# THE WHITE DWARF DISTANCE TO THE GLOBULAR CLUSTER NGC 6752 (AND ITS AGE) WITH THE $HUBBLE\ SPACE\ TELESCOPE^1$

ALVIO RENZINI,<sup>2, 3</sup> ANGELA BRAGAGLIA,<sup>4</sup> FRANCESCO R. FERRARO,<sup>4</sup> ROBERTO GILMOZZI,<sup>2</sup> SERGIO ORTOLANI,<sup>5</sup>
J. B. HOLBERG,<sup>6</sup> JAMES LIEBERT,<sup>7</sup> F. WESEMAEL,<sup>8</sup> AND RALPH C. BOHLIN<sup>9</sup>
Received 1996 February 21; accepted 1996 April 17

### **ABSTRACT**

Deep *Hubble Space Telescope* (*HST*) observations with WFPC2 of the nearby globular cluster NGC 6752 have allowed us to obtain accurate photometry for the cluster white dwarfs (WDs). A sample of local WDs of known trigonometric parallax and mass close to that of the cluster WDs have also been observed with WFPC2. Matching the cluster and the local WD sequences provides a direct measure of the distance to the cluster:  $(m-M)_0=13.05$ , with an uncertainty less than  $\pm 0.1$  mag, which allows a substantial reduction in the uncertainty in the age of the cluster. Indeed, coupling this value of the cluster distance to the cluster metallicity, helium abundance, and  $\alpha$ -element enhancement  $[\alpha/\text{Fe}]=0.5$  yields an age of 15.5 and 14.5 Gyr using evolutionary models that do not include or do include helium diffusion, respectively. The uncertainty affecting these age determinations is  $\sim 10\%$ . The majority of the cluster WDs appear to be of the DA variety, while the color-magnitude location of two WDs is consistent with the DB type. This suggests a cluster DB/DA ratio similar to that of WDs in the solar neighborhood.

Subject headings: distance scale — globular clusters: general — globular clusters: individual (NGC 6752)

#### 1. INTRODUCTION

The age of the universe  $t_0$  is the obvious partner of the Hubble constant  $H_0$ . Together they set a constraint on  $\Omega_0$  if we believe the cosmological constant  $\Lambda$  to be zero, or on a combination of  $\Omega_0$  and  $\Lambda$  if one is willing to accept  $\Lambda \neq 0$  cosmologies. By general consensus, globular cluster ages provide potentially the most accurate estimate of  $t_0 = t_{GF} + t_{GC}$ ,  $t_{GF}$ being the age of the universe when the Galaxy formed and  $t_{GC}$ being the present age of Galactic globular clusters. Since presumably  $t_{\rm GF} \simeq 1\text{--}2$  Gyr  $\ll t_0$ , then  $t_0 \approx t_{\rm GC}$ , and  $t_{\rm GC}$  provides a strict lower bound to  $t_0$ . The age of Galactic globular clusters can be most accurately estimated by using the theoretical relation between age and the luminosity of the mainsequence turnoff (TO), other methods being undermined by uncontrollable systematic errors (Renzini 1991, 1993). For example, one can use a relation that fits the isochrones of VandenBerg & Bell (1985):

$$\text{Log } t_9 \simeq -0.41 + 0.37 M_V^{\text{TO}} - 0.43 Y$$

$$-0.13 [\text{Fe/H}], \qquad (1)$$

where  $t_9$  is the age in Gyr units, Y is the helium abundance, [Fe/H] is the iron abundance in standard notations, and  $M_V^{\text{TO}}$  is the TO absolute visual magnitude. In turn,  $M_V^{\text{TO}}$  =

- $^{\rm 1}$  Based on observations made with the NASA/ESA  $\it Hubble$   $\it Space$   $\it Telescope$  (HST).
- <sup>2</sup> Éuropean Southern Observatory, D-85748 Garching bei München, Germany.
- <sup>3</sup> Università di Bologna, Dipartimento di Astronomia, I-40126 Bologna, Italy.
  - <sup>4</sup> Osservatorio Astronomico, I-40126 Bologna, Italy.
  - <sup>5</sup> Università di Padova, Dipartimento di Astronomia, I-35122 Padova, Italy.
- 6 University of Arizona, Lunar and Planetary Laboratory, Tucson, AZ 85721.
- <sup>7</sup> University of Arizona, Steward Observatory, Tucson, AZ 85721.
- 8 Université de Montréal, Département de Physique, Montréal, Québec, H3C 3J7, Canada.
  - <sup>9</sup> Space Telescope Science Institute, Baltimore, MD 21218.

 $V^{\rm TO} - \text{mod}$ , where  $V^{\rm TO}$  (the TO apparent magnitude) is the directly observable quantity, and mod is the cluster distance modulus. This relation allows one to estimate the relative importance of the uncertainty in each of the four input quantities ( $V^{\text{TO}}$ , mod, Y, and [Fe/H]) in establishing the final uncertainty in the age determination. The current distances are typically affected by a ~1/4 mag error in the modulus—  $\sigma(\text{mod}) \simeq 0.25$  mag—which immediately translates into a ~22% error in the derived cluster age (~3 Gyr for an age of 15 Gyr). All other input quantities convey substantially smaller errors. The high photometric accuracy of CCDs now allows one to determine a cluster's  $V^{\rm TO}$  with an accuracy better than 0.1 mag, which translates into a  $\sim 9\%$  error in age. The helium abundance is very well known, from either the R method, primordial nucleosynthesis, or empirical determinations of the pregalactic abundance, which all indicate Y = 0.23-0.24 (e.g., Boesgaard & Steigman 1985), and even a  $\pm 0.02$  uncertainty in Y gives a negligible 2% error in age. The metal content of the best studied clusters is uncertain by  $\sim 0.3$  dex (most of it being systematic), which translates into a ~9% uncertainty in age. There is a problem with the composition of metallicity (e.g., enhanced [O/Fe], or  $[\alpha/Fe]$ ), a point to which we shall return in § 4. Clearly the first concern is the error in the distance of the clusters, and it is therefore instructive to recognize that distance determinations dominate the error budget not just of the kinematical age of the universe (via  $H_0$ ), but also of globular cluster ages. For the comparison of the two ages to be unambiguous, the error in each of them must be reduced as much as possible. The *Hubble Space Telescope (HST)* Key Project is aimed at achieving  $\sim 10\%$  accuracy on  $H_0$  (Kennicut, Freedman, & Mould 1995). We report here our own attempt at using HST observations to achieve similar accuracy on  $t_{GC}$ .

Using ground-based observations, the distance to globular clusters has been estimated with either the RR Lyrae or the subdwarf methods. Their limitations are extensively discussed by, e.g., Sandage & Cacciari (1990) and Renzini (1991, 1993). Suffice it to mention here that both methods are semiempirical

in nature, relying heavily on theoretical models (e.g., pulsational, atmosphere, and stellar models), and both require the metallicity of the calibrating stars and of the clusters to be measured. Hence, the resulting estimate of the distance is affected by both systematic errors that are difficult to quantify and by errors in metallicity that can dominate the age error budget. In this paper we present the first attempt at determining the distance to a globular cluster by using the white dwarf (WD) method (Renzini 1991, and references therein), which is essentially free from these limitations.

## 2. THE WHITE DWARF METHOD FOR GLOBULAR CLUSTER DISTANCES

The basic idea of using WDs as standard candles is very simple: to fit the WD cooling sequence of a globular cluster to an appropriate empirical cooling sequence constructed using local WDs with well-determined trigonometric parallaxes. The procedure is analogous to the classical main-sequence fitting to the local subdwarfs (e.g., Sandage 1970), but with some nontrivial advantages: the method does not involve metallicity determinations, which inevitably come with their uncertainties, and there are no complications with convection. In fact, WDs have virtually metal-free atmospheres, coming either in the DA or non-DA varieties (nearly pure hydrogen or pure helium, respectively). Moreover, WDs are locally much more abundant than subdwarfs, and therefore accurate trigonometric parallaxes can be obtained for a potentially much larger sample of calibrators. However, cluster WDs are very faint, with  $V \gtrsim 24$  even in the closest globular clusters (De Marchi, Paresce, & Romaniello 1995; Richer et al. 1995; Cool, Piotto, & King 1996). HST is therefore required to detect them and to obtain photometric data of adequate accuracy.

However, the location of the WD cooling sequence is sensitive to the WD mass. The mass of currently forming WDs in globular clusters should therefore be estimated, and the local calibrating WDs must be chosen among those matching cluster WDs. Theoretical WD models (e.g., Wood 1995) give  $\delta(\text{mag}) \simeq 2.4 \delta M_{\text{WD}}$  for the mass dependence of WD magnitude at any given temperature (or color), and therefore WD masses need to be determined with high accuracy for the method to provide competitive distances. On the one hand, the cluster  $M_{\rm WD}$  is very effectively constrained by four independent observations, namely: the luminosities of (1) the red giant branch tip, (2) the horizontal branch, (3) the AGB termination, and (4) the post-AGB stars, which are all very sensitive to the mass of the hydrogen exhausted core, and which consistently indicate  $0.51 \lesssim M_{\rm WD} \lesssim 0.55 \ M_{\odot}$ , or  $M_{\rm WD} = 0.53 \pm 0.02 \ M_{\odot}$ , virtually independent of metallicity (Renzini & Fusi Pecci 1988). Therefore, also the WD method makes some use of theoretical models, but the quantities involved are the least model-dependent—essentially, the core mass-luminosity relation. All in all, the cluster  $M_{\rm WD}$  is perhaps the most robust prediction of stellar evolution theory applied to globular cluster stars. In practice, the  $0.02 M_{\odot}$  uncertainty in the cluster  $M_{\rm WD}$  implies an uncertainty in the distance modulus of only  $\sim 0.05$  mag, or a 5% uncertainty in age, which determines the superiority of WD method.

Local WDs are characterized by a very narrow mass distribution  $(1-\sigma \simeq 0.1~M_{\odot})$ , with  $\langle M_{\rm WD} \rangle = 0.59~M_{\odot}$  (Bergeron, Saffer, & Liebert 1992; Bragaglia, Renzini, & Bergeron 1995), yielding a cooling sequence on the color-magnitude diagram having an intrinsically low dispersion (the cluster WD cooling

TABLE 1
THE LOCAL CALIBRATING WHITE DWARFS

π	Reference	$M/M_{\odot}$	Reference
DA WDs			
$0.1123 \pm 0.0072$	1	$0.553 \pm 0.063$	3
$0.0561 \pm 0.0029$	1	$0.512 \pm 0.013$	5
$0.0611 \pm 0.0028$	1	$0.502 \pm 0.017$	3
$0.0559 \pm 0.0017$	1	$0.494 \pm 0.021$	5
$0.0433 \pm 0.0035$	2	$0.513 \pm 0.012$	5
DB WDs			
$0.0291 \pm 0.0047$	1	$0.60 \pm 0.03$	6
$0.1010 \pm 0.0026$	1	$0.55 \pm 0.05$	7
	$\begin{array}{c} 0.1123 \pm 0.0072 \\ 0.0561 \pm 0.0029 \\ 0.0611 \pm 0.0028 \\ 0.0559 \pm 0.0017 \\ 0.0433 \pm 0.0035 \\ \end{array}$	$\begin{array}{c ccccc} & DA \ WDs \\ \hline 0.1123 \pm 0.0072 & 1 \\ 0.0561 \pm 0.0029 & 1 \\ 0.0611 \pm 0.0028 & 1 \\ 0.0559 \pm 0.0017 & 1 \\ 0.0433 \pm 0.0035 & 2 \\ \hline & DB \ WDs \\ \hline \\ 0.0291 \pm 0.0047 & 1 \\ \hline \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

REFERENCES—(1) Van Altena et al. 1991; (2) Harrington & Dahn 1980; (3) Bragaglia et al. 1995; (4) Bergeron et al. 1995; (5) Bragaglia & Bergeron 1996; (6) Beauchamp 1995; (7) Oswalt et al. 1991.

sequence is expected to be even narrower, given the virtually identical masses of the current progenitors). For this project the local calibrating WDs have been chosen according to the following criteria: (1) an accurate parallax and an accurate spectroscopic mass being available, as close as possible to the cluster WD's mass; and (2)  $10,000 \lesssim T_{\rm eff} \lesssim 20,000$  K, so as to match the temperature range of the cluster WDs expected to be detected with our *HST* observations. Table 1 lists the local WDs that have been used in the present experiment.

### 3. THE HST OBSERVATIONS AND DATA ANALYSIS

For this experiment the cluster NGC 6752 was selected as being the closest of the low-reddening clusters. A field about 2' SE from the center of NGC 6752 was observed with WFPC2, through the F336W, F439W, F555W, and F814W filters. Preliminary reductions have been performed on all the data, but we use here data only for the WDs in the less crowded of the four chips (WF4), and only for the two best exposed bands, F439W and F555W, with total exposure time 10,000 and 6000 s, respectively. The complete analysis will be published elsewhere (Bragaglia et al. 1996). Each exposure was processed through the standard HST-WFPC2 pipeline, including bias subtraction, dark correction, and flat-fielding. All images taken with the same filter have been aligned and averaged using a MIDAS standard task to remove cosmic-ray events. We then used the final, averaged F439W image to inventory automatically all the stars  $\sim$ 5  $\sigma$  above background. The stellar positions determined on this frame were then used as input centers for the point spread function (PSF) fitting procedure for the averaged F555W image.

Preliminary photometry of individual stars was performed on the averaged F439W and F555W frames using ROMAFOT (Buonanno et al. 1983) in a version specifically developed for handling *HST* data. In particular, the *HST* point spread function is modeled by a Moffat function plus a numerical map of residuals. The PSF parameters have been determined analyzing the brightest uncrowded stars in each field. On this preliminary color-magnitude diagram (including ~1500 objects) we made a first selection in color, choosing all objects bluer than the main sequence. We then eliminated obvious mistakes from this sample (e.g., remaining cosmic rays, blends, etc.), narrowing it to about 40 objects forming a fairly narrow sequence at the position expected to be populated by the cluster WDs. In this way the candidate WDs have been singled

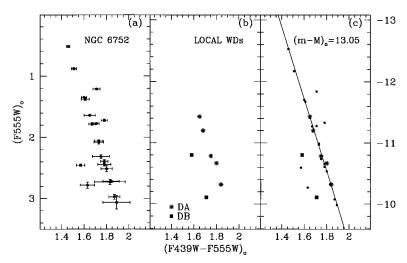


Fig. 1.—(a) Instrumental color-magnitude diagram for the cluster white dwarfs detected on the CCD chip 4 of the WFPC2; (b) the instrumental absolute color-magnitude diagram for the local, calibrating white dwarfs (from WF4 data only) of known trig parallax that are listed in Table 1 (in order of decreasing F555W luminosity to allow cross identification). WDs of the DA and DB varieties are represented by different symbols; (c) the instrumental color-magnitude diagram of the cluster and local WDs, with the former ones having been shifted in magnitude to match the local sequence. This operation delivers the distance modulus of the cluster:  $(m-M)_0 = 13.05$ . The straight line is a linear fit to the cluster WD sequence.

out for further study in order to achieve the best photometric accuracy.

To this end, the two-dimensional fitting was performed separately on each individual frame, obtaining up to five independent measurements. Each candidate WD has been examined by eye, and those contaminated by diffraction spikes or light from bright nearby stars have been rejected. Only the best 21 objects have been retained and, for them, each possibly compromised measure (from cosmic-ray hits) has been excluded for the average.

Aperture corrections to instrumental magnitudes have been applied using an aperture radius of 0.5, as suggested by Holtzman et al. (1995). For each filter, about a dozen of the brightest, unsaturated, and isolated stars were examined on one single frame; the differences between the 0".5 aperture magnitudes and the fitting magnitudes for these reference stars were averaged, and the resulting aperture correction applied to all WD candidates in the WF4 chip. The final instrumental magnitudes, obtained by averaging the independent measures in each filter, have been reduced to 1 s exposures. The resulting CMD for the 21 WDs is presented in Figure 1a, which also shows the individual photometric errors for each WD candidate. The errors have been computed as the root mean square of the frame-to-frame scatter of the instrumental magnitudes of each star. The photometric errors in each filter have then been added in quadrature to produce the error in color. Reddening corrections have been applied to the cluster WDs, adopting  $E(B-V) = 0.04 \pm 0.02$  (Penny & Dickens 1986). This corresponds to E(F436W - F555W) = 0.036 and A(F555W) = 0.13(Holtzman et al. 1995).

The local calibrating WDs have also been observed with WFPC2, exposing each of them in each of the four CCD chips through the same four filters as the cluster, thus totaling 16,  $S/N \simeq 100$  WFPC2 observations per star, though only WF4 data have been used here. On these frames aperture photometry of each WD image was obtained using a 0.5 aperture radius. The resulting magnitudes have also been reduced to 1 s, so that cluster and field WDs magnitudes are completely homogeneous. The absolute instrumental magni-

tudes of these WDs have then been obtained from their trig parallaxes (Table 1), and their location in the absolute color-magnitude diagram is displayed in Figure 1b.

### 4. THE WD COOLING SEQUENCE OF NGC 6752, ITS DISTANCE AND AGE

The WD cooling sequence in Figure 1a appears as a straight line in the diagram. Four stars lie definitely outside this main WD sequence. The two stars above the sequence might overlap with a lower main-sequence star in either a physical or a projection binary; this is confirmed by their separation from the main WD sequence being much larger in diagrams involving the I band photometry (F814W). A comparison with Figure 1b suggests that the two stars lying below the main WD sequence belong to the DB variety, and this is further reinforced by their behavior in all diagrams involving also the F336W and F814W colors (Bragaglia et al. 1996). Therefore, we identify the main WD sequence with the sequence of WDs of the DA variety, and notice that the DB/DA ratio of the cluster (roughly  $\sim 10\%$ ) appears to be consistent with that of the WD population in the solar neighborhood (e.g., Sion 1984). Having excluded these four outliers, we then proceed to obtain the distance of the cluster.

A vertical shift by  $\delta(F555W) = -13.05$  brings the cluster WD sequence to overlap the local calibrating WDs of the DA variety, as displayed in Figure 1c, and we conclude that the distance modulus of NGC 6752 is  $(m - M)_0 = 13.05$  (cf. mod = 13.12 as reported by Djorgovski 1993). The formal uncertainty of the fit is very low ( $\sim 0.025$  mag). When taking into account uncertainties in the relative cluster and local WD photometry, reddening, parallax, and average mass offset of the calibrating WDs, we conservatively estimate the overall uncertainty in the distance modulus to be less than  $\pm 0.1$  mag. It is worth emphasizing that this determination of the distance modulus does not require absolute photometric calibrations, and for this reason we prefer to stick to the instrumental magnitude scale. With this measure of the distance modulus we proceed to determine the absolute Johnson V magnitude of the main-sequence turnoff, and then the L26 RENZINI ET AL.

cluster age. For the cluster parameters we adopt:  $V^{TO} = 17.4 \pm 0.07$  and  $A_V = 0.12 \pm 0.06$  (Penny & Dickens 1986),  $[Fe/H] = -1.54 \pm 0.3$  (Zinn 1985), and  $Y = 0.23 \pm 0.02$  (Boesgaard & Steigman 1985). Thus,  $M_V^{TO} = 4.23$ , and entering equation (1), one gets a cluster age of 18.0 Gyr. This assumes solar proportions for the cluster heavy elements. Under the same assumption, the models of Salaris, Chieffi, & Straniero (1993; see their eq. [6]) yield an age of 17.8 Gyr. However, there is now ample evidence that the  $\alpha$ -elements (i.e., O, Ne, Mg, Si) are enhanced relative to iron in metal-poor halo stars and clusters, with  $[\alpha/Fe] = 0.4-0.6$  at the metallicity of NGC 6752 (e.g., Bessell, Sutherland, & Ruan 1991).

Salaris et al. have shown that what matters in the age- $M_{\nu}^{\text{TO}}$ relation is the overall heavy element abundance [M/H], rather than the detailed distribution, with [M/H] = [Fe/H] +log (0.638 $f_{\alpha}$  + 0.362), where  $f_{\alpha}$  = dex[ $\alpha$ /Fe]. Thus, for  $[\alpha/\text{Fe}] = 0.4$  and 0.6 the models of Salaris et al. give a cluster age of 16.1 and 15.3 Gyr, respectively. For  $[\alpha/\text{Fe}] = 0.6$  (i.e., [M/H] = -1.1) we estimate an age of 15.2 Gyr from Figure 7 in Bergbush & VandenBerg (1992), once more showing the good agreement between different sets of stellar models when identical cluster parameters and assumptions are adopted. This latter estimate assumes no helium diffusion inside stars during their main-sequence lifetime. When helium diffusion is allowed, Bergbush & VandenBerg models yield an age of 14.0 Gyr for [M/H] = -1.1. The error to attach to these age determinations can be estimated from the propagation of the errors in the various input parameters  $V_0^{\rm TO}$ ,  $(m-M)_0$ , [M/H], and Y. We estimate the overall error to be ~10% (a detailed error analysis will be presented along with the full data set in Bragaglia et al. 1996), with a systematic uncertainty related to helium diffusion of  $\sim \pm 0.5$  Gyr. In summary, adopting the central value  $\left[\alpha/\text{Fe}\right] = 0.5$ , one obtains an age of  $15 \pm 1.5 \pm 0.5$  Gyr (random plus systematic uncertainties).

Unlike the quest for  $H_0$ , different groups have always estimated globular cluster ages that are in substantial agreement with each other. Not surprisingly, the cluster age we have derived is in tight agreement with all other recent estimates (e.g., Bolte & Hogan 1995, and references therein). Yet, we claim to have achieved a sizable decrease of the error affecting this determination, from  $\sim 25\%$  to below  $\sim 10\%$ . When the age of the universe at the epoch of the formation of NGC 6752 is taken into account (1–2 Gyr), we end up with a present age of

the universe that can hardly be lower than 15 Gyr. When coupled with current estimates of the Hubble constant the well-known age problem is encountered, with high- $\Omega_0$  cosmological models being disfavored, and the cosmological constant  $\Lambda$  making an entrance many would prefer not witnessing. We restrain from embellishing further on this issue and rather focus on what can still be done to put globular cluster ages on even firmer grounds.

Further improvements in all steps involved in the age determination may include tightening down further the accuracy of cluster photometry and stellar abundances. Improved trig parallaxes of nearby WDs and a wider number of such calibrators to be observed with WFPC2 would also be of interest. All this together may somewhat reduce the random error below ~10%. Yet the main surviving uncertainty is perhaps of rather systematic nature. After all, the use of stellar models is unavoidable in the age dating process, and they still need to be thoroughly tested before the resulting globular cluster ages can be regarded as definitively established. However, in spite of the pressure on this issue no obvious shortcoming of stellar models has yet emerged that is able to significantly affect the derived globular cluster ages. The most crucial test to be adequately performed is perhaps one in which theoretical and empirical luminosity functions are compared, especially for the luminosity range going from the turnoff to the lower red giant branch (e.g., Renzini & Fusi Pecci 1988; Renzini 1991). Any effect able to accelerate central hydrogen exhaustion and/or an early expansion and cooling of the envelope (hence our stellar clock readings) should leave its imprint in the luminosity function that would show up as an excess of stars in the luminosity range just above turnoff. Very extensive, complete, uncontaminated, and photometrically accurate samples of cluster stars are needed for this fundamental check.

We are grateful to G. Piotto for extensive discussions on the best strategies to obtain accurate stellar photometry from *HST* data, to R. Lucas, M. Mutchler, and A. Suchkov at STScI for their invaluable help in getting our *HST* program properly set up, and to the referee for helpful comments. This project was supported in part by the Italian Space Agency (ASI), by NASA through grant GO-05439.01-93A from STScI (to J. B. H. and J. L.), and by NSERC Canada and FCAR Québec (to F. W.).

### REFERENCES

Beauchamp, A. 1995, Ph.D. thesis, Univ. Montréal
Bergbush, P. A., & VandenBerg, D. A. 1992, ApJS, 81, 163
Bergeron, P., Saffer, R., & Liebert, J. 1992, ApJ, 394, 228
Bergeron, P., Wesemael, F., Lamontagne, R., Fontaine, G., Saffer, R. A., & Allard, N. F. 1995, ApJ, 449, 258
Bessell, M. S., Sutherland, R. S., & Ruan, K. 1991, ApJ, 383, L71
Boesgaard, A. M., & Steigman, G. 1985, ARA&A, 23, 319
Bolte, M., & Hogan, C. J. 1995, Nature, 376, 399
Bragaglia, A., & Bergeron, P. 1996, in preparation
Bragaglia, A., Renzini, A., & Bergeron, P. 1995, ApJ, 443, 735
Bragaglia, A., et al. 1996, in preparation
Buonanno, R., Buscema, G., Corsi, C. E., Ferraro, I., & Iannicola, G. 1983, A&A, 126, 278
Cool, A. M., Piotto, G., & King, I. R. 1996, preprint
De Marchi, G., Paresce, F., & Romaniello, M. 1995, ApJ, 440, 216
Djorgovski, S. 1993, in ASP Conf. Ser. 50, Structure and Dynamics of Globular Clusters, ed. S. G. Djorgovski & G. Meylan (San Francisco: ASP), 373
Harrington, T. S., & Dahn, C. C. 1980, AJ, 85, 454
Holtzman, J. A., Burrows, C. J., Casertano, S., Hester, J. J., Trauger, J. T.,
Watson, A. M., & Worthey, G. 1995, PASP, 107, 1065

Kennicutt, R. C., Freedman, W. L., & Mould, J. R. 1995, AJ, 110, 1476
Oswalt, T. D., et al. 1991, AJ, 101, 583
Penny, A. J., & Dickens, R. J. 1986, MNRAS, 220, 845
Renzini, A. 1991, in Observational Tests of Cosmological Inflation, ed. T.
Shanks et al. (Dordrecht: Kluwer), 131
——. 1993, Ann. NY Acad. Sci., 688, Relativistic Astrophysics and Particle
Cosmology, ed. C. W. Akevlof & M. A. Srednicki, 124
Renzini, A., & Fusi Pecci, F. 1988, ARA&A, 26, 199
Richer, H. B., et al. 1995, ApJ, 451, L17
Salaris, M., Chieffi, A., & Straniero, O. 1993, ApJ, 414, 580
Sandage, A., 1970, ApJ, 162, 841
Sandage, A., & Cacciari, C. 1990, ApJ, 350, 645
Sion, E. M., 1984, ApJ, 282, 612
Van Altena, W. F., Lee, J. T., & Hoffleit, D. 1991, The General Catalogue of
Trigonometric Stellar Parallaxes, Preliminary Version (New Haven: Yale
Univ. Obs.)
VandenBerg, D. A., & Bell, R. A. 1985, ApJS, 58, 561
Wood, M. A. 1995, in White Dwarfs, ed. D. Koester & K. Werner (Berlin:
Springer), 41
Zinn, R. 1985, ApJ, 293, 424