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A RELATIONSHIP BETWEEN MEAN ROTATION PERIOD IN LOWER MAIN-SEQUENCE STARS AND ITS OBSERVED RANGE[1]

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ABSTRACT

Chromospheric Ca II H and K fluxes have been measured in a sample of ∼100 stars on or near the lower main sequence at Mount Wilson Observatory. Observations were made several times a week and span more than 10 years. Within an observing season, many stars show periodic variations due to rotation. Thirty-six of the stars have highly significant periods in at least five seasons. We compute the range in the observed period, ΔP, and suggest that it is a measure of, and a lower limit to, the surface differential rotation.

Several physical and selection effects can affect the measured ΔP value. An analysis of the cumulative variance distribution at various timescales, however, demonstrates that Ca II variations due to active region growth and decay are generally of longer period and smaller amplitude than those due to rotation. We argue that other effects (e.g., multiple active regions, latitude bands) either are small or primarily act to reduce the measured ΔP relative to its true value.

Including results for the Sun, we find that ΔP depends on the mean seasonal rotation period $\langle P \rangle$ such that $\Delta P \propto \langle P \rangle^{1.3 \pm 0.1}$, independent of mass. We briefly discuss this result in the context of dynamo models, and other observations of surface differential rotation and active region structure.

Subject headings: stars: activity — stars: chromospheres — stars: late-type — stars: rotation

1. INTRODUCTION

The solar surface differential rotation (SDR) was discovered independently by Carrington (1858, 1863) and Spörer (1861) through observations of the mean latitude of sunspots over the course of the 11 yr cycle. Maunder's "butterfly diagram" (Maunder 1913; Yallop & Hohenkerk 1980) shows that, at the start of a new sunspot cycle, active regions (ARs) form at midlatitudes and, as activity rises and then declines, ARs subsequently form at latitudes closer to the equator. At the same time, the mean rotation period marked by sunspots progresses from longer to shorter periods.

Measurements of SDR set an observable boundary condition on the magnitude of the internal differential rotation. In the case of the hydromagnetic dynamo, the interaction of internal differential rotation, convection, and turbulence is responsible for generating and sustaining stellar magnetic fields. However, no dynamo model successfully recreates the spatial and temporal properties of magnetic field emergence on the Sun (e.g., Levy 1992; Krause 1993). Stellar SDR data may provide useful information in the models and so help in improving them.

It is not yet possible to resolve stellar disks and thus directly observe SDR from the migration of starspots. However, the presence of SDR can be inferred from changes in Doppler-imaging maps or changes in rotation period. The first technique is best suited to rapidly rotating stars (Vogt & Penrod 1983; Vogt & Hatzes 1991). In the second technique, broadband photometric measurements have been used successfully to infer SDR (e.g., Hall 1991), but its applicability is generally restricted to stars with significant spot coverage (again, rapidly rotating stars).

Chromospheric variations usually have a higher amplitude than photometric variations in the slowly rotating stars. A way to survey SDR in slowly rotating stars may come from spectrophotometric records of Ca II H and K fluxes, which are sensitive to the presence of chromospheric activity. If other lower main-sequence stars behave like the Sun, we would expect them to show changes in the mean latitude of activity with time, presumably in phase with the activity cycle, as in the solar butterfly diagram. Changes in the mean latitude of H- and K- emitting regions should therefore produce changes in the observed rotation period linked to changes in the phase of long-term activity, if present.

Analysis of over a decade of Ca II H and K spectrophotometric measurements made several times a week at Mount Wilson Observatory is well suited for this kind of study. The seasonal periods derived from Ca II data are related to their systematic change over the timescale of an activity cycle. We interpret the periods as rotation and their systematic change to indicate the range of SDR. We base our interpretation in part on the successful determination of SDR by the same analysis of solar records of Ca II K fluxes.

2. OBSERVATIONS

Nearly 100 lower main-sequence stars have been observed since 1966 in an effort to monitor long-term changes in chromospheric activity (Wilson 1968, 1978; Baliunas et al. 1995). Frequent monitoring of these stars (two to five nights per week) began on the 60 inch telescope in 1980 (Vaughan et al. 1981; Baliunas et al. 1985) and thus far has yielded 12 seasons of data from which rotation periods might be determined. The exceptional sky conditions at Mount Wilson for spectrophotometry plus nearly full use of the 60 inch telescope permit observations on as

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many as 300 nights per year and yearly observing windows ("seasons") for individual stars that are 150–200 days in length.

The current instrument is described in detail by Vaughan, Preston, & Wilson (1978). Briefly, at the exit slit of the spectrometer a chopper wheel is used to measure the flux in four channels at a frequency of about 30 Hz using a single photomultiplier. Two of the channels are 0.109 nm wide (FWHM) and centered on the Ca II H and K lines. The other two are 2.0 nm wide continuum passbands centered at 389.1 nm and 400.1 nm (labeled V and R). The exit slit is translated to compensate for each star's radial velocity and for the terrestrial motion around the Sun.

The observed quantity, S, is the ratio of the H and K data to the continuum passbands V and R, $S = \alpha(\mathrm{H} + \mathrm{K})/(V + R)$. The nightly calibration factor, α, is determined from observations of a standard lamp and of several "standard" stars (which show little or no apparent H and K variation), and applied ex post facto to the database. Generally, a set of three sequential observations is made for each star. An integration continues until ≈ 2000 counts are detected in the K channel, usually 1–5 minutes. Background readings of the nearby sky are taken several times during the night and are subtracted from each channel's raw counts.

Solar disk-integrated Ca II K-line measurements from Sacramento Peak Observatory (SPO; Keil & Worden 1984; Keil 1992) were analyzed to compare the results from the stars to those from the Sun (Donahue 1993; Donahue & Keil 1995). The data are described by Keil & Worden (1984). Since the solar data are nearly continuous (i.e., without seasonal gaps), the data were split into 200 day "seasons" so that they could be treated in the same way as the stellar data. This allowed 19 seasons of solar activity (with ≥ 30 days of observations each), spanning 8 yr, to be analyzed for rotation. In addition, as discussed below, the choice of a seasonal window that is shorter than 1 yr optimizes the detection of rotation (and SDR) by minimizing the effects of AR growth and decay (ARGD). Little overlap between the SPO K-line index (1977–present) and the MWO S index (lunar observations, 1966–1977) required that the two be linked by using the 10.7 cm solar radio flux as an intermediate transformation (Baliunas et al. 1995). We find $\langle S_{\odot} \rangle = 0.179$ averaged over cycles 20, 21, and 22 (the Wilson 1978 value $\langle S_{\odot} \rangle = 0.171$ was derived from cycle 20 alone).

3. ANALYSIS

Seasonal periods were inferred from periodograms suited to the analysis of unevenly spaced data (Scargle 1982; Horne & Baliunas 1986). Periodograms were calculated for each observing season for the SPO solar data and the 111 lower main-sequence stars from the HK program that had the longest time series. Any significant peak found in the periodogram was analyzed and then subtracted to check for additional peaks. Filtering the first period alters the interpretation of the periodogram for any residual peak, since the variance of the data has been changed. However, the peak height and false-alarm probability are still good estimates of the validity of secondary periods *provided the initial period determination is correct* (Baliunas et al. 1985; Horne & Baliunas 1986). The error in the *determination* of each period (Kovács 1981) is almost always 0.25%–1.0% for $P < 20$ days, and typically 0.5%–2.0% for $P > 20$ days, with a few poorly determined periods in the 30–60 days range reaching almost 4%.

Two examples of periodograms showing double peaks are shown in Figure 1. The left three panels show the chromospheric time series from the 1981 observing season for the G0 V star HD 206860 (HN Peg, *top left panel*), the associated periodogram (*left center panel*), with two clear peaks at 4.67 ± 0.01 and 4.91 ± 0.01 days, and the residual periodogram after filtering out the primary frequency (*bottom left panel*). The panels on the right side of Figure 1 show similar behavior for the time series of HD 152391 (G8 V) during the 1981 observing season (*top right panel*), with a blended peak at 10.93 ± 0.01 days (*right center panel*) that can be further resolved to show a secondary peak at 11.60 ± 0.05 days (*bottom right panel*). In each of these cases, the signals are very strong and have highly significant peaks.

3.1. *Detection of Solar and Stellar Rotation*

In the case of the Sun, the progression of seasonal rotation periods correctly follows the decline of cycle 21 and increases to the maximum of cycle 22 (Donahue 1993; Donahue & Keil 1995). The successful detection of the solar SDR from disk-integrated chromospheric data at first appears to be at odds with previous efforts (LaBonte 1982, 1984; Harvey 1984; Gilliland & Fisher 1985) that did not successfully detect the solar SDR pattern from a variety of activity indicators, including Ca II. The primary difference between our effort and previous attempts appears to be in our choice of the shorter seasonal window length, which for all previous efforts was on the order of 1 year.

The length of the seasonal window is a factor in detecting SDR from the SPO records. The pooled variance of the solar data (Dobson et al. 1990) shows that there is a marked difference in the variance at ~180 days and ~1 yr, suggesting an increase in the contribution of ARGD to the time series between the two timescales. A 1 yr baseline contains a significant amount of ARGD within the observing window. Since the average lifetime of a solar active longitude is on the order of 180 days (Gaizauskas et al. 1983), a baseline of 1 yr could have significant shifts in phase caused by the change in mean longitude of ARs marking rotation. Those shifts will blur the precision of the observed rotation period. A shorter (200 day) observing window lessens this effect, yet is long enough for several rotations to be included.

Use of the shorter baseline results in a positive detection of the solar SDR. However, in comparison to other stars in the sample, the success rate of detecting rotation from the solar data is below average. Out of 19 observing seasons for the Sun of length 200 days, rotation was measured in only eight seasons, one of the lowest success rates in the sample. If the Sun were measured as frequently as the stars in the sample, it is likely that only about five successful rotation detections would have occurred. Since a minimum of five rotation determinations was (arbitrarily) required for a star to be included as an SDR candidate, it is possible that a star with properties similar to the Sun would *not* have made the cutoff.

3.2. *Range of Observed Rotation Periods, ΔP*

Thirty-six stars from the MWO chromospheric data show five or more seasons with detectable periodic modulation. Stars with less than five determinations of seasonal

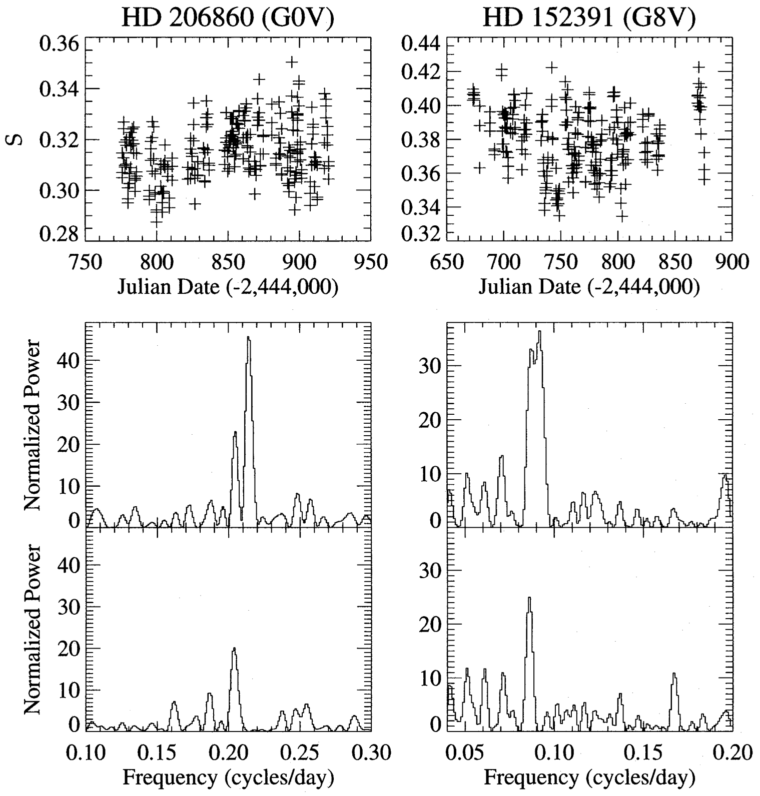

FIG. 1.—Two examples of multiple periods observed within a single observing season, suggestive of the presence of surface differential rotation. *Left panels:* the 1981 observing season for HD 206860 (G0 V, *top*) has two peaks in the primary periodogram (*center*) at 4.67 ± 0.01 and 4.91 ± 0.01 days, respectively. The secondary period is still noticeable in the filtered periodogram (*bottom*). *Right panels*: the 1981 observing season for HD 152391 (G8 V, *top*) has a broad-blended peak with a maximum at 10.93 days (*center*). Filtering the primary period resolves the secondary 11.60 day peak.

rotation were discarded from further analysis. These intraseasonal variations are interpreted to indicate rotational modulation, as they correspond to rotational velocities that are consistent with available $v \sin i$ measurements and in some cases (e.g., HD 1835, G3 V) match previously measured periods obtained through both chromospheric (Baliunas et al. 1983, 1985) and photometric (Dorren & Guinan 1994) measurements. We observe a range in P over the 12 yr interval of observation, which includes in some cases complete coverage of an activity cycle (Baliunas et al. 1995). We then compute $\Delta P \equiv P_{\max} - P_{\min}$. We will assume that the range of P may be interpreted as the change of the mean latitude of ARs due to SDR.

An example showing the entire set of periodograms for one of the program stars appears in Figure 2 for HD 115383 (59 Vir, G0 V). Rotation was detected in five seasons, with a mean period of $\langle P_{\rm rot} \rangle = 3.33$ days and a range of 0.18 days (5%). This star is listed in Table 2 of Baliunas et al. (1985) under the heading "No Detection of Differential Rotation," although rotation was detected in both 1982 and 1983 (the top two sets of periodograms in Fig. 2). While some detections of rotation are clear (the 1982, 1983, and 1984 observing seasons), others are more speculative; for example the cluster of power near the mean rotational frequency in 1988 only presents a single peak if low-frequency modulations are filtered from the data (presumably due to active region evolution). The persistent secondary peak in the 1983 observing season was noted by Baliunas et al. (1985) as an alias frequency arising from the season's window function. Unlike the strong peaks seen in Figure 1, several of the peaks used to determine rotation for this star are weak and approximately indicate the limit to which we can reliably determine a rotation period from the available chromospheric time series through periodogram analysis.

Table 1 lists the identification of each star (HD number, col. [1]), the $(B-V)$ color and mean activity index $\langle S \rangle$ (cols. [2] and [3]), the mean rotation period measured over all seasons (col. [4]), the number of rotation periods found in the data (N_P, col. [5]), and the minimum and maximum periods used to calculate ΔP (cols. [6] and [7]). For some stars, N_P exceeds the number of seasons because, in several cases, multiple periods were found in which case the relative significance is not certain (Baliunas et al. 1985). In a few cases, the values of ΔP are influenced by a single, discrepant period. In order to consider the accuracy of ΔP, those outlying periods were discarded and ΔP was recalculated. However, these discarded periods are included as footnotes to Table 1.

Four stars (HD 206860 = HN Peg, HD 101501 = 61 UMa, HD 190406 = 15 Sge, and HD 114710 = β Com) were previously noted by Baliunas et al. (1985) to have a likely detection of SDR. An additional eight were also listed as possibly showing SDR, of which six appear in Table 1. With 12 yr of rotational data (eight or nine more than available to Baliunas et al. 1985), it is possible to monitor the change in P for several stars over the course of most of an activity cycle.

Analysis of the record of the G0 V star HD 114710 (β Com; Donahue & Baliunas 1992) showed a variation of P in phase with its 17 yr activity cycle, but opposite to the Sun's with time, i.e., the rotation period of β Com *increases* with the activity cycle phase. While some of the stars in

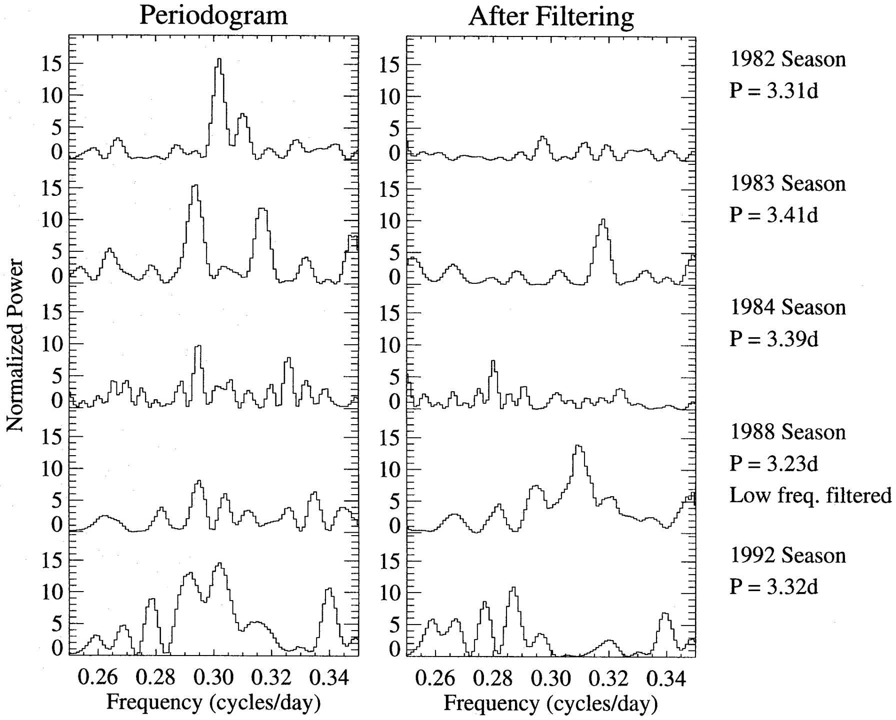

FIG. 2.—Rotation determinations for HD 115383 (G0 V, 59 Vir) in five observing seasons. *Left panel*: the primary periodograms at frequencies near the mean rotation period $\langle P_{\rm rot} \rangle = 3.33$ days. *Right panel*: the residual periodograms after filtering of the primary frequency. In the 1988 season, a period was measured from a secondary peak only after a low-frequency signal had been removed (presumably arising from active region evolution). The apparently strong residual peak in the 1983 season was previously identified by Baliunas et al. (1985) as an alias arising from the window function.

Table 1 show either solar-like or β Com-like trends of period with cycle phase, others have a varied set of correlations between $\langle P \rangle$, mean activity level, and activity cycle phase (if a cycle is present). Further investigation of SDR patterns and the variation of rotation period with activity cycle phase is discussed in related papers (Donahue 1993; Donahue & Baliunas 1994; Donahue 1996).

3.3. *Systematic Biases That Affect* ΔP

The criteria of selecting stars from the analysis introduce several biases. First, only stars that have repeated (≥ 5) high-quality measurements of a seasonal rotation period are chosen. Therefore, a star lacking one dominant AR, or with several ARs spaced in longitude, might not show a clear rotational signal. The possible reasons for a lack of one dominant AR are several: a uniform AR distribution, confusion with multiple regions or measurable AR evolution, a nearly pole-on inclination, or sparse data. Stars with such problems will produce significant rotation periods infrequently. The Sun sometimes shows a single dominant AR and at other times several ARs. SDR will most easily be detected among stars that have relatively stable modulation over several rotations within a season from a dominant group of ARs that experience a noticeable change in mean AR latitude (corresponding to a change in mean rotational period) between consecutive observing seasons.

Ideal values of ΔP would indicate the full range of rotation periods exhibited by active regions on the surface. However, selection effects may affect the analysis and interpretation of ΔP.

In the case of the Sun, the observed rotation period of activity tracers depends on the phase of the sunspot cycle. Therefore, it is important to observe P throughout the course of at least one cycle, particularly if, like the Sun, stars also show an abrupt change in the mean latitude of active regions (and therefore P) near activity minimum. Observed activity cycle periods for lower main-sequence stars (Wilson 1978; Baliunas & Vaughan 1985; Baliunas et al. 1995) range from 2 to 25 yr, with some showing only long-term trends in activity. Since the success rate of detecting a seasonal rotation period for the stars in Table 1 ranges from 36% to 85%, it is possible that some latitudes at which ARs exist during a star's activity cycle have not been sampled, leading to an underestimate of ΔP.

A bias in ΔP may arise if trends exist in mean AR latitude with spectral type. Coriolis effects will tend to force flux to rise parallel to the rotation axis for stars with more rapid rotation (Schüssler & Solanki 1992). The shallower convection zones of F and G stars may permit flux concentration at lower latitudes than K stars. Then, given the random distribution of sin *i*, it is possible that active latitudes will be sampled differently for some spectral types than for others. At all phases of the solar cycle, ARs are seen at a range of latitudes, and the location of the active-latitude band is influenced by the cycle's intensity (see Maunder's "butterfly diagram" in Maunder 1913; Yallop & Hohenkerk 1980). For example, β Com (Donahue & Baliunas 1992), may have more than one active latitude. Multiple periods for ARs at different latitudes could affect the determination of P from the periodogram analysis and hence, ΔP. On very active stars with large filling factors, ΔP may be minimized because the periods determined might only result from a limited latitude band where enough "gaps" in the plage exist to permit detection of rotation modulation.

The periodogram technique assumes that one primary complex of active regions at a latitude ϕ is responsible for marking out $P(\phi)$ and that $P(\phi)$ can be mapped to a unique $\pm\phi$. However, images of the Sun frequently show several

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TABLE 1

LIST OF SDR CANDIDATE STARS

HD (1)	$(B-V)$ (2)	$\langle S \rangle$ (3)	$\langle P \rangle$ (days) (4)	N_P (5)	$P_{\rm min}$ (days) (6)	$P_{\rm max}$ (days) (7)
Sun[a]	0.66	0.179	26.09	8	24.5	28.5
1835	0.66	0.349	7.78	14	7.23	8.30
4628	0.88	0.230	38.5	8	37.2	41.4
10476[b]	0.84	0.198	35.2	7	34.0	37.3
16160	0.98	0.226	48.0	5	42.2	51.5
17925	0.87	0.653	6.76	10	6.56	7.20
20630	0.68	0.366	9.24	9	9.01	9.48
22049[c]	0.88	0.496	11.68	9	11.04	12.18
26913	0.70	0.396	7.15	5	6.93	7.26
35296	0.53	0.332	3.56	7	3.44	3.66
39587	0.59	0.325	5.36	14	5.07	5.89
72905	0.62	0.367	4.69	6	4.60	4.82
78366	0.60	0.248	9.67	9	9.12	10.19
81809	0.64	0.172	40.2	6	37.0	43.0
82885	0.77	0.284	18.60	11	17.93	19.73
101501[d]	0.72	0.311	16.68	10	16.13	17.62
106516	0.46	0.208	6.91	5	6.75	7.13
114378	0.45	0.244	3.02	9	2.76	3.23
114710[d]	0.57	0.201	12.35	12	11.43	13.54
115383	0.58	0.313	3.33	5	3.23	3.41
115404[e]	0.93	0.535	18.47	7	17.24	19.90
129333[f]	0.61	0.544	2.80	5	2.50	3.12
131156A[e]	0.76	0.461	6.31	9	6.06	6.62
131156B[g]	1.17	1.381	11.94	7	10.92	13.19
141004	0.60	0.155	25.8	6	23.6	28.8
149661[e]	0.82	0.339	21.07	9	20.6	22.9
152391[e]	0.76	0.393	11.43	11	10.20	12.97
154417[e,h]	0.57	0.269	7.78	13	7.49	8.41
155885[i]	0.86	0.384	21.11	5	19.46	22.91
155886	0.86	0.375	20.69	7	18.07	22.47
160346[e]	0.96	0.300	36.4	5	35.4	37.8
166620	0.87	0.190	42.4	7	33.4	50.8
190007	1.17	0.746	28.95	9	25.72	32.83
190406[d]	0.61	0.194	13.94	10	12.67	15.57
201091	1.18	0.658	35.37	13	26.82	45.25
201092	1.37	0.986	37.84	12	31.78	46.57
206860[d,j]	0.59	0.330	4.86	18	4.57	5.30

[a] Excludes a $22^{\rm d}.8$ period observed in 1990. The longest periods observed ($\gtrsim$28 days; see Donahue & Keil 1995) correspond to latitudes on the Sun that are beyond the range where ARs are usually found. These periods are seen at phases just after cycle minimum are likely influenced by the evolution of small ARs.

[b] Excludes a $31^{\rm d}.6$ period observed in 1985.

[c] Excludes a weak secondary $13^{\rm d}.9$ period observed in 1980.

[d] Likely SDR candidate of Baliunas et al. 1985.

[e] Possible SDR candidate of Baliunas et al. 1985.

[f] Excludes two detections of rotation that disagree significantly from periods derived from contemporaneous photometry.

[g] Excludes a weak secondary $14^{\rm d}.6$ period observed in 1984.

[h] Excludes a $6^{\rm d}.8$ period observed in 1992.

[i] Excludes a $32^{\rm d}.0$ period observed in 1981.

[j] Excludes a secondary $5^{\rm d}.7$ period observed in 1984.

ARs on the disk at the same time. In the worst case, this will completely destroy the coherence of the rotation signal (in which case a seasonal rotation period will not likely be measured). In other cases, for example, two ARs at nearly the same latitude but widely separated in longitude, the periodogram and resulting P can be altered by splitting the periodogram peak into two peaks about P and with deviations from P up to a few percent (Donahue 1993).

ARGD can also affect the period determination. For the majority of the 36 stars analyzed here, Donahue (1993) showed that the mean contribution of ARGD to the pooled variance over an observing season (Dobson et al. 1990) is below the contribution from rotational modulation. In such cases, the variability caused by AR evolution is greatly reduced in the presence of persistent active longitudes (Gaizauskas et al. 1983). For example, Donahue (1993) finds the star HD 206860 (HN Peg) has a noticeable excess in pooled variance at the timescale of the observing season compared to the contribution by the rotational amplitude to the total variance, suggestive of extensive AR evolution. However, Baliunas et al. (1985) infer the presence of an active longitude lasting at least 3 yr from an observed phase constancy at $\langle P \rangle$. The apparent contradiction is resolved if a dominant longitude of activity persisted but individual AR areas changed dramatically.

In general, ΔP will be underestimated by varying amounts because of these selection effects, and thus ΔP represents a *lower limit* on the true SDR rate of a star. Overestimates of ΔP will primarily be the result of errors in the individual determinations of P alone, which we have endeavored to reduce by eliminating extreme outliers (§ 3.2).

The Sun is among the few older and more slowly rotating G stars in the sample as a result of a selection bias that favors more active stars. While Wilson (1978) attempted to sample the chromospheric activity of both active and inactive stars, younger (more rapidly rotating) stars have rotational amplitudes that are easier to detect and therefore are more readily satisfactory under the selection criteria for investigating the presence of SDR. Conversely, stars which are inactive either because of age or interim suppression of magnetic activity, i.e., "Maunder minima" (Baliunas & Jastrow 1990; Baliunas et al. 1995), less frequently provide a detectable rotation signal and therefore do not meet the criteria for further study.

In 1992, the HK project began observing approximately 50 G-type dwarfs with activity levels similar to the Sun. Therefore, with time the SDR characteristic of stars with similar to the mass and age/rotation of the Sun can be examined more closely.

We have explored the relationship between ΔP, spectral type, and rotation. Figure 3 shows the power-law relationship for ΔP versus mean rotation P. The relation, $\Delta P \propto \langle P \rangle^{1.3 \pm 0.1}$ (with correlation coefficient $r = 0.90$), shows a power law that is distinctly nonlinear. For our sample, the same coefficient of power is obtained if $\langle P \rangle$ or $P_{\rm min}$ is used. We obtain the same power-law relationship (within the statistical errors) when we separately analyze the G and K stars, suggesting ΔP is primarily a function of $\langle P \rangle$,

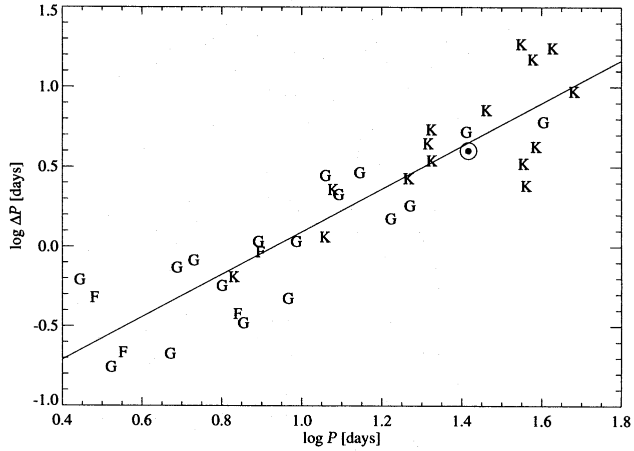

FIG. 3.—Range of observed rotation periods vs. log $\langle P \rangle$. Least-squares fit of these data yields $\Delta P \propto \langle P \rangle^{1.3 \pm 0.1}$ (correlation coefficient $r = 0.90$).

and not spectral type. The Sun agrees well with the overall relationship.

4. POWER-LAW RELATIONSHIP AND DISCUSSION

Simple dynamo theories often assume that internal differential rotation ($d\Omega/dr$) scales linearly with rotation ($d\Omega/dr \propto \Omega$; see, e.g., Steenbeck & Krause 1969; Durney & Robinson 1982; Montesinos, Fernández-Figueroa, & Castro 1987). If SDR reflects internal differential rotation, our nonlinear result for ΔP versus $\langle P \rangle$ disagrees with that assumption. Early attempts at theoretical estimations of stellar SDR (Belvedere, Paternò, & Stix 1980; Moss & Vilhu 1983) yielded conflicting results. Küker, Rüdiger, & Kitchatinov (1993) examine the effects of altering their theoretical α-Ω model of the solar differential rotation for different mean rotation rates. They predict that for a rapidly rotating Sun ($\Omega = 7.4\ \Omega_\odot$), the SDR *increases* and that, in the very slowly rotating case ($\Omega = 0.09\ \Omega_\odot$), it is nearly zero. This behavior is exactly the opposite of Figure 3 and suggests a power law the exponent of which is negative. In contrast, our result implies that SDR increases as a star ages and Ω decreases. New results by Kitchatinov & Rüdiger (1995) predict $\Delta P \propto P^{2.6}$, with an exponent which has the same sign but is much larger than our empirical result.

An example of the effects of a nonlinear relationship between mean rotation and SDR is shown in Figure 4. Figure 4 shows the classification of the long-term chromospheric activity variations for several stars in the Wilson survey (Baliunas et al. 1995) that either have activity cycles (*squares*) or variable behavior (*asterisks*). Both are characterized as a function of a dimensionless mixing-length parameter, $\xi \equiv \ell/R$, and the $\alpha - \Omega$ dynamo number (N_D) as a function of ξ, the Rossby number, and differential rotation (Brandenburg et al. 1990),

$$N_D = \frac{9}{4}\,\xi^{-3}\,\mathrm{Ro}^{-2}\,\frac{\Delta\Omega}{\Omega}\,. \qquad (1)$$

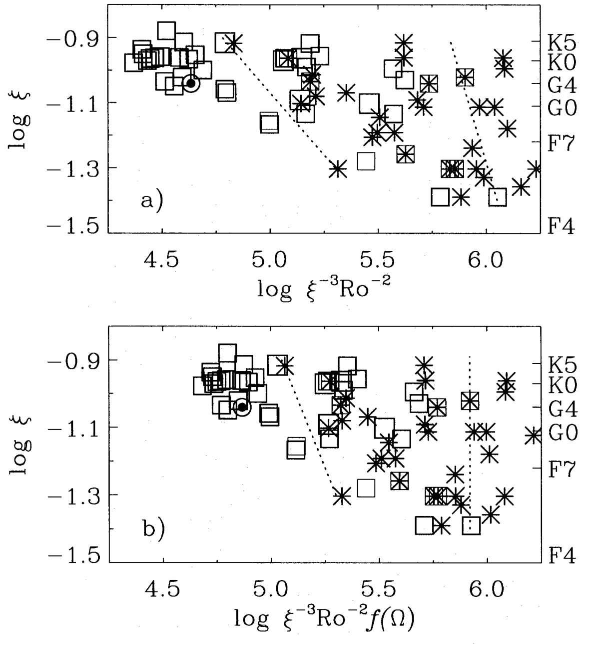

FIG. 4.—Activity cycle classification (Baliunas et al. 1995) for stars with activity cycles (*squares*) and variable, nonperiodic activity (*asterisks*) shown as a function of normalized mixing length, ξ, and $\alpha - \Omega$ dynamo number (as defined by Brandenburg et al. 1990), assuming $\Delta P \propto P$ ($\Delta\Omega \propto \Omega$) (*top panel*) and with $\Delta P \propto \langle P \rangle^{1.3}$ (*bottom panel*). The two dashed lines are the approximate boundaries where cycles begin (*right*) and chaotic behavior ends (*left*). Note that in the bottom panel the boundaries for the onset of activity cycles for different masses occurs at nearly a constant dynamo number.

To construct the top panel of Figure 4 we assumed that $\Delta P \propto \langle P \rangle$, while in the bottom panel we used the empirically derived power-law relation of $\Delta P \propto \langle P \rangle^{1.3}$. N_D should mark significant boundaries in dynamo behavior. Figure 4 indicates that the onset of cyclic activity (*right dashed line*) does not occur at fixed N_D if one assumes $\Delta\Omega \propto \Omega$. If dynamo number is a physically significant quantity important to the understanding of stellar activity, then a properly constructed N_D should define significant events in the evolution of stellar activity. Figure 4 suggests that the empirically derived power law for ΔP may improve the agreement between simple dynamo theory and observed changes in characteristics of magnetic activity variations as a star ages.

There is some theoretical evidence that ΔP might be smaller for fast rotators. Schüssler & Solanki (1992) compare models of the convective zone velocity patterns of stars with rapid rotation, in which the Coriolis force dominates over magnetic buoyancy. As a result, they predict that rapidly rotating stars should have high-latitude active regions, and they suggest that this is responsible for the many cases of polar or near-polar starspots seen in Doppler images of chromospherically active stars. Furthermore, since the convective zone depth increases substantially from F to K stars, the projected latitude range from Coriolis-influenced flux tubes should be smaller for K stars than for F or G stars. If this is the case, it would imply an underestimate of the equator-to-pole SDR, exaggerating the variation of an observed ΔP versus $\langle P \rangle$ power law with spectral type.

Studies of chromospherically active stars have been done by Hall & Henry (1990), Hall & Busby (1990), and Henry et al. (1995) in which measurements of SDR have been made from the changes of starspot latitudes over time. They examine the differential rotation coefficient, k, as a function of the equatorial rotation period, P_{eq}, and the latitude of the active region, ϕ. Since, in general, neither the latitude nor the equatorial rotation period is known, they also use the range of observed rotation periods, and an estimated range of latitudes, $f \equiv \sin^2 \phi_{\max} - \sin^2 \phi_{\min}$, or

$$(P_{\max} - P_{\min})/P_{\mathrm{eq}} = kf\,, \qquad (2)$$

where they assume values of $\phi_{\min}$ and $\phi_{\max}$.

Henry et al. (1995) arrive at a power-law relationship, $k \propto P^{\sim 0.8}$ (where f is assumed to be 1.0, implying $P_{\mathrm{eq}} \approx P_{\min}$) for a sample of mostly magnetically active binaries. This in turn implies $\Delta P \propto \langle P \rangle^{\sim 1.8}$, a steeper relation than found from the chromospheric data. The range of P_{eq} included in their sample extends beyond the sample of lower main-sequence stars presented here.

Hall & Busby (1990) assume that f is constant for their sample but k can vary from star to star. If one accepts the hypothesis of Schüssler & Solanki (1992), then as the effects of the Coriolis force are lessened over the course of rotational spin-down, the mean latitude moves equatorward (and the size of the active-latitude band probably broadens) with a greater effect seen for K stars than for F stars. The log-inverse Rossby number versus log ΔP diagram (Fig. 5)

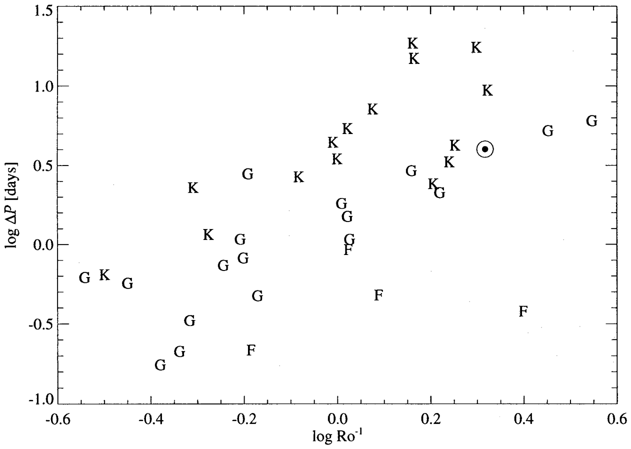

FIG. 5.—ΔP vs. inverse Rossby number

shows that at any value of Ro^{-1}, ΔP is largest for K stars and decreases for G and F stars, although the number of F stars is small and the derived convective turnover times (Noyes et al. 1984) are probably less accurate. Thus, the assumption that f is constant may not be valid, and it is likely that both f and k vary. Since Ro^{-1} scales with age through rotation, this implies that at a fixed age, K stars have greater ΔP than G or F stars.

The difference in power-law exponents for the two samples is interesting. However, the samples are different in two ways. First, the stars studied by Hall and collaborators are largely active, close, and tidally locked binaries which may well exhibit a different behavior involving rotation and SDR. The Wilson sample stars in Table 1 are all either single stars or widely separated binaries. Second, the two sets of measurements refer to different layers of the stellar atmosphere. However, this does not appear to affect determinations of the mean rotation period; photospheric periods measured photometrically by Lockwood & Skiff (1988) and Dorren & Guinan (1982) agree with chromospheric periods measured by Baliunas et al. (1983).

The systematic effects (§ 3.3) that cause us to mostly underestimate ΔP may also change the slope of the power law. However, photometric ΔP measurements are subject to the same systematic effects (incomplete latitude sampling, spectral type–dependent activity bands, etc.), and so *both* power laws may be affected similarly. Thus, the difference between the photometric and chromospheric ΔP relations may be real. If real, the differences in the power-law relations derived from photometry (exponent ~ 1.8) and from Ca II emission (exponent 1.3 ± 0.1) may reveal differences in the mean latitudes of the respective surface features, spot and plage, that these proxies measure. On the Sun, spots and plage have roughly the same latitude ranges. Our results thus suggest the possibility that spots may be confined to smaller latitude ranges than plage on rapid rotators.

Very rapidly rotating single dwarfs may also obey the power law. Doppler imaging has been done for the single star LQ Hya (HD 82558, K2 V), which has a rotation period of only 1.601 days (Jetsu 1993). Saar, Piskunov, & Tuominen (1994) have measured an upper limit of ΔP for this star ($\lesssim 0.05$ days), which in the Doppler imaging maps covers latitudes nearly from equator to pole. LQ Hya lies only 0.3 dex (1.3 σ) below the derived power-law relationship for a star with a 1.6 day rotation period.

5. SUMMARY

We have attempted over an interval of about a decade to infer seasonal mean rotational periods and period changes for stars from the Mount Wilson survey of ~ 100 lower main-sequence stars. We assume the range of P, ΔP, is a measure of the stellar surface differential rotation rate. After discarding obvious outliers, we find $\Delta P \propto \langle P \rangle^{1.3 \pm 0.1}$, independent of stellar mass. While ΔP is subject to numerous selection effects (stellar inclination angle, interference from ARGD, limited latitude range of activity, multiple active regions, etc.), we argue that it will generally be a valid lower limit to the true SDR rate. A similar analysis of solar data successfully detects SDR, unlike previous studies that analyzed yearly data sets. The empirical value of $\Delta P_{\odot}$ that we find (4 days) is close to its directly observed value over the range of latitudes for which most active regions are seen, reassuring us that the selection effects and biases have a small effect (at least for older G stars).

The best-fit power law is nonlinear, in contrast to a typical assumption of mean field dynamo theory that $\Delta\Omega \propto \Omega$. A dynamo number constructed using mean field theory plus our ΔP relation suggests that the onset of cyclic dynamo variability occurs at roughly a constant N_D, which is *not* the case if $\Delta P \propto P$ is assumed. The observed correlation between SDR and rotation implies that SDR increases as a star ages and $\langle P \rangle$ increases. At a fixed age, lower mass stars tend to have higher ΔP values.

Our relation is less steep than that found from photometry of active, evolved binaries. This may be due to the differences in the sample of stars observed, but could also be due to real differences in SDR as measured in plage (Ca II) or spots (photometry). If the latter is true, the SDR ΔP increases more rapidly for spots than plage as stars age and P increases.

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