

MICROLENSING VARIABILITY OF THE GRAVITATIONALLY LENSED QUASAR Q0957+561 A,B

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Received 1995 September 5; accepted 1995 December 28

ABSTRACT

The brightness of the two image components of the first discovered gravitational lens, Q0957+561 A,B has been monitored nightly for three years, and occasionally over the 15 years since its discovery in 1979. Historical data give a record back to 1903, so the brightness has been sampled on timescales from 1 day to 90 years. Large amplitude microlensing predicted for an unresolved source has not been observed. Instead, we find microlensing to be limited to 0.3 mag on the longest timescales, and 0.05 mag on timescales of months. We interpret the low amplitude of the observed long-term microlensing in terms of a statistical theory and conclude that the quasar has a luminous source 6 times larger than the Einstein ring for a half-solar mass microlens, or a quasar luminous source size of 0.3 light years. We discuss some implications of the more rapid microlensing features also observed.

Subject headings: galaxies: photometry — gravitational lensing — quasars: individual (0957+561)

1. INTRODUCTION

In an era when astronomers are studying a universe which is mostly unseen, so that the missing mass is a factor of 10 greater than the observed mass, any probe which might identify the missing mass must be exploited. Gravitational microlensing offers such a probe, since the individual masses in the lens galaxy will leave their signature in brightness fluctuations of the quasar shining through the lens galaxy (Chang & Refsdal 1979). These microlensing brightness fluctuations are readily identified by comparing the two images of a lensed quasar in which the pattern of brightness fluctuations intrinsic to the quasar is subtracted out after correction for the time delay between arrival of the two images.

Determination of the Q0957+561 A,B time delay has by now benefitted from 975 nights of observation, principally with the Mount Hopkins 0.61 m and 1.2 m telescopes, with data published in Schild & Thomson (1995a). We adopt a time delay value of 404 days (Schild 1990; Schild & Thomson 1994) although the Pelt et al. (1996) value of 423 days is not precluded. The radio value of 540 days (Lehar et al. 1992) is no longer tenable, given the new radio determination of around 440 days by the same group (Harsma et al. 1996). Discussion of the cosmologically interesting time delay has been hindered by the complication that numerous microlensing effects have been encountered, including a sustained 10 year drift, numerous cusp-profiled bursts with quarter-year timescale, periodic effects on 1 yr timescales, and a network of cusp-shaped bursts with a time scale of a few days. The principal features have been described by Thomson & Schild (1996), and in this report we comment upon the low amplitudes of the observed microlensing.

We have attempted to create a daily microlensing record to confirm the existence of the daily brightness microlensing structure. At the Whipple Observatory 1.2 m telescope, all observers are asked to make a nightly observation, consisting of three image pairs with the telescope moved between each pair. This strategy is to prevent some cosmic-ray or CCD defect from spoiling all the data from a single night. The six images are reduced separately and the nightly mean, with 1σ error estimates, is reported in Schild & Thomson (1995a). In the first year of this campaign, data were

obtained for 109 nights, on 124 nights in the second year, and on 148 nights in the third. Because of clouds and other problems, large gaps in one record or the other occur for the autumn and winter seasons, but long strings of good weather in the springs of all years produced excellent records of 50 days overlap for a 1.1 yr time delay. The autumn-winter data will provide a powerful record of the 90 day timescale events, and the 50 day well-sampled record is illustrated in Figure 1.

The data in Figure 1 show the observed A image brightness as open symbols with 1σ error bars. The B data were observed the following year, 1995, and are plotted for a 404 day time delay as filled squares. The intrinsic quasar brightness fluctuations should be subtracted away, and the many differences between the two curves are attributed to microlensing. The fact that the time delay corrected data disagree so much accounts for the difficulty in determining the time delay. If the data are plotted for the 423 day time delay of Pelt et al. (1995), the results look the same.

Several kinds of features are found in the comparison of the A and B brightness curves. In general, both images show both positive and negative spikes having a timescale of days to a week or two. Near JD 2449455, the A image had a small cluster of these spikes, with amplitudes of 0.04 mag. We have seen a similar cluster of these features for image B around JD 2449480 (not illustrated here). Another cluster of spikes was observed in the A image brightness record around JD 2449840.

Another unexpected feature is the well-observed 0.04 mag drift between the two images at JD 2449440. The event lasted about 30 days.

We have noticed and called attention to this rapid microlensing variability before (Schild & Thomson 1994), and with new data of higher quality we are more convinced that the effect is real and significant. While the data in Figure 1 may not be entirely convincing by themselves, we find from a wavelet analysis of the entire Schild & Thomson (1995a) data set that there is no value of time delay for which the rapid variability is the same or similar in the two image components. It is largely for this reason that the time delay is so difficult to determine; even with 975 nights of data available as shown in Schild & Thomson (1996), time delay

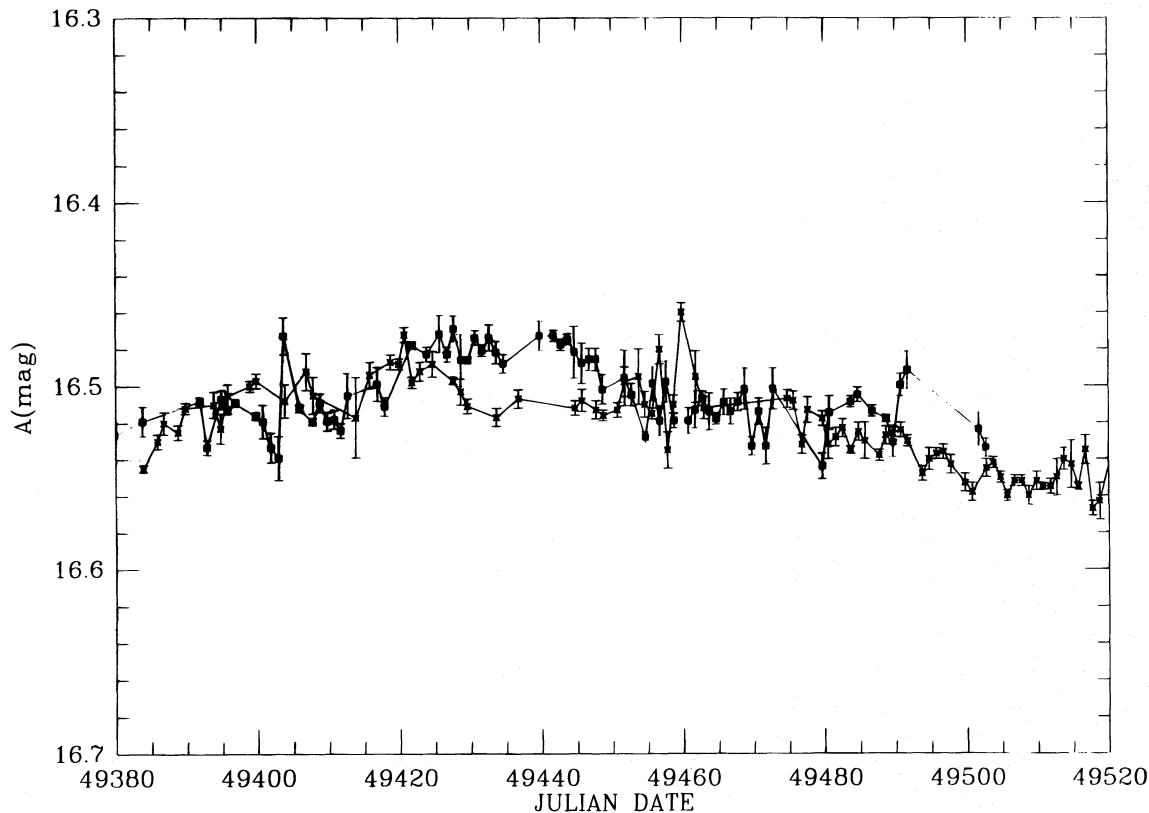


FIG. 1.—Comparison of the A and B image brightness fluctuations, shown for a 404 day time delay and with a 0.04 mag increase in the *B*-magnitude to improve the registration. Open symbols are for the A image, with 1 σ error bars. Stronger fluctuations are seen in the A than in the B image around JD 2449460, and for no value of time delay do the fluctuations align.

estimates from independent calculations range from 387 to 430 days. The predominance of the microlensing can also be seen in the results of Pelt et al. (1996); inspection of their figures 10 and 11 shows that the high-frequency filtered part

of the brightness record does not show the cosmological time delay, so the observed high-frequency fluctuations probably do not originate in the source quasar. The 10 yr microlensing history is shown in Figure 2.

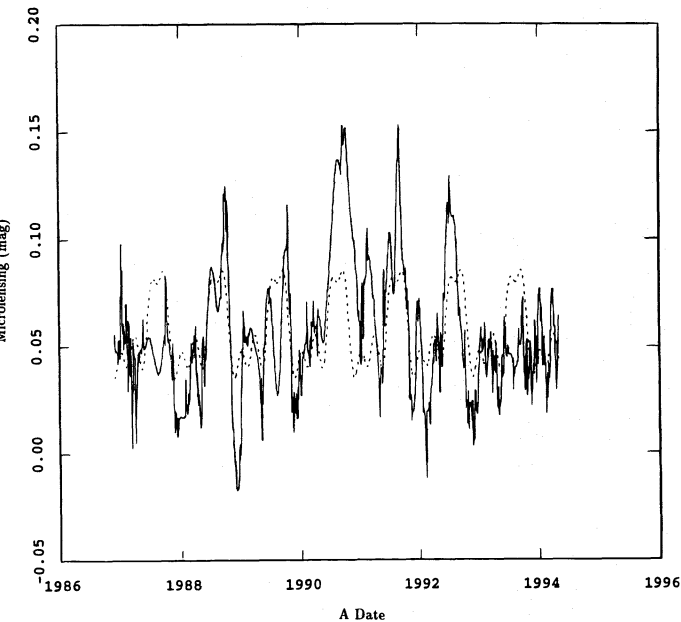


FIG. 2.—Microlensing record since 1986. Because the observational sampling exceeded 100 nights yr^{-1} since 1986, the fluctuations are well-sampled, particularly in the past 3 years when the source was monitored continuously. A fit to the curve having frequencies 1 yr + $\frac{1}{2}$ yr + $\frac{1}{4}$ yr is shown as a dotted line.

2. THE MICROLENSING HISTORY

Microlensing brightness changes for the two image components resulting from solar mass stars in the lens galaxy G1 have been predicted to be 3 mag for an unresolved source (Chang & Refsdal 1979; Young 1981; Gott 1981). The timescale would be 30 yr for an event. Both of the quasar images have been measured during three of these timescales with photometry reported for 1903 and 1915 by Keel (1982a), for 1955 by Walsh et al. (1979), and by Schild and colleagues since 1979. We summarize the observed estimates in Table 1.

Note that the historic measurements have been *B* magnitudes, whereas we used an *R* filter for the CCD moni-

TABLE 1		
HISTORIC Q0957+561		
<i>R</i> MAGNITUDES		
Year	A	B
1903	16.0	16.7
1915	15.6	16.2
1955	16.4	16.8
1980	16.7	16.8
1995	16.5	16.5
mean	16.2	16.6
rms	0.44	0.26

toring; the two photometries have been combined using the $B-R$ color difference measured by Vanderriest et al. (1989). The estimates for 1955 given by Walsh et al. (1979) are clearly incompatible with this color index, and we have assumed that the errors of the estimates are the same; we determine mean values by converting the B filter estimates to R using the measured $B-R$ color and averaging with the corresponding R magnitude published estimates.

At all epochs the brightnesses of the two components were approximately equal to the modern value. And although the optical depths of the two images are significantly different, the two components have always had nearly equal brightness. The largest difference from the modern value was observed in 1915, when image A was approximately 0.6 mag brighter and image B was 0.4 mag brighter than the means. The fact that both images were brighter suggests that the QSO was intrinsically brighter at this earlier time. Thus the microlensing is probably somewhat overestimated in our Table 1 rms estimates, and the accretion disk size estimated below is underestimated.

In our discussion of microlensing to follow, we adopt a mass of $0.5 M_{\odot}$ for an average star in lens elliptical galaxy G1. Although microlens timescale values in the literature are consistently given for a solar mass microlens, an “average” star in an elliptical galaxy is probably less massive by an unknown amount. The question of the low end of the mass function was extensively investigated 15 yr ago, because evolutionary corrections to the luminosities of elliptical galaxies were being sought. Careful analysis of the near-IR CO features led to the conclusion by Frogel et al. (1978) that giants dominate the light of ellipticals and dwarfs dominate the mass; even though the measured CO bands give good giant-dwarf discrimination, the observations do not show where the rising initial mass function turns over. For our purposes we take the mass from the timescale of the single microlensing brightness increase observed over 1979–1990. We presume that the 10 yr rise is half a microlensing profile, whose full duration would be 20 yr, corresponding to a microlensing mass of $0.5 M_{\odot}$. We take the “average” microlensing mass to be uncertain by a factor of 2.

Because the brightness variability observed on timescales appropriate for microlensing by half-solar mass stars in G1 is so much smaller than the value predicted for an unresolved source, we use the statistical theory of Refsdal & Stabell (1993) to describe it. This statistical theory applies when the projected Einstein ring diameter D_0 of the microlens is 10 times smaller than the diameter of the source. Following the discussion of Table 1, we consider that the B component brightness variability about the mean value sampled during three 20 yr microlensing timescales is 0.26 mag. Estimates of the surface optical depth (defined in Schneider et al. 1992) in lens galaxy G1 have been stable in the last several generations of lens models, with (0.32, 1.2) estimates for the (A,B) image components in both the Falco et al. (1991) and Grogan & Narayan (1996) models. From equation (1) in Refsdal & Stabell (1991) and a surface optical depth of 1.2, we compute a quasar luminous source diameter of $D = 9.2D_0$, where D_0 is the source plane projected diameter of the Einstein ring for an average microlensing star. Similarly, for image A we calculate $D = 2.8D_0$. We adopt $D = 6 \pm 3D_0$ as a reasonable mean value. For $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega = 1$, $\Lambda = 0$, $D_0 = 4 \times 10^{16} \text{ cm}$ for a half-solar mass microlens, so the diameter of the lumi-

nous quasar source $D = 6D_0 = 3 \times 10^{17} \text{ cm} = 0.3 \text{ lt-yr} = 2 \times 10^4 \text{ AU}$. Note that this size follows from an assumption that the luminous source is a reasonably uniform surface of constant surface brightness; we shall see below that we are already detecting departures from this assumed simple model.

This value of the luminous source size is much larger than the value normally adopted for a QSO accretion disk. Is there other evidence for larger QSO structure? Such evidence is available from spectropolarimetry, where optically luminous sources of size $0.1 \text{ pc} = 0.3 \text{ lt-yr}$ are reported by Antonucci (1982), and from our own autocorrelation analysis of the TwQSO images, which suggest multipath reflections over a light year size scale (Thomson & Schild 1996). Radio observations (Miyoshi et al. 1995) suggest significant AGN structure on a 0.2 pc scale in Seyfert galaxy NGC 4258. Absence of large microlensing brightness fluctuations in quasars seen along lines of sight to foreground galaxies have already been noted as a general feature of quasars by Keel (1982); a large quasar source size would explain this observation.

New evidence for large source structure indicated by multipath reflections may be seen in the autocorrelation calculation illustrated in Figure 3, where we plot a simple autocorrelation of the A and B images separately. This plot has been shown in Thomson & Schild (1996). It is seen in Figure 3 that the autocorrelations differ considerably for the two image components, even though both images are from the same QSO. We attribute this apparent discrepancy to effects of microlensing as we presume that the network of cusps originating in lens galaxy G1 is different for the two images. We conclude that the QSO probably has structure on the scales of the autocorrelation peaks, which are found at 125, 190, and 540 days in both image components. For a QSO redshift of 1.41, the corresponding proper times are 52, 80, and 224 days. We conclude that Q0957 has luminous source structure on the scale of a light year. The diameter of the luminous source determined from observed microlensing brightness fluctuations is somewhat smaller than this value, though with large uncertainty. Most likely the

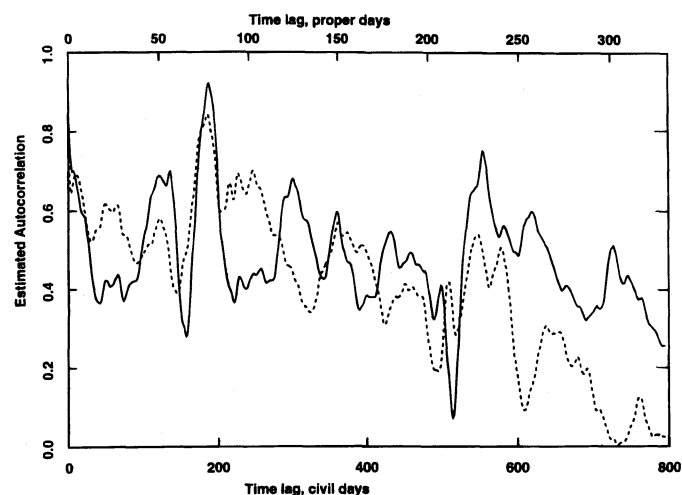


FIG. 3.—Autocovariance plot for the A (dashed line) and B (solid line) light curves. The cross terms are binned at 1 day resolution, and the result smoothed with a taper having an 11 day total width. Note the coincidence of the 187 day peaks and the broad extra peak in the B light curve between 100 and 140 days. Subtracting the two curves gives a crudely sinusoidal pattern with a period of about 3.4 yr.

quasar's luminous source is structured, and consists of rings or clouds with a 10% filling factor. Such structure may need to be taken into account to explain the observed high-frequency microlensing.

Note that for an alternative explanation of the autocorrelation peaks as due to some process of reverberation within the central black hole, the microlensing could not explain the differences seen in the autocorrelation properties of the two image components. We believe that the most straightforward explanation of the observations is that the autocorrelation peaks represent real reflecting or fluorescing regions exterior to the QSO's accretion disk, and that the strengths of these reflecting regions vary in time due to microlensing of the luminous regions by stars in lens galaxy G1.

The source size determined from the statistical microlensing test, $D = 6D_0$, is seen to be perhaps smaller than the size inferred from multipath reflections. We infer that the QSO internal structure is patchy, perhaps consisting of a network of clouds or rings, and the filling factor is only 10%. What is the characteristic size of these QSO structures? We refer to the two-dimensional plot of time delay given in Figure 6 of Thomson & Schild (1996) and interpret the contours of time delay as revealing the radial size of the structures in the QSO. From the average FWHM of these contours we infer a structure size of 100 light days observed, or a proper size of 40 light days. Of course, smaller substructures may also be present.

The large quasar source size inferred here should not be confused with the microlensing complexity caused by frequent existence of more than one microlens in any one line of sight. Because of the large quasar luminous source size, many lines of sight are microlensed at any one time, and each line of sight, on average the size of an Einstein ring for a half-solar mass microlens, may contain more than one microlens. Calculations of microlensing by Wambsganss et al. (1992) show that as the number of microlenses increases, the microlensing does not damp out, but rather becomes more complex, with somewhat finer structure. The case of a highly resolved luminous source has not been considered with detailed calculations, however.

3. MICROLENSING ON SHORTER TIMESCALES

In the previous section, microlensing on a timescale of 20–30 yr has been analyzed to find information about the quasar accretion disk. However, microlensing on much shorter timescales has been reported by Schild & Thomson (1994). This rapid microlensing seems to have two components. The slower component consists of a network of cusps having a nominal width of 90 days, and an amplitude of 0.05 mag. These features are readily seen in the microlensing records published in Schild & Thomson (1994, 1995b, 1996). We have shown that these features, if interpreted as resulting from a population of microlenses in lens galaxy G1 imply a microlensing mass of only $10^{-5} M_{\odot}$.

Even more rapid microlensing has also been noted by Schild & Thomson (1994) to occur on timescales of a week and with an amplitude of only 0.03 mag. A really convincing case for the existence of these events remains to be made. Although many such features can be seen in Figure 1 and elsewhere throughout the CCD brightness record, and although the accuracy of the observations has apparently been slightly conservatively estimated (Schild & Thomson 1995a), these subtle features may need to be confirmed by

independent observations. The existence of brightness fluctuation on timescales of days was first noted by Vanderriest et al. (1982) and the daily brightness fluctuations can already be seen in the data of Schild & Weekes (1984) and Schild & Cholfin (1986), although they were not specifically commented upon. The more complete CCD data records from recent years show that these events occur in a nearly continuous pattern, and an excellent example of a 90 day event with superimposed weekly fluctuations has been shown by Schild & Thomson (1995b; see also Fig. 5 below). Questions of the accuracy of the CCD data have been addressed by Schild & Thomson (1995a), who find that the errors have been accurately or slightly conservatively estimated. This conclusion is based upon a statistical test that shows that brightness fluctuations observed are coherent over timescales of several days and therefore probably not the result of nightly errors. If the observed brightness fluctuations on week timescales originated in intrinsic quasar fluctuations, then the gravitational lens time delay should be easy to measure, which has not been the case. However, the best argument that the week-long fluctuations are microlensing comes from a wavelet analysis to the image components; for no value of time delay do the fitted wavelets for A and B images agree. This will be the subject of a forthcoming paper.

The Fourier power spectrum of the microlensing (Fig. 4 and Schild & Thomson 1994) offers a clue about the origin of these brightness fluctuations. The 90 day features are continuous with the spectrum of the lower frequency microlensing events, and probably reflect the mass spectrum of

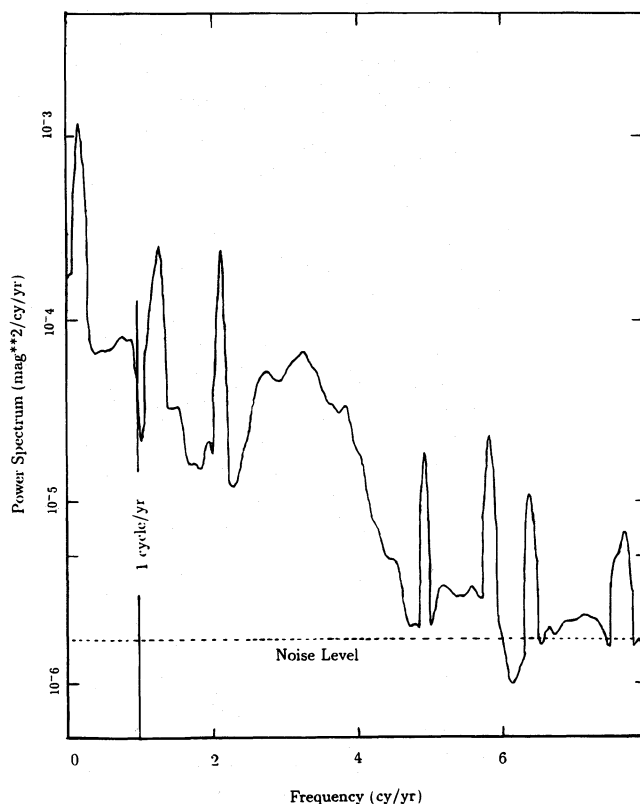


FIG. 4.—Fourier power spectrum for the microlensing data record illustrated in Fig. 2. Several discrete frequencies around a year are found, suggesting orbital motion. A broad secondary peak around 4 cycles yr^{-1} is caused by the microlensing spikes of 0.04 mag amplitude seen throughout the microlensing record.

microlensing masses in lens galaxy G1. As pointed out by Schild & Thomson (1995b), this interpretation literally means that a substantial fraction of the mass in the lens galaxy must have planetary masses around $10^{-5} M_{\odot}$. We shall refer to these as the rogue planet microlenses. Although other explanations of the observed fluctuations may be possible, none has yet been offered in the several years since the rapid microlensing was reported in 1993 (Schild & Thomson 1994). Of course, if the interpretation of the 90 day events as microlensing is true, the existence of very fine luminous quasar structure, on size scales of an astronomical unit, is implied; we return to this point below. Since the 90 day fluctuations occur nearly continuously, it follows that the rogue planets are the mass component having unit surface mass density and that the missing mass is rogue planets.

The more rapid fluctuations on weekly scales seen in Figure 5 superimposed upon the broad 90 day profile are more likely to originate in the halo of our own Galaxy. If we presume the existence of such a population of halo rogue planets, based upon their probable detection in lens galaxy G1, would they be visible as MACHOs?

To estimate the probability of detecting such a planetary mass MACHO, we estimate the magnifications involved with reference to the Einstein ring diameters calculated for various lens configurations. If d is the Einstein ring angular diameter corresponding to a $10^{-5} M_{\odot}$ particle in the lens galaxy G1, then from simple scaling of the standard formula

$d = 2[4GM/c^2][Dl/Dl*Ds]^{1/2}$ we can calculate D , the angular diameter for the same particle in the halo, and DS , the angular diameter for a half-solar mass particle in lens galaxy G1. We calculate $D = 100d$ and $DS = 200d$. It is interesting and significant that these numbers are comparable.

It is well known from calculations by Paczyński (1987) and Griest (1991) that the probability of detecting a microlensing event originating in the halo of our Galaxy for a cosmologically interesting density of MACHOs is 10^{-6} . How is this probability affected by the background source being resolved, as found in the previous section, and by the low masses of the particles considered here?

The size of the luminous source region of the quasar found in § 2 is $6DS$, where DS is the diameter of the Einstein ring for a solar mass microlens in lens galaxy G1. We have computed above that $DS = 200d$, and that the Einstein ring for a rogue planet in the halo is $100d$. Thus, the quasar's luminous source region has a diameter of $1200d = 12D$, so at any moment there are $12^2 = 144$ sight lines to the luminous quasar structure, each having a 10^{-6} probability of microlensing or a net probability of 10^{-4} that microlensing of the quasar luminosity is microlensed by halo rogue planets at any moment. This probability becomes one for a cosmologically significant population of halo microlenses with $10^{-9} M_{\odot}$. For such microlenses, the quasar is highly resolved and at first it seems unlikely that rapid microlensing events would be observed. The same is true even for the $10^{-5} M_{\odot}$ microlenses in G1; they would be seen projected in front of a highly resolved luminous source.

On the other hand, it appears to be an observational fact that the rapid brightness fluctuations are observed. We conclude that either the QSO source is highly structured, with cloudlets or some other features on 0.01 pc and 0.00001 pc size scales, or perhaps the microlenses see a complex network of cusps that originate closer to the quasar. The halo rogue planet microlens would see cusps due to the rogue planets and solar mass stars in G1, and the G1 rogue planet microlenses would see the cusps originating in solar mass stars in the halo of the quasar host galaxy or an intervening galaxy.

We caution that no $10^{-5} M_{\odot}$ particles have been reported in the MACHO (Alcock et al. 1995) or EROS (Aubourg et al. 1995) searches. However, a calculation of the size of the Einstein ring for a rogue planet MACHO shows that a red giant in the Large Magellanic Cloud would be resolved, and the microlensing probability calculation breaks down for such a low mass. Refsdal & Stabell (1993) suggest that the microlensing observed in Q2237+0305 can also be explained by a population of planetary-mass microlenses.

4. SUMMARY AND CONCLUSIONS

Gravitational microlensing can reveal the presence of masses of any stellar or planetary size in lens galaxy G1. The optical spectrum of the lens galaxy has long been interpreted to reveal the presence of a population of approximately solar mass stars. These stars must have sufficient surface mass density to cause microlensing events on a timescale of 20–30 yr. We find that the two image components each show only small brightness fluctuations on this timescale, and we conclude that the quasar's luminous size must be a factor of 6 larger than the Einstein ring of the microlensing stars.

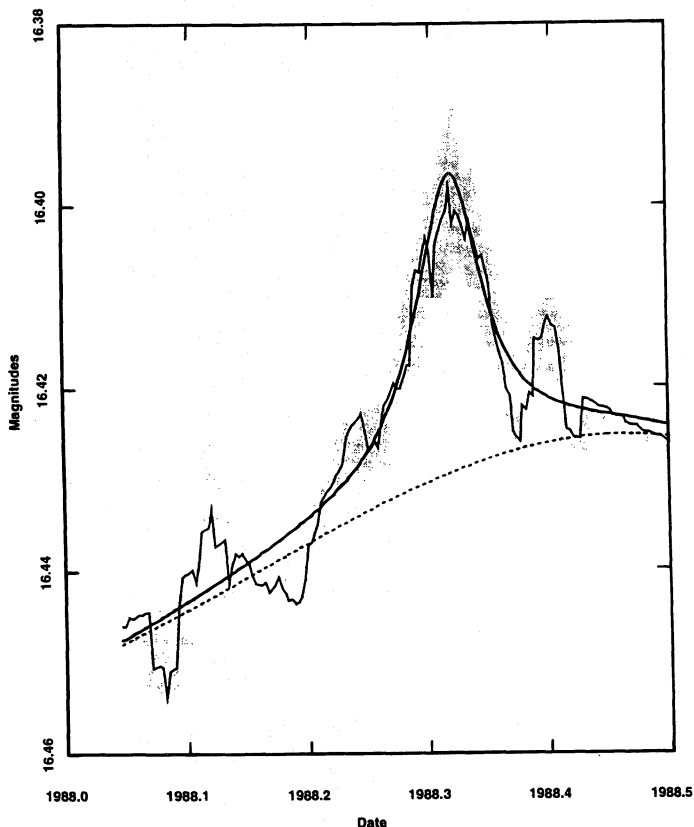


FIG. 5.—Profile of a single 90 day event, from a simple plot of the 90 nights of data for component A in 1988. The event was not seen in B 1.1 yr later, and is presumably due to microlensing. The shaded area shows the error per data point. Note that in addition to the main event, several low-amplitude events with positive and with negative magnification and timescale of a week are found.

Much more rapid events are also found in the microlensing record. We interpret the 90 day spikes as indicating the presence of a large population of objects having approximately $10^{-5} M_{\odot}$. These are likely to be the missing mass. An even more rapid network of cusps at 0.03 mag amplitude and week timescale could indicate microlensing events in the halo of our galaxy. Other explanations should be sought, since the low-mass population has not been seen in the MACHO searches. We note that the frequency of the periodic term at 3.4 yr found by Thomson & Schild (1996) in the microlensing record corresponds to the predicted frequency of gravitational waves (Rajagopal & Romani 1995), which would result if the radio source lens galaxy G1 had a

binary black hole; the more rapid cusps observed have the timescale corresponding to expected chirps from inspiral at cosmological distances. We do not have an amplitude estimate for these effects because the microlensing is very sensitive to the exact alignments of the microlensing particles, and would create a microlensing amplifier of the gravitational waves.

We thank Hans Witt, Shude Mao, Norman Grogin, and Emilio Falco for helpful discussions. Many of the illustrations were prepared by David J. Thomson for our earlier conference reports, and we thank him for continued guidance.

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