaries with low-mass main-sequence and subgiant components (CVs) or helium-rich stars losing mass via Roche lobe overflow, and detached binaries (symbiotic stars) with (super)giants losing mass via a Reimers-type stellar wind.

In the second step, we model the mass exchange history for each "newborn" potential SSS. For semidetached systems, the evolution is followed until the exhaustion of the reservoir of nuclear fuel for mass transfer (i.e., transformation of donor into a WD) or until the stage in which the mass transfer rate becomes so low that the system does not contribute any more to the population of SSSs. For detached stars, we model the stellar wind mass exchange through the giant and asymptotic giant branch stages until the formation of a WD or the termination of these stages by unstable RLOF prior to exhaustion of the envelope. We allow not only for stable hydrogen burning, but also examine the implications of unstable burning (i.e., of nova explosions) for SSSs. The latter is suggested by the fact that observations detected SSSs in objects that experienced classical or symbiotic nova explosions. In fact, the possibility that some fraction of the SSSs are hot objects in a post-explosion state was suggested previously (e.g., RDS; Sion & Starrfield 1994; Beuermann et al. 1995). Because of the recurrent nature of thermonuclear runaways, postnovae produce a class of *recurrent* SSSs. A combination of data on the length of the mass exchange stages, the accreted mass, and the critical masses for thermonuclear runaways allows us to estimate the numbers and properties in all the subclasses of the total population of SSSs.

In the third step, we make a rough estimate of the selection effects involved and thus obtain the properties of the "observed" population of SSSs.

2.1. "*Zero-Age*" *SSS Population Synthesis*

For the generation of the population of binary systems that may become candidates for SSSs, we applied the population synthesis code, which was already used for the investigation of Wolf-Rayet stars, binary nuclei of planetary nebulae, white dwarfs in binaries, supernovae in binary systems, merger rates of binary neutron stars, low- and high-mass X-ray sources, and symbiotic stars (see Yungelson et al. 1995 for a description of the most important ingredients of the code and for references). All the computations were performed with a common envelope (CE) parameter $\alpha_{ce} = 1$ (Livio & Soker 1988) and assuming a flat distribution of zero-age main-sequence binaries over the mass ratio of the components. Taking $\alpha_{ce} = 1$ recently received observational support with the discovery of six close double degenerates (Marsh, Dhillon, & Duck 1995; Marsh 1995) just at the maximum of the distribution over orbital periods predicted with the same population synthesis code for $\alpha_{ce} = 1$ (Yungelson et al. 1994). Also, a recent theoretical calculation of the CE phase indicated a value of $\alpha_{ce} \sim 1$ (Rasio & Livio 1995). Below we describe briefly the evolutionary channels that result in the formation of candidate SSS objects.

1. The formation of cataclysmic variables involves two phases, which have already been explored by several authors (e.g., Politano 1988; Tutukov & Yungelson 1989; de Kool 1992; Han, Podsiadlowski, & Eggleton 1995). In the first phase, a star with an initial mass between 0.8 and 11.4 $M_\odot$ (in our code) and a low-mass companion ($\lesssim 2.5$ $M_\odot$) evolves to the asymptotic giant branch (AGB) and then overflows its Roche lobe. The ensuing common envelope stage results in the formation of a "precataclysmic binary." If the mass of the secondary is less than ~ 1.2 $M_\odot$, angular momentum loss (AML) via a magnetic stellar wind (MSW) is effective (Verbunt & Zwaan 1981; the upper limit for the action of MSW used by us is somewhat lower than the usually quoted value of 1.5 $M_\odot$, because the former limit is suggested by an analysis of the rotation velocities in the Hyades; Fedorova & Tutukov 1994). This angular momentum loss may bring into contact systems with post common envelope separations $\lesssim 10$ $R_\odot$. Main-sequence secondaries that are more massive than ~ 0.8 $M_\odot$ may fill their Roche lobes because of expansion driven by nuclear evolution. If the conditions for stable mass exchange controlled by nuclear burning or angular momentum loss (Tutukov, Fedorova, & Yungelson 1982; Hjellming 1989; Hjellming & Webbink 1987) are satisfied, such a system becomes a CV. These conditions were taken as

$$q = M_2/M_1 \lesssim \begin{cases} 0.6 & \text{for } M_2 \lesssim 0.8\ M_\odot\ , \\ 2.5 & \text{for } M_2 \gtrsim 0.8\ M_\odot\ . \end{cases} \quad (1)$$

For $1.2 \lesssim q \lesssim 2.5$, mass exchange occurs on a thermal timescale. For higher values of q, one may expect mass loss from the donor on a dynamical timescale and the formation of a common envelope. In fact, the critical value of q may depend on the mass of the donor in a more complicated way (see, e.g., Fig. 2 in de Kool 1992). In § 3.5 below we explore the influence of the assumed upper limit of the critical q for stable mass exchange on our results.

2. If the system (after the formation of the white dwarf) is wider than the limit that allows the secondary to make contact with its Roche lobe during the main-sequence (via the action of MSW), it may fill its Roche lobe in the hydrogen shell burning stage. If the donors do not have deep convective envelopes (in fact, this is equivalent to having degenerate helium cores of mass $M_{He} \lesssim 0.25$ $M_\odot$), stable mass exchange in such systems is possible (provided that the conditions described by eq. [1] are fulfilled; Iben & Tutukov 1984; Kraicheva, Tutukov, & Yungelson 1986).

3. Symbiotic stars with carbon-oxygen WD accretors may be formed via two channels (Yungelson et al. 1995). In the first, occurring in relatively wide systems but still "close" in the evolutionary sense, the primary component overfills its Roche lobe when it is an AGB star. After the CE stage, the system remains relatively wide, and the secondary is able to lose a substantial part of its mass via a Reimers-type stellar wind before RLOF (it may also never experience a RLOF). In the wind mass-loss stage, such a system will be observed as a symbiotic star. In the second channel, the system is initially so wide that the primary becomes a white dwarf in a way similar to that of single stars, without RLOF. Such systems are also able to produce a symbiotic star phenomenon when the secondary component becomes first a giant and then a supergiant (Tutukov & Yungelson 1976).

4. In (1)–(3) we described the channels that result (in our model) in the formation of most SSSs. A smaller contribution comes from other channels, which still may be considered as viable ones. The first is the channel of "helium Algols" (Iben & Tutukov 1994). The progenitors of these systems have main-sequence masses of the components in the range 5–11 $M_\odot$. The primaries become CO or ONe white dwarfs after experiencing RLOF following helium exhaustion in their cores (case C of mass exchange). The secondaries fill their Roche lobes in the hydrogen shell burning phase and become helium stars. After the exhaustion of helium in the core, they expand, and if the mass ratio of the components is again favorable, they may experience stable RLOF with a mass transfer rate that allows stationary burning of He (the so-called BB case of mass exchange).

5. The next channel is represented by semidetached double degenerate systems (some of which may be similar to the AM CVn–type stars). The progenitors of these systems have typically primaries with main-sequence masses between 2.5 and 6 $M_\odot$, and secondaries below 2.5 $M_\odot$. The primaries become carbon-oxygen WDs after experiencing

case C mass exchange. The secondaries experience case B mass exchange and become helium white dwarfs. After two CE episodes, the system may become so tight that the AML via the radiation of gravitational waves is able to bring the components into contact in less than the Hubble time. The Roche lobe is first filled by the He WD, which is the lighter component. A candidate SSS emerges, if two conditions are fulfilled. The first is the condition for dynamically stable mass exchange: $(d\ln R_2/d\ln M_2)_s > (\partial \ln R_{cr}/\partial \ln M_2)_J$, where $(d\ln R_2/d\ln M_2)_s$ is the adiabatic mass radius exponent of the secondary, and J is the angular momentum (e.g., Ritter 1988). The second condition is the requirement for the mass exchange rate to be lower than the critical mass exchange rate $\dot{M}_{RG}$, above which white dwarfs accreting He develop extended envelopes. $\dot{M}_{RG}$ ranges from $\sim 10^{-6}\ M_\odot$ yr^{-1} for $M_{wd} \approx 0.6\ M_\odot$ yr^{-1} for $M_{wd} \approx 1.3\ M_\odot$ (Iben & Tutukov 1989). Typically, the mass of the accretor in successful systems is $\sim 0.6\ M_\odot$ and the mass of the donor is ~ 0.13–$0.20\ M_\odot$.

6. Finally, since among the observed SSSs there exists an object that has been provisionally identified with a planetary nebula nucleus, we computed the birthrate of white dwarfs, both in wide and close binaries, that may emit supersoft X-rays during the short hottest phase of their way to becoming cooling white dwarfs. Using Paczyński's (1971a) data, we found that white dwarfs more massive than $\sim 0.7\ M_\odot$ may spend a time $t \approx 100(M/M_\odot)^{-5.13}$ yr in the region of the H-R diagram in which $T_{eff} \gtrsim 250{,}000$ K and $L \gtrsim 5 \times 10^3\ L_\odot$.

2.2. *Model for the Evolution of Candidate SSSs*

For every zero-age candidate SSS, which was generated as described in the previous section, the history of mass exchange was followed. This was done in order to estimate the mass of the hydrogen or helium available for burning and the time dependence of the mass exchange rate and, consequently, the lifetime in the SSS state.

1. The evolution of CVs was considered as follows. For masses of donors above $\sim 0.8\ M_\odot$ and mass ratios of the components between 2.5 and 1.2, the evolution initially proceeds on the thermal timescale of the donor $\tau_{2,th}$ (see, e.g., Paczyński 1971b). As trial full-scale evolutionary computations show, in this stage, the mass-loss rate by the donor may be represented with a reasonable accuracy as being linearly decreasing with time, from the peak value of

$$\dot{M}_2 = (M_2/M_\odot)/(\tau_{2,th}/\text{yr})\ M_\odot\ \text{yr}^{-1} \quad (2)$$

to

$$\dot{M}_2 = 3 \times 10^{-8}(M_2'/M_\odot)^{2.5}\ M_\odot\ \text{yr}^{-1}\ , \quad (3)$$

where M_2' is the mass of the secondary when $M_2'/M_1 = 1.2$. Later on, the mass-loss rate may be approximated by equation (3), with M_2' being the instantaneous mass of the donor. Equation (3) also approximates quite well the mass-loss rate in systems in which initially $M_2/M_1 < 1.2$ and/or $M_2 < 0.8\ M_\odot$ and in systems evolving on the timescale of angular momentum loss via MSW.

When M_2/M_1 decreases to 1.2, M_2 is typically $\leq 1\ M_\odot$ and angular momentum loss via MSW dominates the evolution. The evolution continues on the timescale of MSW until M_2 decreases to $\sim 0.27\ M_\odot$. This limit corresponds to the upper edge of the "gap" in the distribution of CVs over orbital periods. Systems with donor masses between 0.27 and 0.07 $M_\odot$ evolve on the timescale of AML via gravitational wave radiation (GWR). In this stage, the mass-loss rate by the donor is approximately constant: $\dot{M} \approx 10^{-10.2}\ M_\odot$ yr^{-1}. We did not follow the evolution of CVs beyond $M_2 = 0.07\ M_\odot$, because systems with donors below this limit have such low $\dot{M}$ that they almost do not contribute to the observed population of CVs.

Clearly, equation (3) merely represents a rough approximation to the real mass transfer rate, which for every particular system depends on the masses of the components, the AML, and the degree to which the donor is evolved at the instant of RLOF. However, evolutionary tracks tend to converge to a single uniform solution despite different initial conditions (see, e.g., discussion in Stehle, Ritter, & Kolb 1995). In Figure 1, as an example, we show the relation between the mass-loss rate and the mass of the donor for a system with initial masses of both components equal to 0.8 $M_\odot$ for a case of conservative mass transfer (controlled initially by AML via MSW and later by AML via GRW) and under the assumption that all the mass accreted by the white dwarf is lost in nova eruptions. In the latter case, it was assumed that the specific angular momentum of the ejected matter is equal to the specific orbital angular momentum of the white dwarf. Trial evolutionary calculations were accomplished by a Henyey-type code, which was previously used, e.g., for the modeling of binary star evolution (Fedorova & Ergma 1989; Sarna & Fedorova 1989) and for seismic solar models (Kosovichev & Fedorova 1991). As can be seen in the figure, the mass-loss rate suggested by equation (3) reproduces the two computed curves to within a factor ~ 2.

Mass-loss rates given by equation (3) also reproduces well the *average* observed mass transfer rates in CVs (see, e.g., Patterson 1984). The rather high scatter of the observed rates around this value may be explained as an oscillation of the mass transfer rate around its secular mean, possibly

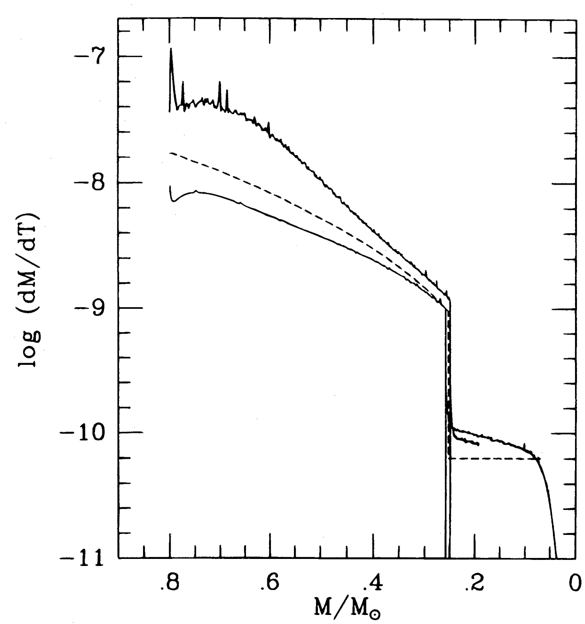

FIG. 1.—Mass-loss rate vs. donor mass in a system with initial masses of both components equal to 0.8 $M_\odot$. The upper solid line is for the case in which angular momentum is lost via a magnetically coupled stellar wind and all matter transferred by the donor is ejected by accretor, taking away the specific orbital angular momentum of the latter. The lower solid line describes the conservative case. The dashed line shows the approximation to the mass transfer rate adopted in the present paper.

caused by the effects of irradiation on the secondary (King et al. 1995; see, however, Ritter 1995).

2. The evolution of systems in which a donor with a degenerate helium core overflows its Roche lobe was in fact investigated to a very little extent. These systems attracted some attention when the first simulations of mass exchange in binaries were made (e.g., Kippenhahn, Kohl, & Weigert 1967; Ziolkowski 1970; Giannone & Giannuzzi 1970, 1972). Again, two phases of mass exchange were discovered: a "fast" one, which lasts until the mass ratio of the components becomes somewhat less than 1, and a subsequent "slow" one. In the fast stage, the star strongly deviates from thermal equilibrium; for deep convective envelopes and/or extreme initial mass ratios of the components, mass loss by the donors proceeds on a dynamical timescale (e.g., Paczyński & Sienkiewicz 1972 and references therein). In the slow stage, the star is in thermal equilibrium, mass loss is controlled by the expansion of the donor due to nuclear burning of hydrogen in a shell. The slow stage lasts until only several percent of the hydrogen envelope remains at the surface of the donor; then the envelope contracts and a helium white dwarf forms. In the aforementioned simulations, the initial phases of mass transfer were usually followed in a very rough manner: in most cases mass was removed from the donor on the thermal timescale, until the radii of the models became smaller than the respective Roche lobe radii. In this way, one may estimate the amount of mass lost on a thermal timescale and the average mass-loss rate in the initial phase of mass transfer, but the actual (time-dependent) behavior of the mass-loss rate is poorly known. In most cases, the curves describing the mass loss were not published. Subsequently, it became clear that in this case B evolution for low-mass stars, the overwhelming majority of the systems have to evolve not via stable RFOF, but rather via a common envelope phase. This is a consequence of the fact that the mass transfer rates are usually higher than the critical accretion rates for white dwarf companions ($\sim 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$) or the rate at which hydrogen can burn stably at the surface of the accreting white dwarf ($\sim 10^{-7}\ M_\odot\ \mathrm{yr}^{-1}$). Since common envelope evolution results in the formation of a white dwarf with a mass almost equal to the mass of the donor core at the instant of RLOF, the interest in models of this type subsided (despite the fact that the closest "cousins" of such systems, ordinary low-mass Algols with main-sequence primaries, still lack a quantitative explanation).

Refsdal & Weigert (1971) suggested an algorithm that allows one to follow the mass-loss rate by *thermally stable* subgiant donors with degenerate helium cores in the stage controlled by nuclear burning (of course, if they are able to reach it). This algorithm was applied by Yungelson (1973a), Webbink, Rappaport, & Savonije (1983), Iben & Tutukov (1984), Kraicheva et al. (1986), and others to a number of problems related to mass exchange in low-mass systems. The algorithm makes use of the fact that for such stars, the radius, luminosity, and core growth rate are functions mainly of the core mass M_{He} itself (see a thorough discussion in Refsdal & Weigert 1970). The essence of the algorithm, as used by us, is the following. The condition for continuous Roche lobe filling is

$$R_2 = R_{\mathrm{cr}} \quad \text{and} \quad \frac{d\ln R_2}{d\ln M_2} = \frac{d\ln R_{\mathrm{cr}}}{d\ln M_2} \tag{4}$$

(e.g., Paczyński 1967). In the situation under consideration, we deal with donors that have close companions and have relatively deep convective envelopes. Therefore, one may expect that they, like their main-sequence progenitors, have magnetic stellar winds, and AML via MSW may influence their evolution. Also, some of the matter that is initially accreted by the white dwarf may be later lost, with the specific angular momentum of the white dwarf (either due to nova explosions or due to a radiative stellar wind, if hydrogen burns stably). If the fraction of the matter retained by the WD is α, and the (logarithmic) rate of angular momentum loss via MSW is expressed by $\dot{J}/J$, from equation (4) one obtains the following equation for the mass-loss rate by the donor:

$$\frac{\dot{M}_2}{M_2} = \left(\frac{\dot{J}}{J} - \frac{1}{2}\frac{\dot{R}}{R}\right)\left[1 - \alpha q - \frac{1}{2}\frac{1-\alpha}{1+q}\,q - \frac{1-\alpha}{1+q}\,q^2 - \frac{1}{2}(1+\alpha q)f'\right]^{-1} . \tag{5}$$

In equation (5), $q = M_2/M_1$ and f' is the logarithmic derivative of the relative radius of the Roche lobe, for which we use the approximation given by Eggleton (1983).

We have used the following approximations for the dependences of the radius, luminosity, and core growth rate on the mass of the core (in solar units and years), as derived by Iben & Tutukov (1984) for solar chemical composition stars:

$$R \approx 10^{3.5} M_{\mathrm{He}}^{4} , \quad L \approx 10^{5.6} M_{\mathrm{He}}^{5.6} , \quad \dot{M}_{\mathrm{He}} \approx 10^{-5.36} M_{\mathrm{He}}^{6.6} . \tag{6}$$

Using the first and the third expressions in (6), one may integrate equation (5) over time until $M_2 = M_{\mathrm{He}}$, thus obtaining the mass-loss rate and timescale of the evolution controlled by nuclear burning. For the mass conservative case, equation (5) may be applied if $q \leq 0.78$ (and starting from a slightly larger q if α is smaller than 1).

For the initial phase of mass transfer, which does not allow (with the present knowledge) formalization, we proceeded in the following manner. A few published mass-loss curves (e.g., Ziolkowski 1970; see especially Kolb & Ritter 1990) and our own experience with calculations of close binaries (Yungelson 1973b) show that immediately after RLOF, the mass-loss rate grows sharply. If the mass-loss process does not become dynamic, i.e., if $\dot{M}$ does not attain values as high as 0.01–0.1 $M_\odot\ \mathrm{yr}^{-1}$, where model iterations in Henyey-type codes usually cease to converge, $\dot{M}$ grows to a certain maximum value, and then gradually decreases to much lower $\dot{M}$, which may be approximated by equation (5). Therefore, if initially q exceeded 0.78, we interpolated smoothly the transition from the initial peak value of $\dot{M}$ (given for the thermal timescale by eq. [1]) to the $\dot{M}$ given by equation (5) for $q = 0.78$ and the respective α, over two to three thermal timescales. The values of α were obtained using the results of Prialnik & Kovetz (1995; see below). In systems with either main-sequence components or subgiant components, if the initial $\dot{M}$ exceeded the upper limit for stable hydrogen burning $\dot{M}_{\mathrm{RG}}$, or equation (1) suggested a dynamical mass exchange, such systems were rejected from the "zero-age" population of potential SSSs. Since $\dot{M}_{\mathrm{RG}}$ is of the order of only $10^{-7}\ M_\odot\ \mathrm{yr}^{-1}$, this procedure effectively restricted the radii of the donors in candidate SSSs and prevented systems that may have evolved into a common envelope from entering the sample. In the sample of "zero-age" SSSs in systems with subgiant donors, which is described in § 3.1 below, almost all the donors experienced RLOF when roughly $R_d \lesssim 5R_\odot$, i.e., in the gap of the H-R diagram or close to the base of the red giant branch. Admittedly, if there actually exists a class of systems that can evolve through a common envelope phase retaining most of their hydrogen envelopes, then such systems will be absent from our sample.

3. The algorithm for the computation of mass loss in symbiotic binaries is described in detail in Yungelson et al. (1995). Therefore, we describe here very briefly only a few essential points. In these systems, accretion is assumed to occur from a Reimers-type stellar wind. We use the dependences of the radius, luminosity, and core growth rate on the core mass for stars in the hydrogen shell burning and AGB stages of evolution, to estimate the time dependence of the ingredients of the Reimers equation for the rate of mass loss in the stellar wind. The Bondi (1952) formula is used to estimate the accretion rate. An important point, which strongly influences the number of stars exhibiting the symbiotic phenomenon, is the possibility that the accretor is located in the zone of acceleration of the stellar wind, where the velocity of the latter is still far from the terminal one. This factor enhances the efficiency of accretion considerably, making it close to 100% when the donor is close to the Roche lobe. The evolution of the donors in symbiotic binaries was followed up to RLOF or up to the exhaustion of the hydrogen envelope and the transformation of the donor into a white dwarf. In almost all cases, RLOF in symbiotic stars probably results in mass loss on a dynamical timescale, because the donors at this stage typically have deep convective envelopes and the mass ratio of the components exceeds 1 (Yungelson et al. 1995).

4. "Helium Algols" (stars with CO cores and thick helium mantles), after RLOF, may support mass transfer at an almost constant rate. From Iben & Tutukov (1994), we derive

$$\dot{M} \approx 6.3 \times 10^{-13} M_{\rm MS}^{7.5}\ M_\odot\ {\rm yr}^{-1}\ , \tag{7}$$

where $M_{\rm MS}$ is the mass of the progenitor of the helium star in $M_\odot$ on the main sequence.

5. For the systems with helium WD donors, we again use the condition of stable mass transfer (eq. [4]). The radii of the WDs were approximated by the mass-radius relation for zero-temperature degenerate objects (Nauenberg 1972). Tutukov & Yungelson (1995a) have shown that the application of more accurate mass-radius relations (e.g., Zapolsky & Salpeter 1969) does not influence the mass-loss rates for the initial stages of mass exchange, which are important for the present study.

As it was noticed by Yungelson & Tutukov (1995) and Tutukov & Yungelson (1995a), systems with very high mass accretion rates may manifest themselves as SSSs because of the radiation of their accretion disks. The temperature of the disk in the standard, blackbody approximation is

$$T(x) = \left(\frac{3GM_{\rm wd}\dot{M}}{8\pi\sigma R_{\rm wd}^3}\right)^{0.25} x^{0.75}(1 - x^{0.5})^{0.25} \tag{8}$$

(Shakura & Sunyaev 1973), where $x = R_{\rm wd}/r$. The maximum of $T(r)$ exceeds 250,000 K if $\dot{M} \gtrsim 4 \times 10^{-6}\ M_\odot$ yr^{-1} for $M_{\rm wd} = 0.6\ M_\odot$ and $\dot{M} \gtrsim 6 \times 10^{-7}\ M_\odot$ yr^{-1} for $M_{\rm wd} = 1.0\ M_\odot$. This means that some of the systems that are burning hydrogen or helium stably may have a contribution to their supersoft X-ray emission from their accretion disks.

2.3. *Lifetime in the SSS State*

For any of the systems described above, two possible situations may be realized.

1. The WD may burn the accreted hydrogen stably if the accretion rate exceeds

$$\log \dot{M}_{\rm st}(M_\odot\ {\rm yr}^{-1}) \approx -9.31 + 4.12 M_{\rm wd} - 1.42 M_{\rm wd}^2 \tag{9}$$

(Iben & Tutukov 1989), where $M_{\rm wd}$ is in $M_\odot$. However, even in this "stable burning" situation, not all the accreted hydrogen is available for burning, because some of it can be lost via a radiatively driven wind. Iben & Tutukov (1996) estimated, using a single scattering approximation (with a solar abundance of the heavy elements in the envelopes), that about 20% of the hydrogen may be lost via the wind (see also Fig. 2 in Yungelson et al. 1995). If the heavy elements mass fraction Z is higher, the retention efficiency may be reduced, because the stellar wind mass-loss rate scales roughly as $Z^{0.5}$ (Kudritzki, Pauldrach, & Puls 1987). (The presence of a wind may also be important for the suppression of accretion, thus reducing the number of sources; see § 3). Stably burning WDs have effective temperatures in excess of 250,000 K if their mass is higher than $\sim 0.54\ M_\odot$ (Iben & Tutukov 1989). In fact, this means that all the CO or ONe WD accretors generated by our scenario code may be candidate SSSs. Thus, all the systems with WDs that were accreting at a rate higher than that given by equation (9) were considered as steady hydrogen-burning SSSs (with a correction for the lifetimes due to wind mass loss).

2. The situation with WDs accreting below the stability limit (eq. [9]) was treated as follows. These WDs were assumed to experience hydrogen-burning flashes. For a hydrogen flash to occur, a critical layer of hydrogen has to be accumulated. This critical mass is a function of the mass of the WD, its temperature and the accretion rate. Concerning the temperature of the WDs, simulations of the formation rates of CVs (de Kool 1992; Tutukov & Yungelson 1995b) show that in CVs with WD masses below 1 $M_\odot$, the ages of the WDs are typically higher than 3×10^8 yr. At that age their temperatures have to decrease below 10^7 K (e.g., D'Antona & Mazzitelli 1990). White dwarfs with $M \gtrsim 1\ M_\odot$ may be typically as young as 1×10^8 yr, but they cool faster than the less massive objects. Consequently, one probably has to use for the approximation of the masses of the critical layers, $\Delta M_{\rm crit}$, the results of numerical simulations for *cold* dwarfs. We have used (as in Yungelson et al. 1995) the following average expression for $\Delta M_{\rm crit}$, derived from the *cold* models of Schwartzman, Kovetz, & Prialnik (1994):

$$\frac{\Delta M_{\rm crit}}{M_\odot} \approx 2 \times 10^{-6}\left(\frac{M_{\rm wd}}{R_{\rm wd}^4}\right)^{-0.8}\ , \tag{10}$$

where $M_{\rm wd}$ is the mass of the white dwarfs in grams and $R_{\rm wd}$ in centimeters is given by the Nauenberg (1972) approximation. We neglected the dependence of $\Delta M_{\rm crit}$ on $\dot{M}$, which is relatively weak ($\sim \dot{M}^{-0.3}$).

Observations of nova abundances show that the accreted matter mixes with WD material, to form a layer $\Delta M_{\rm env}$, which is enriched in helium or heavier elements (Livio & Truran 1994). Once the WD undergoes a flash, it loses material through a combination of dynamical ejection, an optically thick wind phase, and mass loss due to formation of a common envelope (see Livio 1994 for a review). Irrespective of the mechanism of mass loss, after the mass of the hydrogen layer atop the WD decreases below a certain limit, the envelope recedes, the WD gradually returns to the region of the H-R diagram that corresponds to stable burning and there most of the remainder of $\Delta M_{\rm env}$ is consumed. During most of this stage of evolution, the bolometric luminosity is nearly constant, the spectrum of the emergent radiation hardens, and for most of this "residual hydrogen-burning time," postflash objects have to show up as supersoft X-ray sources (e.g., Iben 1982; Fujimoto 1982; MacDonald, Fujimoto, & Truran 1985). The luminosity of the WDs in this stage may be estimated by the mass-luminosity relation for WDs with stationary hydrogen-burning shells:

$$L/L_\odot \approx 4.6 \times 10^4(M_{\rm wd}/M_\odot - 0.26) \qquad (11)$$

(Iben & Tutukov 1989). Since $M_{\rm wd} \gtrsim 0.54\ M_\odot$, all the SSSs in our model have $L \gtrsim 5 \times 10^{37}$ ergs s^{-1}.

The basic question is now, how to estimate the time $T_{\rm X}$, during which the WD is hot enough to emit in the supersoft X-ray range.

An upper limit to $T_{\rm X}$ may be taken to be $t_{3\rm bol}$—the time it takes the WD to decline by 3 mag in its bolometric luminosity. We consider $t_{3\rm bol}$ to be an upper limit because, as the published $L_{\rm bol}$ light curves show (e.g., Kovetz & Prialnik 1994), at $t_{3\rm bol}$ the luminosity of the WD may already be below the threshold of $\sim 4 \times 10^{37}$ ergs s^{-1}, which is usually applied as a criterion for the classification of an object as an SSS (e.g., Motch et al. 1994). From the grid of nova models generated by Prialnik & Kovetz (1995), we derived the following rough approximations for $t_{3\rm bol}$ in years for *cold* white dwarfs:

$$\log t_{3\rm bol} \approx \begin{cases} 2.04 - 3.92(M_{\rm wd} - 0.65) \\ \quad \text{for } M_{\rm wd} \le 1.25\ M_\odot\,, \\ -0.31 - 9.33(M_{\rm wd} - 1.25) \\ \quad \text{for } M_{\rm wd} > 1.25\ M_\odot\,, \end{cases} \qquad (12)$$

where $M_{\rm wd}$ is in $M_\odot$. In fact, $t_{3\rm bol}$ also depends on the accretion rate, changing in an irregular fashion for $10^{-10} \le \dot M \le 10^{-7}\ M_\odot$ yr^{-1}, being typically lower than the value given by equation (12) for lower $\dot M$. Equation (12) reproduces $t_{3\rm bol}$ to within a factor ~ 2, and we neglected the dependence on $\dot M$ for the purposes of the present study.

As another estimate for $T_{\rm X}$, one may use the relation between the computed masses of the envelopes of steady hydrogen-burning WDs and their effective temperatures. From Iben & Tutukov (1989), the envelope masses for those WDs that have effective temperatures above 250,000 K are given by

$$\Delta M_{\rm X}/M_\odot \approx \begin{cases} (10^{1.5M_{\rm wd}/M_\odot - 0.8} - 1) \times 10^{-2.53 - 3.25M_{\rm wd}/M_\odot} \\ \quad \text{for } 0.54 \le M_{\rm wd}/M_\odot \le 0.7\,, \\ 4.3 \times 10^{-6} - 6.25 \times 10^{-5} \log M_{\rm wd}/M_\odot \\ \quad \text{for } M_{\rm wd}/M_\odot > 0.7\,. \end{cases} \qquad (13)$$

With this $\Delta M_{\rm X}$ and equation (11), we can estimate the lifetime in the SSS stage after every hydrogen-burning flash as

$$T_{\rm X} \approx \frac{1.5 \times 10^6 \Delta M_{\rm X}}{(M_{\rm wd} - 0.26)}\ {\rm yr}\,, \qquad (14)$$

where $\Delta M_{\rm X}$ and the mass of the WD are in $M_\odot$. This estimate of $T_{\rm X}$ may be considered as being close to the lower bound of the "on" time for SSSs, because it is based on the mass of hydrogen that is available for burning when the star recedes to the high-temperature region of the H-R diagram. When this reservoir of mass is exhausted, the stars typically have luminosities close to the lower limit of that for SSSs. The value of $T_{\rm X}$ may be reduced, if the star loses a substantial amount of matter via a stellar wind when it is hot. The relations between $M_{\rm wd}$ and $T_{\rm X}$ for the two cases described by equations (12) and (14) are plotted in Figure 12 and the differences between the two sets of models are discussed in § 3.

The lower limit for the accretion rate for which stable helium burning is obtained (Iben & Tutukov 1989, Fig. 3*a*) may be reasonably well approximated by

$$\log \dot M^{\rm cr}_{\rm He}(M_\odot\ {\rm yr}^{-1}) \approx 1.325 M_{\rm wd}/M_\odot - 7.06\,. \qquad (15)$$

In systems with donors having CO cores and thick He mantles, the mass transfer rates are always close to the limit given by equation (15). Therefore, these systems are stable burners or experience weak flashes, and we considered their lifetimes as SSS to be equal to the lifetimes of the systems in the semidetached state.

For systems with helium degenerate donors, the lifetime in the SSS state was estimated as the period of time during which $\dot M \gtrsim 10^{-6.5}\ M_\odot$ yr^{-1}, or equal to the period of time during which the maximum of $T_{\rm eff}$ given by equation (8) exceeded 250,000 K.

2.4. *Evolution of the Accretor Mass*

Hydrogen-burning flashes may certainly be accompanied by mass loss. In order to follow the secular evolution of the WD accretors we used the data given by Prialnik & Kovetz (1995), who list for each combination of accretor mass and mass accretion rate the relation between the amounts of accreted and ejected mass (after a limit cycle of flashes has been established). In this way, we were able to follow the changes in the accretor mass with time and their influence on the frequency of hydrogen-burning flashes, and hence, on the number of SSSs.

Depending on the secular evolution of the accretion rate, the mass of the WD may grow or decrease (Fig. 2). An increase in the mass of the WD means the accumulation of a He-rich layer beneath the H-rich one (also, CO WDs are born with a He layer at their surface). For He there also exists a critical mass for ignition, which depends on the mass of the WD and the accretion rate. If $\dot M \lesssim (2\text{–}3) \times 10^{-8}\ M_\odot$ yr^{-1}, the density of the He envelope at the instant of ignition is so high that He detonates (Fujimoto & Taam 1982; Kawai, Saio, & Nomoto 1987; Limongi & Tornambé 1991; Iben et al. 1987; Iben & Tutukov 1991; Tutukov & Khokhlov 1992; Woosley & Weaver 1994). Helium detonation can possibly result in the detonation of the core and a supernova (e.g., Livne & Glasner 1990; Woosley & Weaver 1994; Livne & Arnett 1995). At higher $\dot M$, He flashes are less violent, but most of the He layer is probably lost dynamically or via a common envelope. The estimates for the critical mass of He that is necessary for ignition, given by the above authors, are uncertain to within a factor of ~ 2. An uncertainty also exists in the masses of accretors for which detonation is possible. Therefore, in the situations in which helium has accumulated, we assumed

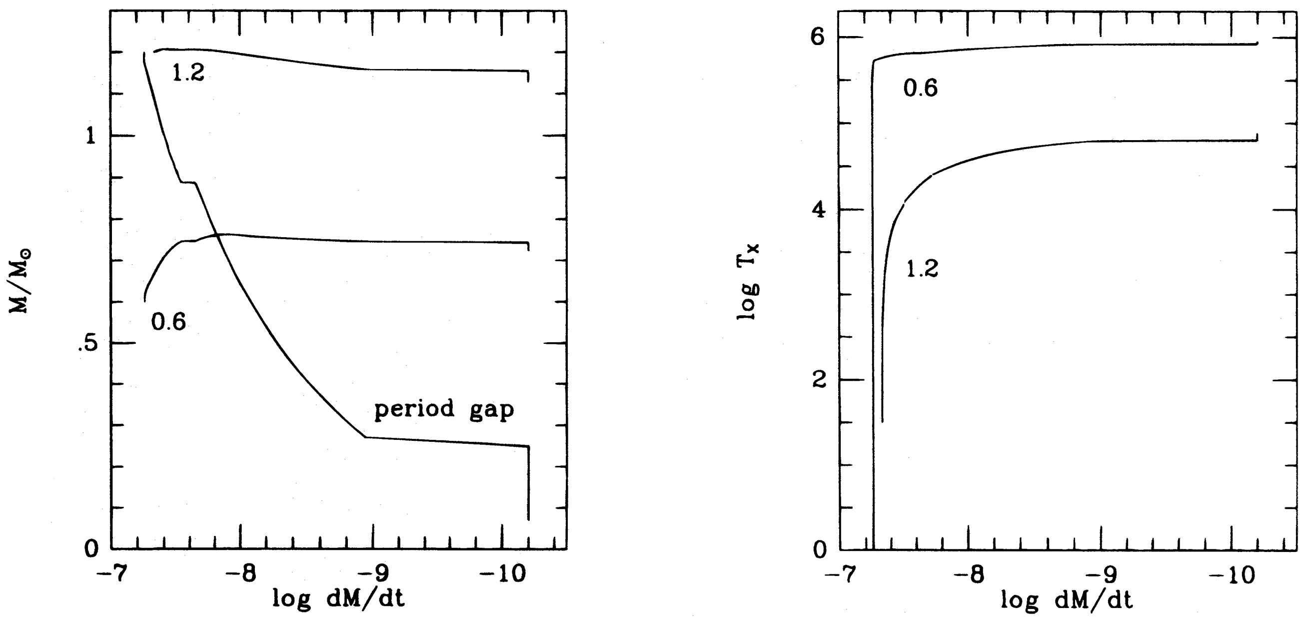

FIG. 2a FIG. 2b

FIG. 2.—(a) Relation between the mass exchange rate and the mass of the components. The initial mass of the donor was 1.2 $M_\odot$, and the initial mass of accretor 0.6 $M_\odot$ or 1.2 $M_\odot$. Accumulation or erosion of the WD were taken into account as functions of M_{wd} and $\dot{M}$ after Prialnik & Kovetz (1995). The part of the "donor" line marked as "period gap" corresponds to a discontinuity in $\dot{M}$ caused by the detached stage of evolution over the gap in the distribution of CVs over orbital periods. (b) The dependence of the time spent in the supersoft source state on the accretion rate and mass of the WD (in fact, secular evolution of T_X).

that the rate of its accumulation is equal to the hydrogen-burning or accretion rate and estimated the critical He-layer mass by means of a simple "one-zone" approximation given by Iben & Tutukov (1989, eq. [11a]), which, for $0.6 \lesssim M_{wd}/M_\odot \lesssim 0.8$ and $\dot{M} \sim 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, reproduces reasonably well the results of more sophisticated numerical modeling:

$$\Delta M_{He} \approx 10^{6.65} R_{wd}^{3.75} M_{wd}^{-0.3} \dot{M}_{-8}^{-0.57} , \quad (16)$$

where ΔM_{He}, M_{wd}, and R_{wd} are given in solar units, and $\dot{M}_{-8}$ is the accretion rate in units of $10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$. If a helium flash occurred, we assumed that the helium layer was completely lost. If the accretion rate at which this occurred was lower than $3 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, the system was excluded from further consideration (and entered our statistics of "shell supernovae"). In fact, this assumption may yield an upper bound to the actual rate of explosions, because most of the "critical" layer is accumulated at accretion rates $\dot{M} > 3 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$. If the accumulation of mass resulted in growth of the CO white dwarf mass to M_{Ch}, such an event was considered as a Type Ia supernova or as an "accretion-induced collapse (AIC)," if the white dwarf was ONe.

In post–helium flash systems, accretion occurs onto a "bare" CO core. One may expect in this case a significant enrichment of matter by heavy elements, resulting in increased violence of the outbursts and consequently in the erosion of the WD.

Figure 2a shows examples of the secular evolution of the masses of the donor and accretor in two cataclysmic binaries with initial masses of the donor of 1.2 $M_\odot$ and two different masses of the accretor: 1.2 and 0.6 $M_\odot$. While the mass-loss rate by the donor is similar in both cases (therefore, only one curve for the donor is drawn), there is a pronounced difference in the evolution of the accretors. In the case of the 1.2 $M_\odot$ WD, the lower limit for stable hydrogen burning is $\sim 6.6 \times 10^{-7}\ M_\odot\ \mathrm{yr}^{-1}$. Therefore, from the very beginning of the accretion process, the WD experiences hydrogen-burning flashes, which are probably relatively weak. At $\dot{M} \sim (3\text{–}5) \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ it is possible that the flashes will not even manifest themselves in the optical range of the spectrum. (The existence of "novae" that never become optically bright and radiate mainly in the extreme UV was suggested by Shara, Prialnik, & Shaviv 1977; see discussion of models of these objects and relevant references in Truran, Glasner, & Yungelson 1995). Despite weak flashes, the WD slowly increases in mass; in $\sim 2.5 \times 10^6$ yr it accumulates $\sim 0.008\ M_\odot$ of He and experiences a helium flash, resulting in the loss of the whole accumulated envelope. Because at that instant $\dot{M}$ is still relatively high ($\sim 4 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$), the mass of the WD grows by 0.006 $M_\odot$ in the next 5×10^6 yr, and then the WD begins to erode, decreasing its mass finally to 1.13 $M_\odot$. Thus, in spite of the relatively high initial masses of both the accretor and the donor, and the apparent good chances to accumulate the mass necessary for an SN Ia or an AIC, this appears to be impossible. The example of a 0.6 $M_\odot$ WD accretor demonstrates another pattern of evolution. For this mass, the critical accretion rate for stable hydrogen burning is much lower than in the former case: $\sim 4.5 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$. Therefore, the WD spends the first 900,000 yr of accretion in the regime of stable burning, and afterward enters the regime of weak flashes with a high efficiency of retention of the accreted material. It therefore succeeds in accumulating $\sim 0.16\ M_\odot$ before entering the regime of erosion. For such low-mass WDs, the critical mass for He ignition is high and therefore in this particular case the accretor does not experience a He flash. This means that ejecta of such a WD, when it explodes as a nova, may be enriched in He, but not in C and O. The final mass of the WD is 0.724 $M_\odot$.

The difference in the evolutionary patterns of accretors of different masses is reflected in their appearance as SSSs. The total time T_X spent by both WDs as SSSs is shown in Figure 2*b* (for the case in which the duration of the SSS stage is approximated by $t_{3\text{bol}}$). The 1.2 $M_\odot$ WD, which oscillates between "on" and "off" states, experiences altogether ~90,000 flashes, but because it spends less than a year in the "on" state after each flash, it is unable to contribute significantly to the population of SSSs. In contrast, the 0.6 $M_\odot$ WD "accumulates" 60% of the 900,000 yr of its total lifetime as an SSS in the stage of stable burning, and another 40% following 7500 nova outbursts, during the rest of the evolutionary lifetime. In the stage of evolution controlled by GWR, T_X almost does not grow.

3. POPULATION OF SUPERSOFT SOURCES

3.1. *Underlying Populations*

It is evident from the above discussion that there exist three *main* groups of candidate objects. We shall refer to them, for brevity, as CVs, Algols (systems with subgiant donors), and SySs (symbiotic systems).

First, we have to note that our model produces a population of CVs with CO or ONe accretors, the space density of which (~3×10^{-5} pc^{-3}) is within the range of current observational estimates (e.g., Patterson 1984; Ritter & Burkert 1986; Della Valle & Duerbeck 1993; Shara et al. 1993). The predicted total Galactic nova rate (~40 yr^{-1}) is also within the range of the observational estimates, which themselves vary from 11–46 yr^{-1} (Ciardullo et al. 1990) through 15–50 yr^{-1} (Della Valle & Livio 1994) to 73±24 yr^{-1} (Liller & Mayer 1987). We estimate the rate of "detectable" Galactic novae as ~1 yr^{-1}, also consistent with observations (Duerbeck 1990). For the latter estimate, we use the fact that the sample of detected novae begins to decline quite rapidly beyond $V \approx 8$ mag (Duerbeck 1987, 1990), and, using equation (11), we can construct a sample of novae with $V \leq 8$ mag in the plateau stage.

Somewhat unexpectedly, we find a relatively high rate of WD flashes in systems that in our model have subgiant cold components. However, most of these occur in systems with orbital periods below 1 day (see Figs. 8*b* and 9*b*, in which their position has to be the same as the position of the recurrent SSSs in Algols). This means that, typically, the radii of the subgiants in these systems do not exceed ~$2R_\odot$. Consequently, it would be hard to distinguish between these objects and evolved main-sequence stars. Another observational selection may be related to the fact that orbital periods that are close to 1 day are usually difficult to detect, and this may have prevented the discovery of some subgiant companions to both recent and old novae. A possible overproduction of novae in Algols (in the model) may also be the result of a relatively poor approximation for the mass transfer in Algols. At the same time, we should note that CVs with orbital periods of about a day or more do exist (e.g., V394 CrA [~0.76 days], U Sco [~1.23 days], GK Per [~2.0 days], V1017 Sgr [~5.7 days], V630 Cas [~6.0 days]; Ritter 1990; Warner 1994). Finally, we should note that in quiescence, a typical difference between the absolute magnitude of the subgiant donor and the accretion disk is ~3 mag (see Fig. 3, in which we plotted the distributions over M_v for the donors and accretion disks in the Algol subpopulation underlying the respective group of SSSs). The values of M_v were computed from the luminosities of the Algols (eq. [6]) and applying bolometric corrections from Allen (1973). For the average absolute magnitude of the disk, we used the expression from Webbink et al. (1987):

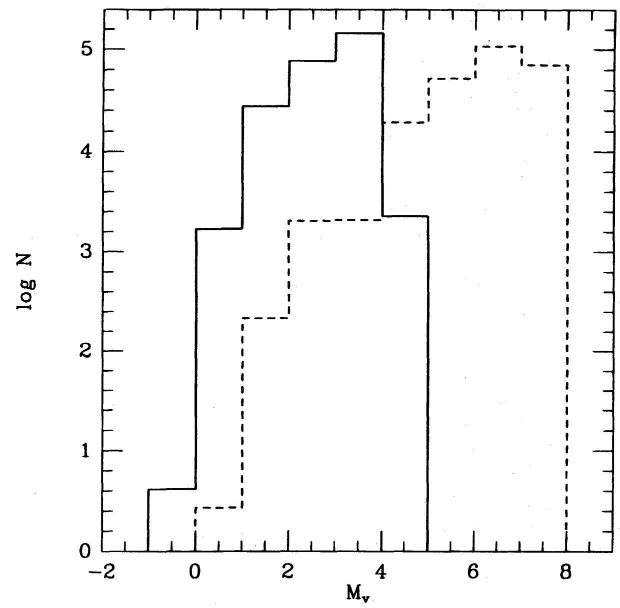

FIG. 3.—Distribution of donors (*solid line*) and accretion disks (*dashed line*) over absolute stellar magnitude in systems with subgiant donors.

$$M_v = -9.48 - \tfrac{5}{3} \log (M_{\text{wd}} \dot{M}) , \qquad (17)$$

where M_{wd} is in $M_\odot$ and $\dot{M}$ in $M_\odot$ yr^{-1}. The difference between the magnitudes of the disk and the donor may be even larger, because equation (17) was derived under the assumption of an infinite disk size, while in real CVs, the effect of a finite size of the disk may increase the value of M_v by ~1 mag (e.g., Paczyński & Schwarzenberg-Czerny 1980; Smak 1989). Dwarf novae with known distances have M_v between 4 and 6.5 mag in eruption (Patterson 1984). Hence, most of the donors have to be much brighter than the disks, even if the latter are unstable and manifest themselves in dwarf novae eruptions.

We do not consider the possible contribution of the accretion disks to the X-ray flux from CVs and Algols, because, as it can be seen from equation (8), this is important only for a very limited range of the highest mass and/or $\dot{M}$ systems. While extremely important in its own right, it is beyond the scope of the present paper to model disk accretion and to calculate the spectrum of the emergent radiation.

Concerning SySs, Yungelson et al. (1995) have shown that the model for their Galactic population (which is the parent population for the relevant subset of SSSs) is consistent with the observations.

3.2. *General Properties of the Population of SSSs*

The main results of our study are summarized in Tables 1 and 2 and in Figures 4–12.

Table 1 gives the results for models obtained under the assumption that the lifetime in the SSS state may be approximated by $t_{3\text{bol}}$. Table 2 gives the numbers for the

TABLE 1

NUMBERS OF SUPERSOFT X-RAY SOURCES AND THEIR PARENT SYSTEMS (t_{3bol} APPROXIMATION)

Parameter	Cataclysmic Variables	Subgiant Systems	Symbiotic Stars	Double Degenerates	Helium Algols	Planetary Nebulae
Parent Population						
Birthrate (yr^{-1})	0.39×10^{-2}	0.57×10^{-3}	0.47×10^{-1}	0.13×10^{-1}	0.85×10^{-3}	0.11
Total number	9.5×10^6	2.4×10^5	1.6×10^3	1.4×10^8	200	40
Novae (yr^{-1})	30	10	1	...	...	...
Permanent SSSs	130	400	450	70	≤ 200	40
Recurrent SSSs	720	60	160	...	...	...
Number of Sources with Correction for Shielding						
Permanent SSSs	130	400	0 (450)	...	...	...
Recurrent SSSs	670	45	7 (160)	...	...	...
Number of "Detectable" Sources						
Novae (yr^{-1})	0.9	0.3	0.1	...	...	...
Permanent SSSs ($r \leq 2$ kpc)	2	7	0 (9)	1	≤ 4	≤ 1
Recurrent SSSs ($V \leq 8$ mag)	14	1	7 (4)	...	...	...

model in which the lifetime in the SSS state was determined using equations (13) and (14). In Table 1 we also list some properties of the parent populations.

We separate all the SSSs into two main classes: "permanent" sources, in which hydrogen (helium) burns steadily, and "recurrent" sources, in which supersoft X-rays are expected to be emitted for short (up to ~200 yr, Fig. 6) intervals of time, following hydrogen shell burning flashes (these are not to be confused with the known class of X-ray transients which contains neutron star or black hole accretors).

It is important to note that the "recurrence" of supersoft X-ray sources, as considered here, is different from "transience" on a timescale of days to years, which is observed in some SSSs. The latter may be caused, for example, by variations in $\dot{M}$ (van den Heuvel et al. 1992; Southwell et al. 1996) or by variable attenuation of the X-ray emission by circumbinary matter (RDS). Only for *very hot* and/or *very massive* white dwarfs accreting at $\dot{M}$ close to the lower limit of stable H burning, one may expect a limit cycle behavior on a timescale of ~10 yr.

TABLE 2

NUMBERS OF SUPERSOFT X-RAY SOURCES AND THEIR PARENT SYSTEMS (IN THE APPROXIMATION OF HYDROGEN-BURNING TIME)

Parameter	Cataclysmic Variables	Subgiant Systems	Symbiotic Stars
Total number	9.5×10^6	2.4×10^5	1.6×10^3
Novae (yr^{-1})	30	10	1
Permanent SSSs	130	400	450
Recurrent SSSs	350	70	45
Number of Sources with Correction for Shielding			
Permanent SSSs	130	400	0 (450)
Recurrent SSSs	300	50	2 (40)
Number of "Detectable" Sources			
Novae (yr^{-1})	0.9	0.3	0.1
Permanent SSSs ($r \leq 2$ kpc)	2	7	0 (9)
Recurrent SSSs ($V \leq 8$ mag)	5	1	2 (1)

The different parent groups differ in terms of the number of objects that they contain of the two classes. This can be understood as a result of the difference in the masses of the accretors and the mass exchange rates in the parent populations (Figs. 4 and 5).

1. CVs combine predominantly low-mass WDs with a very small fraction of the systems with $\dot{M} \gtrsim 10^{-7}\ M_\odot\ yr^{-1}$ and therefore, in CVs, recurrent sources dominate over permanent sources. About 60% of all CVs initially experience mass exchange on a thermal timescale and afterward enter a mass transfer phase controlled by AML, when q decreases to 1.2. The rest have $q \leq 1.2$ initially.

2. In Algols, the fraction of systems with $\dot{M} \gtrsim 10^{-7}\ M_\odot\ yr^{-1}$ is relatively higher than in CVs, and their WDs are of moderate and high masses. Consequently, the conditions

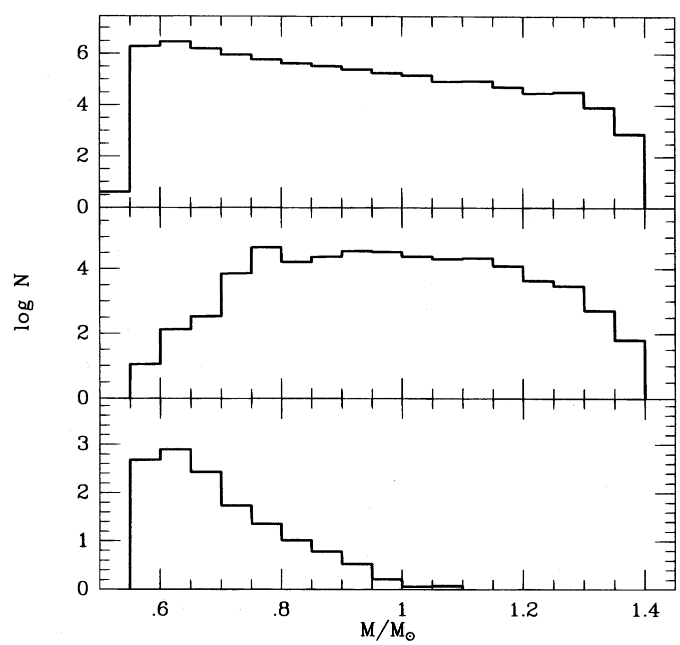

FIG. 4.—Distribution of WDs over mass in cataclysmic variables (*top panel*), systems with subgiant donors (*middle panel*), and in symbiotic binaries (*bottom panel*).

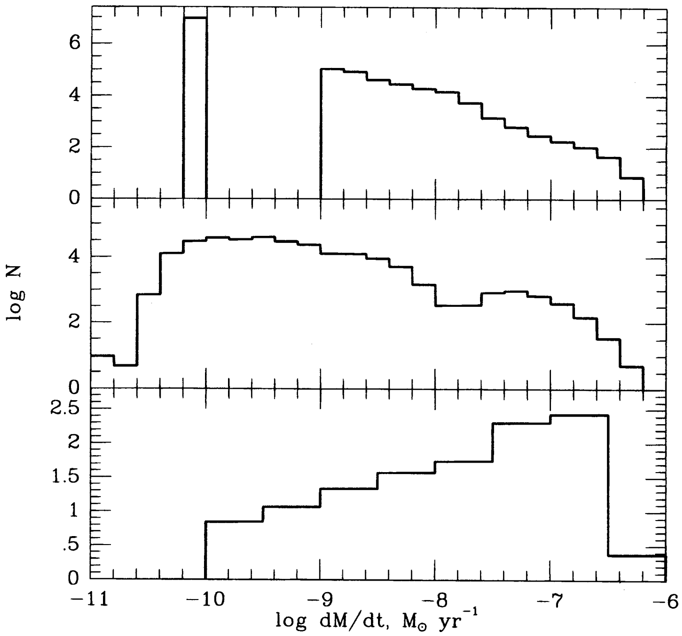

FIG. 5.—Distribution of systems over the mass-loss rates by the donors in cataclysmic variables (*top panel*), systems with subgiant donors (*middle panel*), and over the accretion rate in symbiotic binaries (*bottom panel*).

are more favorable for stable burning, which is also much more efficient in the production of SSSs (see Fig. 2*b*). Therefore, Algols produce predominantly permanent SSSs. The low mass transfer rate "tail" in Fig. 5*b* produces recurrent sources rather efficiently because of the high masses of the accretors.

3. Symbiotic stars almost do not contain massive WDs (Yungelson et al. 1995). This is because of mass loss, which moves the components apart. Large separations, and hence low accretion rates, prevent systems with the most massive white dwarfs from becoming efficient producers of the symbiotic phenomenon. High accretion rates result in a high proportion of permanent sources, while the low $\dot{M}$ "tail" again produces novae, and consequently, recurrent sources.

To summarize, a relatively small number of systems with high $\dot{M}$ are able to produce permanent SSSs very effectively. Low-$\dot{M}$ systems are always responsible for novae (and thus for recurrent sources).

Figures 6 and 7 show the numbers of systems versus lifetimes as SSSs ($T_{\rm on}$) for all three basic groups of binary sources, for the two approximations for the lifetimes in the SSS state after eruption (*thick solid lines*). The lifetimes of the recurrent sources in the active stage are determined by the mass spectrum of the accretors: SySs (with the lowest masses of the accretors) have the longest $T_{\rm on}$, typically longer than 100 yr, then follow CVs with $T_{\rm on} \sim 100$ yr, and Agols with $T_{\rm on}$ between 30 and 100 yr. Concerning the lifetimes of the permanent sources, the most slowly evolving CVs have the longest $T_{\rm on}$, then follow the Algols, and SySs. The latter have relatively shorter $T_{\rm on} \sim 10^5$ yr, because the efficient producers of SySs have to be the fastest evolving AGB stars.

A comparison of the results for the two approximations for $T_{\rm on}$ shows, as expected, that the approximation based on the mass of the hydrogen envelope gives much shorter $T_{\rm on}$ for the least and the most massive accretors. Therefore, (1) the total number of recurrent sources is lower, and (2) the

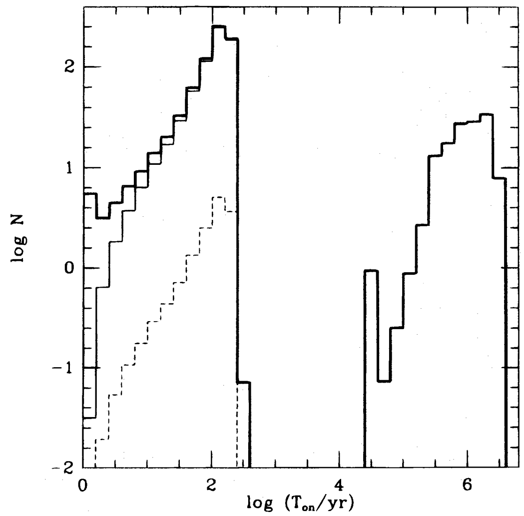

FIG. 6*a*

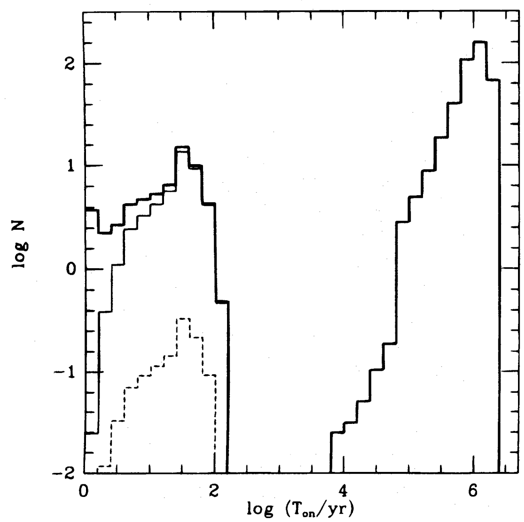

FIG. 6*b*

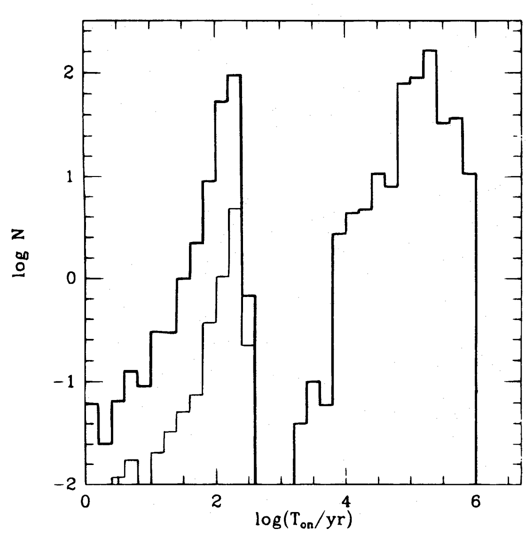

FIG. 6*c*

FIG. 6.—Distribution of SSSs over the "on" times (in the $t_{3\rm bol}$ approximation for recurrent sources, eq. [12]). (*a*) Cataclysmic variables; (*b*) systems with subgiant donors; (*c*) SySs. *Thick solid line*, All sources; *thin solid line*, recurrent sources with shielding effects taken into account; *thick broken line*, recurrent sources in the "detectable" sample.

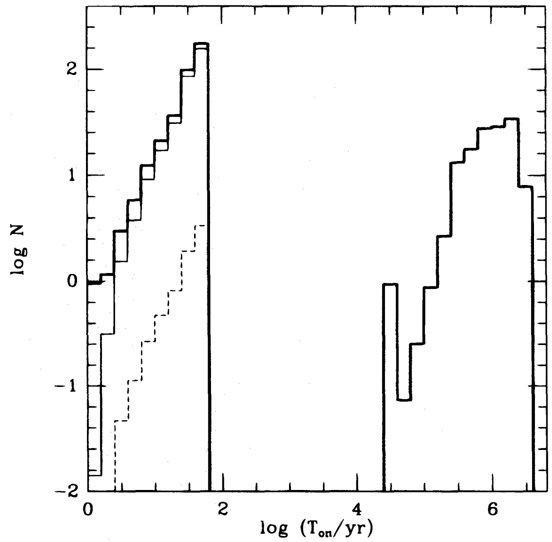

FIG. 7*a*

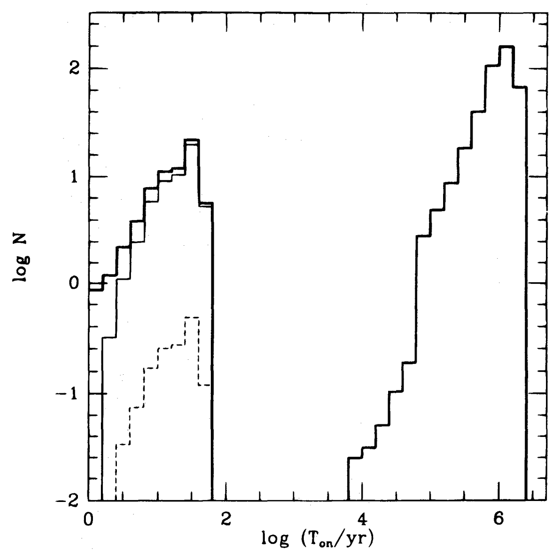

FIG. 7*b*

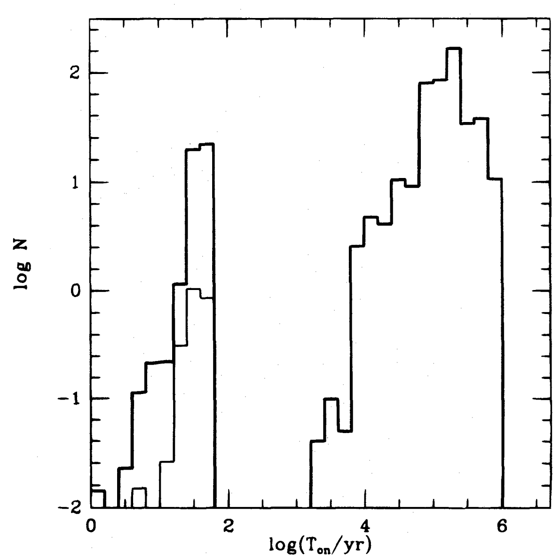

FIG. 7*c*

FIG. 7.—Same as in Fig. 6 for the approximation of hydrogen-burning shell (eq. [14]).

least influenced are the Algols, which contain WDs of intermediate mass.

The following points hould be noted. It is possible that in using the calculated t_{3bol} for *all* systems, we overestimate the relative number of SSSs resulting from CVs as compared to the number of SSSs originating in SySs. The reason for this would be that while CVs experience a CE phase following nova explosions, which could lead to enhanced mass ejection and a decrease in the amount of hydrogen available for burning, SySs do not pass through a CE phase.

Figures 8 and 9 show the distribution of model SSSs over orbital periods for the two approximations for the lifetimes of SSSs. Permanent sources appear in two distinct ranges of periods: 6–60 hr, with a pronounced peak close to 10–20 hr, and 300–20,000 days, with a strong peak at 500–3000 days.

Recurrent sources also form several well separated groups. In CVs, they are almost evenly (within factor ~ 2) distributed between the "minimum" period of CVs (slightly over 1 hr) and 8 hr. Our assumption of a constant mass-loss rate below the period gap may be the reason for a certain overproduction of the shortest period systems. Algols cover the period range between 10 hr and 100 hr (with a "tail" to 300 hr), and in SySs the recurrent sources have periods between 100 and 10,000 days. In general, the recurrent sources span a wider range of orbital periods than the permanent ones. Interestingly, in CVs the range of the recurrent sources is almost not overlapping with the range of the permanent sources. In Algols and SySs, permanent and recurrent sources may be found in the same interval of periods. This happens because the periods and mass transfer rates in these systems are not a function of the donor mass only, as they are in CVs.

The two different approximations for the lifetimes of the SSSs result only in differences in their numbers, but do not influence their period distribution.

Figures 10 and 11 show the distribution of the WDs in SSSs over masses. A comparison with Figure 4 shows, again, that SSSs are produced mainly by the lower mass members of both groups. Permanent sources typically have masses in the 0.6–0.7 $M_\odot$ range, because the combination of a low-mass accretor with a high mass transfer rate favors their formation in all three types of systems. The peak of the distribution for the recurrent sources in CVs and SySs is at 0.60–0.65 $M_\odot$, more pronounced in the former systems, because of the existence of numerous CVs with very low $\dot{M}$ values below the gap (Fig. 5). In Algols the distribution is more even, because of the more homogeneous distributions of parent systems both over M_{wd} and $\dot{M}$ (Figs. 4 and 5). The approximation that uses the hydrogen envelopes for the lifetimes favors the formation of low-mass SSSs.

There exists a certain "evolutionary" difference between the SSSs in CVs and Algols and in SySs: in the two former classes, sources may be initially stable and afterward enter the regime of unstable hydrogen burning, while in SySs, sources are initially always unstable. It is not clear whether this difference in the evolutionary histories may influence the properties of sources.

3.3. *The "Detectable" Sample of SSSs*

Interstellar absorption of X-rays effectively prevents the discovery of Galactic SSSs at distances exceeding $r \approx 2$ kpc (Motch et al. 1994). Di Stefano & Rappaport (1994) accomplished a detailed modeling of the effect of interstellar absorption on the detectability of Galactic and extra-

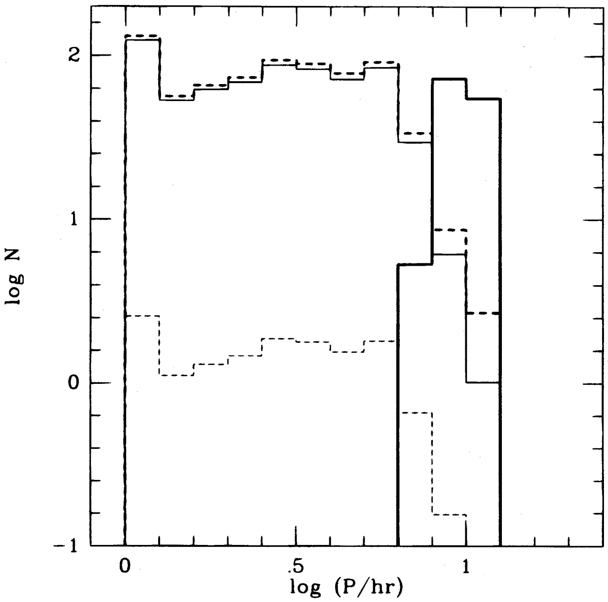

FIG. 8*a*

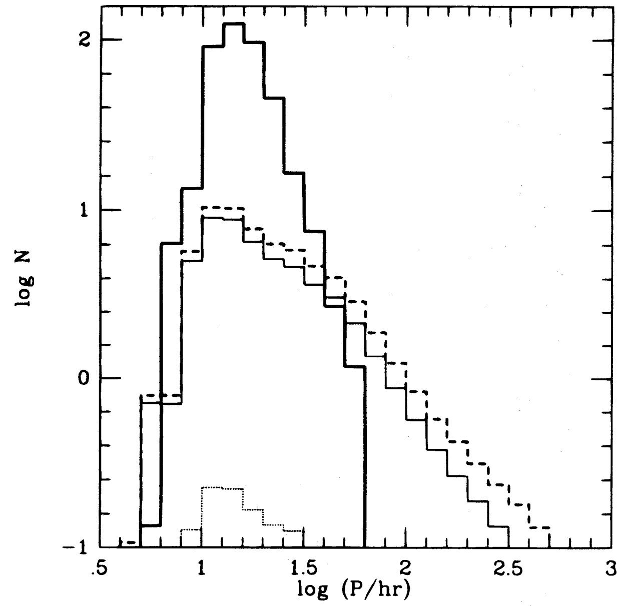

FIG. 8*b*

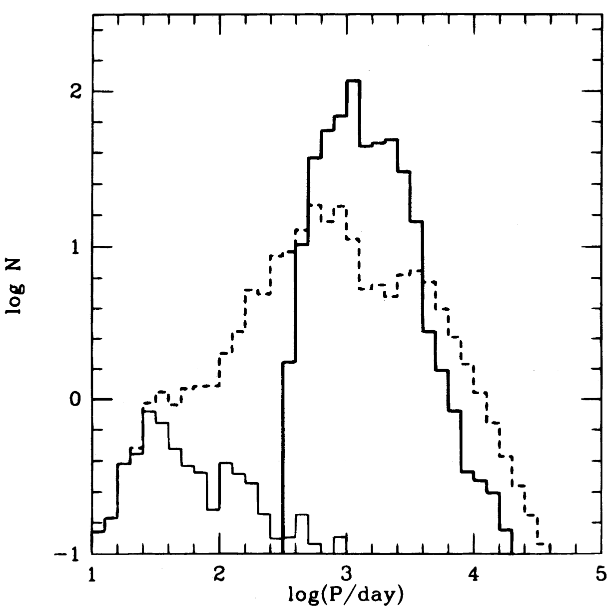

FIG. 8*c*

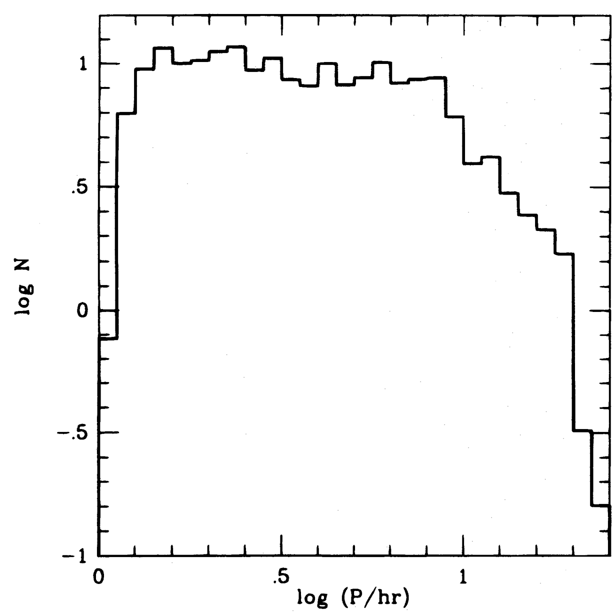

FIG. 8*d*

FIG. 8.—Distribution of SSSs over orbital periods (in the $t_{3\mathrm{bol}}$ approximation for recurrent sources, eq. [12]). (*a*) Cataclysmic variables; (*b*) systems with subgiant donors; (*c*) symbiotic stars; (*d*) helium Algols. *Thick solid line*, Permanent sources; *thick broken line*, recurrent sources; *thin solid line*, shielded sources; *thin broken line*, "detectable" sources.

galactic SSSs in the *ROSAT* all-sky survey. Even in their least restrictive models, the calculated fraction of possible detections does not exceed 0.01.

It should also be recognized that in most sources intrinsic absorption is present. Having this in mind, we attempted to construct a "detectable" sample of the SSSs, taking into account possible selection effects.

For the recurrent sources in CVs and Algols, the most obvious source of intrinsic absorption may be the shielding of sources by the envelopes ejected during novae eruptions. In order to calculate the influence of this shielding properly, one has to model the propagation of the X-ray radiation through the expanding nova shell, a task that is beyond the scope of the present study. Therefore, we performed an exercise that shows the effect, at least qualitatively. We assumed that the *spherical* nova shell effectively shields the source until the optical depth to radiation at 400 eV becomes less than unity (for the classification of a source as an SSS, all the energy has to be radiated below this threshold; e.g., Hasinger 1994; Kahabka & Trümper 1995). Then, we computed the corresponding reduction in "on" time.

We estimated the optical depths of novae shells using the values of the ejected mass ΔM_{ej} as a function of the mass of the WD and the accretion rate (§§ 2.3 and 2.4). For the opacity of the ejected matter, we used the analytical fit to the interstellar photoelectric absorption cross sections in the 284–400 eV range (Morrison & McCammon 1983). As a typical ejection velocity, we assumed $v = 1000$ km s^{-1},

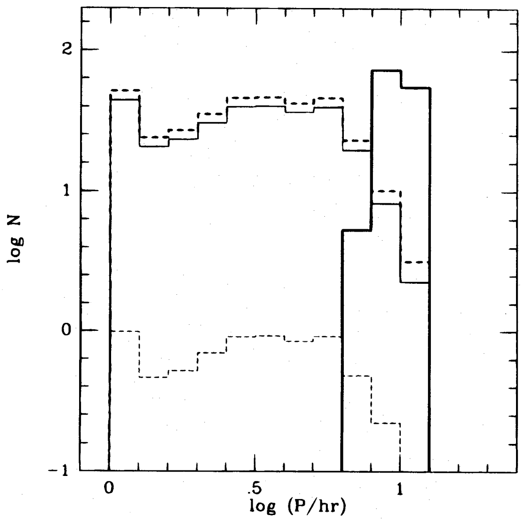

FIG. 9*a*

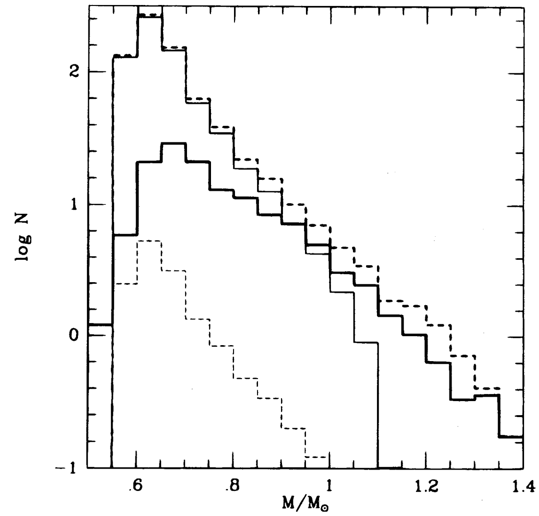

FIG. 10*a*

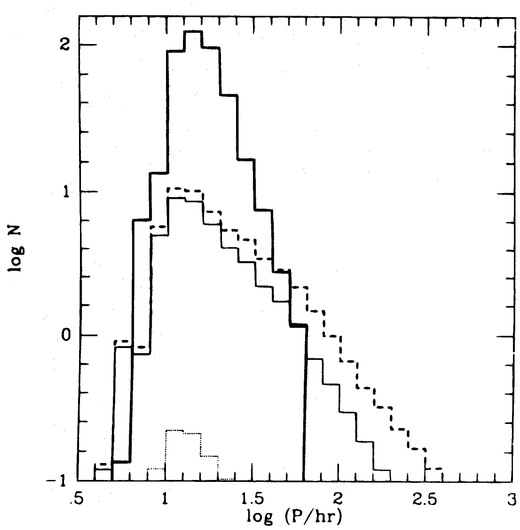

FIG. 9*b*

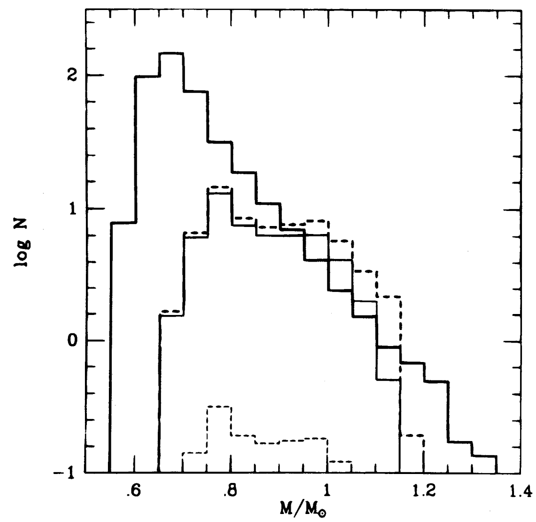

FIG. 10*b*

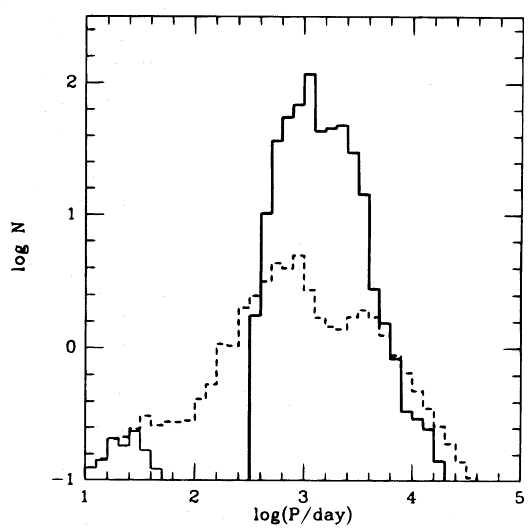

FIG. 9*c*

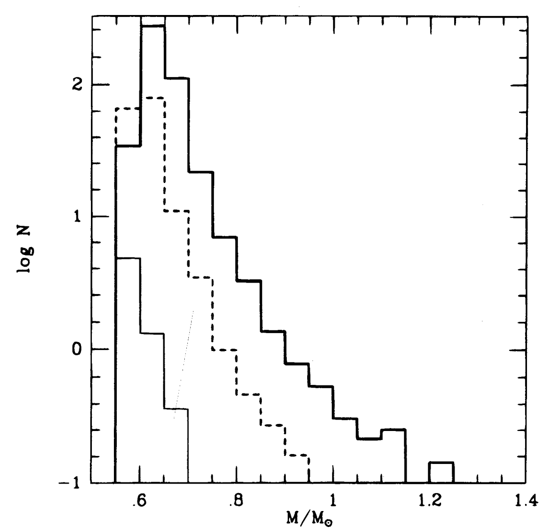

FIG. 10*c*

FIG. 9.—Distribution of SSSs over orbital periods (in the approximation of hydrogen-burning shell, eq. [14]). (*a*) Cataclysmic variables; (*b*) systems with subgiant donors; (*c*) symbiotic binaries. *Thick solid line*, Permanent sources; *thick broken line*, recurrent sources; *thin solid line*, shielded sources; *thin broken line*, "detectable" sources.

FIG. 10.—Distribution of SSSs over the masses of the accretors (in the t_{3bol} approximation for recurrent sources, eq. [12]). (*a*) Cataclysmic variables; (*b*) systems with subgiant donors; (*c*) symbiotic binaries. *Thick solid line*, Permanent sources; *thick broken line*, recurrent sources; *thin solid line*, shielded sources; *thin broken line*, "detectable" sources.

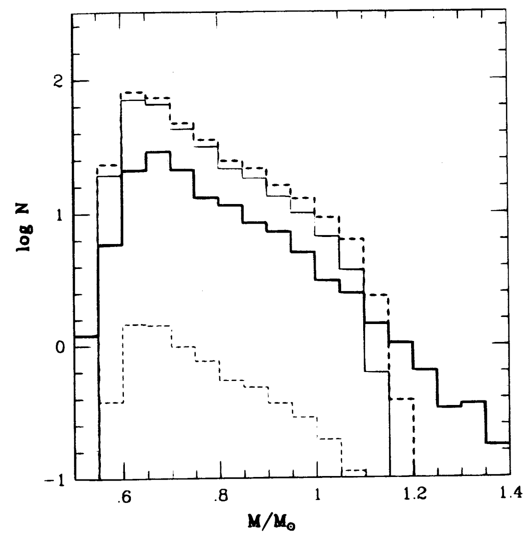

FIG. 11*a*

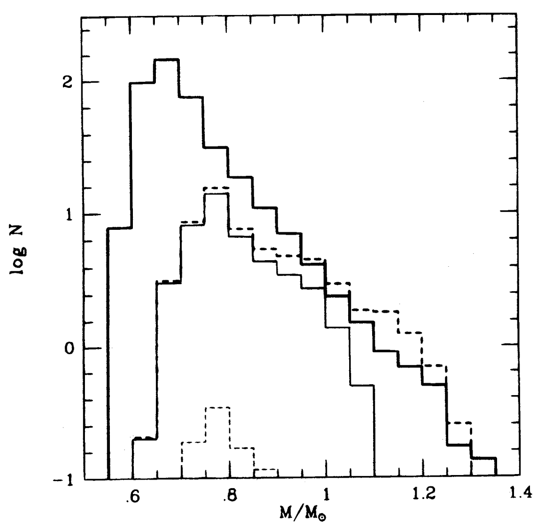

FIG. 11*b*

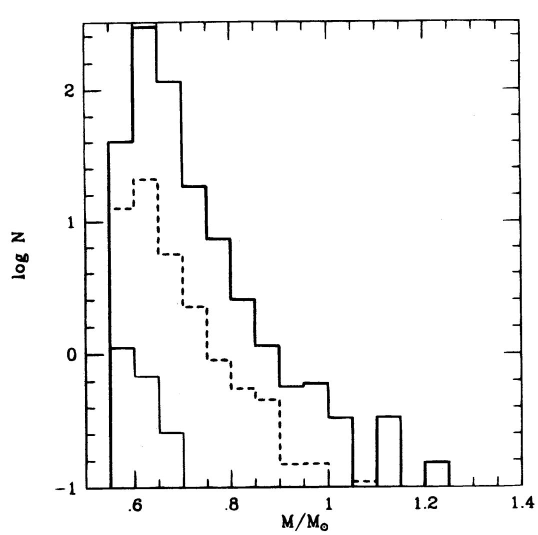

FIG. 11*c*

FIG. 11.—Distribution of SSSs over the masses of the accretors (in the approximation of hydrogen-burning shell, eq. [14]). (*a*) Cataclysmic variables; (*b*) systems with subgiant donors; (*c*) symbiotic binaries. *Thick solid line*, Permanent sources; *thick broken line*, recurrent sources; *thin solid line*, shielded sources; *thin broken line*, " detectable " sources.

probably on the high side of the real range of observed v. The ejection was considered as an instantaneous event (Fig. 12 illustrates the effects of different values of v, see below).

Figure 12 shows the effects of intrinsic shielding on the possibility of detection of SSSs. The solid curve represents the dependence of $t_{3\text{bol}}$ on the mass of the WD (eq. [12]), while the thick broken line shows the dependence of the lifetime of the source on the mass of the WD computed by means of equations (13) and (14). The thin solid lines show the " shielding " times t_{sh}, as a function of the mass of the WD, under the assumption that a critical ignition mass (eq. [10]) is ejected. Plotted are the shielding times for $v = 500$ and 1000 km s^{-1} (upper and lower curves, respectively). The " observed " lifetime of a source with a given WD mass after each explosion is limited by the interval between the thin line t_{sh} and the thick " lifetime " lines. It appears that intrinsic absorption may be very important for high-mass WDs, which have short lifetimes in the hot state and are eroded by explosions. In the given example, a source with a mass exceeding $\sim 1.15\ M_\odot$ will never be observed. Even at 1 $M_\odot$, shielding effectively cuts off the observed lifetimes of all the systems by 25%–100%. Shielding becomes unimportant only for lower mass (0.6–0.8 $M_\odot$) WDs.

Figure 12 may explain why only one SSS was found in a survey of 26 novae, which exploded in the last 10 yr (Orio 1993). Short lifetimes plus shielding exclude from the

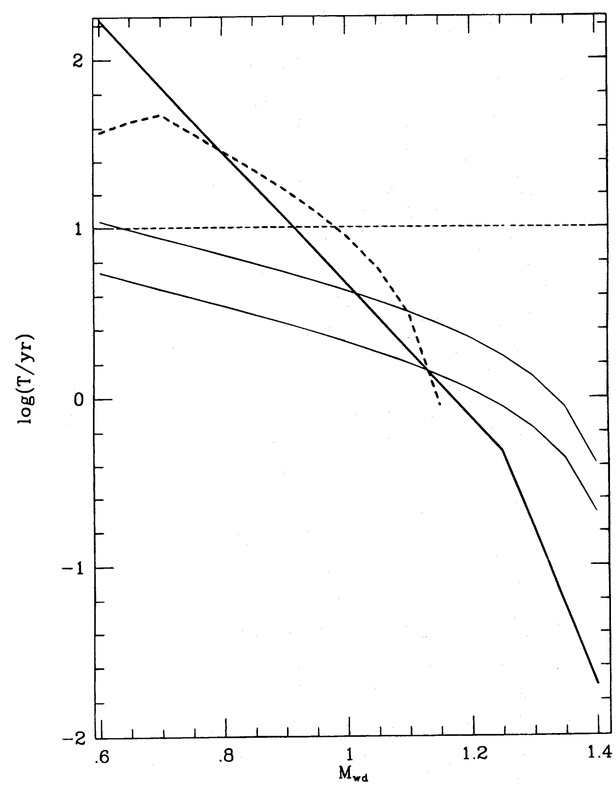

FIG. 12.—Dependence of $t_{3\text{bol}}$ (eq. [12]) on the mass of the WD (*thick solid line*); the dependence of the lifetime of an SSS on the mass of the accretor in the " hydrogen shell approximation (eq. [14]) " (*thick broken line*); shielding times for $E = 400$ eV and $v = 500$ km s^{-1} and 1000 km s^{-1} (*upper and lower thin continuous curves*, respectively). For the latter estimate, it is assumed that a mass equal to the critical ignition mass of the layer is instantaneously ejected and that the ejectum is spherically symmetric. The thin broken line is drawn to illustrate that most of the SSSs were possibly lost in the survey of novae, restricted to objects that exploded over the recent 10 yr.

sample the most massive objects, which dominate the sample of the observed novae because of selection effects (Ritter et al. 1991). While a typical WD mass in observed nova systems is $\sim 1\ M_\odot$, in detectable model SSSs it is between 0.6 and 0.8 $M_\odot$ (see Figs. 10 and 11). Thus, there may be a higher chance to find an SSS in a survey of relatively old novae. Limiting the survey to the most recent 10 yr may result in missing most SSSs. The high-mass systems would be missed because they switch off before the envelope becomes transparent to supersoft X-rays, and the low-mass ones would be missed because it takes ~ 10 yr for their envelopes to become transparent.

The estimated numbers of SSSs in the model samples which took into account the shielding appear in Tables 1 and 2 under the heading "Number of Sources with Correction for Shielding." In Figures 8–11 they are shown by the thin solid lines. Obviously, only the recurrent sources are influenced by the considered effect. For the permanent sources, obscuration by the accretion disks may influence their detectability. We do not consider this effect here.

Shielding cuts off mostly Algols, with more massive white dwarfs (by about 20%), and to a lesser extent (7%–15%) CVs. Interstellar absorption reduces the detectability further, in a way discussed by Di Stefano & Rappaport (1994).

For symbiotic systems, the possibility of discovery of the SSSs is more complicated, because both the nova ejecta and winds from the cool components act as shielding agents. We treat these shielding causes separately, to illustrate their potential effects. Again, we assume that the ejected shells or donor wind are spherically symmetric.

For shielding by the wind, we computed the optical depth at 400 eV in the vicinity of accretor, treating the wind according to the algorithm described in Yungelson et al. (1995). The outcome is quite dramatic: all the permanent sources have an optical depth $\tau > 1$ at the distance of the accretor from the donor and appear to be completely shielded. Only about 4% of the recurrent sources remain observable. The numbers of sources are given in Tables 1 and 2 (without parentheses).

As another extreme, we considered only shielding by the nova ejecta. The effect of shielding appears unimportant in the $t_{3\rm bol}$ approximation and quite weak in the hydrogen shell burning approximation. This is a consequence of the dominance of low-mass WDs in SySs. The relevant numbers are given in the tables in parentheses.

The situation with symbiotic systems obscured by winds is somewhat paradoxical. For steady hydrogen burning, the accretion rate onto the dwarf has to be $\sim 10^{-7}\ M_\odot\ {\rm yr}^{-1}$. This implies a high rate of mass loss by the donor, because, typically, the efficiency of accretion from a wind is $\sim$1%–10%. Thus, steady burners in symbiotics are produced mainly by donors that are at the top of the RG or AGB branches. On the other hand, for these high mass-loss rates the absorption is most efficient! From Bondi's (1952) formula for the (supersonic) accretion rate, it follows that the number density of matter in the vicinity of the accretor is $n_{\rm H} \approx 2 \times 10^7\ \dot{M}_a V^3 M_{\rm wd}^{-2}\ {\rm cm}^{-3}$, where $\dot{M}_a$ is in units of $10^{-7}\ M_\odot\ {\rm yr}^{-1}$, the relative velocity V is in 10 km s^{-1}, and the mass of the white dwarf in $M_\odot$. It is thus easily seen that a column density of $\sim 10^{21}$–10^{22} cm^{-2}, which is sufficient for a total obscuration of the source, may be readily accumulated. More sophisticated estimates by Nussbaumer & Vogel (1987) show that when the mass-loss rate from the donor $\dot{M}_d$ exceeds a certain limit, the ionization zone around the hot star becomes "closed," with a characteristic size less than the orbital separation. Such circumstances do not give the X-ray photons much chance to leak out of the system. Trial calculations with a velocity law of the form $V = V_\infty(1 - R/r)$, where r is the distance from the donor and R is the radius of the donor, show that for $T_{\rm wd} = 300{,}000$ K, the limiting $\dot{M}_d$ is $\sim 10^{-6}\ M_\odot\ {\rm yr}^{-1}$ (D. Bisikalo & O. Kuznetsov 1996, private communication). This limit depends weakly on R, but it is a sensitive function of $T_{\rm wd}$. On the other hand, one may expect the distribution of matter in symbiotic systems to be highly inhomogeneous (Bisikalo et al. 1994), and so the radiation may escape nonuniformly.

Moreover, the actual picture of symbiotic systems is probably complicated by the interaction between the winds of the hot and cool components (e.g., Dgani, Walder, & Nussbaumer 1993; Stevens, Blondin, & Pollock 1992). In particular, the wind from the WD may occasionally suppress accretion (e.g., Inaguchi, Matsuda, & Shima 1986). Consequently, the following pattern of evolution may emerge. At relatively low values of $\dot{M}$, a recurrent source will be obtained. The flashes will become less violent with increasing $\dot{M}$. At some point, steady burning will be obtained, however, further accretion may be suppressed at that stage. The accreted layer will burn until its mass is reduced below the extinction threshold (Iben 1982). Accretion will resume at that point and the process will repeat itself after the accretion of a critical layer for H ignition. The outcome of a picture of this type would be that SySs do not produce truly permanent sources; however, the recurrent sources described above may be indistinguishable from the permanent ones from an observational point of view (unless the turn-on phase happens to be observed).

As the next step toward the generation of the sample of "detectable" sources, we modeled the sample that may be optically identified. The relevant numbers enter Tables 1 and 2 under the heading "Number of Detectable Sources."

For the detection of supersoft X-ray emission, the source has to be closer than ≈ 2 kpc (Motch et al. 1994) to the Sun. Hence, only about 4/225 of the sample with shielding may be observed. The resulting numbers for the permanent sources are in the single digits. At the distance of 2 kpc, the accretion disk in a persistent source (for $M_{\rm wd} = 0.6\ M_\odot$, $\dot{M} = 10^{-7}\ M_\odot\ {\rm yr}^{-1}$) will have an apparent stellar magnitude of about 16.25 mag. (We assumed $A_V = 1.1$ mag kpc^{-1} after Duerbeck 1990). This is sufficient for optical identification. Algol-type systems may be somewhat brighter, because the subgiant also contributes to the total luminosity of the system.

For the identification of a recurrent source with a nova, we apply the requirement for the nova to have $V \leq 8$ mag in the plateau luminosity stage. This restriction results from the observation that the number of annually detected novae rapidly declines beyond 8 mag (Duerbeck 1990). The numbers again are in the single digits, but the number of "detectable" postnova SSSs is perhaps somewhat higher than is suggested by the observations. This is probably a consequence of the following two factors. First, the values of $t_{3\rm bol}$ used should definitely be regarded as upper limits. This is particularly true if the CE phase enhances mass loss considerably and the wind from the hot star is effective. Second, in some of the systems, a particular combination of the mass of the WD and $\dot{M}$ may result in a flash that never shows up in the optical part of the spectrum, while it still produces an X-ray source (EUV/soft X-ray novae; e.g., Shara et al. 1977; Sion & Starrfield 1994; Truran et al. 1995).

For the SSSs in symbiotic systems, the "detectable" sample depends more on the assumptions about the shielding of the source. However, if the winds of the giants are really the dominant shielding factor, one may expect to observe a considerable fraction of all SSSs in symbiotic novae.

In Figures 7–11 we present the distributions for these "detectable" sources (with the exception of symbiotic stars, because of the ambiguity in the treatment of shielding in these sources).

3.4. *Minor Contributors to the Population of SSSs*

In addition to the main families discussed so far, we list in Table 1 other possible contributors to the Galactic population of SSSs. These other contributors may add $\sim 20\%$ of the total number of SSSs.

1. There may exist semidetached systems with He WD donors, stably transferring mass. In the initial phase of mass transfer, when $\dot{M} \sim 10^{-6}$ to 10^{-5} $M_{\odot}$ yr^{-1} (Tutukov & Yungelson 1979), these systems may manifest themselves as SSSs. The model suggests that there may be ~ 70 systems with mass exchange rates above $10^{-6.5}$ $M_{\odot}$ yr^{-1}, close to the limit given by equation (15), which may burn He stably or in mild flashes. Additionally, there are 35 sources that simultaneously have accretion disks that are hot enough to emit supersoft X-rays (eq. [8]). Both numbers are listed in Table 1. The orbital periods predicted for these systems are between 2 and 4 minutes. No observational counterparts to these model systems have been detected yet.

2. The next class of potential SSSs consists of systems in which the donors are descendants of stars initially more massive than 5 $M_{\odot}$, which experience stable mass exchange in the so-called BB case, when they have CO cores and thick helium mantles (Iben & Tutukov 1994). In fact, when equations (7) and (15) are applied to model systems generated by our code (which experience RLOF in the BB case), it appears that $\dot{M}$ never exceeds the stability limit given by equation (15), being always slightly below it. One may expect nevertheless that He will burn in relatively weak flashes, and we list in Table 1 the upper limits on the birthrates and numbers of these "helium Algols." The predicted orbital periods of these systems are between 1 and 20 hr (Fig. 8*d*). Again, no observed candidates for their counterparts are known as yet.

3. Finally, we list in Table 1 the number of planetary nebula nuclei more massive than 0.7 $M_{\odot}$, which may spend in the course of their evolution some time at a temperature in excess of 250,000 K and have at that time $L \gtrsim 5 \times 10^3$ $L_{\odot}$. These PNNs are mostly members of wide binaries or products of mergers of the components of close binaries in the common envelope phase. Thus, effectively all of them are single stars. One candidate PNN SSS is known among the extragalactic objects (Hasinger 1994).

In addition to the above families, some supersoft sources may clearly be associated with hot pre–white dwarfs (PG 1159 stars; Cowley et al. 1995b) and with a variety of chromospherically active stars.

3.5. *Discussion of Some Model Assumptions*

There are many uncertainties in the input parameters of the model, which may influence the results. RDS tested the effects of variations of the initial distribution of binaries over the mass ratio of the components q and of the common envelope parameter α_{ce}. These parameters influence the population of SSSs via the properties of the underlying population. A relative increase in the number of initial systems with comparable masses of the components increases the proportion of systems that are able to exchange mass within the limits of stable hydrogen burning. A reduction of α_{ce} results in an increase in the birthrate and number of SSSs in CVs and Algols, mainly because the strip of their progenitors in the "initial mass of the primary–initial separation of the components" plane becomes wider. Test runs with our code confirm these conclusions: e.g., a variation of α_{ce} from 1 to 0.5 approximately doubles the number of sources in CV systems. Similar results were obtained by RDS. In symbiotic systems on the contrary, a reduction of α_{ce} results in a decrease in the number of SSSs via a reduction in the number of objects that can undergo efficient accretion after the common envelope episode (see Fig. 4 and Table 3 in Yungelson et al. 1995).

In order to compare our results with those of RDS, we examined several of the other assumptions that may influence the model population of SSSs. A certain difference may arise from the difference in the assumed conditions for dynamically stable mass exchange and the algorithm for the description of mass exchange (time dependent vs. constant). The reservoirs of mass available for exchange in the two models also differ somewhat. Another source of difference is in the different ranges of $\dot{M}$ for steady H burning. In the RDS model, the upper limit of $\dot{M}$ is ~ 1.3–2 times lower (for M_{wd} between 1.0 and 0.6 $M_{\odot}$). The lower limit is also in RDS ~ 2 times lower. In both models these assumptions are certainly within the range allowed by the current understanding of the theory of stellar (and binary) evolution.

Table 3 summarizes the results of several trial runs of our code for CVs and Algols, under different dynamically stable mass exchange conditions, and different $\dot{M}$ algorithms and $\dot{M}_{steady}$ ranges. The algorithms describing the mass transfer are the one used in our code and the one used by RDS. The latter was applied until $M_1 = M_2$ and assuming 100% efficient conversion of the accreted H into He. Also, no limit on q_{crit} for dynamical mass transfer was applied, because RDS required the donors in candidate systems only not to have radii exceeding the radii at the base of the AGB. The ranges of $\dot{M}_{steady}$ are again the one used by us (from the Iben & Tutukov models) and the one of RDS. (The numbers in the cols. (2) and (8) differ slightly from those given in Table 1 because for the trial runs we used a coarser grid). The numbers are given for a case in which the emitting phase was estimated by t_{3bol}.

Columns (2) and (3) compare the influence of the reduction of the upper limit of q for dynamically stable mass exchange in equation (1) from 2.5 to 2.1 for our $\dot{M}$ algorithm and the range of $\dot{M}_{steady}$ given by Iben & Tutukov. More systems that evolve into RLOF after the common envelope stage are rejected because of high values of q in the second case. Therefore, there are fewer steady burners and fewer recurrent sources that descend from them. The change in q_{crit} does not influence the systems that enter the recurrent regime immediately after RLOF.

Columns (2) and (4) compare for the same q_{crit} and our $\dot{M}$ algorithm the influence of the range of $\dot{M}_{steady}$. For the RDS range of $\dot{M}_{steady}$, fewer systems (which survive the cut by q_{crit}) are rejected because their initial $\dot{M}$ exceeds the upper limit of $\dot{M}_{steady}$. Comparison of runs (3) and (4) with different values of q_{crit} shows that with the RDS range of $\dot{M}_{steady}$, the change in q_{crit} does not introduce significant differences

TABLE 3

COMPARISON OF MODELS WITH DIFFERENT ASSUMPTIONS ON MASS LOSS AND STEADY H BURNING

	CVs						ALGOLS			
PARAMETER (1)	Y; 2.5; I (2)	Y; 2.1; I (3)	Y; 2.5; R (4)	Y; 2.1; R (5)	R; ∞; I (6)	R; ∞; R (7)	Y; 2.5; I (8)	Y; 2.5; R (9)	R; ∞; I (10)	R; ∞; R (11)
ν_{SSS}[a] × 10^3, yr^{-1}	3.9	3.8	3.8	3.8	0.15	0.18	0.51	0.18	3	1.6
Permanent SSSs	130	50	340	340	100	210	350	120	2450	2260
Recurrent SSSs	720	680	680	660	10	10	50	10	140	50
Novae (yr^{-1})	30	24	23	23	12	13	12	8	24	16
$\nu_{SN\,Ia}$[b] × 10^5 (yr^{-1})......	3	3	3	3	4	5	0.9	0.8	8	6

NOTE.—The combinations of symbols in the headings of the columns have the following meanings: first symbol, mode of mass loss simulation—Y = present model, R = RDS model; second symbol, upper limit of q for thermally stable mass transfer—∞ means no limits on q; third symbol, range of accretion rates that allow steady hydrogen burning—I = Iben & Tutukov 1989, R = RDS.

[a] ν_{SSS} = birthrate of SSSs.

[b] ν_{SNIa} = the rate of SN Ia's.

in the results: their upper limit on $\dot{M}_{steady}$ cuts systems with high $\dot{M}$ more efficiently.

The RDS algorithm for $\dot{M}$ implies lower values of $\dot{M}$ than ours. Therefore, for the same (Iben & Tutukov) range of $\dot{M}_{steady}$, the runs presented in columns (2) and (6) produced comparable numbers of steady burners. If we apply the RDS algorithm for $\dot{M}$ and switch from the Iben & Tutukov range of $\dot{M}_{steady}$ to the RDS range (cols. [6] and [7]), the birthrate and number of steady burning SSSs more than double, similar to the effect produced with our $\dot{M}$ algorithm (cols. [2] and [4]). Test runs for the RDS algorithm of $\dot{M}$ showed that it is influenced neither by q_{crit} nor by the limit imposed by the existence of deep convective envelopes. The most stringent restriction for the RDS models results from the existence of an upper limit on $\dot{M}_{steady}$.

A comparison of the models presented in columns (8) and (9) for Algols, computed with our algorithm for $\dot{M}$, but for different ranges of $\dot{M}_{steady}$, shows that for higher initial $\dot{M}$ values than in CVs, we are able to obtain more steady burners for the Iben & Tutukov range than for the RDS range. This is a consequence of the fact that fewer systems are cut at the upper border of the $\dot{M}_{steady}$ strip. This result is not influenced by variation in q_{crit} or the existence of deep convective envelopes.

Finally, columns (10) and (11) compare model samples computed with the RDS algorithm for $\dot{M}$ and different ranges of $\dot{M}_{steady}$. In fact, model (11) combined with model (7) is the closest in its assumptions to the RDS models with a flat distribution over the initial mass ratios. For $\dot{M}$ values that are typically a few times lower than in our model, but still relatively high, models (10) and (11) produce a similar number of permanent SSSs. These numbers are much higher than in models (8) and (9), because fewer candidate systems are cut at the upper border of the $\dot{M}_{steady}$ strip.

At variance with RDS, we considered accreting CO or ONe white dwarfs as potential sources of supersoft X-rays, irrespective of their mass. In our model samples, typically 75%–80% of the systems had $M_{wd} \geq 0.65\ M_\odot$.

There is a difference in the normalization of the two models. RDS derive from the constant stellar birthrate model of Miller & Scalo (1979) a rate of formation of binaries with $1.8 \leq M_1/M_\odot \leq 8$ of 0.3 yr^{-1}. They assume that 50% of all stars are binaries. The percentage of close binaries in their sample is 0.36. With these numbers, the birthrate of steady burning SSSs in the RDS model is (0.7–1.0) × 10^{-3} yr^{-1} (for the "standard" model and model with magnetic braking). Our model assumes that all the stars are born in binaries. The percentage of close binaries in it is similar to that of RDS: ∼0.39. If we reduce by a factor of 2 the birthrate and number of systems in the sum of our model samples listed in columns (7) and (11) of Table 3, which are the closest to the RDS model, we arrive at a more than satisfactory agreement with their results on the number of permanent sources. The period ranges covered by the steady burners in both models are similar. The rate of AICs in the steady burning SSSs in models (10) and (11) is lower, but still comparable to that in the most restrictive RDS model: (3–4) × 10^{-5} yr^{-1} (after a renormalization by a factor of 2). The difference may be due to the different percentage of the most massive white dwarfs in the two models.

Interestingly, the data in Table 3 show that systems that exchange mass on a thermal timescale are able to produce a nonnegligible rate of novae, even if they are restricted by $q > 1$.

4. SUMMARY AND CONCLUSIONS

1. We have modeled the Galactic population of supersoft X-ray sources with white dwarf accretors. To achieve this goal, we investigated systematically possible situations in which a steady or unsteady hydrogen or helium burning at the surface of an accreting white dwarf may be expected.

2. We have shown that there exist three main sub-populations of such systems: cataclysmic variables with main-sequence donors, cataclysmic variable–like objects with subgiant donors, and symbiotic stars. Other systems, such as central stars of planetary nebulae or semidetached systems with helium donors, may also contribute to the observed sources, although less significantly.

3. Each of the main classes of SSSs contains both white dwarfs that burn hydrogen steadily at their surface and white dwarfs that experience recurrent hydrogen shell flashes. Due to differences in the initial state of the systems upon formation, the proportion of "steady" and "unsteady" burners (respectively, "permanent" and "recurrent" sources) are different in every class of objects. The number of recurrent sources in an "on" state is a sensitive function of assumptions about the duration T_X of the stage in which, after a thermonuclear runaway and a possible common envelope stage, the white dwarf still has $T_{eff} \gtrsim 250{,}000$ K, favorable for the emission of supersoft X-rays. Our estimates, based on two different approaches for the determination of T_X, may be considered as upper and lower bounds to the number of SSSs.

4. The ranges of orbital periods covered by the main three classes of objects overlap with the period ranges of observed SSSs with known orbital periods. Every model class of objects has its counterparts among the discovered Galactic SSSs. For example, GQ Mus ($P_{orb} \approx 85$ minutes), which was detected in supersoft X-rays nine years after outburst (Ögelman et al. 1993) and shortly afterward switched off (Shanley et al. 1995), and V 1974 Cyg ($P_{orb} \approx 1.95$ hr; Krautter et al. 1995; DeYoung & Schmidt 1994) are clearly short-period cataclysmic systems, and indeed in the CVs the "recurrent" sources have to dominate. The sources RX J095.7−4756 with $P = 3.5$ days (Motch et al. 1994), RX J0019+21 with $P = 15.85$ hr (Reinsch, Beuermann, & Thomas 1993; Beuermann et al. 1995), and RX J0513−69 with $P \simeq 18.3$ hr (Southwell et al. 1996) may be identified with systems with subgiant donors. The symbiotic stars RR Tel (Jordan, Mürset, & Werner 1994) and AG Dra (Hasinger 1994), again, confirm the contribution of this class. The nature of the globular cluster source 1E 1339.8−2837 (Hertz, Grindlay, & Bailyn 1993) is less clear, but its variability and high intrinsic absorption suggest that it may be a recurrent EUV/soft X-ray source (Kahabka & Trümper 1995). Therefore, it may correspond in our model to CVs or Algols with relatively massive accretors and high $\dot{M}$ (but below the limit of steady burning) or may contain a very hot accretor (Sion & Starrfield 1994).

The relatively low fraction of probable recurrent sources among the detected SSSs (only one postnova) may indicate that a significant fraction of the matter accumulated by the WDs between outbursts is lost because of the action of frictional energy deposition during the CE phase, thus seriously reducing their lifetimes in the "on" state. Another factor that may reduce the lifetime in the SSS state is the stellar wind of the hot star. Since the wind mass-loss rate can constitute a significant fraction of the rate at which hydrogen is being burnt, its effect could be considerable.

5. We have shown that, while interstellar absorption plays the major role in determining the number of detectable SSSs (Motch et al. 1994; Di Stefano & Rappaport 1994), intrinsic absorption in the binary system may also seriously limit the number of observed sources, especially in symbiotic stars.

6. The total predicted number of Galactic SSSs (~ 2000) is consistent with the estimates inferred by scaling the numbers of SSSs in other galaxies. If this population is convolved with the limitations on the observability of SSSs imposed by interstellar (and intrinsic) absorption, it appears that 20%–30% (depending on model assumptions) of all the Galactic SSSs that can be potentially detected have already been found. Uncertainties in the input parameters and assumptions may influence the estimated incidence of SSSs to within a factor ~ 2–3, still leaving it compatible with observational estimates, which themselves are highly uncertain.

7. The model predicts that $\lesssim 20\%$ of all SSSs may have helium-rich donors (Table 1). While no such source has been detected yet, it is too early to conclude that these sources do not exist.

8. Our model allows us to estimate the rate of expected SN Ia's in the systems considered. The rate at which the WDs reach the Chandrasekhar mass in CVs is 2.2×10^{-5} yr^{-1}, in Algols 0.6×10^{-5} yr^{-1}, and in helium Algols 0.7×10^{-5} yr^{-1}. Thus, all of these systems do not appear to contribute significantly to the estimated Galactic rate of SN Ia's (~ 0.003 yr^{-1}). Furthermore, some of these events may in fact result in AICs, if the accretors are ONe white dwarfs (e.g., Livio 1995). The only systems in which the accumulation of a critical He shell occurs under conditions favorable for detonation are the Algols. In our highly simplified model, detonations of He in a shell occur at the rate of 0.3×10^{-3} yr^{-1}. Thus, the model predicts a rate of potential SN Ia events that is lower than the observational estimate. However, since large uncertainties exist in both the calculations and in fact in the estimate for the Galactic SN Ia rate (e.g., Branch et al. 1995), SSSs cannot be excluded as potential contributors to SN Ia's. It is worthwhile in this respect to emphasize the importance of studying the conditions under which accreting white dwarfs can accumulate helium and the conditions for explosive helium burning.

9. The main differences between our calculations and those of RDS are in the following points.

a) We model *several* potential classes of systems, accreting either via RLOF or via a stellar wind, and we consider both stable and unstable hydrogen-burning, thus including recurrent sources. This is achieved by following all of the subpopulations self-consistently, using the same code. Since we follow the evolution of the donors until the exhaustion of the reservoir of matter available for mass transfer, we are able to discuss both systems with very short orbital periods (postnovae in cataclysmic binaries) and very long ones (symbiotic systems).

b) We model a time-dependent mass transfer/accretion rate and allow for the accumulation of matter in the phases of stable hydrogen burning and for either growth or erosion of the accretor in the phases of unstable hydrogen burning. RDS used a constant mass transfer rate and assumed a 100% efficient conversion of the accreted hydrogen into helium.

c) We apply somewhat different criteria for dynamically stable mass transfer and steady hydrogen burning than those used by RDS. There is, in general, reasonable agreement between our model and that of RDS on the permanent sources with main-sequence and sub-giant donors, which are the only type of SSSs included in the RDS calculation. In our model, CVs and Algols combined, produce steady burners in the same period range 10 hr–3 days, as in model (7) of RDS, which is similar to ours, because it includes magnetic braking.

Trial runs, in which we simulated the RDS model as closely as possible, produce, after a renormalization of the assumed Galactic incidence of binaries, a number of permanent sources that is in good agreement with the RDS model. It appears that the lower occurrence rate of SSSs in our model results mainly from the more restrictive constraints on the mass ratios in candidate systems, the different algorithms for the mass exchange rates, and the higher assumed accretion rates that allow for steady burning. The present understanding of the theory of stellar evolution and of the evolution of binary systems does not allow a clear choice between the models. Both models are, at present, in general agreement with the observations.

Another source of potential discrepancy, though less important, lies in the different treatment of pre-SSS evolution, e.g., somewhat different prescriptions for common envelope evolution and/or initial-final mass relations for stars that experience RLOF.

In relation to the above discussion, it may be worthwhile to renew the interest in computations of the mass exchange process in low-mass, close binaries, experiencing RLOF in the main-sequence stage or immediately following it, i.e., the so-called A, AB, and early-B cases of mass exchange.

10. We did not investigate in our study the influence of the value used for the CE parameter α_{ce} or the assumed distribution over mass ratios on our model results. These influences for CVs and Algols may be inferred from the models of RDS, and for symbiotic systems from our earlier work. A variation in these parameters within reasonable limits results in variation in the number of SSSs to within a factor ~ 2. At the present stage in the study of SSSs, when only the very basic properties of SSSs are known, and their estimated Galactic number can vary by an order of magnitude (Di Stefano & Rappaport 1994), it is premature to attempt to use the observations to constrain theoretical models significantly.

11. Our model assumes that 100% of the stars are initially in binaries. Proportional scalings may be made for other assumptions on the percentage of binaries.

12. The number of SSSs may be higher than estimated under the assumption of a constant star formation rate, if the latter was considerably higher than at present in the first several billion years of Galactic life. An enhanced star formation rate in the early epochs could create a reservoir of systems with masses of secondaries below $\sim 0.8\ M_{\odot}$, which experience RLOF because of the action of systemic angular momentum loss. One may expect this effect to influence recurrent sources in the short-period systems.

13. An important factor, yet unexplored, which may influence the number of SSSs, is the X-ray irradiation–induced stellar wind from the donor. For hard X-rays (~ 10 keV), this problem has been considered by, e.g., Podsiadlowski 1991, Harpaz & Rappaport 1991, Hameury, King, & Lasota 1993, Tavani & London 1993, and Iben, Tutukov, & Yungelson 1995. All the estimates in this paper were obtained without taking this possibility into account. If a part of the donor mass is lost via an induced wind, the total number of SSSs may decrease. An induced wind may also influence the distribution of sources over orbital periods (by carrying away angular momentum from the system). The importance of this effect can be studied once the problem of the interaction of supersoft X-rays with the donor's atmosphere is clarified

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