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RADIATION-DRIVEN MAGNETOHYDRODYNAMIC WIND SOLUTIONS FOR HOT LUMINOUS STARS

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ABSTRACT

Solutions for the stellar winds of hot luminous stars are obtained by solving the magnetohydrodynamic (MHD) equations combined with the radiatively driven outflow formalism given by Castor, Abbott, & Klein. We consider the interaction of radiation pressure and collimated magnetic fields for a nonrotating star. The streamline shape has been prescribed from a phenomenological point of view. In order to decouple the MHD equations, we assume all the lines contributing to the radiation pressure are optically thick. Spherically and nonspherically symmetric mass distributions are considered. In both cases, the dependence of the terminal velocity on the star luminosity and magnetic strength is discussed. It is found that the terminal flow speed is strongly affected by slight variations of both the luminosity and the mass distribution asymmetry but shows a weak correlation with the magnetic field intensity. The present formalism can be thought of as a wide generalization of the MHD solutions introduced by Low & Tsinganos. Highly collimated magnetic structures, as well as rotation, are considered in another paper.

Subject headings: MHD — stars: magnetic fields — stars: mass loss

1. INTRODUCTION

Although it is an event common to all stars along the evolutionary sequence, the mass ejection phenomenon is particularly severe in hot luminous objects, namely, O, OB, and Wolf-Rayet stars. The theory of radiation-driven winds is successful in explaining with some detail the gross properties of the winds from early-type objects in terms of mass-loss rates and terminal velocities. As is well established, hot stars have an extremely intense radiation field that is absorbed by thousands of UV metal lines in the outer atmospheric shells. This absorption yields an outward accelerating force (typically stronger than gravity by a large factor) that is sufficient to start and sustain the stellar wind (Lucy & Solomon 1970).

Spectroscopically, this behavior results in the presence of broad violet-displaced absorption lines and/or prominent P Cygni profiles along the UV spectrum, as observed by the *Copernicus* and *IUE* satellites (Morton 1967; Morton, Jenkins, & Brooks 1969). Even though strongly model dependent, some limiting values for the mass-loss rate, $\dot{M}_*$, and terminal velocity, V_∞, of hot stars can be derived from the available observational information. It is widely accepted that for the objects we are dealing with, $\dot{M}_*$ has values within the range 10^{-7} to 10^{-5} $M_\odot$ yr^{-1} (Knapp & Morris 1985) or still greater (Kwan & Webster 1993). In turn, V_∞ belongs to the range 1000–3000 km s^{-1} for early-type objects (Lamers & Leitherer 1993), while it runs from 1500 to 4000 km s^{-1} for W-R stars (Abbott & Conti 1987).

The stellar material, however, does not freely flow because of its interaction with magnetic structures originating beneath the stellar surface. In principle, the magnetic field sets up the flow channels along which the wind flows. But owing to the frozen-in condition and the relatively high initial kinetic energy, the outflowing stellar plasma, driven by the intense radiation pressure, drags out the field lines and stretches them continuously. Thus, magnetic fields of hot stars for which the radiative push is dominant would adopt open, nearly purely radial configurations.

Another important feature that seems to be common to both OB and W-R stars is their colatitude-dependent mass distribution. In fact, aspherical mass densities are rather the rule and not the exception among astrophysical bodies. The nearest and best-documented example of asymmetric mass outflows is our star, the Sun. The bulk of the solar wind emerges from discrete magnetically open zones and has two components: low-density streams originating in the coronal holes and a slow wind coming from the neighboring denser helmets (Withbroe 1989).

Many other examples of nonisotropic outflows can be found along the evolutionary sequence: the CO bipolar flows associated with young stellar objects (Fukui 1989), the elliptical or "butterfly-shaped" planetary nebulae (Kwock 1982), or the noncircular disks, presumably resulting from previous ejections, that surround many luminous blue variables (Davidson 1989). Nonisotropic outflows were reported to emerge from Be stars (Friend 1990; Taylor 1992), while there is growing evidence that winds from W-R stars have their origin at the poles, which are expected to be regions of lower density than the equatorial zone (Zickgraf & Schulte-Ladbeck 1989; Eenens 1992).

Nevertheless, outflows from OB or W-R stars take place under different dynamical circumstances. Actually, there is no evidence of azimuthal velocity in W-R stars, perhaps because their spectra are dominated by the wind and no reliable rotational features can be found. On the other hand, mass outflows from O and B stars, which are known to be fast rotators, are strongly affected (and eventually governed) by the high azimuthal velocity.

If the radiative term is neglected (which seems to be plausible for late B-type and A-type stars), the problem under analysis reduces to a well-known family of analytic magnetohydrodynamic solutions (Tsinganos & Trussoni 1991; Trussoni & Tsinganos 1993; Rotstein & Ferro Fontán 1995a, b). However, it becomes highly difficult when the radiation pressure governs the flow. In order to solve this problem, some attempts have been made since the pioneer work of Lucy & Solomon (1970) for a rotationless, nonmagnetic, radiatively driven wind from an early-type star. Among these are the works of Castor, Abbott, & Klein (1975), Abbott (1978), and Pauldrach et al. (1990).

The equatorial wind solutions for luminous magnetic rotators have been treated by Friend & MacGregor (1984), Poe & Friend (1986), Casinelli et al. (1989) and Casinelli (1992). In turn, the influence of Alfvén waves on the terminal velocity of non-rotating, magnetized winds was exhaustively analyzed by Casinelli (1982) and dos Santos, Jatenco-Pereira, & Opher (1993).

The aim of this work is to extend the analytic class of MHD wind solutions introduced by Low & Tsinganos (1986) in order to include the effect of the radiation pressure. Needless to say, some idealizations are necessary to keep the problem at a solvable level. For this reason, we will assume that all UV lines are optically thick. In § 2 we will discuss the implications and plausibility of this assumption and the functional form that the radiative term must adopt in order for the MHD equations to be decoupled.

As in Hu & Low (1989), we will simulate the asymmetry in the mass density distribution through a mass assymetry parameter, while assuming spherically symmetric Mach-Alfvén surfaces. Although the general formalism is presented, we shall concentrate on the solutions for a purely radial, nonrotating magnetic field, postponing to another paper the treatment of highly collimated, rotational configurations. In this way, we shall end up with a first-order differential equation with no critical points (owing to the rotationless hypothesis), which will allow us to interpret in a simpler manner the physical mechanisms that drive the wind. Section 3 is devoted to this task, while in § 4 we discuss the results obtained.

2. GENERAL FORMALISM

2.1. *The Mathematical Framework*

The steady dynamical interaction between an inviscid, compressible fluid of high electrical conductivity, which has magnetic field lines originating in a central object that generates a spherically symmetric gravitational field, and a radiation field is described within the MHD framework by the following equations:

$$\nabla \cdot (\rho \boldsymbol{v}) = 0 , \tag{2.1}$$

$$\nabla \cdot \boldsymbol{B} = 0 , \tag{2.2}$$

$$\nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = 0 , \tag{2.3}$$

$$\rho(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla \mathscr{P} + \mu_0^{-1}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \rho g \hat{\boldsymbol{e}}_r + \boldsymbol{f}_{\rm rad} , \tag{2.4}$$

$$\frac{3}{2}\frac{k_{\rm B}}{m}\rho(\boldsymbol{v} \cdot \nabla)T + \mathscr{P}\nabla \cdot \boldsymbol{v} = \mathscr{Q}(r, \theta) , \tag{2.5}$$

where $\boldsymbol{v}$ represents the bulk velocity of the flow, $\boldsymbol{B}$ the magnetic field intensity, ρ the mean density, $\mathscr{P} = k_B \rho T/m$ the gas pressure, $g = GM_*/r^2$ the gravity acceleration, and $\boldsymbol{f}_{\rm rad}$ the force exerted on the fluid by the radiation field. The remaining parameters, $k_{\rm B}$, G, M_*, m, and T, are the Boltzmann constant, gravitational constant, stellar mass, mean molecular weight of the fluid, and the temperature of the gas, respectively. $\mathscr{Q}(r, \theta)$ represents, as in Low & Tsinganos (1986), the volumetric rate of energy that must be added to the flow in order to reach the terminal regime. It may be interpreted as the particular distribution of heating and/or cooling sources along the outflow that consistently closes the system of dynamic equations (2.1)–(2.4).

Note that the thermodynamics of the outflow could be fixed through a polytropic relation between pressure and mass density via the specification of a constant polytropic index γ; in this case, the magnetic field structure would be determined once the dynamical equations are solved. But it seems at least unrealistic to specify the heating mechanisms along the entire wind region at which the outflow takes place. What is more, this procedure could lead to unsuitable streamline shapes. For this reason, we will take on the opposite approach; based on plausibility criteria, we will prescribe the magnetic field configuration, and then we will calculate the self-consistent heating distribution that allows the wind to take place along the flow region.

2.2. *The Magnetohydrodynamic Field*

As we have already mentioned, a certain number of hypotheses are needed to keep the set of equations (2.1)–(2.4) at a workable level. Therefore, it must be noted that we are mainly interested in the average values of the dynamic variables that may be related to observations. Average large-scale structures can be considered by neglecting any localized configuration in a two-dimensional formalism that assumes rotational symmetry. Thus, with $\partial_\phi = 0$, note that equations (2.1)–(2.2) can be written in terms of two scalar functions as

$$\boldsymbol{B} = \frac{1}{r \sin \theta} \nabla A(r, \theta) \times \hat{\boldsymbol{e}}_\phi + B_\phi \hat{\boldsymbol{e}}_\phi \tag{2.6}$$

and

$$\rho \boldsymbol{v} = \frac{1}{r \sin \theta} \nabla \Lambda_0(r, \theta) \times \hat{\boldsymbol{e}}_\phi + \rho v_\phi \hat{\boldsymbol{e}}_\phi , \tag{2.7}$$

where $A(r, \theta)$ and $\Lambda_0(r, \theta)$ are defined as the magnetic flux function and the mass flux function, respectively.

In turn, the $\hat{\boldsymbol{e}}_r$ and $\hat{\boldsymbol{e}}_\theta$ components of equation (2.3) lead to the general result that Λ_0 can be expressed as a function of the magnetic flux function (Tsinganos 1982). As for the rest, $\Lambda_0(A)$ is a free function of $A(r, \theta)$. In turn, the $\hat{\boldsymbol{e}}_\phi$ component of equation (2.3) yields another free function of $A(r, \theta)$, namely, the electrostatic potential Ω_0:

$$\boldsymbol{v} \times \boldsymbol{B} = \nabla \Omega_0(A) = \frac{\partial \Omega_0}{\partial A} \nabla A(r, \theta) . \tag{2.8}$$

Provided we define for convenience

$$\Omega = \frac{\partial \Omega_0}{\partial A} \quad \text{and} \quad \Lambda = \frac{\partial \Lambda_0}{\partial A} ,$$

we get from equation (2.8), by means of equations (2.6) and (2.7), a general relation between the magnetic and velocity fields, which is

$$\boldsymbol{v} = \mu_0^{1/2} \frac{\Lambda(A)}{\rho} \boldsymbol{B} + \Omega(A) r^2 \sin \theta \hat{\boldsymbol{e}}_\phi . \tag{2.9}$$

We will analyze the rotationless regime in which the magnetic field lines are pushed by the intense radiation field and adopt a purely radial configuration. By definition, a field line is the region of the meridional plane for which $A(r, \theta)$ is constant; for our purpose, it is sufficient to define

$$A(r, \theta) = af(\theta) , \tag{2.10}$$

where a is a constant and $f(\theta)$ is, in principle, an arbitrary function of colatitude. Moreover, note that from equation (2.9) the poloidal Mach-Alfvén function can be defined:

$$M_A^2(r, \theta) \equiv M(r, \theta) = \frac{\mu_0 \Lambda^2}{\rho} = \frac{\rho v_p^2}{B_p^2} , \quad (2.11)$$

where the subindex p denotes the poloidal component. Note that the easiest way to introduce any deviation from sphericity in the mass distribution is through the mass flux function, assuming spherically symmetric Mach-Alfvén surfaces, that is, $\partial_\theta M(x, \theta) = 0$ (see eq. [2.11] above). As has already been mentioned, for the objects we are dealing with, there is growing evidence of the existence of an equatorial static region, denser than the polar zones at which the wind takes place. In order to simulate such mass density distribution, we will set

$$\Lambda(A) = \lambda(1 + jA)^{1/2} = \lambda(1 + \zeta \sin^2 \theta)^{1/2} , \quad (2.12)$$

where j is an arbitrary constant, $\zeta = ja$, and the constant λ defines the polar mass density at the Alfvénic point. A nonrotational regime requires $v_\phi = 0$ and $\Omega_0(A) =$ constant (see eqs. [2.7] and [2.9]). Thus, by means of equation (2.12), the magnetohydrodynamic field becomes

$$B_r = \frac{2a}{r^2} \cos \theta , \quad B_\theta = B_\phi = 0 , \quad (2.13)$$

$$v_r = \frac{V_0}{M_0} \frac{M(x)}{x^2} \frac{|\cos \theta|}{(1 + \zeta \sin^2 \theta)^{1/2}} , \quad v_\theta = v_\phi = 0 , \quad (2.14)$$

where M_0 refers to the initial Mach-Alfvén number; besides, the dimensionless distance $x = r/R_*$ has been defined, as well as the initial velocity of the outflow ,

$$V_0 = \frac{2M_0 \Psi}{\mu_0 \lambda} ,$$

and the polar magnetic flux at the photosphere,

$$\Psi = B(x = 1, \theta = 0) = 2 \frac{a}{R_*^2} .$$

2.3. *The Radiation Field*

Let us consider the radiative force $f_{\rm rad}$. Following Castor et al. (1976, hereafter CAK), we will assume a spherically symmetric radiation field and Sobolev approximation, which in turn depends on the velocity gradient. Given that the velocity field reduces to its radial component, it is easy to see that the radiation force will act only in the radial direction. Bearing this result in mind, as in the CAK model, we will consider the continuum and line contributions to the radiation pressure through an expression of the form

$$f_{\rm rad} = \rho\left(\frac{\sigma_e L_*}{4\pi r^2 c} + \sum_i g_i\right) , \quad (2.15)$$

where σ_e is the mass scattering coefficient of the free electrons and L_* the star luminosity. The first term on the right-hand side of equation (2.15) represents the continuum contribution to the radiation pressure, while the second term adds the effect of momentum transfer through UV lines.

The contribution of each strong line ($\tau_i \gg 1$) to the radiation field is (Cassinelli 1979)

$$g_i = \frac{\pi F_{\nu 0}}{c} \frac{\Delta \nu_D}{\rho v_{\rm th}} \frac{dv}{dx} , \quad (2.16)$$

so that the acceleration on strong lines is independent of the line strength, and the total acceleration is proportional to the number of strong lines. Moreover, if only strong lines existed, the line acceleration would depend linearly on dv/dx.

We will adopt the line opacity–independent depth scale defined by CAK:

$$t \equiv \frac{\sigma_e \tau_i}{\kappa_i} = \int_r^\infty \sigma_e \rho\, dr \qquad \text{for a static atmosphere} ,$$
$$= \sigma_e \rho v_{{\rm th},i} \left|\frac{dv}{dx}\right|^{-1} \qquad \text{for an expanding atmosphere} . \quad (2.17)$$

In this way, the contribution of all lines, $\sum_i g_i$, can be expressed in a convenient manner so that equation (2.15) can be written in the form

$$f_{\rm rad} = \rho \frac{\sigma_e L_*}{4\pi r^2 c} [1 + F(t)] ,$$

where $F(t)$ is defined as the force multiplier. Taking into account the most abundant species in the stellar atmosphere, the force multiplier $F(t)$ can be computed as a function of the normalized optical depth t (CAK; Abbott 1982). Traditionally, the resulting curve is fitted by means of exponential functions of the form

$$F(t) = kt^{-\alpha} , \quad (2.18)$$

where α and k are two parameters to be adjusted in order to reproduce the resulting data. It must be mentioned that both constants depend upon the particular characteristics of the star. Roughly, for the typical ranges of interest of the normalized optical depth, k is a number within the interval 0.01–0.6, and α is of the order of 0.5–0.7.

In view of equation (2.16), and noting that the mass conservation law (eq. [2.1]) imposes

$$4\pi r^2 \rho v_r = \text{constant} = \dot{M}_*$$

for a purely radial velocity field, equation (2.18) can be written as

$$F(t) = k\left(\frac{4\pi R_*}{\sigma_e v_{\rm th} \dot{M}_*}\right)^\alpha \left(x^2 v_r \left|\frac{dv_r}{dx}\right|\right)^\alpha . \quad (2.19)$$

2.4. *Two-dimensional Equations with Radiation and Magnetic Field*

Taking into account equations (2.12)–(2.19), after some rather cumbersome algebra, the $\hat{e}_r$ and $\hat{e}_\theta$ components of equation (2.4) can be written in terms of the Mach-Alfvén function as

$$-\partial_x P = \left[\frac{4}{x^2} \partial_x\left(\frac{M}{x^2}\right) + \frac{Z^2}{Mx^2} \times \left\{(1 - \Gamma) - k\left[\epsilon M \left|\partial_x\left(\frac{M}{x^2}\right)\right|\right]^\alpha\right\}\right] - \left[\frac{4}{x^2} \partial_x\left(\frac{M}{x^2}\right) - \frac{Z^2}{Mx^2} \times \left\{\zeta(1 - \Gamma) + k\left[\epsilon M \left|\partial_x\left(\frac{M}{x^2}\right)\right|\right]^\alpha\right\}\right] \sin^2 \phi , \quad (2.20)$$

$$\partial_\theta P = \frac{4}{x^4} \sin\theta \cos\theta , \tag{2.21}$$

where we define the dimensionless gas pressure

$$P = \mathscr{P} \frac{\mu_0}{\Psi^2} ,$$

as well as

$$Z^2 = \frac{GM_*}{R_*} \left(\frac{\lambda\mu_0}{\Psi}\right)^2 = 2\left(\frac{V_e}{V_0}\right)^2 M_0^2 = 4 \frac{\mathscr{E}_e}{\mathscr{E}_M} ,$$

V_e being the escape velocity from the central object (but with no appreciable radiation), $V_0 = v_r(x = 1, \theta = 0)$, $\mathscr{E}_e$ the escape energy, and $\mathscr{E}_M$ the magnetic energy. We have also introduced the parameter ϵ defined as

$$\epsilon = \Gamma \frac{4\pi R_* V_0^2}{\sigma_e v_{\rm th} \dot{M}_* M_0^2} \equiv \frac{\mathscr{E}_L}{\mathscr{E}_e} \frac{V_0}{v_{\rm th} M_0^2} = \frac{\mathscr{E}_L \mathscr{E}_M}{\mathscr{E}_e \rho v_0 v_{\rm th}} ,$$

where Γ is defined as $\Gamma = \sigma_e L_*/4\pi c G M_*$ and $\mathscr{E}_L$ is the luminous energy.

It can be easily seen that the set of equations (2.20)–(2.21) can be reduced to a decoupled pair of equations provided we set $\alpha = 0$ or $\alpha = 1$, which are mathematical restrictions equivalent to assuming all lines are either optically thin or optically thick, respectively. The approximation $\alpha = 0$ brings no line contribution and was already treated by Tsinganos, Trussoni, & Sauty (1993). Thus, we will discuss the case $\alpha = 1$.

As has already been mentioned, α and k are parameters used to fit the computational data to an exponential function along a wide range of values of the variable t. However, we have checked that in the wind region, the parameter t takes on values of the same order. Thus, within the range of interest of t, the force multiplier can be considered a linear function of the velocity gradient. For each given set of initial parameters, the value of the parameter k was iteratively determined in order to intersect the $\alpha = 1$ and Abbott (1982) curves for the mean value of the parameter t. We started each integration of equations (2.20)–(2.21) with an estimated value of the parameter k; once these equations were solved, we evaluated the interval of values of t to adjust the value of k; after no more than two or three iterations, the method converges. Figure 1 displays one of the curves obtained from Abbott (1982) and one constructed with $\alpha = 1$. It can be seen that the force multiplier results overestimated (underestimated) for the lower (higher) values of the interesting normalized optical depth by no more than 10%.

With $\alpha = 1$, equation (2.21) can be immediately integrated to obtain

$$P(x, \theta) = \frac{2}{x^4} \sin^2\theta + \Pi_1(x) , \tag{2.22}$$

where $\Pi_1(x)$ is the pressure component that depends only on the radial distance to the star. Setting equation (2.20) equal to the derivative of equation (2.22) with respect to x allows us to obtain an equation for $\Pi_1(x)$ and another for $M(x)$, namely,

$$\partial_x \Pi_1(x) = -\frac{4}{x^2} \partial_x U(x) + \frac{\beta k}{x^2} |\partial_x U(x)| - \frac{Z^2(1-\Gamma)}{U(x) x^4} , \tag{2.23}$$

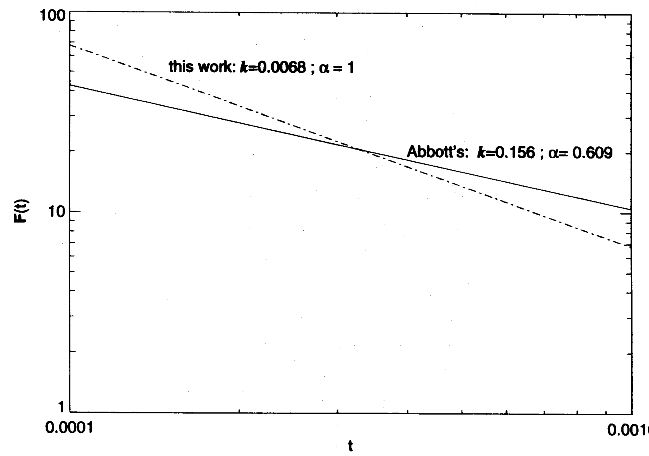

FIG. 1.—Force multiplier, $F(t)$, is shown as a function of the depth scale–independent line opacity t. The solid line is the result obtained by Abbott (1982) for a star with $T_{\rm eff} = 30{,}000$ K and Ne/W = 1.0(+11) cm^{-3}, $k = 0.156$, and $\alpha = 0.609$. The dashed-dotted line is a curve with $\alpha = 1$ and $k = 0.0068$ that adjusts the values of Abbott in the center of the region of interest, in this case, $0.0002 \le t \le 0.0004$.

$$\frac{4}{x^2} \partial_x U(x) - \frac{\beta k}{x^2} |\partial_x U(x)| - \frac{Z^2(1-\Gamma)\zeta}{U(x) x^4} = -\frac{8}{x^5} , \tag{2.24}$$

where we have defined

$$U(x) = \frac{M(x)}{x^2} \tag{2.25}$$

and

$$\beta = \epsilon Z^2 \equiv \frac{4\mathscr{E}_L}{\rho V_0 v_{\rm th}} . \tag{2.26}$$

Note that from equation (2.14), up to a constant value, the variable $U(x)$ represents the velocity of the fluid. Given that all the remaining variables are written in terms of $\Pi_1(x)$ and $M(x)$, equations (2.23)–(2.24) are sufficient to determine the dynamic and thermodynamic states of the system. The integration of the basic variables will be accomplished for different boundary conditions in the next section.

3. WIND SOLUTIONS

3.1. *Solutions for $\zeta = 0$*

In order to clarify the role played by the radiative force within the present framework, let us solve the set of equations (2.23)–(2.24) assuming that the mass density is spherically distributed. An additional advantage of the $\zeta = 0$ hypothesis is the fact that solutions become analytical. Under such circumstance, equation (2.24) can be written in the form

$$(1 + \delta\omega) U'(x) = -\frac{2}{x^3} , \tag{3.1}$$

where we have defined $\omega = \beta k/4$, a prime denotes differentiation with respect to the variable x, and δ takes into account the sign of the derivative of $U(x)$; $\delta = -1(+1)$ corresponds to an atmosphere that expands with a velocity that monotonically increases (decreases to a nonzero final value).

Notice that this formalism is an extension of the one introduced by Low & Tsinganos (1986), to which it reduces for $\omega = 0$. The inclusion of a radiative term substantially modifies the wind solutions: on the one hand, it allows for the outflow to be initially sub-Alfvénic; on the other hand, there now exists a subfamily of outflow solutions with increasing velocity. In fact, we are mainly interested in this last class of solutions, so we will directly integrate equation (3.1) for $\delta = -1$. Note that the condition $\omega > 1$ is necessary and sufficient for the derivative of $U(x)$ to be positive definite along the entire wind region. In this case, it can be immediately obtained:

$$U(x) = C - \frac{1}{(\omega - 1)x^2}\,, \tag{3.2}$$

where C is an integration constant. From its definition (eq. [2.25], we have

$$M(x) = Cx^2 - \frac{1}{\omega - 1}\,, \tag{3.3}$$

and, given that C fits the Mach-Alfvén number at the origin,

$$C = M_0 + \frac{1}{\omega - 1}\,, \tag{3.4}$$

so that the remaining variables are easily obtained as

$$v(x, \theta) = \frac{V_0}{M_0}\left[M_0 + \frac{1}{\omega - 1} - \frac{1}{(\omega - 1)x^2}\right]|\cos\theta| \tag{3.5}$$

and, by direct integration of equation (2.23),

$$\Pi_1(x) = -\frac{2}{x^4} + 2\left(\frac{V_e}{V_0}\right)^2(1 - \Gamma)M_0\left(\frac{1}{a^2} - 1\right) \times \left[\frac{1}{2a}\left|\ln\left(\frac{x - a}{x + a}\right)\right| - \frac{1}{x^2}\right], \tag{3.6}$$

where we have defined

$$a^{-2} = (\omega - 1)C = M_0(\omega - 1) + 1\,. \tag{3.7}$$

It must be noted that, because of the condition $\omega > 1$, $a^2 < 1$, so that $x > a$ along the entire wind region. Two points deserve to be discussed in view of equation (3.5): on the one hand, for values of ω very close to unity (albeit greater), it seems that the terminal velocity at the poles,

$$v_\infty = V_0\left[1 + \frac{1}{M_0(\omega - 1)}\right], \tag{3.8}$$

can reach arbitrarily high values; and, on the other hand, given that $\omega \propto L_*$ (see eq. [2.26]), it seems that the terminal velocity decreases for increasing star luminosities. We must bear in mind that ω actually depends on a parameter, k, that is introduced to fit observational data, so that the relation between v, ω, and L_* is more complicated than it would seem from equations (2.26) and (3.5). If the star luminosity is slightly increased, the resulting force multiplier $F(t)$ will be also enhanced; thus, k must take a lower value in order to again intersect the Abbott (1982) curve at the same value of t (see Fig. 1). In fact, we can show that for a slight increase in L_*, ω actually decreases. In doing that, let us write expression (2.19), taking into account equation (3.3), as

$$F(t) = \frac{k\eta}{\mathscr{L}}\frac{1}{k\mathscr{L}^7 - \nu}\,, \tag{3.9}$$

where we have defined $\mathscr{L} = L_*^{1/8}$, and we have introduced

$$\eta = \frac{\pi R_* V_0^2 c}{\sigma_e M_0^2} \tag{3.10}$$

and

$$\nu = c\dot{M}_*\left(\frac{k_B}{m_H}\right)^{1/2}\frac{1}{(4\pi R_*^2\sigma_B)^{1/8}}\,, \tag{3.11}$$

where k_B and σ_B are the Boltzmann and Stefan-Boltzmann constants, respectively. Suppose we now have a certain solution with a given value L_1 of star luminosity, to which corresponds a given value of the parameter k, say k_1. Let us increase the luminosity by a factor $m > 1$ (although $m \sim 1$); in order to keep fixed the value of $F(t)$, k must take on a new value given by

$$k_2 = \frac{k_1 \nu m^{1/8}}{k_1\mathscr{L}^7(m - 1) + \nu}\,. \tag{3.12}$$

From its definition (eq. [2.26]), it is easy to see that we can write the variable ω in terms of $\mathscr{L}$ and ν as

$$\omega = \frac{k\mathscr{L}^7}{4\nu}\,. \tag{3.13}$$

Provided ω_1 and ω_2 respectively denote the values for L_1 and $L_2 = mL_1$,

$$\frac{\omega_1}{\omega_2} = \frac{k_1}{k_2}m^{7/8} = m^{3/4}\frac{k_1\mathscr{L}^7(m - 1) + \nu}{\nu}\,. \tag{3.14}$$

Given that $m > 1$, $\omega_1 > \omega_2$. Thus, the terminal velocity V_∞ increases with luminosity. Figure 2 displays such behavior for a given set of initial parameters.

Moreover, if the luminosity were increased in such a way that ω could take a value very close to unity, the velocity gradient would be strongly enhanced. Under such circumstances, the parameter k must be diminished in order to keep fixed the value of $F(t)$. Then, $\omega < 1$, violating the hypothesis of a positive velocity gradient. In brief, within this framework, only values of ω sufficiently higher than unity allow for changes in the values of the photospheric parameters with no variation in the physical behavior of the solutions.

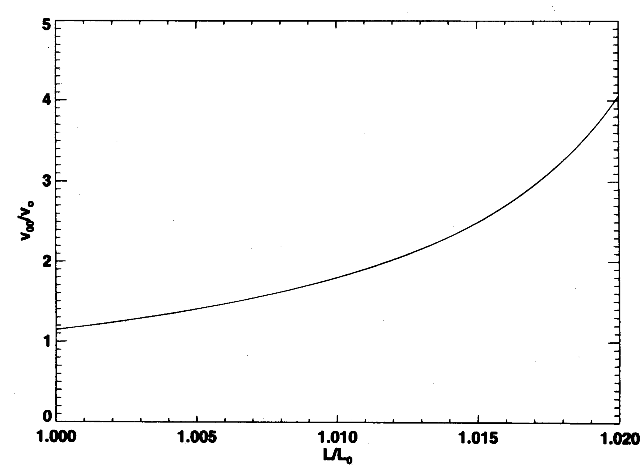

FIG. 2.—Terminal velocity v_∞ in terms of the initial velocity V_0 as a function of the star luminosity in terms of a given initial star luminosity. The dependence is so high that an increase of 2% in the luminosity increases the terminal velocity by a factor of 4.

Let us return now to equation (3.6) to note that there exists another boundary condition. In doing that, let us first note that, from equation (2.23) and in view of solution (3.3), if $\Pi_1(x)$ has an extreme, it must be a maximum. In fact, pressure is initially an increasing function provided the condition

$$\Pi_1'(x = 1) > 0 \equiv 4 > (1 - \Gamma)\left(\frac{V_e}{V_0}\right)^2 M_0 \qquad (3.15)$$

is fulfilled. Then, given that $\Pi_1(x) \to 0$ for $x \to \infty$, the conditions at the origin,

$$M_0(1 - \Gamma)\left(\frac{1}{a^2} - 1\right)\left[\frac{1}{2a}\left|\ln\left(\frac{1-a}{1+a}\right)\right| - 1\right] > \left(\frac{V_0}{V_e}\right)^2 , \qquad (3.16)$$

is sufficient for the pressure $\Pi_1(x)$ to be positive along the wind region. That is, in view of definition (3.7), the higher the luminosity, the lower the initial velocity V_0 must be for the inequality (3.16) to hold. This result is almost to be expected from equation (3.8), if the resulting terminal velocity is not to reach an arbitrarily high value. Consequently, the pressure needed to push the flow against the gravitational pull decreases for increasing luminosities. This fact has to do with the existence of an equatorial static region; in this zone, the only force acting inward is gravitation, which is reduced by the continuum radiation by a factor of $1 - \Gamma$. Given that Γ increases as the luminosity does, the equatorial equilibrium is sustained by a lower pressure in this case. Moreover, the energy needed to start and sustain the outflow will be consistently lower.

Another important point emerges from equation (3.6). The initial regime is governed by expression (3.15). In fact, provided this inequality is fulfilled, the temperature profile shows a maximum (located close to the photosphere) resembling the temperature distribution of a chromosphere. But far from the star, as can be easily seen, pressure drops to a zero final value as x^{-2}. In turn, the Mach-Alfvén function behaves as x^2; thus, temperature asymptotically decreases to a constant value.

This fact is related to the behavior of the thermodynamics of the flow. An effective polytropic function, say γ, can be defined in the usual way as

$$d \ln T = (\gamma - 1) d \ln \rho , \qquad (3.17)$$

in such a way that, at the pole, its expression reduces to

$$\gamma = 1 + \frac{\rho}{T}\frac{dT}{d\rho} = -\frac{M(x)\Pi_1'(x)}{M'(x)\Pi_1(x)} . \qquad (3.18)$$

In view of equations (3.3) and (3.6), we conclude that far from the star, the wind reaches a nearly isothermal regime; thus, the internal energy of the outflow is entirely absorbed by the expansion and $\gamma \to 1$. Let us analyze in the remaining part of this section the role played by the asymmetric mass distribution.

3.2. *Solutions for* $\zeta \neq 0$

Before numerically integrating equation (2.24), let us discuss some interesting physical properties of this model. In doing this, let us notice that for the case $L_* \to 0$ (that is, $\omega \to 0$ and $\Gamma \to 0$), the resulting equation (2.24) was studied by Hu & Low (1989) for a purely radial configuration, while Trussoni & Tsinganos (1993) and Rotstein & Ferro Fontán (1995b) have analyzed the resulting outflow for curved magnetic fields. Provided $L_* \neq 0$, once the radiative force is fixed at the base of the wind through the intrinsic luminous parameters of the star, the two formalisms must look similar.

In fact, it must be emphasized that the equatorial balance of forces is not affected by the radiative push (except for the continuum contribution, as mentioned above) because of its $\cos\theta$ dependence. Then, as in the case $L_* \to 0$, if the weight of the plasma equatorial column is enhanced via the parameter ζ, pressure must also be increased in order to keep the equatorial region in static equilibrium.

The θ-dependent component of the total pressure, $\Pi_2(x)$, merely compensates for the Lorentz force in order for a fluid element to outflow on a given field line. Then, the frozen-in condition, under which the wind spatially evolves, fixes the equatorial value of $\Pi_2(x)$, no matter what the value of ζ is. As a result, the higher value of ζ, the higher the value of the component $\Pi_1(x)$. At the pole, this enhanced value of $\Pi_1(x)$ leads to a higher acceleration and, consequently, to a higher terminal velocity.

We must bear in mind that the radiation force does not depend on the parameter ζ, so that the mass asymmetry effect does not compete with the radiative action but is superimposed on it. As a result, for a given set of boundary values, the terminal velocity within this framework is higher than in the $L_* \to 0$ formalism.

However, a slight difference exists between the two formalisms. While in the $L_* \to 0$ framework, wind solutions that do not monotonically increase to the terminal regime can be found (Hu & Low 1989), in our case we must claim as a necessary condition $U'(x) > 0$ for all x (otherwise, the Sobolev approximation would not be valid). It is easy to see, from equation (2.24), that this condition reduces to

$$U'(x) > 0 \equiv \zeta > \frac{4V_0^2}{V_e^2 M_0(1 - \Gamma)} \quad \text{for } 1 > \omega ,$$
$$\equiv \zeta < \frac{4V_0^2}{V_e^2 M_0(1 - \Gamma)} \quad \text{for } \omega > 1 . \qquad (3.19)$$

Figures 3–5 show the behavior of the variables $v(x)$, $\Pi_1(x)$, and the required energy at the pole $Q_0(x) = \mathcal{Q}(x)R_* V_0^{-1}(\mu_0/\psi^2)$. For the given values of the parameter ζ, the terminal velocity of the wind remains within 1.5–3 V_e, as measured for hot luminous stars (Abbott 1982). Unlike the

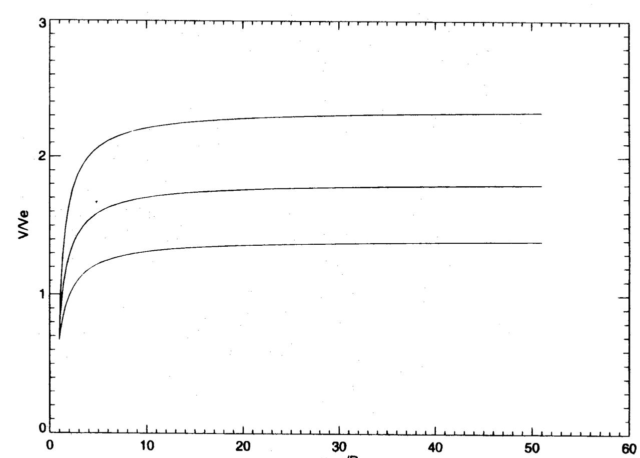

FIG. 3.—Terminal velocity v_∞ is shown in terms of the escape velocity V_e as a function of the dimensionless distance x. We show the results obtained for different values of the parameter ζ. From top to bottom, $\zeta = 8, 5$, and 3.

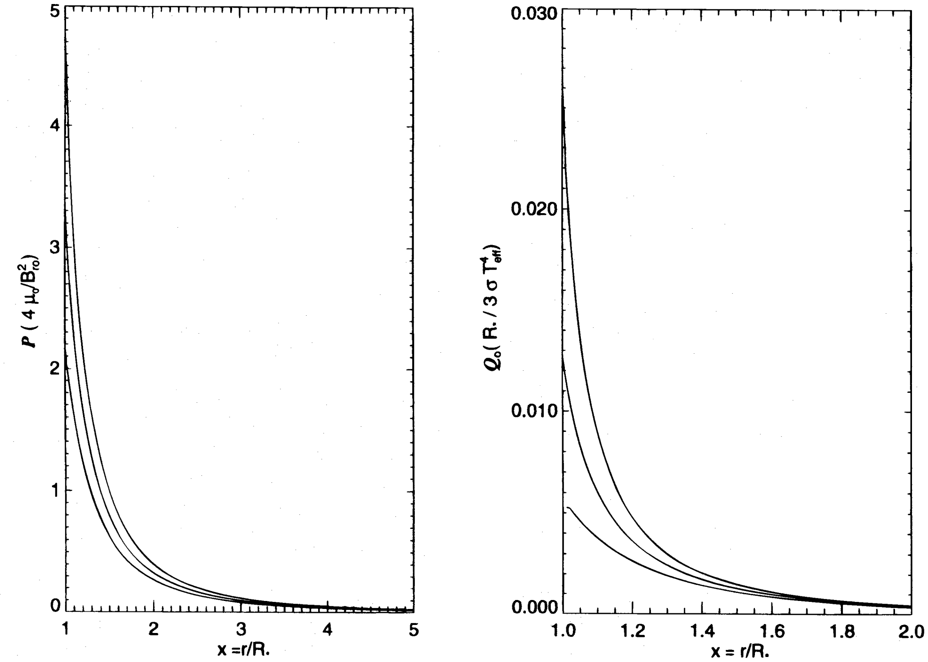

FIG. 4.—*Left-hand panel*: Gas pressure in terms of the magnetic pressure as a function of the dimensionless distance x for the three cases of Fig. 3. From top to bottom, $\zeta = 8$, 5, and 3. *Right-hand panel*: Energy at the pole in terms of the energy radiated by the star as a blackbody as a function of the dimensionless distance x for the three cases of Fig. 3. From top to bottom, $\zeta = 8$, 5, and 3.

$\zeta = 0$ case, pressure is now a monotonically decreasing function that quickly drops to interstellar values. Notice that inequality (3.15) is no longer valid for $\zeta \neq 0$; instead, the high initial acceleration requires a decreasing pressure function in order for the terminal velocity to remain bounded within the observed values.

In all cases, the heat distribution reduces to a heating source practically attached to the photosphere. From the discussion at the end of the previous section, it is expected that the higher the value of ζ, the higher the value of Q_0, because more energy is required to sustain the flow until the higher terminal velocity is reached. But in all cases, the necessary power is only a low percentage of the blackbody power radiated by the star. Moreover, as has already been mentioned, the Lorentz force (acting in the θ direction) is only counterbalanced by the component $\Pi_2(x)$ of the total pressure. For a given mass distribution, any increment in the value of the photospheric magnetic field leads to an enhancement of the θ-dependent part of the gas pressure. In turn, it leads to a decrement in the component $\Pi_1(x)$ (via the equatorial balance of forces), so that the terminal velocity decreases for growing values of B_0. This feature is shown in Figure 5. It must be mentioned that this is the way in which the magnetic field acts in the dynamics of the stellar wind. It neither directly governs the outflow nor explicitly in its thermodynamics. The magnetic structure imposes the geometry of the flow channels along which the wind develops, and through this geometry, participates in the balance of forces. However, it must be noted that, compared with luminosity or mass distribution, variations in the magnetic field only slightly affect the terminal velocity of the wind.

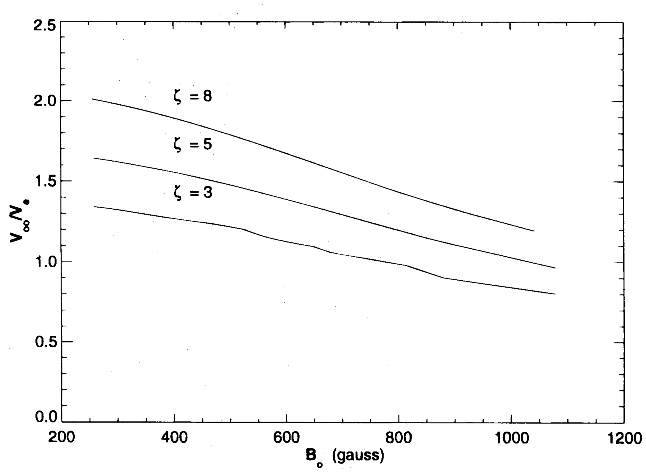

FIG. 5.—Terminal velocity v_∞ in terms of the escape velocity V_e as a function of the modulus of the photospheric magnetic field at the surface of the star for the three cases of Fig. 3.

4. SUMMARY AND CONCLUSIONS

In this paper we have analyzed the properties of rotationless, magnetized stellar outflows initialized by the radiative force. The streamlines' shape has been prescribed from a phenomenological point of view. It is suspected that, owing to the dragging action of the fluid, accelerated by the intense radiation field, the magnetic lines are pushed away and stretched outward, adopting a radial, bicycle wheel-like configuration.

The magnetic field sets up the flow tubes along which the plasma flows. At the same time, the flow tubes are carried away by the outflowing plasma; this is a direct consequence of the frozen-in condition and plays an important role in the dynamics of the wind. As we have already mentioned, in our formalism we did not *explicitly* include any magnetic energetic mechanisms. However, the energy $Q(x)$ added to the outflow could include a magnetic mechanism, e.g., dissipation of Alfvén waves, that could be efficient in driving the wind, as suggested by dos Santos et al. (1993) and others. By means of the frozen-in condition, an increase in the magnetic field intensity leads to a decrease in terminal velocity, for the following reason: the Lorentz force, acting in the $\hat{e}_\theta$ direction, is counterbalanced only by the colatitude-dependent part of the gas pressure $\Pi_2(x)$, so that both increase together. While at the equator, the plasma column is supported in static equilibrium by the gas pressure gradient, at the pole $\Pi_2(x)$ does not act, because of its $\sin\theta$ dependence. Thus, a lower polar gas pressure contributes to accelerating the flow, and it overcomes the gravitational pull with a smaller terminal speed.

As expected, the terminal velocity of the flow increases with the radiation pressure. It must be remembered that we have considered all lines as optically thick by assuming $\alpha = 1$. At first sight, it could look rather arbitrary, but the flow takes place within a narrow interval of the normalized optical depth. In this way, the force multiplier can be adequately fitted through a linear dependence on the velocity gradient.

Through the mechanisms discussed in § 3.2, the asymmetric mass distribution leads to the enhancement of the terminal velocity. As we have pointed out, with a spherically symmetric mass distribution, wind solutions can be found in which the terminal velocity is lower than the initial speed. These kinds of solutions cannot be found for $\zeta \neq 0$. The combined action of both the luminosity and the balance of gas pressure coming from mass asymmetry accelerates the wind until the terminal regime is reached. Within this formalism, unlike the CAK model, the radiative push contributes to, but does not completely govern, the dynamics of the outflow.

Related to this is the distribution of energy sources. Basically, it consists in a heating source attached to the base of the wind, close to the stellar surface. Roughly, the power needed to sustain the mass ejection does not exceed about 10% of the blackbody power radiated by the star. Such energy rates can be supplied by several meaningful mechanisms, such as wave dissipation or resistive heating. They have been suggested as the driving mechanisms of the outflows from cool and hot stars (Hartmann & MacGregor 1982; dos Santos et al. 1993).

Many hypotheses have been employed in order to keep the problem at a solvable level. In particular, the spherical symmetry of the Alfvénic surfaces, though it has an unclear physical meaning, has basically been assumed in order to reduce the mathematical complexity of the problem. Nevertheless, the resulting behavior and the values obtained for the different wind variables adjust quite well to fit the observational data, and in addition, the basic characteristics of the kind of outflows we have studied are well described within this framework. In conclusion, we think that a further effort is worthwhile in order to improve the present models.

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