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DO THE MAGELLANIC CEPHEIDS POSE A NEW PUZZLE?

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ABSTRACT

The observed Magellanic Cloud (MC) Cepheid data pose a new challenge to both stellar structure and stellar evolution. The Fourier analysis of the LMC and SMC data indicate that the 2:1 resonance ($P_2/P_0 = 0.5$) between the fundamental mode of pulsation and the second overtone occurs with a period P_0 of $\approx$10 days, just as in our Galaxy. The implications of this resonance are difficult to reconcile with linear radiative Cepheid models computed for the low metallicity of the MC. They also require a large upward shift in the mass-luminosity relation (MLR), seemingly larger than is comfortable for evolutionary calculations. The disagreement worsens if we interpret the structure in the first overtone Fourier parameters near 3 days as the result of an alleged resonance with the fourth overtone. In contrast to earlier expectations, the beat Cepheid data with $P_1/P_0 \approx 0.7$, as well as those with $P_2/P_1 \approx 0.8$, impose weak constraints, at best, on the MLRs.

Subject headings: Cepheids — Magellanic Clouds — stars: oscillations

One of the exciting by-products of the search for dark matter has been the collection of a large number of good quality data on tens of thousands of stars in the Magellanic Clouds (MCs). In particular, hundreds of Cepheids have been observed and Fourier analyzed in the last year (Beaulieu 1995; Beaulieu et al. 1995, 1996; Alcock et al. 1995; Welch et al. 1995). These efforts greatly increase the number and especially the diversity of known Cepheids. Indeed, it has been known for some time that the Magellanic Clouds, LMC and SMC, are observed to have metallicities $Z \approx 0.01$ and 0.005, respectively (Luck & Lambert 1992), and are thus metal poor compared with the Galaxy, for which $X \approx 0.70$, $Z \approx 0.02$. Both MCs are expected to have a slightly larger hydrogen content. Observations indicate that the Galaxy as well as the MCs have a substantial dispersion in composition, although the latter seems to be somewhat smaller in the SMC. The time seems ripe to examine whether observations, stellar pulsation, and stellar evolution can provide a self-consistent picture of the Cepheids.

It is now well known that the so-called Hertzsprung progression of the bump in the Cepheid light curves is related to a resonance ($P_{20} \equiv P_2/P_0 = 0.5$) between the self-excited fundamental mode of pulsation and the second overtone. In addition to such asteroseismological constraints, which are based on observed periods and inferred resonances, there also exist similar constraints based on observed period ratios, as in the case of beat Cepheids. The pulsation calculations have to be consistent with these observed constraints, on the one hand, and with the mass-luminosity relations (MLRs) that arise from stellar evolutionary tracks, on the other.

Moskalik, Buchler, & Marom (1992, hereafter MBM; see also Kanbur & Simon 1993) performed a survey of pulsation models with the new Livermore opacities (Iglesias, Rogers, & Wilson 1992) and found that stellar pulsation calculations, stellar evolution calculations, and observations of *Galactic* Cepheids are basically in agreement. In retrospect, these studies were very limited in their stellar parameter ranges because, at the time, nobody expected the LMC and SMC Cepheids to be so different from their Galactic siblings. In this Letter we examine the constraints that the new MC observations impose on Cepheid models.

Our hydrostatic models and their linear stability analyses are computed with a relatively coarse 200 point mesh. We have checked that this mesh gives a reasonable resolution for the eigenvectors and for the work integrands (except in the vicinity of the sharp partial hydrogen ionization front). Models have been computed for the compositions $X = 0.70$ and 0.80, each with $Z = 0.004$, 0.01, 0.02, and 0.03 for which OPAL (Iglesias et al. 1992) and Alexander & Ferguson (1994) opacities are available. Convection has been ignored.

The constraints that we have to deal with here all involve finding, by iteration, the M and L that are compatible with a given period, period ratio, and composition. The effective temperature, of course, also enters the picture. In order to accommodate the finite width of the instability strip, we need to allow for a range of $T_{\rm eff}$, which we define in terms of the distance ΔT from the linear blue edges (for the corresponding M and L). In order to reduce the model dependence of our results, we deliberately refrain from using $T_{\rm eff}$ directly (its definition is somewhat sensitive to the treatment of radiative transport in the surface layer). Furthermore, its observational uncertainty is large.

Let us first examine the constraints imposed by the 2:1 "bump" resonance ($P_{20} = 0.5$) near $P_0 = 10$ days. We are faced with several problems when we try to quantify the constraints. First, the instability strip has a finite width, and the pulsators that are in exact resonance are therefore spread over a range in $T_{\rm eff}$, and thus also in P_0. In the Galaxy, this

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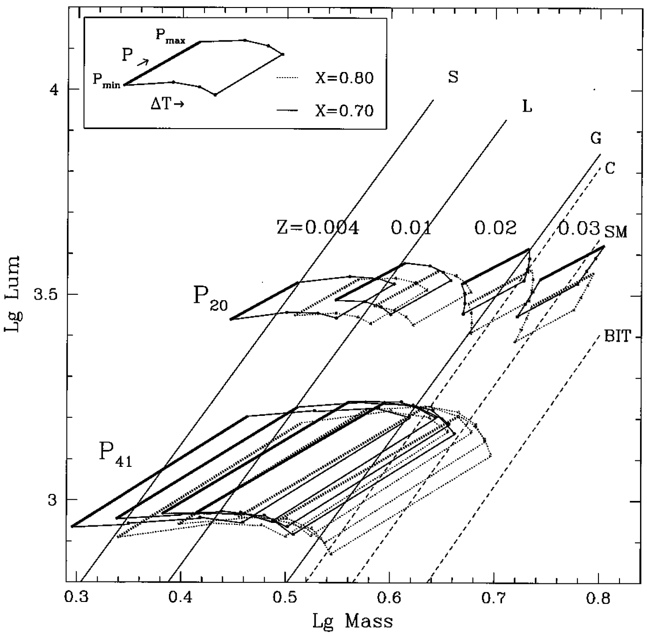

FIG. 1.—*Boxes*: resonance constraints, P_{20} = 0.5 and P_{41} = 0.5, *thin lines*: MLRs; cf. text.

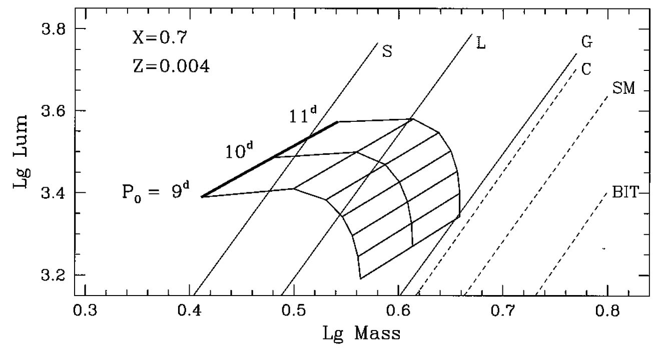

FIG. 2.—P_{20} resonance constraints; ΔT = 0, 1200 K in steps of 200 K

resonance period range $P_{\rm min}$–$P_{\rm max}$ has been estimated to be 9.5–10.5 days (MBM). For further use, we note that the direct determinations of the temperature width of the instability strip are quite uncertain, with various estimates ranging from 500 to 1000 K (e.g., MBM). This temperature width corresponds to our definition of ΔT (because it is defined in an H-R diagram, i.e., at constant L, and, the evolutionary tracks being almost horizontal, therefore also at constant M). Second, the observations indicate the location of the *nonlinear* resonance center, rather than the linear center that is obtained from our linear calculations. However, hydrodynamical modeling of Cepheids (e.g., Buchler, Moskalik, & Kovács 1990; MBM; Buchler 1993) has shown that the nonlinear shift is expected to be relatively small for this particular resonance. Third, the first overtone is linearly unstable, so that the actual fundamental blue edge is shifted to lower $T_{\rm eff}$ due to nonlinear effects (see Buchler & Kovács 1986 for a simple explanation of this nonlinear shift). An inspection of the hydrodynamical models of MBM shows that this shift is small, however, $\lesssim$100 K. Fourth, the models ignore convection, so that our periods become less trustworthy farther from the blue edge (large ΔT). Finally, when comparing our results with observations, one has to keep in mind that in all three galaxies the Cepheids display a spread in composition.

The results of our study are reported in the M-L plot of Figure 1. The thick lines denote the linear blue edges of the resonant models $[P_{20} = P_{20}(L, M, T_e = T_e^{\rm BE}(L, M)) = 0.5]$ as a function of period $P_0 = P_0(L, M, T_e)$, ranging from $P_{\rm min}$ = 9.5 days to $P_{\rm max}$ = 10.5 days. The emerging lines represent the resonant models that have respective periods $P_{\rm min}$ and $P_{\rm max}$, and that are located a distance ΔT from the linear blue edges, i.e., with $T_e = T_e^{\rm BE}(L, M) + \Delta T$. The plotted range is ΔT = 0–300 K, in steps of 100 K (*dots*). The inset in the upper left-hand corner shows the direction of variation of P and of ΔT.

Consider first the Galactic Cepheids and, for illustration, assume that they obey a narrow single MLR. If everything were consistent, the actual MLR should cut the resonant fundamental blue edge for the appropriate Z (= 0.02) at the upper period, i.e., $P_{\rm max} \approx 10.5$ days. But the same MLR also needs to cut the $P_{\rm min} \approx 9.5$ days line at the temperature appropriate for the resonant red edge.

The MLR, denoted by "G" in Figure 1, has been chosen with a slope (= 3.502) similar to the ones derived from stellar evolutionary calculations and has been made to go through the resonant fundamental blue edge at the upper period, i.e., $P_{\rm max} \approx 10.5$ days. Unfortunately, the temperature appropriate for the resonant red edge (or ΔT) cannot be computed with our radiative models but must be guessed from other considerations, as noted above. We can conclude from the shape of the Z = 0.02 loci in Figure 1 that the "G" MLR comes close to satisfying the second constraint for a $\Delta T \approx 1000$ K. For reference to previous work, we have shown as dashed lines the two MLRs with Z = 0.02, "BIT" and Chiosi "C" that were used in MBM, as well as a more recent one with OPAL opacities (Schaller et al. 1992), "SM": log L = 0.79 + 3.56 log M. We thus recover the conclusion of MBM that, for the Galaxy, a MLR, with an upward shift of ≈0.05 compared with Chiosi, essentially gives agreement between observations, pulsation, and evolution.

We turn now to the LMC and SMC. In both galaxies, the Fourier decomposition data indicate that the same resonance occurs in the vicinity of 10 days. The shape of the Fourier decomposition curves of the LMC data (Welch et al. 1995) indicate that the resonance range $\Delta P_R = P_{\rm min}$–$P_{\rm max}$ is comparable to that of the Galactic Cepheids, viz., $\Delta P_R \approx 1$ day. In the SMC, there are far too few observed stars to enable us to constrain ΔP_R. In Figure 1 we have plotted the constant-period loci for resonant models with appropriate metallicities Z = 0.01 and 0.004. The two values of X = 0.7 and 0.8 bracket the observed ones. One notes a small decrease of the luminosity with Z and a small increase with X for a resonant Cepheid with given period of, say, 10 days. This trend is in agreement with the observed period-luminosity relations. A blowup of the Z = 0.004, X = 0.7 case (Fig. 2) displays the typical behavior of the constant-period lines with larger ΔT up to 1200 K. We are faced here with the following difficulty: a MLR, such as the one denoted "S," that goes through the resonant blue edge at $P_{\rm max}$ = 10.5 days cuts the $P_{\rm min}$ curve at a $\Delta T \approx 200$ K. This, however, would imply an unacceptably narrow instability strip. Are there escapes from this dilemma? We have tacitly taken the slope of the Chiosi "C" MLR here. It is clear from the figure, though, that the results do not depend sensitively on the precise value within the range of slopes that various evolutionary calculations have determined.

We also note that, following common practice, we have ignored the curvature of the MLRs, but the curvature can at best provide a little help in giving better agreement. It is true that the resonant blue edge is shifted by nonlinear effects, but this shift is unlikely to exceed $\Delta T = 100$ K (cf. Buchler et al. 1990). Furthermore, the problem would be alleviated or eliminated if the loci bent downward more rapidly, but whether convection could produce this effect is unlikely. Finally, it might seem that we could get out of this dilemma if the blueward loops of the evolutionary tracks do not extend far enough to reach the fundamental resonant linear blue edge for the metal-deficient Cepheids, in other words, if the observational blue edge were determined not by the changeover in vibrational stability but by the reversal of the evolutionary tracks.

Recently, Chiosi, Wood, & Capitano (1993) have derived new MLRs (unfortunately using LA opacities), $\log L = 3.125 + f - 3.14X - 13.14Z + 3.502 \log M$, which have the merit of indicating the dependence on X, Z and on the convective overshoot factor f. Figure 1 indicates that such a large X and Z dependence is completely incompatible with our linear models, even when we take into account all the uncertainties that we have discussed.

Are there additional resonances to tie down the MLRs? It has been conjectured that the sharp feature of the ϕ_{21} Fourier phase in the Galactic s Cepheids is due to the resonance $P_{41} = 0.5$ in the vicinity of 3.2 days (Antonello, Poretti, & Reduzzi 1990). A similar sharp feature occurs both in the LMC and the SMC overtone Cepheids at ≈2.7 and 2.2 days, respectively (Beaulieu et al. 1995, 1996; Welch et al. 1995). In all three galaxies, the sharpness of the structure would limit the period range to $\Delta P_R \lesssim 1$ day, and it suggests that the first overtone blue edge has approximately the same period width. A curt hydrodynamical study of first overtone Cepheid models by Antonello & Aikawa (1993) failed to reproduce the observed features, although these authors claimed good agreement. A subsequent, fairly extensive hydrodynamical survey (Schaller & Buchler 1993) found that there is a serious problem with our ability to understand the pronounced features in the observed Fourier decomposition for the overtone Cepheids. This study also indicated that nonlinear effects cause a substantial shift in this resonance that makes it difficult to infer the location of the *linear* resonance center from the observations.

Notwithstanding, when we use this alleged resonance as a constraint, we find the elongated "lozenges" with $\log L \approx 2.8$–3.2 in Figure 1. We have allowed for an uncertainty in the resonance center of 2.5–3.5 days (*upward from left to right*). The width of the lozenges is again associated with the proximity ΔT of the model, now to the first overtone resonant blue edge (here we show 200 K steps, from 0 to 600 K). Figure 1 reveals that, if we accept the "G" MLR for the Galaxy (and we implicitly accept a reasonable slope), the resonance at the linear blue edge would have to occur at a period $P_1 = P_{max} \approx 4.3$ days. This requires a nonlinear shift in the resonance center of ≈1.3 days, which is rather excessive. If we consider a nonlinear shift $\Delta T = 100$ K in the resonant blue edge, this would put $P_{max} \approx 3.7$ days, which is still large but in agreement with Schaller & Buchler (1993). The almost horizontal nature of the constant P_{min} loci thus causes the same problem as it did for the bump resonance in the MCs, namely, that the resonant red edge is too close to the blue edge in terms of periods, or that the instability strip is wider (in ΔT) than expected. It is therefore very hard to reconcile the narrowness of the structure in the observational Fourier phases with the alleged resonance. We might add that on physical grounds it is also difficult to understand how such a strongly damped fourth overtone could be so strongly entrained in the resonance.

The same problems arise for the LMC and SMC. The MLRs that cut the resonant blue edge at P_{max} will cut the P_{min} locus at a ΔT that is far too small, as Figure 1 shows. It is thus impossible to reconcile a reasonable period width of the instability strip with the models. We note, however, that if we take the "G," "L," and "S" MLRs at face value, and we determine their intersections with the respective resonant blue edges, $P_{max} \approx 4.3$, 3.5, 3.0 days, we see the same trend to lower values as the already mentioned trend in the observational Fourier decomposition parameters for the three galaxies.

It is of course interesting to note the implications for the evolutionary tracks in an H-R diagram. Figure 1 shows that the blueward loops in which the bump Cepheid variables lie should have a luminosity that decreases only very slowly with Z and with Y. In contrast, the masses of the resonant models depend very sensitively on composition.

We now turn to the constraints imposed by beat Cepheids, which are the stars that pulsate in two modes simultaneously and with constant amplitudes. We recall that the theoretical status of the nonlinear period ratios of the beat Cepheids remains unsolved: numerical hydrodynamical modeling has not been able to produce steady beat pulsations in realistic Cepheid models. This, unfortunately, introduces unknown but significant uncertainties estimated to be of the 1% level when we use linear theoretical period ratios to compare with observed ones.

The LMC beat Cepheids happen to fall into a small- and a large-period group (Alcock et al. 1995). The linear model code yields the periods and growth rates as a function of M, L, T_{eff}, X, and Z. Eliminating L in favor of P_0, we can thus obtain the relations $P_{10} = P_{10}(M, P_0; \Delta T, X, Z)$. This allows us to examine the constraints that the beat Cepheids impose on the masses, as illustrated in Figure 3 in the left panels for two typical stars in each group. The horizontal lines represent the observed period ratio P_{10} and a ±1% shift to allow for unknown nonlinear effects. For the longer period Cepheids, the mass is very poorly determined and can span almost the whole range 2–7 $M_\odot$, depending mostly on the unknown nonlinear shift in P_{10}, X, Z, and to a much smaller extent on ΔT. The upper and lower paired curves correspond to $\Delta T = 400$ and 0 K in the P_{10} cases. (Note that even when we get a theoretical handle on the nonlinear period shifts, the horizontal nature of the curves will still leave a large uncertainty on the mass.) For the lower period Cepheids, the mass is a little better constrained. It is comforting that a $Z = 0.01$, typical of the LMC, is in best agreement with the observational data (Fig. 3). The situation for two typical P_{21} LMC beat Cepheids is illustrated in Figure 3 in the right panels. Here ΔT plays essentially no role. Again, from the nature of the curves, one concludes that the mass cannot be accurately determined. Furthermore, there are no solutions for larger P_{21}, which seems to imply that we should expect that the nonlinear pulsational effects increase P_{21} compared with its linear value.

In an M-L plot similar to Figure 1, the loci for the beat Cepheids, both for P_{10} and P_{21}, thus appear as elongated boxes that stretch horizontally across the figure and are compatible with just about any MLR. This generalizes the conclusion of

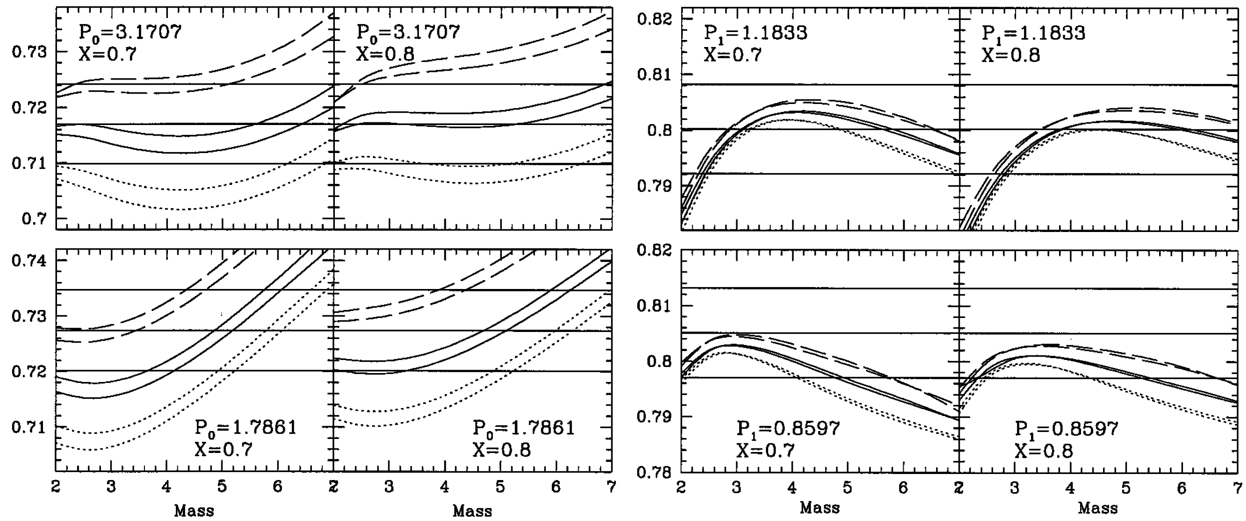

FIG. 3.—Linear period ratio vs. period for four typical LMC stars; P_{10} (*left panels*), P_{21} (*right panels*); $Z = 0.01$ (*solid curves*), $Z = 0.02$ (*dotted curves*), $Z = 0.004$ (*dashed curves*); the paired curves have $\Delta T = 0$ and 400 K, respectively.

MBM and Christensen-Dalsgaard & Petersen (1995), who examined only two specific MLRs. The examination of the beat Cepheid data, both Galactic and SMC, led to the same conclusion, viz., that because of this insensitivity the beat Cepheids do not provide very useful constraints for stellar structure and evolution.

What are the deficiencies of our calculations and how can they affect our overall conclusions? First, we recall that we have disregarded the effects of convection on stellar structure and on the periods. We show that a problem already arises in the vicinity of the blue edge where convection is expected to be unimportant. Second, our results are based on *linear periods*, and there is some sensitivity to nonlinear period shifts, small in the case of the bump Cepheids but important for the beat Cepheids. Third, we have not examined the implications of a spread in composition in each galaxy, and, finally, "metals" are all lumped into a single parameter Z. MBM found that there are small but noticeable changes associated with different abundance mixtures, and it may become necessary to assess these effects more carefully. Perhaps additional small improvements in the opacities may also be necessary.

With these caveats in mind, we are led to conclude that there is a problem with reconciling the MC data with stellar pulsation models. We have also noted that there is a discrepancy with stellar evolution calculations. In the latter, it appears that (1) a large "convective overshoot" fudge factor is required in stellar evolution calculations to bring the low-Z tracks into agreement, (2) the sensitivity of the evolutionary tracks to composition needs to be reexamined, and (3) the precise extent of the blueward loops in the evolutionary tracks may play an important role.

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