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SPECTROSCOPIC ORBITS FOR THREE BINARIES WITH LOW-MASS COMPANIONS AND THE DISTRIBUTION OF SECONDARY MASSES NEAR THE SUBSTELLAR LIMIT[1]

TSEVI MAZEH,[2,3,4] DAVID W. LATHAM,[3] AND ROBERT P. STEFANIK[3]

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ABSTRACT

We present orbital solutions for three low-amplitude spectroscopic binaries discovered in a sample of 20 solar-type IAU radial velocity standard stars observed with the Digital Speedometers at the Harvard-Smithsonian Center for Astrophysics. We update the orbital solutions for HD 114762 and HD 140913, and present a preliminary new solution for HD 29587. For all three orbits, the *minimum* mass for the secondary is less than 0.08 $M_\odot$, the borderline between stellar and substellar masses.

We consider the probability that all three binaries have small enough inclination angles so that their companions are above the substellar limit. To do so, we treat the 20 IAU standards as a sample drawn from a population of binaries with a mass-ratio distribution that does not allow any substellar companions. We calculate the probability that such a sample could still have three binaries, with the low-amplitude orbits actually found within the IAU sample. We show that this probability is small, depending on the specific mass-ratio distribution. For example, a flat mass-ratio distribution that assumes there are no substellar companions can be excluded at a high confidence level, 99.7%. We further show that our three detections may imply that the secondary-mass distribution rises near the substellar limit. However, the observations do not yet allow us to distinguish whether the unseen companions of HD 114762, HD 140913, and HD 29587 have stellar or substellar masses. In particular, recent attempts to estimate the mass of the companion of HD 114762 based on assumptions about the intrinsic rotation of the primary are inconclusive, and the companion could easily have a mass as low as 0.02 $M_\odot$.

We compare our three detections with the null results of four very precise radial velocity searches for substellar companions. The difference is indeed puzzling but can be accounted for if just a small fraction of the solar-type stars, of the order of a few percent, have companions with masses near the substellar limit.

Subject headings: binaries: spectroscopic — stars: individual (HD 29587, HD 114762, HD 140913) — stars: low-mass, brown dwarfs

1. INTRODUCTION

Stefanik et al. (1994) announced recently the discovery of a *periodic* low-amplitude radial velocity modulation of the G0 dwarf HD 140913, which they interpreted as orbital motion due to an unseen low-mass companion (LMC). Together with the F9 dwarf HD 114762 (Latham et al. 1989), HD 140913 was the second spectroscopic binary discovered among the 20 solar-type IAU radial velocity standard stars monitored with the Digital Speedometers operated by the Harvard-Smithsonian Center for Astrophysics (CfA). In this paper, we update the orbital solutions for HD 114762 and HD 140913 and present a preliminary new orbit for the G2 dwarf HD 29587, a third low-amplitude spectroscopic binary discovered in our sample.

The discovery of radial velocity variables among the IAU standard stars should not be surprising. The improved precision of modern radial velocity measurements and the ability to accumulate rich data sets have enabled astronomers to explore new regimes of variability that were difficult to reach when those standards were originally chosen. Indeed, several of the stars from the original lists of IAU standards were subsequently noted as variables (e.g., Batten 1985). In fact, the IAU standards may offer the best hunting ground for low-amplitude variables among the various samples of stars monitored with the CfA Digital Speedometers (Latham 1992), because the standards have been monitored more frequently, for a longer span of time, and with better quality spectra than any of the other samples.

Our detections of low-amplitude velocity variations for HD 114762, HD 140913, and HD 29587 are especially interesting, because they may indicate the presence of substellar companions. Unfortunately, the exact mass of the unseen companion cannot be derived for any of the three stars, because their orbital inclinations relative to our line of sight, i, are unknown. Only the *minimum* mass of the secondary, $M_2 \sin i$, can be derived from a single-lined orbit, assuming a primary mass consistent with its spectral type. The minimum masses are 0.009, 0.05, and 0.06 $M_\odot$ for HD 114762, HD 140913, and HD 29587, respectively. In all three cases, the minimum mass is less than 0.08 $M_\odot$, the approximate theoretical borderline between stellar and substellar objects (cf. Burrows, Hubbard, & Lunine 1989; Nelson, Rappaport, & Joss 1993).

The possibility that one or more of these companions might prove to be less massive than the substellar limit could be very important, especially to the search for brown dwarfs (e.g., Stevenson 1992). This is very intriguing because

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[1] Some of the observations reported here were obtained with the Multiple Mirror Telescope, a joint facility of the Smithsonian Institution and the University of Arizona.

[2] School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel; mazeh@wise7.tau.ac.il.

[3] Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138; dlatham@cfa.harvard.edu, rstefanik@cfa.harvard.edu.

[4] Laboratory for Astrophysics, National Air and Space Museum, Smithsonian Institution, Washington, DC 20560.

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the expected value for sin *i* for randomly oriented planes of motion is 0.79. Adopting this value for the three binaries yields masses of 0.01, 0.06, 0.08 $M_\odot$, still in the substellar region. On the other hand, the low-amplitude modulations might be due simply to very small orbital inclination angles, allowing each of the three secondaries to be more massive than the substellar limit.

We consider here the full sample of 20 solar-type IAU radial velocity standards observed extensively at the CfA, out of which three members show low-amplitude orbital motion. The three binaries comprise 15% of the sample, a fraction that is much higher than the frequency expected for low-amplitude spectroscopic binaries if we assume, for example, a flat mass-ratio distribution. In fact, the fraction of detected low-amplitude binaries is almost as large as the *total* frequency expected for spectroscopic binaries, which is about 20% (e.g., Duquennoy & Mayor 1991). On the other hand, a sample of 20 stars is extremely small, so the statistical significance of the three detections is not obvious. Therefore, we consider the *null* hypothesis that there are no substellar companions orbiting nearby solar-type stars, and that all three low-amplitude binaries have stellar companions viewed at very low inclination angles. We assess the statistical confidence with which this hypothesis can be rejected, assuming different mass-ratio distributions and random binary orientations.

We do not include giants in our statistical analysis of the frequency of substellar companions, because giants can exhibit low-amplitude velocity variations that do not arise from orbital motion. In their pioneering radial velocity survey of more than 100 giants in the globular cluster M3, Gunn & Griffin (1979) reported that the most luminous giants exhibited a velocity "jitter" of about 1 km s^{-1}. Subsequent work has confirmed that cool giants often show pseudoperiodic velocity variations with periods up to about 1000 days and amplitudes up to a few km s^{-1} (cf. Mayor et al. 1984; Walker et al. 1989, 1992; Hatzes & Cochran 1993; Murdoch, Clark, & Hearnshaw 1992). No clear-cut explanation for these variations has yet been established, with the main contenders being stellar pulsations, stellar rotation coupled with surface features, stellar activity cycles, and orbital motion due to unseen low-mass companions.

In contrast to the cool giants, for the solar-type dwarfs there is no known astrophysical effect that can produce observed velocity variations as large as 100 m s^{-1} when weak absorption lines are the main source of velocity information (Latham et al. 1989; see also McMillan et al. 1993, 1994). Orbital motion appears to be the only viable explanation for strictly periodic velocity variations larger than about 100 m s^{-1} among solar-type dwarfs. Thus, in this paper we restrict our attention to F and G dwarfs and subgiants among the IAU standards observed at the CfA.

Our statistical approach is similar to that of Mayor et al. (1992) and Marcy & Butler (1995). Both of those studies considered the fact that Mayor et al. (1992) reported nine stars with "a lower limit for the mass of the companion less than 0.08 solar masses," within a sample of 540 nearby G and K dwarfs. Both studies concluded that the hypothesis that there are no substellar companions can be rejected, if a flat mass-ratio distribution was assumed. Our approach is different in some of the details of the statistical analysis. The difference stems from the fact that our sample of standards was previously subjected to a coarse search for binaries, as part of the process that led to their selection as standards.

We describe briefly in § 2 the process that was used to select the IAU radial velocity standards and summarize the status of the 20 solar-type standards observed extensively at the CfA. In § 3 we update the orbital solutions for HD 114762 and HD 140913, and present a preliminary new orbit for HD 29587. Section 4 outlines the statistical approach we use, and § 5 describes the application of this approach to our IAU sample. Section 6 confronts our results with the null results of four other radial velocity searches for low-mass companions, and § 7 points out some other observational techniques that might be able to pin down the masses of the secondaries. Section 8 summarizes our conclusions.

2. THE SOLAR-TYPE IAU STANDARDS OBSERVED AT THE CfA

2.1. *A Brief History of the IAU Standards*

Forty years ago Pearce (1955) proposed 60 stars as candidates for IAU radial velocity standards, none of which had shown obvious velocity variations up to that point. It was recommended that these stars be observed routinely by every observatory performing radial velocity work, to allow comparisons between the velocity systems of the various observatories. Pearce's list of standards was divided into two subsets: 25 "bright" standards, which were observed extensively with high-dispersion coudé spectrograms, and 35 "faint" standards observed only with lower dispersion. Within a decade it had been established that four stars from the original list of 60 had velocity variations larger than 2 km s^{-1}. Three supergiants (HD 20902, HD 45348, and HD 156014) in the bright sample were identified as variable (Heard 1958; Deutsch 1956), while a K1 dwarf (HD 184467) in the faint sample was shown to be a double-lined spectroscopic binary with period $P = 492$ days and semi-amplitudes $K_1 = 9.4$ and $K_2 = 10.5$ km s^{-1} (McClure 1983).

In 1967, Heard (1968) proposed 24 additional candidate standards, drawn from a large survey using moderate dispersion at the David Dunlap Observatory, with only four or five velocities per star. Subsequent observations at higher dispersion (Heard & Fehrenbach 1972) showed that three of Heard's stars had variable velocities. Spectroscopic orbits were eventually published for two of the three stars: HD 204934, a K1 giant with period $P = 144$ days and semi-amplitude $K = 5.9$ km s^{-1} (Radford & Griffin 1975; Bassett 1978), and HD 160952, a G8 giant with $P = 181$ days and $K = 2.6$ km s^{-1} (Radford & Griffin 1976). An orbit has not yet been published for the third star, the G0 subgiant BD +29°1553, but there is little doubt that the star is a spectroscopic binary. A handful of recent CfA observations confirms the original report (Heard & Fehrenbach 1972) that the velocity varies by several km s^{-1}. In 1980, Griffin (1980) showed that a fourth Heard star, the K3 giant HD 14969, is a spectroscopic binary with $P = 1935$ days and $K = 4.4$ km s^{-1}. Finally, Scarfe (1992) has reported that the G0 subgiant HD 42397 is a double-lined spectroscopic binary. No orbital solution is available yet, but the period appears to be long. This brief history of the selection of the IAU radial velocity standard stars and the early efforts to identify low-amplitude variables lurking in the sample illustrates the process that we characterize as an initial coarse search for spectroscopic binaries.

In the 15 years since 1980, the IAU standards have been

TABLE 1

THE CfA RADIAL-VELOCITY OBSERVATIONS OF THE SOLAR-TYPE IAU STANDARD STARS

Star (HD/BD)	IAU	α(J2000)	δ(J2000)	V (mag)	$B-V$ (mag)	Spectrum	T_{eff} (K)	log g	V_{rot} (km s^{-1})	[m/H]	N_{obs}	Span (days)	V_{rad} (km s^{-1})	rms (km s^{-1})
693	F	$00^{\mathrm{h}}11^{\mathrm{m}}15^{\mathrm{s}}.8$	$-15°28'05''$	4.89	0.49	F7 V	6750	4.5	10	0.0	62	5008	+14.54	0.41
22484	F	03 36 52.3	+00 24 06	4.28	0.58	F9 V	6000	4.0	0	0.0	136	5201	+27.97	0.36
29587[a]	F	04 41 36.3	+42 07 06	7.29	0.64	G2 V	6000	4.5	0	0.0	81	5205	+112.32	0.96
65583	F	08 00 32.2	+29 12 44	7.00	0.71	G8 V	5750	4.5	0	0.0	213	5106	+14.71	0.57
89449	F	10 19 44.1	+19 28 15	4.79	0.45	F6 IV	6500	4.0	20	0.0	237	5319	+5.94	0.58
102494	H	11 47 56.4	+27 20 25	7.50	0.87	G9 IV	5250	3.5	0	0.0	142	4322	−21.88	0.62
102870	B	11 50 41.6	+01 45 53	3.61	0.55	F8 V	6000	4.0	0	0.0	61	2567	+4.41	0.44
103095	F	11 52 58.7	+37 43 07	6.45	0.75	G8 Vp	5000	4.5	0	−1.5	269	4965	−98.15	0.44
112299	H	12 55 28.1	+25 44 23	8.42	0.52	F8 V	6250	4.5	0	0.0	86	4284	+3.94	0.62
114762[a]	F	13 12 19.7	+17 31 00	7.31	0.54	F9 V	5750	4.5	0	−1.0	495	4953	+49.30	0.63
122693	H	14 02 52.1	+24 33 48	8.06	0.57	F8 V	6250	4.5	0	0.0	61	4166	−5.52	0.57
126053	F	14 23 15.2	+01 14 30	6.27	0.63	G1 V	6000	4.5	0	0.0	77	5160	−19.38	0.47
140913[a]	H	15 45 07.7	+28 28 10	8.08	0.54	G0 V	6250	4.5	10	0.0	124	4216	−20.08	1.12
144579	F	16 04 56.7	+39 09 23	6.66	0.73	G8 V	5500	4.5	0	0.0	51	4992	−59.46	0.38
149803	H	16 35 54.2	+29 44 44	8.54	0.49	F7 V	6500	4.0	10	0.0	172	4250	−7.54	0.65
154417	F	17 05 16.7	+00 42 09	6.01	0.58	F8.5 IV–V	6250	4.5	10	0.0	103	5248	−16.78	0.57
182572	F	19 24 58.0	+11 56 39	5.16	0.77	G8 IV	5250	4.0	0	0.0	308	5197	−100.16	0.42
+28°3402......	H	19 35 00.2	+29 05 13	8.88	0.44	F5 V	6500	4.0	10	0.0	111	4881	−36.38	0.70
187691	F	19 51 01.5	+10 24 56	5.11	0.55	F8 V	6000	4.0	0	0.0	157	5321	+0.09	0.43
222368	B	23 39 56.9	+05 37 35	4.13	0.51	F7 V	6250	4.0	10	0.0	103	4895	+5.54	0.46

[a] These are the three stars for which we present orbital solutions implying LMCs. As expected, all three LMCs have rms deviations larger than the median value, 0.54 km s^{-1}.

monitored regularly at several observatories. At the General Assembly of the IAU held in Buenos Aires in 1991, the Working Group on Radial Velocity Standards of Commission 30 reviewed the status of the standards and recommended that four additional stars be rejected as variable. Mayor (1991) reported that the F3 supergiant HD 36673 from Pearce's (1955) bright list showed velocity variations larger than 1 km s^{-1}. That same summer Duquennoy & Mayor (1991) published preliminary new orbital solutions for K larger than 1 km s^{-1} for three giants from Pearce's (1955) faint list: $P = 1492$ days and $K = 1.88$ km s^{-1} for the K0 giant HD 35410, $P = 3393$ days and $K = 1.18$ km s^{-1} for the M1 giant HD 44131, and $P = 509$ days and $K = 1.54$ km s^{-1} for the M2 giant HD 115521. All four of these stars are evolved, so caution must be exercised when interpreting the velocity variations as orbital motion.

In this paper, we restrict our attention to the subset of IAU standards listed by Latham & Stefanik (1991), who attempted to eliminate all the variables with amplitudes larger than 1 km s^{-1} that had been noted up to the time of the IAU General Assembly in Buenos Aires. We invoke three selection criteria. First, we include only dwarfs and subdwarfs, and exclude giants and supergiants, because we want to avoid stars that might exhibit velocity variations for reasons other than orbital motion, as explained in § 1. Second, we include only F and G stars, so that we match very closely the range of spectral types used by Duquennoy & Mayor (1991) to select their sample, because we need to use the results of their survey in our analysis. Third, we include only those stars with more than 50 CfA observations, to ensure that the threshold for detecting binaries is more or less uniform across our sample and is not seriously degraded because some stars have only a few observations. Only 20 stars passed all three criteria, making our sample rather small.

2.2. *CfA Observations of the IAU Standards*

The radial velocities of the 20 stars included in our sample were monitored with the Digital Speedometers (Latham 1985, 1992) operated by the CfA, primarily with the 1.5 m Wyeth reflector at the Oak Ridge Observatory in Harvard, Massachusetts, and occasionally with the 1.5 m Tillinghast reflector at the Whipple Observatory and the Multiple Mirror Telescope (MMT), both on Mount Hopkins, Arizona.

To derive radial velocities, we followed the general approach reported by Nordström et al. (1994) for CfA echelle spectra of F stars. Observed spectra were cross-correlated against calculated spectra, computed by Jon Morse from an extensive grid of Kurucz (1992a, b) model atmospheres. The cross-correlation was performed using XCSAO (Kurtz et al. 1992) running inside the IRAF.[5]

All the target stars observed by Nordström et al. (1994) were originally chosen to lie in a narrow range of colors, so they were able to adopt a single effective temperature, $T_{\mathrm{eff}} = 7000$ K, for all their correlations. They concentrated on determining the rotational velocity that gave the best radial velocities for each star. Our sample of F and G standards covers a range of temperatures and gravities, so we ran grids of correlations in order to determine simultaneously the combination of effective temperature, rotational velocity, and surface gravity that gave the best correlations for each star. Each observed spectrum was correlated against a three-dimensional grid of calculated template spectra, with effective temperatures (5000, 5250, 5500, 5750, 6000, 6250, 6500, 6750, and 7000 K), rotational velocities (0, 10, and 20 km s^{-1}), and log surface gravity (4.5, 4.0, and 3.0 cm s^{-2}). To determine which template gave the best match for each star, we calculated, for every template of the grid, the average, over all the exposures of that star, of the peak

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[5] The Image Reduction and Analysis Facility (IRAF) is distributed by the National Optical Astronomy Observatories, which is operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.

height of the correlation function and chose the template that gave the highest average. For all except two of the stars, we used the solar metallicity for the calculated spectra. The other two, HD 114762 and HD 103095, are metal poor and were included in the Carney-Latham survey of proper-motion stars, so we adopted templates using the temperatures and metallicities previously derived for them (Carney et al. 1994).

The results of the CfA measurements for the 20 F and G dwarfs and subgiants are summarized in Table 1. The second column specifies the source of the standard, with B and F denoting stars from the bright and faint lists of Pearce (1955), and H denoting stars from Heard (1968). The columns labeled $T_{\rm eff}$, log g, $V_{\rm rot}$, and [m/H] give the parameters of the calculated template spectrum chosen for each star. The synthetic template spectra were calculated assuming the inclination angle of the rotational axis of these stars is 90°. The next four columns give the number of velocities, the time span in days from the first to the most recent observation as of 1995 December, the mean radial velocity, and the rms deviation from the mean.

3. ORBITS OF HD 114762, HD 140913, AND HD 29587

Three of the stars included in Table 1 show low-amplitude periodic velocity modulations with an amplitude of the order of 1 km s^{-1}. For the solar-type dwarfs and subgiants, there is no known astrophysical effect that can produce observed radial velocity variations as large as a few tenths of km s^{-1} (Latham et al. 1989). Therefore, we attribute the observed modulations to orbital motion due to unseen low-mass companions.

The first spectroscopic binary discovered among the 20 solar-type IAU standards was the F9 dwarf HD 114762 (Latham et al. 1989). Since then we have continued to monitor the velocity of that star, looking for an additional periodic velocity variation that might be indicative of a second low-mass unseen companion. Although the 495 velocities accumulated at the CfA did not indicate any additional periodicity, they did allow us to improve our orbital solution.

The velocities for HD 114762 are plotted in the top panel of Figure 1, and the corresponding power spectrum (Mazeh, Krymolowski, & Latham 1993) is plotted in the bottom panel. Although no periodic variation is obvious in the time history, the power spectrum shows a strong peak at a frequency corresponding to a period of about 84 days. Figure 2 shows the velocity curve for an orbital solution derived

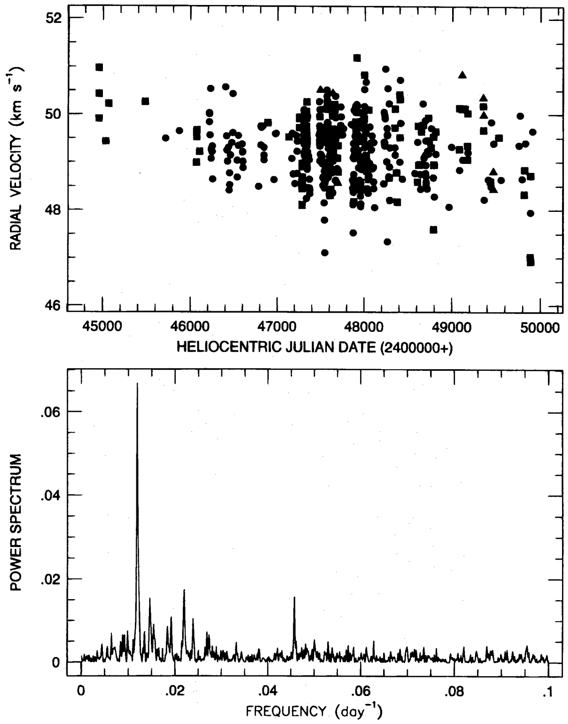

FIG. 1.—CfA radial velocities for HD 114762. The time history of the velocities is plotted in the top panel, the power spectrum in the bottom. The circles, squares, and triangles in the top panel denote velocities from the Wyeth reflector, Tillinghast reflector, and MMT, respectively.

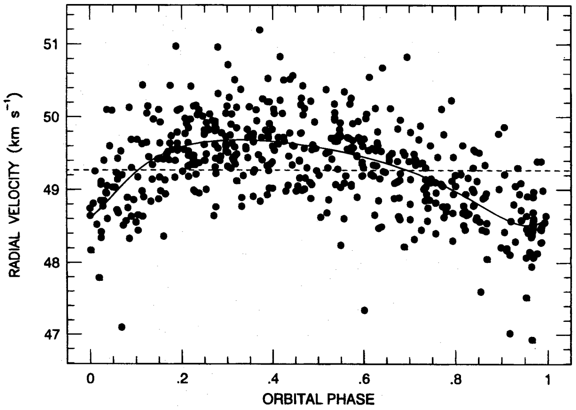

FIG. 2.—The orbital solution for HD 114762

TABLE 2

ORBITAL ELEMENTS

Element	HD 114762	HD 140913	HD 29587
P (days)	83.90 ± 0.08	148.04 ± 0.24	1481 ± 22
γ (km s^{-1})	+49.27 ± 0.02	−20.03 ± 0.05	+112.29 ± 0.08
K (km s^{-1})	0.59 ± 0.04	1.93 ± 0.15	1.02 ± 0.16
e	0.35 ± 0.05	0.54 ± 0.04	0.33 ± 0.15
ω	214° ± 10°	27° ± 6°	104° ± 22°
T (JD 2,400,000)	47710 ± 2	49105 ± 1	49353 ± 76
$N_{\rm obs}$	495	124	81
rms (km s^{-1})	0.50	0.51	0.72
$a \sin i$ (Gm)	0.64 ± 0.04	3.29 ± 0.28	19.6 ± 3.3
$f(M)$ ($M_\odot \times 10^{-6}$)	1.45 ± 0.27	65 ± 16	137 ± 67
M_1 ($M_\odot$)	0.73	1.19	1.04
$M_2 \sin i$ ($M_\odot \times 10^{-3}$)	9 ± 1	46 ± 4	55 ± 7

using ORB18 (Mazeh et al. 1993), together with all the measurements folded onto the orbital period. The parameters for the orbit of HD 114762, together with the orbits of the other two spectroscopic binaries, are listed in Table 2.

Our new orbital elements for HD 114762 are very similar to the ones published 6 years ago, based on a much more limited set of data. The new eccentricity, $e = 0.35 \pm 0.05$, is somewhat larger, but within the errors of the old value of $e = 0.26 \pm 0.07$. This is in better agreement with the higher eccentricity of $e = 0.380 \pm 0.015$ reported by Cochran, Hatzes, & Hancock (1991). Their orbital solution was based on 28 velocities with rms velocity deviations of 0.034 km s^{-1}, an order of magnitude more precise than the CfA velocities.

The second spectroscopic binary discovered among the 20 solar-type IAU standards was the G0 dwarf HD 140913 (Stefanik et al. 1994). For the past 2 years, we have monitored the velocity of that star intensively, and as of 1995 December, we had accumulated 124 velocities. Plots of the velocities, the corresponding power spectrum, and the orbital solution are presented in Figures 3 and 4.

A careful review of the CfA radial velocity data for all the IAU standards led to the detection of a third solar-type dwarf with a significant periodic variation, the G2 dwarf HD 29587 (see Figs. 5 and 6). For this star we have only 81 velocities, and only one orbital cycle has been covered well. Thus, we consider our orbital solution to be preliminary. We plan to continue to monitor this star over the next several years, with the goal of improving the preliminary orbit. We have little doubt that this star is a spectroscopic binary with a low-mass companion, and therefore it can be used for an analysis of the frequency of low-amplitude binaries found in our sample of solar-type IAU standards. A table reporting all the individual CfA velocities for HD 114762, HD 140913, and HD 29587 will be submitted for publication in the AAS CD-ROM series.

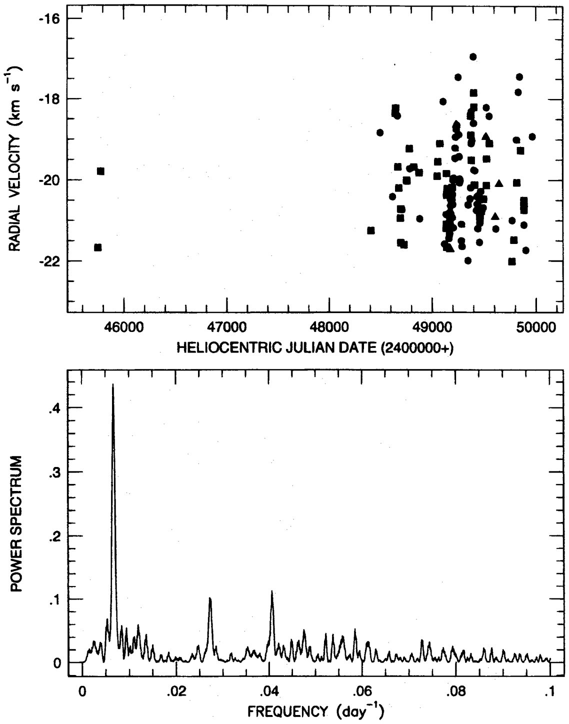

FIG. 3.—CfA radial velocities for HD 140913. See caption for Fig. 1 for details.

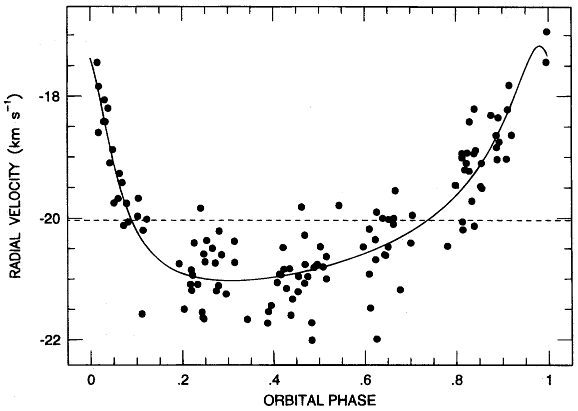

FIG. 4.—The orbital solution for HD 140913

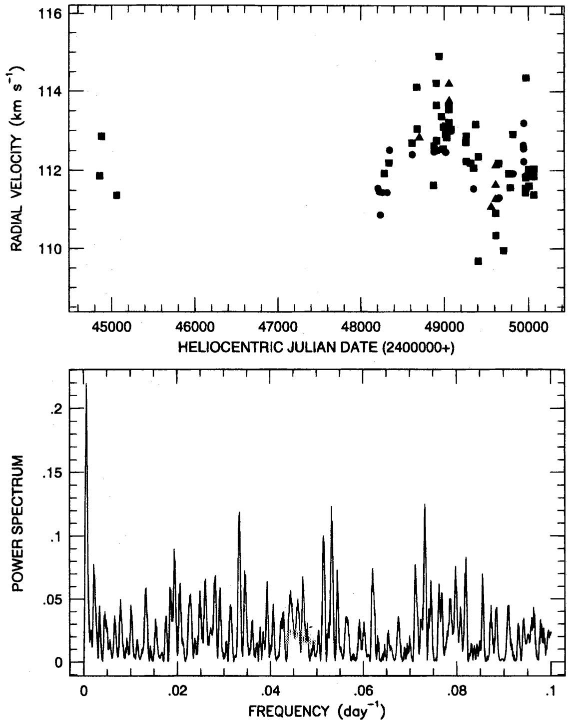

FIG. 5.—CfA radial velocities for HD 29587. See caption for Fig. 1 for details.

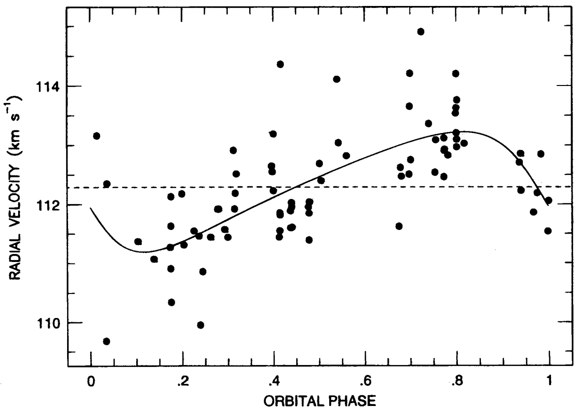

FIG. 6.—The orbital solution for HD 29587

Table 2 includes an estimate for the minimum mass of the companions, $M_2 \sin i$. To calculate these masses, we had to adopt masses for each of the primaries. For HD 140913 and HD 29587, we used the observed $B-V$ color and the calibration of $B-V$ versus mass given by Gray (1992) for normal main-sequence stars. HD 114762 is metal poor with $[\mathrm{m/H}] = -0.8$, which means that the primary mass is considerably smaller than for a normal main-sequence star with the same color or temperature. For this star we adopted the primary mass derived by Carney et al. (1994).

All three minimum masses are less than 0.08 $M_\odot$, the theoretical borderline between stellar and substellar objects. However, as stated in the introduction, it is possible that all three orbits have small inclination angles, thus allowing all three companions to be stars, with masses above the substellar limit. The probability for this to occur by chance is assessed in the next section.

4. STATISTICAL MODEL

In this section, we present our approach for assessing the statistical significance of the detection of three spectroscopic binaries with low-mass companions among a sample of 20 solar-type IAU standard stars. The approach takes into account the fact that the IAU standards were subjected to a coarse search for spectroscopic binaries with a certain threshold for detection. We will estimate the probability that a binary has a small enough mass and/or an orbit with a small enough inclination so that its radial velocity variation would not be detected by the coarse search. In other words, we will estimate the probability for an elusive binary to be included in the IAU standards.

The probability of detecting a previously unnoticed binary within the sample of IAU standards depends, obviously, on the mass-ratio distribution of the parent sample. We will consider a few possible distributions and show that, for some of them, the probability of detecting a new binary is extremely small. We will use this result, together with our three detections, to exclude such distributions.

We concentrate on the probability of detecting a new binary with small semiamplitude K and not on detecting binaries with small $M_2 \sin i$, because K is a directly observable parameter. Our approach is generic, and can be applied to any survey for spectroscopic binaries, to confront an assumed mass-ratio distribution with the actual number of binaries detected within a specified range of semiamplitudes.

4.1. *Probability of Not Detecting a Binary*

We first consider the probability of *not* detecting a binary in a systematic search for spectroscopic binaries. Suppose that the search detects all binaries with orbital period P between P_{min} and P_{max}, with primary semiamplitude K larger than or equal to the search threshold K_{min}. For simplicity, we first consider only circular orbits, using the known formula for K:

$$K(P, q, M_1, \sin i) = 212.9 P^{-1/3} M_1^{1/3} \frac{q}{(1+q)^{2/3}} \times \sin i \ (\mathrm{km\ s^{-1}}) \,, \quad (1)$$

where P is the orbital period in days, M_1 is the primary mass in solar mass units, q is the mass ratio of the system, $q = M_2/M_1$, and i is the inclination angle of the orbital plane relative to our line of sight (e.g., Batten 1973). To separate the dependence of K on $\sin i$ from its dependence on the other parameters, we define K_0 as the semiamplitude of the binary as seen by an observer located in the binary orbital plane:

$$K_0(P, q, M_1) = \frac{K(P, q, M_1, \sin i)}{\sin i} = 212.9 P^{-1/3} M_1^{1/3} \frac{q}{(1+q)^{2/3}} \ (\mathrm{km\ s^{-1}}) \,. \quad (2)$$

For a given period, primary mass, and mass ratio, all systems with an inclination smaller than some minimal inclination, i_0, *cannot* be detected, because K is smaller than K_{min}. We consider, therefore, $U[K_{\mathrm{min}}](P, q, M_1)$—the probability of *not* detecting a binary by a search with a given threshold K_{min}. We get (e.g., Mazeh & Goldberg 1992)

$$U[K_{\mathrm{min}}](P, q, M_1) = \begin{cases} 1 - \sqrt{1 - \left[\dfrac{K_{\mathrm{min}}}{K_0(P, q, M_1)}\right]^2} \,, & \text{if } K_{\mathrm{min}} \le K_0(P, q, M_1) \,, \\ 1 \,, & \text{otherwise} \,. \end{cases} \quad (3)$$

This simple analytical expression is based on geometrical considerations, and the only assumption involved is the random distribution of the binary orbital planes.

To get the probability of *not* detecting a binary taken at random from a given population of binaries, we have to integrate $U[K_{\mathrm{min}}](P, q, M_1)$ over the permitted range of mass ratios and periods,

$$U[K_{\mathrm{min}}](M_1) = \int_{P_{\mathrm{min}}}^{P_{\mathrm{max}}} \int_{q_{\mathrm{min}}}^{q_{\mathrm{max}}} U[K_{\mathrm{min}}](P, q, M_1) \times f_{q,P}(q, P) dq\, dP \,, \quad (4)$$

where $f_{q,P}(q, P) dq\, dP$ is the probability of finding a binary with mass ratio between q and $q + dq$ and with period between P and $P + dP$. Therefore,

$$\int_{q_{\mathrm{min}}}^{q_{\mathrm{max}}} \int_{P_{\mathrm{min}}}^{P_{\mathrm{max}}} f_{q,P}(q, P) dq\, dP = 1 \,. \quad (5)$$

The integral of equation (4) presents the weighted average of $U[K_{\mathrm{min}}](P, q, M_1)$ over the mass-ratio and period domains, and therefore gives the probability of *not* detecting a binary drawn at random from a population of binaries with mass-ratio and period distribution $f_{q,P}(q, P)$.

If the mass-ratio distribution does not depend on the binary period, we can separate the dependence on q and P by writing

$$f_{q,P}(q, P) = f_q(q) \times f_P(P) \,, \quad (6)$$

where

$$\int_{P_{\mathrm{min}}}^{P_{\mathrm{max}}} f_P(P) dP = \int_{q_{\mathrm{min}}}^{q_{\mathrm{max}}} f_q(q) dq = 1 \,. \quad (7)$$

If this is the case, then

$$U[K_{\mathrm{min}}](M_1) = \int_{P_{\mathrm{min}}}^{P_{\mathrm{max}}} \left\{ \int_{q_{\mathrm{min}}}^{q_{\mathrm{max}}} U[K_{\mathrm{min}}](P, q, M_1) \times f_q(q) dq \right\} f_P(P) dP \,. \quad (8)$$

Let us now consider the probability that a *star* drawn at random is an undetected binary in a systematic search, given the characteristics of the binaries in that population and the detection limit of the search. Denoting this probability by U^s, we get

$$U^s[K_{\min}](M_1) = f_b(P_{\min}, P_{\max})U[K_{\min}](M_1) , \quad (9)$$

where $f_b(P_{\min}, P_{\max})$ is the total frequency of binaries with periods between $P_{\min}$ and $P_{\max}$. For a given sample of N stars, which is subject to a systematic search for spectroscopic binaries with a detection threshold $K_{\min}$, the expected number of undiscovered binaries in the sample is $U^s[K_{\min}]N$.

For a given systematic search for spectroscopic binaries, $U^s[K_{\min}]$ can be used to check any assumed set of characteristics of the binary population. This is so because one can predict, a priori, how many binaries will be found with K in a certain range, between, say, $K_{\max}$ and $K_{\min}$. The probability of finding such a binary in a sample is

$$Pr(K_{\min} \leq K \leq K_{\max}) = U^s[K_{\max}] - U^s[K_{\min}] . \quad (10)$$

This probability can be compared with the actual number of binaries found in the survey at that range of K.

The above derivation is based on a few simplifying assumptions. One of them is the assumption that all binaries with $K < K_{\min}$ are not detected, while all binaries with K larger or equal to $K_{\min}$ are detected. This is a very naive assumption. The truth of the matter is that, for K close to $K_{\min}$, there is some probability for detection (e.g., Tokovinin 1992), which we took to assume only the value of 0 or 1. Taking into account this effect will necessitate numerical simulations that will obscure the dependence of the result on the geometry and on the different distribution functions that characterize the binary population, while modifying the results only by a small factor of the order of a few percent. We therefore adopt the simple expression of equation (3).

Another simplifying assumption is that the orbits are circular. Eccentricity introduces two effects, the first of which is the dependence of K on the eccentricity e. Equation (1) should include an additional factor of $(1 - e^2)^{-1/2}$, which causes K to increase for increasing e. The other factor is the dependence of the detection threshold $K_{\min}$ on e. Our simplifying assumption about the constancy of $K_{\min}$ throughout the sample breaks down when we consider eccentric orbits. This is so because, for eccentric orbits, the velocity variation tends to concentrate around the periastron passage, and therefore $K_{\min}$ increases for increasing eccentricity. These two effects, when applied to $U[K_{\min}]$, tend to cancel each other (Fischer & Marcy 1992); the net effect depends on the characteristics of the observational search. By running numerical simulation, we have found that, if the detection limit depends on the rms scatter of the observed radial velocity measurements, the two effects cancel each other for any reasonable eccentricity. If the detection limit depends on the maximum spread of the stellar radial velocities, the probability for nondetection, $U[K_{\min}]$, is slightly smaller for higher eccentricity. For $e = 0.5$, for example, the difference is of the order of 10% for most of the parameter space. We therefore adhere to the simple expression of equation (3).

Equation (6) assumes the independence of the mass-ratio distribution (=MRD) on the period. This assumption is probably the most difficult to defend. Nevertheless, we assume that the dependence of the MRD on the period is weak for the period range of the spectroscopic binaries, which is typically 1–3000 days, and therefore we will ignore this dependence when analyzing spectroscopic binaries.

4.2. *Probability of Detecting a Binary in a Sample of Standard Stars*

To model the detection of the three IAU standards with low-amplitude modulations, we consider here the following statistical experiment. Assume that we have a population of stars, all with the same mass, M_1, and with a given frequency of binaries $f_b(P_{\min}, P_{\max})$. The binaries have known distributions of periods, mass ratios, etc., with random orientation of their orbital planes. Imagine that we choose a sample of stars out of this population and search for binaries. The search is performed in two phases. Through the first coarse phase, every spectroscopic binary with a period between $P_{\min}$ and $P_{\max}$ and radial velocity amplitude *larger* than some $K_{\min,1}$ is detected. Of course, some of the binaries escape detection, because their secondary mass and/or orbital inclination is too small.

Now suppose we go on and perform the next phase of our search. We take all stars in the sample for which *no* variation was detected in the first phase and put them on a careful, precise, radial velocity search for periodic variation, with a lower threshold $K_{\min,2}$. If we know the characteristics of the binary population, we could predict a priori the expected number of additional binaries to be discovered in the second phase, which is a function of observational parameters $K_{\min,1}$ and $K_{\min,2}$, and is very sensitive to the lower end of the MRD. We can then compare the actual number of additional binaries detected in the second phase with the expected number, to test the validity of any assumed binary distribution.

We define the two sets of events as follows:

1. D_2—Detecting orbital motion of a *star* at the second phase, which implies that its radial velocity amplitude K is $K_{\min,2} \leq K < K_{\min,1}$.
2. N_1—Not detecting orbital motion in the first phase, either because the sample star is a single star or because $K < K_{\min,1}$.

Detecting a low-amplitude binary out of the sample of *standard* stars does not have the same probability as $Pr(D_2)$, because, in the case of the standards, all the obvious binaries have been removed from the sample in the first phase. The probability of detecting a binary out of the standard stars is a conditional probability, $Pr(D_2 \mid N_1)$:

$$Pr(D_2 \mid N_1) = \frac{Pr(D_2 \cap N_1)}{Pr(N_1)} . \quad (11)$$

Using the fact that here $D_2 \subseteq N_1$, and therefore $Pr(D_2 \cap N_1) = Pr(D_2)$, we get

$$Pr(D_2 \mid N_1) = \frac{Pr(D_2)}{Pr(N_1)} . \quad (12)$$

Using equation (10), which gives $Pr(D_2)$, we finally get

$$Pr(D_2 \mid N_1) = \frac{U^s[K_{\min,1}] - U^s[K_{\min,2}]}{Pr(N_1)} . \quad (13)$$

5. APPLICATION OF THE STATISTICAL MODEL TO THE SOLAR-TYPE IAU STANDARDS

We turn now to the sample of 20 F and G dwarfs and subgiants observed at the CfA as IAU standards (see Table 1). We limit our analysis to periods in the range 1–3000 days, because the CfA data should provide good coverage of this range. For the period distribution, we use the results of Duquennoy & Mayor (1991), who studied a volume-limited sample of 164 nearby F and G dwarfs and subgiants, and derived a Gaussian distribution in the log P domain. Duquennoy & Mayor did not specify the normalization for their distribution, so we follow Marcy & Butler (1995) and use a normalization factor of 0.11, which implies that $f_b(1, 3000)$ is 0.17. For the sake of simplicity, we assume that all the primaries have a mass of 1 $M_\odot$. In this case, the mass ratio of a binary is equal to the secondary mass, and the mass-ratio distribution is identical to the secondary-mass distribution.

5.1. *Three Mass-Ratio Distributions*

In order to proceed, we now specify some plausible mass-ratio distributions for solar-type binaries. Following Mayor et al. (1992) and Marcy & Butler (1995), we first consider two versions of a flat MRD. One distribution assumes that there are no substellar companions at all, and its value is therefore 0 for $0 \leq q \leq 0.08$, and 1.087 for $0.08 < q \leq 1$. It will be denoted as the cutoff-flat MRD. The other one assumes a flat distribution of unity for $0 \leq q \leq 1$ (Mayor et al. 1992) and will be denoted as the fully flat MRD.

The flat MRD, although the most simple function of the mass ratio, is not necessarily the one nature uses to form binaries. It is true that the findings of Mazeh et al. (1992), who analyzed a sample of spectroscopic binaries out of the Duquennoy & Mayor (1991) survey, are consistent with a flat MRD. However, the Mazeh et al. (1992) sensitivity to the detailed behavior of the distribution, near the low-mass end in particular, was very limited, because of the small number of binaries analyzed, only 23. They could not have detected a moderate rise of the MRD in the region approaching $q = 0.08$. Actually, a moderate rise of the secondary-mass distribution toward the stellar/substellar borderline, with a power law of $M_2^{-0.8}$ (equivalent to $q^{-0.8}$ for solar-mass primaries), has been suggested already by Marcy & Butler (1995). Following Marcy & Butler, we also consider here this MRD, comparing its prediction with the three detections. This third distribution further assumes that there are no substellar companions, so it is 0 for $0 \leq q \leq 0.08$. Therefore, the function has a cutoff at $q = 0.08$ and will be denoted as the cutoff-exponential MRD.

The three distributions are illustrated in Figure 7. We emphasize that, because of our assumption that the primary masses are all 1 $M_\odot$, all MRDs are numerically the same as the corresponding distributions versus secondary masses.

The three MRDs lead to three different probabilities of not detecting a binary, denoted by $U^s_{F,0.08}$, $U^s_{F,0.0}$, and $U^s_{E,0.08}$, corresponding to the cutoff-flat, fully flat and cutoff-exponential MRD, respectively. To calculate the various probabilities, we used equations (8) and (9). The results are summarized in Table 3 for different values of $K_{\min}$.

To apply equation (13) to the CfA sample of 20 solar-type IAU standards, we need to choose a value for $K_{\min,1}$, the threshold for the initial coarse phase of the search for spectroscopic binaries. The lowest amplitude binary found in the initial phase had $K = 2.6$ km s^{-1}, while the preliminary orbits reported recently by Duquennoy & Mayor (1991, their Table 8) for five giants among the IAU standards and the three orbits reported in this paper all have semi-amplitudes in the range $0.5 < K < 2.0$ km s^{-1}. Apparently, the threshold for the coarse phase lies in the range $2.0 < K_{\min} < 2.6$ km s^{-1}, depending, obviously, on the type and number of observations. For our statistical analysis, we adopt the upper limit, $K_{\min} = 2.5$ km s^{-1}, because it is almost the most generous choice for the probability of detecting additional low-amplitude binaries. For the CfA observations, we adopt $K_{\min} = 0.5$ km s^{-1}, which is just below the lowest semiamplitude actually observed, $K = 0.59$ km s^{-1} for HD 114762.

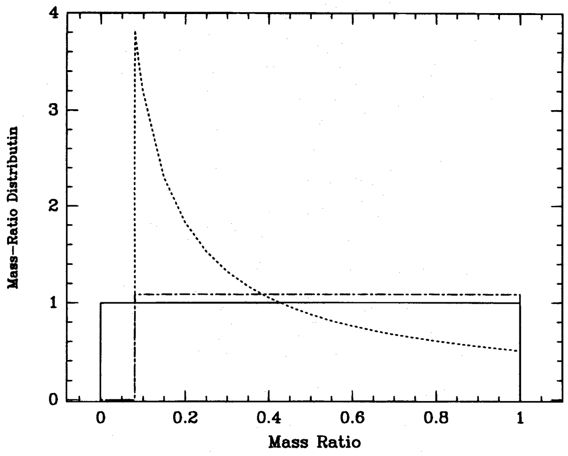

FIG. 7.—Three mass-ratio distributions. The solid line is the fully flat MRD, the dot-dashed line is the cutoff-flat MRD, and the dotted line is the cutoff-exponential MRD.

A key parameter in our statistical approach is $Pr(N_1)$, the probability that a star was not detected as a binary in the initial coarse phase of the search for binaries. We adopt $Pr(N_1) = 0.85$, which corresponds to the frequency of stars not detected as spectroscopic binaries by Duquennoy & Mayor (1991) in a systematic observational survey of a sample of 164 solar-type stars. They report orbits for 23 binaries with periods shorter than 3000 days and semi-amplitudes larger than 2.5 km s^{-1}. Applying equation (13), we derive $Pr(D_2 \mid N_1)$ for the three different MRDs, the results of which are listed on the bottom line of Table 3. $Pr(D_2 \mid N_1)$ is the probability that an IAU standard star will be detected as a new low-amplitude spectroscopic binary. These probabilities are only very weakly sensitive to the actual value of $K_{\min,2}$, since $U^s[K_{\min,2}]$ is small relative to $U^s[K_{\min,1}]$.

5.2. *Statistics of Three*

We now consider the three detections out of the sample of

TABLE 3

PROBABILITY OF UNDETECTION

$K_{\min}$	$U^s_{F,0.08}$	$U^s_{F,0.0}$	$U^s_{E,0.08}$
3.0……	0.015	0.029	0.037
2.5……	0.013	0.023	0.027
2.0……	0.009	0.019	0.018
1.0……	0.002	0.009	0.004
0.5……	0.0005	0.004	0.001

20 solar-type IAU standard stars observed at the CfA. Table 3 implies that, for the cutoff-flat MRD, the number of binaries expected to be detected in the second phase of the search is only 0.3 stars. The probability of getting more than two detections in this phase of the search is 0.0032. *The fact that we detected more than two binaries allows us to reject the cutoff-flat MRD with a significance of 99.7%*. This is equivalent to a 3 σ result for a normal distribution.

For the fully flat MRD, we still expect only 0.4 detections out of the sample of 20 solar-type standards. The probability of getting more than two detections is 0.01. Therefore, we can reject the fully flat MRD with a significance of 99%, which is equivalent to a 2.6 σ result.

The detection of three low-mass companion orbits in the CfA sample therefore argues against the hypothesis that the MRD is flat, with or without a cutoff, and suggests instead that the MRD must rise near the substellar limit. This rise could occur exclusively in the substellar regime, or it could already start in the stellar regime. The rise might even be restricted to the stellar regime, with a sharp drop at the substellar limit, compatible with the assumption of no substellar companions, as suggested by Marcy & Butler (1995). The cutoff-exponential MRD is an example of such a distribution. It predicts 0.6 detections out of the CfA sample of 20 solar-type standards. This is still a small number compared with our three detections, and therefore even the cutoff-exponential MRD is certainly not the best MRD consistent with our results. However, since the probability of getting more than two detections is 0.02, we can reject the cutoff-exponential MRD only with a significance of 98%, which is equivalent to a 2.3 σ result. We therefore conclude that this MRD is unlikely, but it still cannot be ruled out on the basis of the present sample.

Similar conclusions with regard to the flat distributions were reached by Mayor et al. (1992), who considered the G and K dwarfs and subgiants they observed in various CORAVEL projects. Out of 540 stars, they discovered 62 binaries, nine of which have a minimum secondary mass below 0.08 $M_{\odot}$. Mayor et al. considered the hypothesis that all nine binaries have stellar companions and the low-amplitude modulations are due to orbital motion viewed at small inclination angles. Assuming a flat MRD, they rejected this hypothesis with a statistical significance similar to the one we derive here.

6. COMPARISON WITH OTHER RADIAL VELOCITY SURVEYS

In this section, we compare our detection of three LMCs with the results from four other research groups that have been conducting very precise radial velocity searches for giant planets and/or brown dwarfs over periods of several years. So far, these four efforts have failed to derive any orbits with minimum masses below the substellar limit. This is quite surprising, because the typical precision of the four searches is considerably better than for the CfA and the CORAVEL measurements. Therefore, in this section we summarize the results of the four searches and try to reconcile them with the conclusions reached by this paper and by Mayor et al. (1992). A similar summary in a similar context has been published by Marcy & Butler (1995). However, we concentrate on just the F and G dwarfs and subgiants, because we want to compare the results of the very precise searches with ours.

6.1. *Four Very Precise Surveys*

A Canadian team was the first to undertake a very precise radial velocity search for brown dwarfs and giant planets, starting in 1980 (Campbell, Walker & Yang 1988; Walker et al. 1995). They used the coudé spectrograph at the 3.6 m Canada-France-Hawaii Telescope (CFHT) with a hydrogen-fluoride cell to provide a very precise wavelength fiducial by imposing absorption lines on each stellar spectrum. Another team (Cochran & Hatzes 1994) has been using the coudé spectrograph at the 2.7 m telescope at the McDonald Observatory (McD) with an iodine cell (Marcy & Butler 1992). McMillan et al. (1990, 1994), from the Lunar and Planetary Laboratory (LPL), have been using a fiber-fed Fabry-Perot interferometer at the Steward Observatory 0.9 m telescope, with wavelength calibration provided by an iron-argon hollow cathode lamp. We also include in this comparison the results from a fiber-fed echelle spectrograph on the 1 m telescope at the Mount John University Observatory (MJUO), with wavelength calibration provided by a thorium-argon hollow cathode lamp (Murdoch & Hearnshaw 1991; Murdoch, Hearnshaw, & Clark 1993). Although the precision is somewhat poorer for the MJUO survey than the other three, the MJUO survey is valuable for its unique coverage of targets in the southern sky.

The characteristics of the four surveys are summarized in Table 4, which lists the number (N) of F and G dwarfs and subgiants observed, the time span as of 1995 (ΔT), and the precision achieved for a single measurement. We do not consider here the Lick M star survey (Marcy & Benitz 1989) or the Tokovinin (1992) survey, because they included only K and M stars, and we are restricting our attention to the F and G stars. We also do not include the new Lick survey (Marcy & Butler 1995), because no detailed report on the results of that search had been published as of 1995 December.

Only two of the stars monitored by the four very precise surveys showed any hint of a substellar companion. Cochran & Hatzes (1994) report a steady drift down of the velocities for the G5 subgiant μ Herculis, amounting to about 200 m s^{-1} over a period of 5 yr. They concluded that an orbiting companion such as a giant planet or brown dwarf could not be ruled out. Murdoch et al. (1993) note that the G2 subgiant β Hydri may be a low-amplitude variable with a period of 45 days, which might indicate the presence of a substellar companion. For neither of these stars has it yet been possible to derive orbital solutions, so we choose not to include these two candidates as detections.

6.2. *More Statistics of Three*

In order to assess the statistical significance of the failure of the four precise surveys to derive any orbits with minimum masses below the substellar limit, compared with the three LMC orbits detected in the CfA survey, we need to

TABLE 4

FOUR OTHER SEARCHES

Observatory	N	ΔT (yr)	σ (m s^{-1})	Reference
CFHT	12	12	15	Walker et al. 1995
McD	20	8	15	Cochran & Hatzes 1994
LPL.........	13	7	30	McMillan et al. 1994
MJUO......	23	3	60	Murdoch et al. 1993

know how many F and G dwarfs and subgiants there are in the union of the four samples that would have survived the selection process used for the CfA sample.

The CFHT sample includes 12 F and G dwarfs and subgiants (Walker et al. 1995). Three of the 12 stars have clear velocity variations: χ^1 Orionis, Procyon, and ξ Bootis A. Combined spectroscopic/astrometric orbits have been published for χ^1 Ori (Irwin, Yang, & Walker 1992b), with $K = 1.9$ km s^{-1} and $P = 14$ yr, and for Procyon (Irwin et al. 1992a), with $K = 1.7$ km s^{-1} and $P = 40$ yr. For ξ Boo A, the astrometric period is 155 yr, and the CFHT velocities show a clear upward trend amounting to 150 m s^{-1} over 10 yr. All three of these stars should be retained in our merged sample because they would have passed the criterion used to select the CfA sample: the velocity variations have a semiamplitude smaller than 2.5 km s^{-1} and periods longer than 3000 days. One of the CFHT stars is also in the CfA sample of 20 solar-type IAU standards. Thus, the CFHT survey has 11 stars in addition to CfA that would have passed the CfA selection process.

The LPL sample includes 13 F and G dwarfs and subgiants, but detailed results have only been reported for three (McMillan et al. 1994), all of which are included in the CFHT sample.

The McD sample includes 20 F and G dwarfs and subgiants (Cochran & Hatzes 1994). All 12 of the solar-type stars in the CFHT sample are also in the McD sample, including the three radial velocity variables χ^1 Ori, Procyon, and ξ Boo A, which are also found to be variable in the McD observations. One of the eight McD remaining stars is HD 114762, which was added to the McD sample after the original LMC orbit was announced (Latham et al. 1989). Thus, the McD survey has seven stars, in addition to the CFHT and CfA samples, that would have passed the CfA selection process, one of which, μ Her, is a strong candidate to have a low-mass companion.

The MJUO sample includes 21 F and G dwarfs and subgiants (Murdoch et al. 1993). Four of the MJUO solar-type stars are also included in the McD sample: Procyon, β Virginis, 61 Virginis, and β Aquilae. Murdoch et al. (1993) also report a velocity variation for Procyon. In addition, Murdoch et al. report a velocity variation for α Centauri that agrees with the velocity variation expected from the astrometric orbit. This star should be retained in the merged sample because it would have passed the CfA selection criteria. Finally, Murdoch & Hearnshaw (1993) report an orbital solution for HR 3220 with $K = 3.1$ km s^{-1} and $P = 900$ days. This binary would not have passed the CfA selection criteria and therefore should not be included in the merged sample. Thus, the MJUO survey has 16 stars, in addition to the CFHT, McD, and CfA samples, that would have passed the CfA selection process, one of which, β Hydri, is a candidate to have a substellar companion.

We now consider the sample of solar-type stars resulting from the merger of the four very precise surveys with the CfA sample. Obviously, the combined sample is not completely uniform, even though we have eliminated stars that would not have passed the CfA selection criteria, because the five parent samples were not originally selected in the same way. Nevertheless, if we ignore possible remaining differences between the five samples, we are left with a merged sample of $20 + 11 + 7 + 16 = 54$ stars, with three LMC orbits and two candidate LMCs. Suppose that neither of the two candidates turns out to yield an LMC orbit. The probability of getting at random three LMCs in a specific subsample of 20 stars out of the final sample of 54 stars is

$$\frac{20 \times 19 \times 18}{54 \times 53 \times 52} = 0.046 . \qquad (14)$$

Even with this stringent approach, where we ignored the two LMC candidates, the derived probability, although small, is not small enough to reject the underlying hypothesis. We therefore suggest that the fact that the three low-amplitude binaries, with K smaller than 2.5 km s^{-1}, occurred all within the CfA sample of 20 solar-type IAU standards is a result of some statistical fluctuation. We further suggest that the combined sample indicates that a few percent of the solar-type stars have LMCs with periods less than 3000 days and with masses that could be in or near the substellar region.

7. OTHER TECHNIQUES

So far, we have discussed evidence for low-mass companions based solely on orbital solutions for single-lined spectroscopic binaries. This approach is limited, because we cannot infer the exact mass of the companion from the orbital solution, and must rely on statistical arguments.

The ambiguity about the secondary mass can be addressed by using additional information to determine or estimate the inclination angle of the spectroscopic orbit. One possible approach is to determine the inclination of the primary's *axial rotational* and to *assume* the orbital inclination is similar. This assumption is supported by the study of Hale (1994), who found a tendency for coplanarity between the rotational and orbital planes for binaries with periods less than 100 yr. Unfortunately, the inclination of a star's rotational plane cannot be measured directly either. Instead, it can be inferred by estimating the stellar rotational period (or, equivalently, the intrinsic rotational velocity), together with a measured value for the observed projected equatorial rotational velocity, $V \sin i_{\rm rot}$, derived from the width and shape of appropriate spectral lines. The stellar rotational period could be derived either from observed periodic modulations of the star's light curve or from some estimate of a characteristic rotational period of stars with spectral type and age similar to those of the primary.

This approach was taken by Cochran et al. (1991), and more recently by Hale (1995), in an attempt to estimate the mass for the companion of HD 114762. Cochran et al. measured $V \sin i_{\rm rot} = 0$ km s^{-1}, with a "hard upper limit of 1 km s^{-1}," while Hale measured $V \sin i_{\rm rot} = 0.8 \pm 0.7$. The main problem faced by these two studies was what value to adopt for the intrinsic rotational velocity of HD 114762. Cochran et al. used the results of Soderblom (1982) for guidance and adopted 5 km s^{-1} as the representative rotational velocity *observed* for F9 dwarfs. They then corrected for the expected inclination factor and came up with an intrinsic rotational velocity of 6.4 km s^{-1} for HD 114762. Using their "hard upper limit" observed for HD 114762 leads to $\sin i_{\rm rot} \leq 0.16$. If one then assumes an exact alignment, this line of logic almost rules out the possibility that the unseen companion of HD 114762 has a mass below the substellar limit. Cochran et al. noted, however, that the high space motion and low metallicity of HD 114762 indicate that it is older than the F9 stars observed by Soderblom, and therefore the true rotational velocity of the primary could be much less, perhaps as low as 2–3 km s^{-1}. In this case, sin i_{rot} could be as high as 0.5 km s^{-1}, and the mass of the secondary could be as small as 0.02 $M_{\odot}$.

Cochran et al. failed to address two important points. First, the primary of HD 114762 is different from the typical F9 star observed by Soderblom (1982), because HD 114762 is metal poor with [m/H] $= -0.8$ (Carney et al. 1994). Therefore, it is considerably less massive ($M = 0.73$ $M_{\odot}$) and cooler ($T_{\rm eff} = 5790$ K) than normal F9 stars with solar metallicity. If the intrinsic rotation of a star depends more fundamentally on the mass and/or temperature than on spectral type, then the intrinsic rotation of HD 114762 should be slower than that of the Sun, which rotates at 2 km s^{-1}.

Second, Cochran et al. failed to comment on the implication of a possible spread in the intrinsic rotation of F9 stars. The G0 stars observed by Soderblom, for example, have a mean of 4.4 and a spread of 3.5 km s^{-1}, which is much more than the spread expected for random orientations. This introduces additional uncertainty into the value that one should adopt for the intrinsic rotation of HD 114762, and therefore additional uncertainty into the inclination derived for the orbit.

Hale's (1995) analysis is somewhat different. He assumed that an F9 star with log $t = 8.8$, where t is the stellar age in years, has an intrinsic rotational velocity of 8.3 ± 2.2 km s^{-1}, and he adopted the decay law for stellar rotation derived by Skumanich (1972). Hale (1995) then derived a velocity of 2.3 km s^{-1} for the present intrinsic rotation of HD 114762, assuming log $t = 9.9$. He concluded that the best estimate for sin i_{rot} is 0.34, with a corresponding mass for the companion of 0.038 $M_{\odot}$. As in the case of Cochran et al. (1991), Hale's analysis also neglects to take into account the possibility that the primary of HD 114762 may have a much slower intrinsic rotation than normal F9 stars because of its lower mass due to its lower metallicity. In addition, Hale (1995) limited his estimated age for HD 114762 to the lifetime of a 1.17 $M_{\odot}$ star, when in fact the age could be considerably older. For example, Edvardsson et al. (1993) derive an age for HD 114762 of 14 Gyr, based on isochrone comparisons, implying a much slower intrinsic rotation.

Finally, one should not neglect to take into account the effect of the low metallicity of HD 114762 on the primary mass when calculating the secondary mass from the mass function. Latham et al. (1989) adopted 1 $M_{\odot}$ for the primary, but the more recent determination by Carney et al. (1994) gives 0.73 $M_{\odot}$. This means that the companion masses estimated by Latham et al. (1989), Cochran et al. (1991), and Hale (1995) are all too large by a factor of 1.37.

This discussion illustrates the limitations set by uncertainties in the values adopted for the intrinsic rotation of HD 114762. When one considers, in addition, the uncertainty in the misalignment of the rotational and orbital planes, it is clear that the above attempts to establish the mass of the companion of HD 114762 are inconclusive, and the mass of the companion could easily turn out to be as small as 0.02 $M_{\odot}$.

Another technique for deriving the orbital inclination involves the analysis of eclipse light curves, but this can only be applied to those rare cases where eclipses occur. Robinson et al. (1990) searched for eclipses in HD 114762 without success. Based on the CfA velocities available in 1989, they predicted a primary eclipse at JD 2,447,601.1 ± 1.5 days. With the full CfA data set now available, we calculate that an eclipse would have occurred at JD 2,447,603.8 ± 2.2 days. It turns out that Robinson et al. monitored HD 114762 for only 14% of the 1 σ eclipse window predicted by the full data, so their lack of success is inconclusive. Even if eclipses could be ruled out rigorously, it would exclude only a narrow range of inclination angles near 90°.

The secondary mass can be determined directly if the radial velocity of the faint companion can be measured, thus turning a single-lined spectroscopic binary into a double-lined system. Then the mass ratio can be derived from the ratio of the velocity amplitudes for the two components. Detecting the secondary spectrum is very difficult for a low-mass companion, because it is much fainter than the primary. However, algorithms that are very sensitive to the spectrum of a faint companion, such as the two-dimensional correlation TODCOR (Zucker & Mazeh 1994), may be able to measure the radial velocity of the secondary, given enough observed spectral range and high enough signal-to-noise ratio (Mazeh & Zucker 1994). This technique should work best in the infrared, where the contrast between the secondary and the primary should be more favorable than in the optical.

8. CONCLUSIONS

We have analyzed the detection of *three* low-amplitude spectroscopic binaries within a sample of 20 solar-type IAU standards observed at the CfA. Although three is a dangerously small sample for drawing general conclusions, these detections do suggest that the secondary-mass distribution rises near the substellar limit. It is not yet clear whether this rise occurs in the regime of substellar secondaries, which implies that substellar companions are frequent, or whether the rise occurs in the stellar regime. Obviously, these two interpretations are not exclusive, and either or both could be true. If, on the other hand, the mass-ratio distribution is flat above the substellar limit, the assumption that there are no substellar companions is rendered rather improbable.

We have compared our three detections with the results of four very precise radial velocity searches for substellar companions. The failure of the other searches to detect LMC orbits is indeed puzzling but can be accounted for if we assume that only a small fraction of the solar-type stars, of the order of a few percent, have low-mass companions. Over the next few years, the results from larger samples and the application of other techniques for estimating secondary masses should help to uncover the true shape of the mass-ratio distribution near the substellar limit.

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