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SMALL-SCALE POWER SPECTRUM AND CORRELATIONS IN Λ + COLD DARK MATTER MODELS

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ABSTRACT

Cosmological models with a positive cosmological constant ($\Lambda > 0$) and $\Omega_0 < 1$ have a number of attractive features. A larger Hubble constant H_0, which can be compatible with the recent *Hubble Space Telescope* (*HST*) estimate, and a large fraction of baryon density in galaxy clusters make them current favorites. Early galaxy formation also is considered as a welcome feature of these models. But early galaxy formation implies that fluctuations on scales of a few megaparsecs spent more time in the nonlinear regime, as compared with standard cold dark matter (CDM) or cold + hot dark matter (CHDM) models. As has been known for a long time, this results in excessive clustering on small scales. We show that a typical ΛCDM model with $H_0 = 70$ km s^{-1} Mpc^{-1}, $\Omega_0 = 0.3$, and cosmological constant Λ such that $\Omega_\Lambda \equiv \Lambda/(3H_0^2) = 1 - \Omega_0$, normalized to *COBE* on large scales and compatible with the number density of galaxy clusters, predicts a power spectrum of galaxy clustering in real space which is too high: *at least* twice larger than CfA estimates and 3 times larger than estimates for the APM Galaxy Survey for wavenumbers $k = (0.4\text{–}1)h$ Mpc^{-1}. This conclusion holds if we assume either that galaxies trace the dark matter ($\sigma_8 \approx 1.1$ for this model) or just that a region with higher density produces more galaxies than a region with lower density. The only way to reconcile the model with the observed power spectrum $P(k)$ is to assume that regions with high dark matter density produce fewer galaxies than regions with low density. Theoretically this is possible, but it seems very unlikely: X-ray emission from groups and clusters indicates that places with a large density of dark matter produce a large number of galaxies. Since it follows that the low-Ω ΛCDM models are in serious trouble, we discuss which ΛCDM models have the best hope of surviving the confrontation with all available observational data.

Subject headings: cosmology: theory — dark matter — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

Models with density $\rho_0 = \Omega_0 \rho_{\rm crit}$ less then the critical density $\rho_0 < \rho_{\rm crit} = 3H_0^2/8\pi G$ and a cosmological constant $\Lambda \neq 0$ have become popular in recent years for a number of reasons. With an age of the universe $t_0 \geq 13$ Gyr, such models allow a large Hubble constant $H_0 = (60\text{–}80)$ km s^{-1} Mpc^{-1}, which agrees with observational indications for large Hubble constant (e.g., Freedman et al. 1994; Riess, Press, & Kirshner 1995), although the value of H_0 remains uncertain. These models can naturally explain how galaxy clusters could have a large fraction (10%–30%; White et al. 1993b, White & Fabian 1995) of mass in baryons without severe contradictions with amount of baryons compatible with primordial nucleosynthesis (Walker et al. 1991; Krauss & Kernan 1994; Copi, Schramm, & Turner 1995). (Note, however, that the latest results on clusters, taking into account the higher masses from gravitational lensing, decrease the cluster baryon problem for $\Omega = 1$ models; see, e.g., Squires et al. 1995.) Low-Ω ΛCDM models also cure many other problems of the standard CDM model (e.g., Efstathiou, Sutherland, & Maddox 1990; Bahcall & Cen 1992; Kofman, Gnedin, & Bahcall 1993; Cen, Gnedin, & Ostriker 1993; Gnedin 1996; Park et al. 1994; Croft & Efstathiou 1994).

If the universe has $\Omega_0 < 1$ with $\Lambda = 0$, the model is not compatible with standard inflation. This means that we do not have an explanation for such fundamental observed properties of the universe as its homogeneity. The *COBE* measurements of cosmic microwave background anisotropies and other cosmic background measurements put severe constraints on any possible inhomogeneity of the universe. While rather contrived inflationary schemes do appear to be able to generate universes that look from inside like open models (Sasaki et al. 1993; Bucher, Goldhaber, & Turok 1995; Linde & Mezhlumian 1995), it remains uncertain what spectrum of fluctuations to use for structure formation in such models, and also whether they produce a sufficiently homogeneous universe. Thus, open models would solve some problems, but they could open more difficult questions. This is the reason why we consider in the present paper flat cosmological models with cosmological constant Λ: $\Omega_{\rm total} = \Omega + \Omega_\Lambda = 1$. Such models can be compatible with the inflation scenario in its most general form. They also give a larger t_0 than models with the same Ω_0 with $\Lambda = 0$.

Dynamics of cosmological models with Λ constant were discussed in many papers (e.g., Lahav et al. 1991; Carroll, Press, & Turner 1992; Kofman et al. 1993); the latest limits on the cosmological constant from gravitational lensing of quasars are those of Kochanek (1995), who concludes that $\Omega_\Lambda < 0.66$ at 95% confidence in flat models. Already the first *N*-body simulations (Davis et al. 1985; Gramann 1988) revealed a problem with the model: the correlation function is too steep and too large at small scales. For example, Efstathiou et al. (1990) found that the model with $\Omega_0 = 0.20$ matches the APM angular correlation function $w(\theta)$ nicely on large angular scales ($\theta > 1°$) but disagrees by a factor of 3 with APM results on smaller scales. The disagreement was not considered serious because "the models neglect physics that is likely to be important on small scales where $\xi(r) \gg 1$."

While it is true that one cannot reliably take into account all physics of galaxy formation, it does not mean that we can manipulate the "galaxies" in models without any restrictions. For example, one of the ways out of the problem would be to assume that places with high dark matter density somehow are less efficient in producing galaxies. Coles (1993) shows that if the dark matter density has a Gaussian distribution and the number density of galaxies is any function of local dark matter density ("local bias"), then the correlation function of galaxies cannot be flatter then the correlation function of the dark matter. Coles's results are formally applicable only for a Gaussian density distribution function, which is not the case for real models at a nonlinear stage. But it illustrates our point: just introducing antibias does *not* necessarily save the situation. In the case of a Gaussian distribution, it actually makes it worse. Our results presented in § 4 indicate that Coles's results might be true even for realistic non-gaussian distributions. This may give an explanation why we cannot satisfy APM $w(\theta)$ constraints *simultaneously* at large and small scales with any local bias: because the correlation function of dark matter in the ΛCDM model was already too steep at small scales. [So we can reduce small-scale $w(\theta)$, but this will ruin it on large scales.]

Because our numerical and Coles's analytical results were obtained for *local* bias, one might think that in order to reconcile the model with observations we need to appeal to nonlocal effects as used by Babul & White (1991) on small scales or by Bower et al. (1993) on large scales. Those nonlocal effects can result from photoionization of the interstellar medium (ISM) by UV photons produced by quasars, active galactic nuclei (AGNs), and young galaxies. Another source of nonlocality is propagation of shock waves produced by multiple supernovae in active galaxies (Ikeuchi & Ostriker 1986). It should be noted that neither effect is very efficient in suppressing star formation in high-density environments. UV radiation heats gas only to a few tens of thousands of degrees. This does effect the formation of small galaxies and delays the time of formation of the first stars in large galaxies, but it is difficult to see how it can change a large galaxy. If the gas falls into the gravitational potential of a normal size galaxy with effective temperature of about 10^6 K, it does not matter much if it was ionized and preheated. Even if a strong shock is produced by an active galaxy, it is difficult to deliver the shock to a nearby galaxy. Because galaxies are formed in very inhomogeneous environments, a shock produced by one galaxy will have a tendency to damp its energy into a local void, not to propagate into a dense area in which another galaxy is forming. Numerical hydro + *N*-body simulations, which incorporate effects of UV radiation, star formation, and supernovae explosions, could be done to check whether any antibias of luminous matter relative to the dark matter.

In this paper we are trying to avoid complicated questions about effects of star formation. We estimate the nonlinear power spectrum of dark matter and reinforce the old conclusion that it is not compatible with observed clustering of galaxies. Thus, the model has a formal bias parameter $b \sim 1$ must have extra (anti-) bias. Then we put *lower limits on the possible power spectrum of galaxy clustering.* This includes two steps:

1. All regions with mass less than $5 \times 10^9\ h^{-1}\ M_\odot$ are assigned zero luminosity because each such region does not have enough mass to produce a luminous galaxy. This step might remove small galaxies, which do not make their way into the CfA or the APM catalogs. It also raises the power spectrum by a factor of 4 by removing ~50% of mass in low-density regions.

2. We assume that in remaining high-density regions the number of density of galaxies does not depend on the density of the dark matter. This suppresses the fluctuations in groups and clusters, for example. Because galaxies in high-density regions are expected to be more clustered than galaxies in the field, this step gives a lower limit on the galaxy clustering predicted by the model. It also gives a significant antibias for galaxies. But even with this antibias, the lower limit on the power spectrum is 2–3 times higher than CfA estimates (Park et al. 1994) and 3–4 times higher than APM results (Baugh & Efstathiou 1994).

Details of the model and simulations are given in §§ 2 and 3. In § 4.1 we present results on the power spectrum: the nonlinear power spectrum of the dark matter and the lower limit on the power spectrum of galaxies. We try to estimate the correlation function of "galaxies" in real space in § 4.2. Different prescriptions for finding galaxies in simulations are used that are reasonable from our point of view. All of them produce a correlation function that is well above the observational data points. We discuss the implications of our results in § 5 and consider ΛCDM models that could be consistent with observations.

2. MODEL AND NORMALIZATION

We have chosen the following model as the first representative of ΛCDM models to simulate: $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_{\rm bar} = 0.026$, $h = 0.70$. The age of the universe for the model is $t_0 = 13.4$ Gyr. The normalization of the spectrum of fluctuations for the model corresponds to the cosmic microwave anisotropy quadrupole $Q_{\rm rms-PS} = 21.8\ \mu$K, but we actually fix the normalization using the "pivot point" $a_l = 9.5\ \mu$K for the $l = 8$ spherical harmonic amplitude of the temperature fluctuations (Stompor, Gorski, & Banday 1995). This gives the rms mass fluctuation σ_8 for top-hat filter with radius 8 h^{-1} Mpc equal to $\sigma_8 = 1.10$. With this amplitude, the model predicts a bulk velocity $V_{50} = 355$ km s^{-1} for a sphere with 50 h^{-1} Mpc.

Another way of expressing and fixing the normalization is via the number density of rich galaxy clusters. White, Efstathiou, & Frenk (1993a) estimate a cluster abundance of about $n_{\rm Cl} = 4 \times 10^{-6} (h^{-1}\ {\rm Mpc})^{-3}$ for masses exceeding $M_{\rm Cl} \equiv 4.2 \times 10^{14}\ h^{-1}\ M_\odot$. Biviano et al. (1993) give a slightly higher estimate for the same mass limit: $n_{\rm Cl} = 6 \times 10^{-6}\ (h^{-1}\ {\rm Mpc})^{-3}$. The ΛCDM model considered in this paper predicts $n_{\rm Cl} = 4 \times 10^{-6}\ (h^{-1}\ {\rm Mpc})^{-3}$ for $M > M_{\rm Cl}$, which is close to these results. (For further dicussion of

TABLE 1

PARAMETERS OF SIMULATIONS

Model	Box Size (h^{-1} Mpc)	σ_8	Cell Size (h^{-1} kpc)	Mass of Particle (h^{-1} $M_\odot$)	Number of Realizations
ΛCDM$_a$	200	0.89	390	3.9E + 10	1
ΛCDM$_b$	100	1.10	390	3.9E + 10	1
ΛCDM$_c$	80	1.10	100	2.5E + 09	1
ΛCDM$_d$	50	1.10	195	4.9E + 09	1
ΛCDM$_e$	50	1.10	125	4.9E + 09	1
ΛCDM$_f$	50	1.10	62	6.1E + 08	2

cluster masses in different models, see Borgani et al. 1995.) Here we use the Press-Schechter approximation with Gaussian filter and $\delta_c = 1.50$ A top-hat filter with $\delta_c = 1.68$ gives very similar results. White et al. (1993a) found that results from their N-body simulations of clusters are quite well matched by the Press-Schechter approximation with these parameters. These results disagree with estimates of the mass function given by Bahcall & Cen (1993), who find $n_{\rm CI} = 1.8 \times 10^{-6}(h^{-1}\ {\rm Mpc})^{-3}$, which is 2–3 times lower than numbers given by White et al. and by Biviano et al. The number of clusters is extremely sensitive to the amplitude of fluctuations. For example, if instead of $\sigma_8 = 1.10$ we would take $\sigma_8 = 0.80$ (Kofman et al. 1993; Gnedin 1996), the number of clusters with $M > M_{\rm CI}$ for our model would drop to the level $n_{\rm CI} = 4.5 \times 10^{-7}(h^{-1}\ {\rm Mpc})^{-3}$, which makes the model inconsistent with the inferred mass function of galaxy clusters. This means also that there is no room in the model for effects like gravitational waves or a small tilt of the spectrum to reduce the amplitude at small scales in order to ease the problem with the power spectrum and correlation function: lower amplitude on 1–10 Mpc scales makes the model inconsistent with the number of galaxy clusters.

3. SIMULATIONS

In order to estimate the power spectrum in the nonlinear regime, we ran six simulations using standard cloud-in-cell particle-mesh code (Kates, Kotok, & Klypin 1990; Hockney & Eastwood 1981) with various different box sizes and resolutions (see Table 1). These *COBE*-normalized simulations were started at $z = 45$ and run until $z = 0$ with constant step in expansion parameter $\Delta a = 0.003$. By comparing models with different resolution, we can estimate effects of the resolution on the power spectrum in the nonlinear regime.

In order to check our estimates of the number of clusters in the model, we ran a simulation with a much larger box and lower amplitude. In this case, the box size was 200 h^{-1} Mpc, and the resolution was $\Delta x = 390\ h^{-1}$ kpc. The amplitude $\sigma_8 = 0.9$ was chosen above the value $\sigma_8 = 0.75$–0.8 used by Kofman et al. (1993) and Gnedin (1996), but below our amplitude $\sigma_8 = 1.1$. We found that the number density of "clusters" with mass larger than $M = 4.2 \times 10^{14}\ h^{-1}\ M_\odot$ within Abell radius $r_{\rm A} = 1.5\ h^{-1}$ Mpc is $n_{\rm CI} = 1.6 \times 10^{-6}(h^{-1}\ {\rm Mpc})^{-3}$. Just as expected, it is well below Biviano et al. (1993) and White et al. (1993b) estimates.

4. RESULTS

4.1. *Power Spectrum*

Figure 1 presents the power spectrum of dark matter for our ΛCDM model. The bottom panel shows results of different simulations. On small scales ($k > 1\ h$ Mpc^{-1}), results show definite convergence: better resolution leads to a higher power spectrum, but the difference between simulations gets smaller for increasingly better resolution. On scales $0.2\ h$ Mpc$^{-1} < k < 1\ h$ Mpc^{-1}, results are fluctuating because of cosmic variance (small number of large structures in a simulation). The top panel shows the averaged nonlinear power spectrum. The spectrum was averaged over all simulations in the range $0.1\ h$ Mpc$^{-1} < k < 1\ h$ Mpc^{-1}. For larger wavenumbers, we used results from the two simulations with the best resolution. The spectrum matches the linear spectrum at $k \sim 0.2\ h$ Mpc^{-1}, but it goes significantly above it at larger wavenumbers.

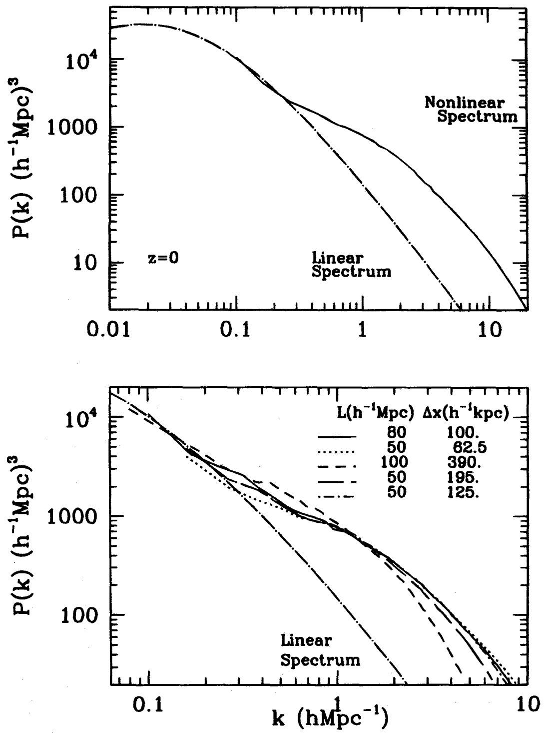

FIG. 1.—Power spectrum of dark matter for ΛCDM model. *Bottom*: Results of different simulations. On small scales ($k > 1\ h$ Mpc^{-1}), results show definite convergence: better resolution leads to higher power spectrum, but the difference between simulations gets smaller for better and better resolution. On scales $0.2\ h$ Mpc$^{-1} < k < 1\ h$ Mpc^{-1}, results are fluctuating because of cosmic variance (small number of large structures in a simulation). *Top*: The averaged power spectrum (see text for details).

In Figure 2 we compare the nonlinear power spectrum with observational results. Dots are results of Baugh & Efstathiou (1993) for the APM Galaxy survey. Open circles and triangles show results for the CfA survey (Park et al. 1994). Formal error bars for each of the surveys are smaller than the difference between the results. For the CfA catalog, the power spectrum was estimated in redshift space, and then a correction was made for velocity distortions (see Park et al. 1994 for details). Because the corrections are large and model dependent (corrections were found by comparing redshift space and real space power spectra of dark matter in a large-box, low resolution ΛCDM PM simulation), these CfA results are probably slightly less reliable then the APM results. The CfA catalog is also more shallow, and it is possible that high values of $P(k)$ in CfA are due to a few nearby large structures like the Great Wall. The full curve represents the power spectrum of the dark matter from Figure 1 (*top panel*). At $k = 0.5\ h$ Mpc^{-1}, it is 3 times larger than the APM estimate and 1.6 times larger

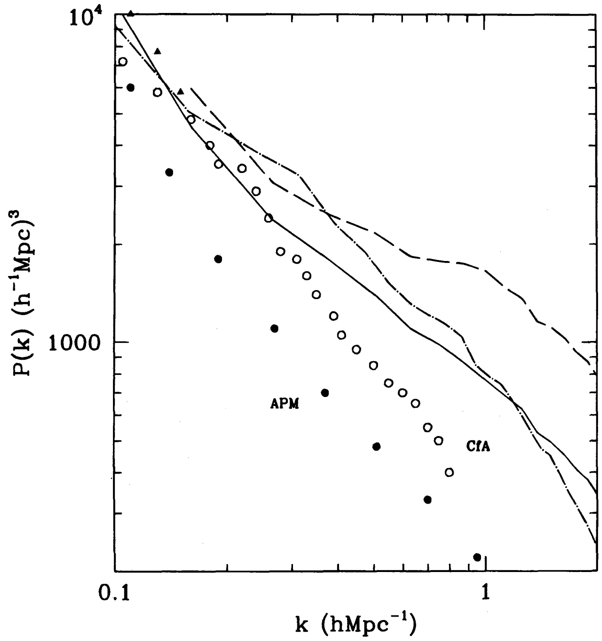

FIG. 2.—Comparison of the nonlinear power spectrum in the ΛCDM model with observational results. Dots are results for the APM galaxy survey. Results for the real space power spectrum for the CfΛ survey are shown as open circles (101 h^{-1} Mpc sample) and triangles (130 h^{-1} Mpc sample). Formal error bars for each of the surveys are smaller than the difference between the results. The full curve represents the power spectrum of the dark matter from Fig. 1 (*top*). Lower limits on the power spectrum of galaxies predicted by the ΛCDM model are shown as the dashed curve (ΛCDM_f simulation) and the dot-dashed curve (ΛCDM_c simulation). Both simulations agree on large scales, but on smaller scales ($k > 0.5$ h Mpc^{-1}) the higher resolution in ΛCDM_f results in a higher power spectrum. Both models indicate that galaxies must be more clustered than the dark matter. Comparison with APM and CfA results indicates contradictions on the level of factor 2–4. A more conventional galaxy identification method (e.g., high peaks) would imply larger discrepancies.

than the CfA value. Note that no simple scale-independent bias can help to reconcile this spectrum with observations because its shape is wrong.

We can place stronger constraints on the model by assuming that the number of galaxies produced in some volume is related to the local dark matter density: $n_{gal} = f(\rho_{dm})$, where f is some function. One would naively expect that f is a monotonically growing function: the larger the density, the larger the number of galaxies. This also agrees with the fact that in places in which we know there are large amounts of dark matter (as indicated, for example, by X-ray emission from clusters and groups), there are more galaxies. (In well-studied clusters such as Abell 2218, the galaxies, gas indicated by X-rays, and dark matter indicated by gravitational lensing or other methods all have essentially the same profile; see Squires et al. 1995) As an extreme case, we can assume that $f =$ constant; if there is enough mass to produce a galaxy, it does not matter what the density is. This is *not* a reasonable assumption for galaxy formation. We do observe that clusters and groups are the places with a higher concentration of galaxies as compared with, say, peripheral parts of cluster or filaments. But this sets a limit on the possible power spectrum of galaxies in this model: if anything, the power spectrum will be larger than our estimate.

Numerically this was realized in two steps: (i) *Remove mass from regions that cannot possibly make a galaxy.* Using our simulations with the best resolution ΛCDM_c and ΛCDM_f, we construct the density field. Then density in all cells (cell sizes are given in Table 1) are set to zero if the mass in the cell is less than 5×10^9 h^{-1} $M_\odot$. This is too small to produce galaxies in the APM or CfA catalogs. Even if we take a mass-to-light ratio 10, the absolute magnitude would be $M_V = -16.7$; only a tiny fraction of galaxies with this magnitude is in the catalogs. (ii) *We assume that all cells with higher density could produce galaxies.* But following our arguments, all of them are assigned constant number density of galaxies. The value of the constant does not affect the result because we estimate the power spectrum of fluctuations: $n_{gal}(x)/\langle n_{gal}\rangle$.

Results for the lower limit on the power spectrum of galaxies obtained in this way are shown in Figure 2 as the dashed curve (ΛCDM_f simulations) and the dot-dashed curve (ΛCDM_c simulation). Both simulations agree on large scales, but on smaller scales ($k > 0.5$ h Mpc^{-1}), the higher resolution in ΛCDM_f resulted in a higher power spectrum. Both simulations indicated that galaxies must be more clustered than the dark matter at least on scales for which we have observational data: 0.1 h Mpc$^{-1} < k < 1$ h Mpc^{-1}. Comparison with APM and CfA results indicates contradictions on the level of a factor of 2–4. A more conventional galaxy identification method (e.g., high peaks) would imply even larger discrepancies.

4.2. *Correlation Function*

Correlation functions provide another way of looking at the same problem. In this case, we cannot place limits on the correlation function as we did for the power spectrum. Instead, we try to construct two reasonable (from our point of view) prescriptions for identifying galaxies in simulations. The first prescription finds dark halos within overdensity 200. Erasing of substructure in groups and clusters ("overmerging") results in too few halos in very high density areas. We address the problem by breaking mass of very large halos into individual halos in a specific way discussed below. We also tried another prescription, which avoids the problem of overmerging by dealing with the luminosity density, not the number density of galaxies. We came to the same conclusion as with the power spectrum: clustering is too large in the cosmological model.

In Figure 3 we plot the real space correlation function of dark matter particles ξ_{dm} as a full curve. For comparison, we show also the predictions of linear theory (*triangles*). The dark matter correlation function is an average of our three simulations with best resolution ($\Lambda CDM_{c,f}$). A small correction was made to take into account waves that are longer than the box size. If ξ_{sim} is the correlation function in a simulation with the size of the box L, then the correction correlation function ξ is

$$\xi(r) = \xi_{sim}(r) + \Delta\xi(r) , \qquad (1)$$

where

$$\Delta\xi(r) = \frac{1}{2\pi^2}\int_0^{2\pi/L} P(k)\,\frac{\sin(kr)}{kr}\,k^2\,dk . \qquad (2)$$

For radii r less than 10 h^{-1} Mpc, the correction depends only slightly on r. For box size $L = 80$ h^{-1} Mpc, is was $\Delta\xi = 0.145$ at $r = 10$ h^{-1} Mpc and $\Delta\xi = 0.154$ at $r < 1$ h^{-1} Mpc. For box size $L = 50$ h^{-1} Mpc, the corrections were 0.344 and 0.393, respectively. The corrections are not important for radii less than 3–4 Mpc.

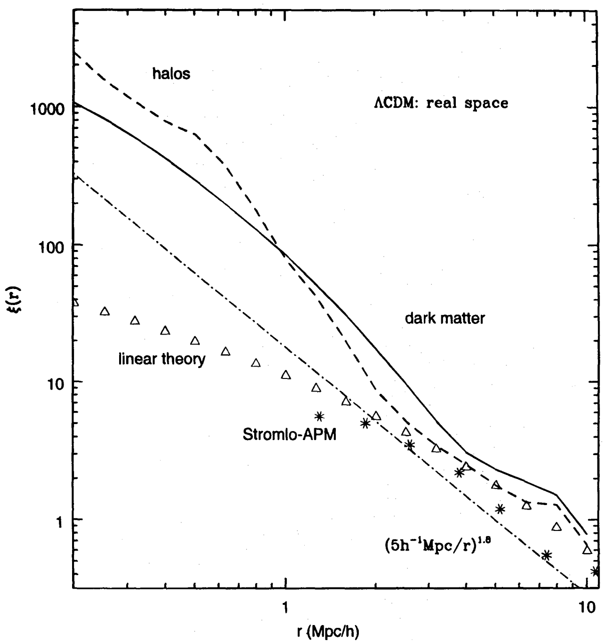

FIG. 3.—Real space correlation function of the dark matter (*full curve*) in the ΛCDM model. For comparison, we show also predictions of the linear theory (*triangles*). The usual power-law approximation $(5\ h^{-1}\ \mathrm{Mpc}/r)^{1.8}$ is shown as the dot-dashed line. Real space results of the Stromlo-APM survey are presented by asterisks. Correlation function of halos with overdensity more than 200 is shown by the dashed curve.

On scales larger than $4\ h^{-1}$ Mpc, the correlation function of the dark matter is only slightly higher than what is predicted by the linear theory. But on smaller scales, the difference is soaring: at $1\ h^{-1}$ Mpc, the nonlinear dark matter correlation function is 8 times higher than the linear theory prediction. For $0.8\ h^{-1}\ \mathrm{Mpc} < r < 4\ h^{-1}$ Mpc, the correlation function can be approximated by the power law:

$$\xi_{\mathrm{dm}}(r) = \left(\frac{6.5\ h^{-1}\ \mathrm{Mpc}}{r}\right)^{2.4} . \tag{3}$$

The slope 2.4 of the correlation function is too steep as compared with the traditional 1.8.

Figure 3 also shows the usual power-law approximation $(5\ h^{-1}\ \mathrm{Mpc}/r)^{1.8}$ (*dot-dashed line*) and real space results of the Stromlo-APM survey (Loveday et al. 1995, Fig. 2*b*) (*asterisks*). The errors of the latter are about 10% for all points (the size of markers on the plot) except for the first data point (smallest *r*), where it is about 20%–30%. It is interesting to note that on scales 1–5 h^{-1} Mpc there is a very good match of the Stromlo-APM correlation function and the correlation function given by the linear theory [i.e., the Fourier transform of the linear power spectrum $P(k)$]. In other words, in this cosmological model *there is no room for nonlinear effects*, which are essential on $\lesssim 5\ h^{-1}$ Mpc scale. In order for the model to survive, galaxies in the model must be severely antibiased with respect to the dark matter, with the biasing parameter $b \approx \frac{1}{3}$ at $r = 1\ h^{-1}$ Mpc.

In an effort to mimic galaxies in the model, we identified all dark matter halos with $\delta\rho/\rho > 200$ (see below) in our three high-resolution simulations ($\Lambda\mathrm{CDM}_{c,f}$). The correlation function of the halos is shown as the dashed curve in Figure 3. The slope of the correlation function of halos is even steeper than the correlation function of the dark matter for $0.5\ h^{-1}\ \mathrm{Mpc} < r < 2\ h^{-1}$ Mpc. Its amplitude at 1 h^{-1} Mpc has not changed. There is small anticorrelation (relative to the dark matter) on large scales $r > 3\ h^{-1}$ Mpc, which is consistent with linear bias $b_{\mathrm{lin}} = 1/\sigma_8 = 1/1.1$. The agreement with the Stromlo-APM data has not improved on $\approx 1\ h^{-1}$ Mpc scale, and it is worse on smaller scales.

The halos were identified in the following way:

1. We started with the density field defined on our 800^3 mesh. All local density maxima above overdensity limit 30 were tagged. This provides a very long list of candidates, which is used to construct a much shorter list of more realistic halos.
2. Then we found positions of density maxima independently of the mesh. For that, we placed a sphere of 70 h^{-1} kpc radius (our resolution) around each tagged maximum. Centers of mass of all dark matter particles within each sphere were found. Then sphere centers were displaced to the centers of mass, and the procedure was iterated until convergence.
3. After that, only halos with maximum overdensity larger than 200 were selected for analysis. The number of selected halos was a factor of 10 smaller than the number of tagged maxima. The radius with mean overdensity 200 was found, and this radius and mass within this radius were assigned as the radius and mass of the halo.
4. Because the procedure of finding all neighbors within large radius is very CPU time consuming, we set a limit of 0.9 h^{-1} Mpc on the maximum possible radius of a halo. Fewer than a dozen extremely large halos were affected by the limit. But those few are very important because they are clusters of galaxies. As the result of the limit, peripheral parts of clusters may be underrepresented in our halo "catalogs." The situation is unclear. While because of overmerging the central parts of clusters ($r \lesssim 300\ h^{-1}$ kpc) are basically structureless with only one halo found by the algorithm, halos are found in very large numbers in peripheral areas of clusters. The typical number of halos with mass larger than $10^{11}\ M_{\odot}$ within 0.9 h^{-1} Mpc radius for a cluster is about 100. It might be too small as compared with real clusters. This indicates also that we might be missing some halos even outside the above radius. For example, we might be missing halos that have already fallen through the cluster and that were destroyed by its tidal field. Because the destruction of halos depends on many details (rate of infall, age of clusters, trajectories of halos, densities of halos and clusters), it is difficult to estimate how important this effect is outside the 0.9 h^{-1} Mpc radius.
5. The destruction of halos in groups and clusters of galaxies ("overmerging") significantly affects the correlation function of halos. Note that destruction is not just due to the lack of resolution (Moore, Katz, & Lake 1995). It happens because dark matter halos that move in a cluster have densities smaller than the density of the halo at the center of the cluster. Any halo that comes close to the center is destroyed by the tidal force. We expect that, in reality, galaxies survive the destruction because gas loses energy and sinks to the center raising the density. But this physics is, of course, not included in dissipationless simulations. In order to overcome this problem, we assume that big halos with mass M above some limit M_{bu} actually represent not a galaxy, but many galaxies. Halos above M_{bu} are broken up into M/M_{bu} "galaxies." which are then distributed randomly inside the big halo in such a way that their number density falls off as r^{-2} from the center. We have chosen $M_{\mathrm{bu}} = 7 \times 10^{12}\ h^{-1}\ M_{\odot}$, but results are not sensitive to the

particular value of M_{bu}. In order to avoid the problem of assigning different masses to broken halos, we simply weight the contribution of every halo to the correlation function by the mass of the halo. Thus, the contribution of a large halo to the correlation function at distances larger than 1 h^{-1} Mpc is not affected at all by the breaking algorithm.

We have tried another, very different approach to "galaxy" identification, in line with what we did from the power spectrum. We tagged dark matter particles with estimated overdensity at the position of the particle above some limit. The density was constructed using our 800^3 mesh. The correlation functions of those "galaxies" is shown in Figure 4 as a long-dashed curve (overdensity larger than 200), short-dashed curved (overdensity larger than 500), and dotted curve (overdensity larger than 1000). Results represent averages over the same three simulations considered above. For comparison, we show also ξ_{dm} (*the full curve*), Stromlo-APM results (*asterisks*), and the 1.8 power law (*dot-dashed line*). On small scales, the correlation function rises with rising overdensity limit. Surprisingly, we found no trend with the overdensity on scales larger than 1 h^{-1} Mpc. The curve for overdensity 200 is almost the same (within 20%) as the curve for halos (Fig. 3).

Comparison of Figures 3 and 4 shows that results are very insensitive to details of galaxy identification. It seems that none of the simplest and most attractive schemes for the distribution of galaxies in the model can give correlation functions that agree with observations. Galaxies cannot follow the dark matter. Neither the power spectrum (§ 4.1) nor the correlation function allows this. The simplest biasing models (halos with overdensity above 200 or density above any reasonable threshold) do not work either: the discrepancy on 2–3 Mpc scales can be reduced, but the situation on smaller scale becomes even worse. We see three possible solutions for the contradiction between observed and predicted correlation functions at $\sim 1\ h^{-1}$ Mpc scale:

1. On scales less than $\sim 1\ h^{-1}$ Mpc, galaxies are 10 times less clustered than dark matter particles. This cannot be achieved by any local bias. In order for the model to succeed, galaxy formation should be significantly suppressed in galaxy clusters. It seems that there is no observational indication of that: the mass-to-light ratio in clusters does not differ much from that in groups. Also, the galaxy distribution follows that of the dark matter.
2. Observational estimates of the correlation function and the power spectrum are significantly wrong. At $k = 1\ h$ Mpc^{-1} is the power spectrum in APM and CfA surveys underestimated by a factor 2–4, and is the Stromlo-APM correlation function 10 times too small at (1–2) h^{-1} Mpc?
3. The model predicts clustering at $\sim 1\ h^{-1}$ Mpc scale, which is not compatible with that in the real universe.

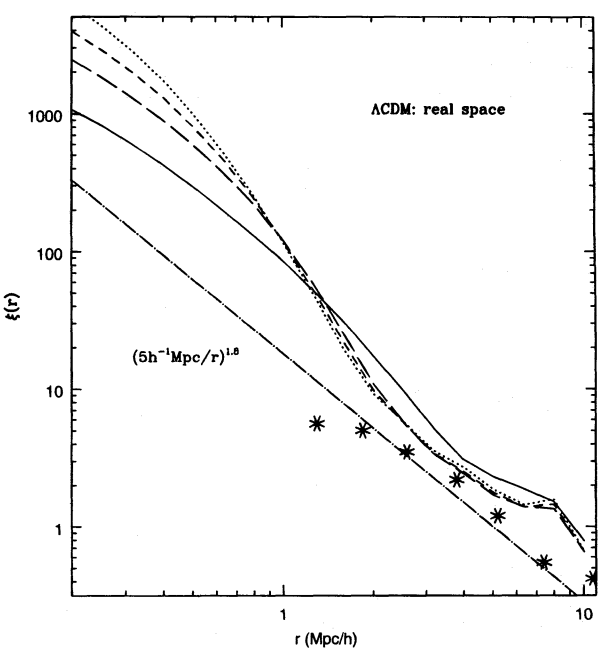

FIG. 4.—The correlation function of dark matter with overdensity more than 200 (*long-dashed curve*), 500 (*short-dashed curve*), and 1000 (*dotted curve*). Other curves and symbols are the same as in Fig. 3. Note that while the correlation function on small scales ($<1\ h^{-1}$ Mpc) depends significantly on the density threshold, it does not depend on the threshold on large scales.

The first alternative, while still possible, does not look very attractive. If we assume it, we would need to admit that there is almost no correlation of dark matter density and number density of galaxies in high-density areas. We would basically need to "paint" galaxies on small scales. This must be done carefully to avoid any disturbances on large scales, which seem to be in accord with observations. One must avoid wrecking the hierarchical scaling observed for galaxies; e.g., reduced skewness S_3 and kurtosis S_4 are independent of scale R, where $S_n(R) = \langle \xi^{(n)} \rangle_R / \langle \xi^{(2)} \rangle_R^{n-1}$; this is nontrivial, since the n-point correlations scale with the bias as $\propto b^n$, thus leading to $S_n(R) \propto b^n/b^{2n-2} = b^{2-n}(R)$ (see, e.g., Bonometto et al. 1995, and references therein; see also Frieman & Gaztanaga 1994, who gave a similar argument against "cooperative galaxy formation" on larger scales at which perturbation theory applies but measurement of S_n is harder). Recent semianalytic models of galaxy formation in the ΛCDM model by Kauffmann et al. (1995) show mild galaxy bias, not antibias on small scales. The second alternative also seems unlikely. While errors are always possible, it seems that the APM and Stromlo-APM surveys must have almost no relevance to the real distribution of galaxies in order for the model to be viable. The third possibility seems to be most likely. It does not mean that all variants of the ΛCDM model are at fault, but it implies that the most attractive variants with large age of the universe, large Hubble constant, and relatively large cosmological constant are very difficult to reconcile simultaneously with the observed clustering of galaxies and with the number of density of galaxy clusters.

5. DISCUSSION AND ALTERNATIVE ΛCDM MODELS

It is instructive to compare the $\Omega_0 = 0.3$, $h = 0.7$ ΛCDM model that we have been considering with standard CDM and with CHDM, which is just CDM with the addition of some light neutrinos whose mass totals about 5–7 eV (Klypin, Nolthenius, & Primack 1995, hereafter KNP95; Primack et al. 1995; and references therein). At $k = 0.5\ h$ Mpc^{-1}, Figures 5 and 6 of KNP95 show that the $\Omega_\nu = 0.3$ CHDM spectrum and that of a biased CDM model with the same $\sigma_8 = 0.67$ are both in good agreement with the values indicated for $P(k)$ by the APM and CfA data, while the CDM spectrum with $\sigma_8 = 1$ is higher by about a factor of 2. CHDM with $\Omega_\nu = 0.2$ (Primack et al. 1995) also gives nonlinear $P(k)$ consistent with the APM and CfA data. In the present paper, we see that for the ΛCDM model we have simulated, $P(k)$ for the same k is at least as high as CDM, and even higher for larger k. Unless the data are misleading, or some sort of complicated galaxy formation physics lead to a $P(k)$ below our estimates, this is a serious problem for the ΛCDM and standard CDM models.

But the ΛCDM model we have considered here is but one of many. So let us look in h-Ω_0 space for alternatives. In Table 2 we have compared predictions of a number of such models with data on several length scales. In this table, all the models (except were noted) are normalized to the 2 yr *COBE* data as recommended by Stompor et al. (1995). The first two lines of numbers give our estimates of several observational quantities and the uncertainties in them, from large to small scales. The bulk velocity at $r = 50\ h^{-1}$ Mpc is derived from the latest POTENT analysis (Dekel 1994; A. Dekel, private communication); the error includes the error from the analysis but not cosmic variance. However, similar constraints come from other data on large scales such as power spectra that may be less affected by cosmic variance, since they probe a larger volume of the universe. We have estimated the current number density of clusters ($N_{\rm clust}$) with $M > 10^{15}\ h^{-1}\ M_\odot$ from comparison of data on the cluster temperature function from X-ray observations with hydrodynamic simulations (Bryan et al. 1994) as well as from number counts of clusters (White et al. 1993b; Biviano et al. 1993). All recent estimates of the cluster correlation function give fairly large values at 30 h^{-1} Mpc (e.g., Olivier et al. 1993; Klypin & Rhee 1994; and references therein); this suggests also that the zero crossing of the correlation function must exceed $\sim 40\ h^{-1}$ Mpc.

The cluster constraint is powerful. The ΛCDM models with Hubble constant h as high as 0.8 cannot have Ω_0 larger than 0.2 and still satisfy the age constraint $t_0 > 13$ Gy, but for such high h and low Ω_0 the *COBE* normalization leads to too low a cluster density. The cluster density is in good agreement with observations for the $\Omega_0 = 0.3$, $h = 0.7$ ΛCDM model that we have considered in this paper, but as we mentioned above this means that we do not have the freedom to lower the normalization in order to lower the value of the nonlinear $P(k)$. Table 2 shows that the linear $P(k)$ is already fairly high at $k = 0.1\ h$ Mpc^{-1}, and as Figure 2 shows, the nonlinear spectrum is considerably higher than the linear one, especially at larger k where the conflict with the APM and CfA data becomes significant. Of course, as we lower h and raise Ω_0, we approach standard CDM for which the cluster abundance is far too high with *COBE* normalization. Thus, for ΛCDM models with intermediate Ω_0 and h correspondingly low to satisfy the t_0 constraint, the cluster abundance goes up. Consider, for example, $\Omega_0 = 0.5$ and $h = 0.6$. With *COBE* normalization, the cluster abundance is about 5 times too high, which means that there is room to lower the spectrum and perhaps resolve the $P(k)$ and $\xi(r)$ discrepancies on small scales without exceeding the *COBE* upper limit on large scales. For example, the line in the table marked "ΛCDM (not normalized)" represents a biased version of the same model, with the spectrum simply lowered by a constant factor. (This could in principle be due to gravity waves contributing on the *COBE* scale. Barrow & Liddle (1993) have an "intermediate inflation" model with a flat spectrum and a large gravity wave contribution. But recent work on inflationary cosmologies in the context of supersymmetric

TABLE 2

COMPARISON OF MODELS: *COBE* NORMALIZATION (except where marked with asterisk)

Model	Ω_0	t_0 (Gyr)	$\Omega_{\rm bar}$ (%)	Ω_ν[a] (%)	$P(0.1)$[b] [10^4(Mpc $h^{-1})^3$]	σ_8[c]	V[d] (50 Mpc)	$N_{\rm clust}$[e] (10^{-7})	r[f] $\xi = 0$	σ_v[g] (1 Mpc)
Observations......	...	≳13	...	...	<1.0	...	375	4.0	>40	<200
Uncertainties......	...	...	...	...	...	...	85	2.0	...	...
ΛCDM Models, $h = 0.8$										
ΛCDM	0.2	13.15	2.0	0	0.92	1.02	321	0.51–1.0	153	95
ΛCDM*	0.2	13.15	2.0	0	1.28	1.20	369	2.4–4.0	153	114
ΛCDM Models, $h = 0.7$										
ΛCDM	0.2	15.03	2.6	0	0.61	0.82	294	0.05–0.15	152	72
ΛCDM*	0.2	15.03	2.6	0	1.21	1.15	416	2.4–4.1	152	103
ΛCDM	0.3	13.47	2.6	0	1.05	1.13	362	3.7–6.2	125	147
ΛCDM/ΛCHDM Models, $h = 0.6$										
ΛCDM	0.4	14.47	3.5	0	0.95	1.09	374	6.3–10.	110	176
ΛCDM	0.5	13.55	3.5	0	1.16	1.25	403	22. –32.	66	253
ΛCDM*	0.5	13.55	3.5	0	0.53	0.85	274	1.0–2.0	66	172
ΛCHDM	0.5	13.55	3.5	7.2	0.88	0.86	390	3.2–5.0	100	122
CDM/CHDM Models, $h = 0.5$										
CDM	1.0	13.04	7.5	0	0.96	1.28	422	100–160	36	479
Cν^2DM	1.0	13.04	7.5	20	0.56	0.78	408	11. –20.	70	169
Cν^2DM$_{n.96}$	1.0	13.04	7.5	20	0.45	0.69	380	4.0–8.0	71	140

[a] Ω_ν is the fraction of critical density in neutrinos. For the ΛCHDM case above, $m_\nu = 2.44$ eV.

[b] $P(0.1)$ is the linear power spectrum evaluated at $k = 0.1\ h$ Mpc^{-1}, in units of (Mpc/$h^{-1})^3$.

[c] $(\Delta M/M)_{\rm rms}$ for $R_{\rm top-hat} = 8\ h^{-1}$ Mpc.

[d] Bulk velocity in top-hat sphere of radius 50 h^{-1} Mpc.

[e] Number density of clusters $N(>M)$ in units of $10^{-7}\ h^3$ Mpc^{-3} above the mass $M = 10^{15}\ h^{-1}\ M_\odot$, calculated using Press-Schechter approximation with Gaussian filter and $\delta_c = 1.50$ (*left column*) and $\delta_c = 1.40$ (*right column*).

[f] Zero crossing [$\xi(r) = 0$] of the correlation function in units of h^{-1} Mpc.

[g] Linear estimate of pairwise velocity at $r = 1\ h^{-1}$ Mpc: $\sigma_v^2 = 2H_0^2\Omega_0^{1.2}\int dkP(k)(1 - \sin kr)/kr$.

models (Dine, Rundall, & Thomas 1995; Ross & Sarkar 1995; Sarkar 1995) suggests that inflation will occur at a lower energy scale in such models, so that gravity wave production will not be cosmologically significant.) Alternatively, we could tilt the model, i.e., assume a primordial spectrum $P_p(k) \approx Ak^{n_p}$ with primordial spectral index n_p less than the Zeldovich value of unity. Or we could consider a ΛCHDM model, i.e., ΛCDM with a small contribution to the density from light neutrinos. The case shown in Table 2 corresponds to a single neutrino species with a mass of 2.44 eV, consistent with the preliminary results from the Los Alamos neutrino oscillation experiment (Athanassopoulos et al. 1995). All these alternatives lower the spectrum significantly and would no doubt result in $P(k)$ in better agreement with the data than the *COBE*-normalized $\Omega_0 = 0.3$, $h = 0.7$ ΛCDM model considered here. The same is true of the CHDM models, represented in Table 2 by two variants of the two-neutrino model with $\Omega_\nu = 0.20$. The first of these is the Harrison-Zeldovich ($n = 1$) version considered in Primack et al. (1995), but with the new *COBE* normalization; now the cluster density is too high. But as the next line shows, a small tilt ($n = 0.96$) with gravity waves corresponding to a $V \propto \phi^2$ chaotic inflationary model gives an excellent fit to all the data considered in the table. All these models would differ in their predictions for other measurable quantities such as small-angle CMB anisotropies in the first Doppler peak region, production of early objects such as damped Lyman-α systems, and small-scale velocities. We regard the further investigation of all these models as an important project for the future.

To summarize our present results: Although ΛCDM models with $\Omega_0 = 1 - \Omega_\Lambda \approx 0.3$ can in principle reconcile Hubble parameter measurements suggesting that $h \approx 0.7$ with globular cluster estimates that $t_0 \gtrsim 13$ Gyr, and also reconcile the baryon content of rich clusters with standard big bang nucleosynthesis, we have shown here that such models probably have too much power on small scales. Although an $h = 0.7$, $\Omega_0 = 0.3$ ΛCDM model normalized to *COBE* fits nicely the data on intermediate scales, including the abundance of rich clusters, the agreement of the linear power spectrum with the data on smaller scales leaves no room for the nonlinear effects that are surely important there. Our minimum *N*-body estimate of the power spectrum in this model for wavenumbers $k = (0.4\text{–}1)\ h^{-1}$ Mpc is at least a factor of 2 too high compared to the CfA results and at least a factor of 3 too high compared to APM. This is true if we assume that the galaxies trace the dark matter without significant bias, but it is also true if we just assume the galaxy density is any monotonically increasing function of dark matter density. It is true even in an extreme model in which we assume that this function is zero for regions of low density and *constant* when the density is above the threshold to produce even a small galaxy. Although our main argument concerns the power spectrum rather than the galaxy correlations, which are harder to estimate reliably from simulations, we find also that our mimimum estimates of the galaxy autocorrelation function for this model are much higher than the Stromlo-APM real space results. The only way this model can be compatible with the data is for the galaxy formation process to result in strongly scale-dependent antibiasing on small scales, but this seems hardly compatible with the observations supporting hierarchical scaling and indicating that the number density of luminous galaxies is highest in regions in which dark matter is densest, such as clusters. We suggest that ΛCDM models will have a better change of agreeing with observations if they have higher Ω_0, lower h, and a tilted ($n_p < 1$) spectrum of primordial fluctuations.

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