

WHAT PLANETARY NEBULAE CAN TELL US ABOUT PLANETARY SYSTEMS

NOAM SOKER

Mathematics-Physics Department, University of Haifa at Oranim, Oranim, Tivon 36006, Israel; soker@phys1.technion.ac.il

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ABSTRACT

We derive the maximal orbital separations allowed for brown dwarfs and massive planets in order to tidally spin up progenitors of planetary nebulae (PNs). We find the maximal orbital separation to be ~ 5 AU. We combine this finding with our basic assumption that the axisymmetric structure of elliptical PNs is due to the interaction of their progenitors with binary companions, being stellar or substellar. This leads us to conclude that substellar objects (brown dwarfs or gas-giant planets) are commonly present within several AU around main-sequence stars. For a substellar object to have a high probability of being present within this orbital radius, on average several substellar objects must be present around most main-sequence stars of masses $\lesssim 5 M_{\odot}$. Our arguments suggest that the presence of four gas-giant planets in the solar system is the generality rather than the exception.

Subject headings: binaries: close — planetary nebulae: general — planetary systems — stars: AGB and post-AGB — stars: horizontal-branch — stars: low-mass, brown dwarfs — stars: rotation

1. INTRODUCTION

It is widely assumed that terrestrial planets are common in our Galaxy. This is based on observations of dust around young stars (e.g., O'Dell, Wen, & Hu 1993; Strom, Strom, & Merrill 1993) and on theoretical calculations. How frequently gas-giant planets have formed seems to be less obvious. In a recent paper, Zuckerman, Forveille, & Kastner (1995) present interesting results and discuss their implications to this question. They observed CO molecules around 20 stars of ages 3×10^6 to $\sim 10^7$ yr and conclude that the total gas mass is much less than the mass of Jupiter. They argue that this implies that gas dissipates rapidly from around young stars. They claim, therefore, that if gas-giant planets are common, then they must form faster than current theoretical models predict, or else they are very rare.

In this Letter we argue that several properties of elliptical planetary nebulae (PNs) hint that gas-giant planets and/or brown dwarfs may commonly exist near (within several AU of) main-sequence stars. We will refer to gas-giant planets and brown dwarfs as *substellar objects*. In § 2, we start with a basic assumption that is based on binary system models for the formation of elliptical PNs and derive the maximal orbital separation for the model to be viable. This orbital separation is the maximal one allowed in order for a substellar secondary to spin up the PN progenitor. It is interesting to note that Peterson, Tarbell, & Carney (1983) mentioned the possibility that planets can spin up red giant stars. Their reasoning was their finding that many horizontal-branch stars rotate quite rapidly. Discussion and summary are in § 3.

2. ORBITAL RADII OF SUBSTELLAR COMPANIONS

We start with the following basic assumption: *The axisymmetric structure of elliptical PNs is due to the interaction of their progenitors with binary companions.* The binary companions can be stellar or substellar objects.

This assumption is the conclusion of several earlier papers (Soker 1996 and references therein). Let us briefly mention some arguments in support of substellar interaction with elliptical PN progenitors. The axisymmetric morphology of elliptical and bipolar PNs results from axisymmetric mass loss

of the asymptotic giant branch (AGB) progenitors (see, e.g., Trammell, Dinerstein, & Goodrich 1994). By “bipolar PNs” we refer to PNs that have two opposed lobes along their symmetry axis and a morphological waist in the equatorial plane (Schwarz, Corradi, & Stanghellini 1992). Elliptical PNs are those that have an elliptical shape and no waist in the equatorial plane. The bipolar PNs compose $\sim 11\%$ of all PNs (Schwarz & Corradi 1995). The similarity of bipolar PNs and symbiotic nebulae suggests that the bipolar morphologies result from AGB progenitors that have massive companions, $M_2 \sim 1 M_{\odot}$, outside their envelopes (Morris 1987; Corradi & Schwarz 1995). The bipolar progenitors have typical masses when on the main sequence of $M_1 \gtrsim 1.5 M_{\odot}$ (Corradi & Schwarz 1995), while the secondary can be a white dwarf or a main-sequence star. Elliptical PNs, on the other hand, tend to be formed from progenitors with main-sequence masses of $M_1 \lesssim 2 M_{\odot}$.

We now estimate the fraction of PN progenitors that are expected to be influenced by substellar companions. About 15 elliptical PNs were found to possess a close binary central star, having orbital periods between a few hours and several days, with the secondaries being low-mass main-sequence stars (Bond 1995; Bond & Livio 1990). The total fraction of PNs expected to contain close binary systems at their centers is $\sim 22\%$ of all PNs (Yungelson, Tutukov, & Livio 1993). At present, binary system models for axisymmetric mass loss on the AGB are the most promising (Livio 1995; but see Kastner & Weintraub 1995, who suggest a model based on surviving protostellar disks). If we accept binary system models for all elliptical PNs, as stated in our basic assumption above, then we must conclude that $\sim \frac{3}{4}$ of all elliptical PNs (or $\sim \frac{2}{3}$ of all axisymmetric PNs) owe their ellipticity to substellar companions. Let us study the implications of this.

In order to influence the mass loss from the progenitor AGB star, the substellar companion must deposit a substantial fraction of its orbital angular momentum in the stellar envelope or interact directly with the envelope, i.e., form a common envelope or accrete mass. The maximal initial orbital separation the companion can have, if it is to interact with the primary, is dictated by tidal forces. It is calculated by equating the tidal interaction timescale to the evolutionary timescale on

the red giant branch (RGB) and AGB. The tidal timescale is proportional to the circularization timescale, which is the timescale for reducing the eccentricity e of the orbit: $\tau_{\text{circ}} \equiv e/(de/dt)$. We discuss two tidal models and start by writing the circularization time according to each model, scaled with typical stellar parameters of the RGB and AGB.

The common tidal model in use is the equilibrium tide mechanism (Zahn 1977, 1989). In this model, the circularization time is given by (Verbunt & Phinney 1995)

$$\tau_{\text{circ}} = 7.3 \times 10^5 \frac{1}{f} \left(\frac{L}{2000 L_{\odot}} \right)^{-1/3} \left(\frac{R}{200 R_{\odot}} \right)^{2/3} \left(\frac{M_{\text{env}}}{0.5 M_1} \right)^{-1} \times \left(\frac{M_{\text{env}}}{0.5 M_{\odot}} \right)^{1/3} \left(\frac{M_2}{M_1} \right)^{-1} \left(1 + \frac{M_2}{M_1} \right)^{-1} \left(\frac{a}{5R} \right)^8 \text{ yr}, \quad (1)$$

where L , R , and M_1 are the luminosity, radius, and total mass of the primary, respectively, M_{env} is the primary's envelope mass, M_2 is the secondary's mass, a is the orbital separation, and $f \approx 1$ is a dimensionless parameter.

A somewhat controversial model for tidal interaction, the hydrodynamical model, was proposed by Tassoul (1987). Approximating the envelope density profile of stars on the upper AGB and RGB by $\rho \propto r^{-2}$, where r is the radial distance from the star's center, we find the primary moment of inertia to be $I = \frac{2}{5} M_{\text{env}} R^2$. Using this in equation (3) of Tassoul (1988), we find the circularization time in the hydrodynamical model to be (see also Soker 1994)

$$\tau_{\text{circ}} = 4.5 \times 10^5 \left(\frac{L}{2000 L_{\odot}} \right)^{-1/4} \left(\frac{R}{200 R_{\odot}} \right)^{9/8} \left(\frac{M_{\text{env}}}{0.5 M_1} \right)^{-1} \times \left(\frac{M_1}{M_{\odot}} \right)^{-1/8} \left(1 + \frac{M_2}{M_1} \right)^{-11/8} \left(\frac{a}{5R} \right)^{49/8} \text{ yr}. \quad (2)$$

The main differences between the two tidal mechanisms are the dependence on a/R and the factor M_2/M_1 . The circularization time of the hydrodynamical model is generally shorter, becoming much shorter for large orbital separations and for $M_2/M_1 \ll 1$.

Verbunt & Phinney (1995) compared theoretical predictions of the equilibrium tide mechanism with observations of binary systems having giant primary stars and found very good agreement. However, as the two equations above show, the hydrodynamical mechanism of Tassoul is likely to provide good agreement as well, especially if we consider the many uncertainties in both theories. These uncertainties include the value of the parameter N in Tassoul's model (we take $N = 10$ for the convective envelope), the exact stellar radii on the RGB, which depend on opacity and assumed mixing length, differential rotations in the envelopes and deviations from synchronous rotation, and the exact mass-loss rates on the RGB and AGB (the mass loss causes the orbital separation to increase).

Because of these uncertainties, and the large differences between the two tidal mechanisms for a substellar companion, i.e., $M_2/M_1 \ll 1$, there is no point in exactly integrating along the stellar evolution, as was done by Verbunt & Phinney. We will instead estimate the orbital separation below which the substellar companion spins up the primary. We deal with substellar secondaries, and so we neglect M_2 with respect to M_1 . The light secondary will never synchronize the primary, and so we can assume that the angular velocity of the primary is much lower than the orbital angular velocity. Since the spiraling-in rate becomes very fast as the orbital separation

decreases even by a small fraction, we may assume that all quantities except the orbital separation a do not change much during the spiraling in of the secondary (Soker 1994). Under these conditions, the secondary loses all its orbital angular momentum and enters the primary's envelope in a time equal to $\tau_{\text{in}} = \tau_{\text{circ}}/2n$, where $n = 8$ in Zahn's equilibrium tide model and $n = 49/8$ in Tassoul's hydrodynamical model (Soker 1994).

The secondary will go through this process only if the primary spends enough time on the RGB and/or AGB. To estimate the relevant evolution time on the RGB, we use the following approximate relations for $0.8 M_{\odot} \leq M \leq 2.2 M_{\odot}$, from Iben & Tutukov (1984; based on Mengel et al. 1979 for Population I composition): $R \approx 10^{3.5} M_c^4$, $L \approx 10^{5.6} M_c^{6.5}$, and $\dot{M}_c \approx 10^{-5.36} M_c^{6.6}$, where R , L , M_c are respectively the radius, luminosity, and core mass in solar units and \dot{M}_c is the rate of core mass increase in solar masses per year. Verbunt & Phinney (1995) used similar relations and analytically derived the amount of tidal circularization while the primary evolves on the RGB. We derive expressions suitable for our purpose in a somewhat different way, though the basic idea is the same. From the above relations, we find that near the RGB tip

$$\frac{R}{\dot{R}} \approx 3.93 \times 10^6 \left(\frac{M_c}{0.47} \right)^{-5.6} \text{ yr}, \quad (3)$$

where we have scaled with a typical core mass of solar-like stars on the RGB tip. Neglecting mass loss, the change in $\ln a$ is given by

$$\int \frac{\dot{a}}{a} dt \propto \int L^q R^m dt \propto \int R^{1.6q+m-2.4} dR, \quad (4)$$

where in the second step we have substituted the dependence of the luminosity on the radius and used $\dot{R} \propto R^{2.4}$, both derived from the RGB relations. In the equilibrium tide mechanism (hydrodynamical mechanism), $m = 22/3$ (5) and $q = \frac{1}{3}$ ($\frac{1}{4}$). Integrating from small radius to the radius at the RGB tip, $R_{\text{RGB-t}}$, we find

$$\int \frac{\dot{a}}{a} dt = \frac{1}{1.6q + m - 1.4} \left(\frac{R}{\dot{R}} \right)_{\text{RGB-t}} \left(\frac{\dot{a}}{a} \right)_{\text{RGB-t}} \equiv \tau_{\text{ev}} \left(\frac{\dot{a}}{a} \right)_{\text{RGB-t}}, \quad (5)$$

where the second equality defines the effective evolutionary time τ_{ev} and the constant in the denominator equals 6.47 and 4 in the equilibrium and hydrodynamical models, respectively.

We find from equation (3), therefore, that the effective tidal interaction time on the RGB tip is $\tau_{\text{ev}}(\text{RGB}) \sim 6 \times 10^5$ yr in the equilibrium model and $\tau_{\text{ev}}(\text{RGB}) \sim 10^6$ yr in the hydrodynamical model. Using equation (1) or equation (2) together with equations (3) and (5) in the condition for significant tidal interaction, $\tau_{\text{ev}} > \tau_{\text{in}} = \tau_{\text{circ}}/2n$, gives the maximal orbital separation allowed for tidal interaction to be of any significance. This maximal tidal-interaction orbital separation in Zahn's equilibrium tide model, with $f = 1$ and for $M_2 \ll M_1$, is given by

$$a_{\text{max}} = 3.9R \left(\frac{\tau_{\text{ev}}}{6 \times 10^5 \text{ yr}} \right)^{1/8} \left(\frac{L}{2000 L_{\odot}} \right)^{1/24} \left(\frac{R}{200 R_{\odot}} \right)^{-1/12} \times \left(\frac{M_{\text{env}}}{0.5 M_1} \right)^{1/8} \left(\frac{M_{\text{env}}}{0.5 M_{\odot}} \right)^{-1/24} \left(\frac{M_2}{0.01 M_1} \right)^{1/8}, \quad (6)$$

where stellar quantities should be taken at the RGB tip for RGB stars and close to the AGB tip for AGB stars. Similarly,

in Tassoul's hydrodynamical model, the maximal tidal-interaction orbital separation for $M_2 \ll M_1$ is given by

$$a_{\max} = 8.6R \left(\frac{\tau_{\text{ev}}}{10^6 \text{ yr}} \right)^{8/49} \left(\frac{L}{2000 L_{\odot}} \right)^{2/49} \times \left(\frac{R}{200 R_{\odot}} \right)^{-9/49} \left(\frac{M_{\text{env}}}{0.5 M_1} \right)^{8/49} \left(\frac{M_1}{M_{\odot}} \right)^{1/49}, \quad (7)$$

where stellar quantities should be taken at the RGB or AGB tip.

The exact maximal radius a star attains on the RGB tip depends strongly on the mass of the star, and to a lesser degree on its composition (see, e.g., Boothroyd & Sackmann 1988; note that they used too short a mixing length, and consequently their radii are overestimated by a factor of ~ 2 on the RGB and AGB). For stars of initial mass $M_1 \approx 1.2 M_{\odot}$, the tidal interaction will take place mainly on the RGB (Verbunt & Phinney 1995). For low-mass stars, we can take a typical RGB tip radius of $R_{\text{AGB-t}} \approx 160 R_{\odot}$. We also consider mass loss, which amounts to $\Delta M_1 \approx -0.15 M_{\odot}$ on the RGB, which will cause the orbital separation to increase by the same ratio as $|\Delta M_1|/M_1$. This reduces a_{\max} by $\sim 10\%$ for low-mass stars. We find from equation (6) that the maximal orbital separation allowed in the equilibrium tide mechanism is $a_{\max} \approx 2, 2.7,$ and 3.3 AU for planets of $M_2 = 0.001 M_{\odot} \approx M_{\text{Jupiter}}$ and brown dwarfs of $0.01 M_{\odot}$ and $0.05 M_{\odot}$, respectively. In Tassoul's hydrodynamical models we find from equation (7) that $a_{\max} \approx 6$ AU for all substellar secondary masses.

We find that according to the hydrodynamical tide mechanism, Jupiter will substantially spin up the Sun as the latter evolves along the RGB, while according to Zahn's equilibrium tide mechanism Jupiter will not influence the Sun at all during the Sun's entire evolution. This was noted in Soker (1994), where it was claimed that most of the interaction will take place while the Sun is on the AGB. More appropriate physical values for solar-like stars used here indicate that most of the tidal interaction will already be occurring on the RGB. Thus, according to the hydrodynamical tidal model, Jupiter will deposit all its angular momentum in the Sun on the RGB.

Stars with initial masses $\gtrsim 1.2 M_{\odot}$ will swell to radii of $R > 160 R_{\odot}$ on the AGB. Their maximal radius on the AGB tip depends mainly on their core mass. This in turn depends on their initial mass and mass-loss rate. The time spent by the star near the AGB tip strongly depends on the mass-loss rate as well. A typical time is $(\tau_{\text{ev}})_{\text{AGB}} \approx 3 \times 10^5$ yr (Boothroyd & Sackmann 1988). Stars of initial mass $3 M_{\odot}$ will expand up to $R \gtrsim 300 R_{\odot}$ (Boothroyd & Sackmann 1988). Thus, the maximal orbital separation allowed can be as high as $\gtrsim 4$ AU in the equilibrium tide mechanism and $\gtrsim 8$ AU in the hydrodynamical mechanism.

If the secondary deposits all its orbital angular momentum in the envelope of mass M_{env} , then the ratio of envelope angular velocity ω to the surface Keplerian angular velocity ω_{K} (critical velocity) is given by

$$\frac{\omega}{\omega_{\text{K}}} \approx 0.1 \left(\frac{M_2}{0.01 M_{\text{env}}} \right) \left(\frac{a}{5R} \right)^{1/2}, \quad (8)$$

where we have approximated the envelope density profiles of AGB stars by $\rho \propto r^{-2}$, so the primary moment of inertia is $I = \frac{2}{9} M_{\text{env}} R^2$, and assumed that the entire envelope rotates with the same angular velocity. At the upper AGB the envelope mass is much smaller than the initial stellar mass

because of the increase in core mass and stellar mass loss. If Jupiter, for example, deposits its orbital angular momentum in the Sun on the RGB, when $M_{\text{env}} \approx 0.5 M_{\odot}$, then the Sun will rotate at $\sim 1\%$ critical while becoming an AGB star. This is likely to be enough to cause a noticeable deviation from spherically symmetric mass loss and hence lead to an elliptical PN (Soker 1994). This process, as noted earlier, will happen in the solar system only if the Tassoul hydrodynamical tide mechanism works. Brown dwarfs and low-mass main-sequence stars may spin up the AGB envelopes to $\sim 20\%$ critical. Rotational velocity of more than $\sim 15\%$ critical will lead to much higher mass-loss rate in the equatorial plane of AGB stars (Ignace, Cassinelli, & Bjorkman 1996). This wind-compressed zone, as termed by Ignace et al., will lead to a high equatorial density region in the descendant PN. This is compatible with morphologies of several PNs having close binary nuclei (Bond & Livio 1990), where the secondaries are of $M_2 \approx \text{several} \times 0.1 M_{\odot}$.

3. DISCUSSION AND SUMMARY

Most PNs are elliptical, and only $\sim \frac{1}{4}$ of them are expected to possess close stellar companions (Yungelson et al. 1993). Since single stars rotate too slowly to explain the large fraction of elliptical PNs, spin-up by substellar companions has been suggested in systems that have no close stellar companions (see Soker 1996 and references therein). This conclusion leads to the following basic assumption, which was the starting point of this Letter: The axisymmetric structure of elliptical PNs is due to the interaction of their progenitors with binary companions, being stellar or substellar. We then derived the maximal orbital separation for substellar secondaries, as dictated by tidal forces, for them to spin up the PN progenitors. The main finding of the previous section is that for substellar secondaries to spin up progenitors of PNs, the initial orbital separation must be less than $\sim 2\text{--}4$ AU, depending on the secondary's mass, according to the equilibrium tide mechanism, and it may be as large as $\sim 5\text{--}10$ AU in the hydrodynamical tide mechanism of Tassoul.

The likelihood of a specific substellar object, i.e., a massive planet or a brown dwarf, orbiting a main-sequence star within ~ 5 AU is small. If substellar objects are commonly present within this distance, as we claim in this Letter, then on average several substellar objects must be present around most main-sequence stars of $M \approx 5 M_{\odot}$. In such a case, the likelihood that one of them will be within ~ 5 AU is much higher than it is if only one substellar object exists around each star. It is very likely according to the proposed scenario that more brown dwarfs or planets exist around the star Gl 229, but at smaller orbital radii than the brown dwarf candidate Gl 229B, which was discovered by Nakajima et al. (1995). It is at a projected orbital separation of 44 AU from the primary, Gl 229. It emerges from our arguments that the presence of four gas-giant planets in the solar system is the generality, not the exception. We expect, though, that in most cases there will be a more massive and a closer planet than Jupiter. It is interesting to note that very recently Mayor & Queloz (1995) reported the discovery of a planet around 51 Peg, with an orbital separation of 0.05 AU.

Some researchers claim that Jupiter was necessary for the formation of intelligent life on Earth since it deflects small bodies in the solar system from hitting Earth. Therefore, they argue, we are lucky to have such a *massive* and *close* planet in

our solar system. Our approach is the opposite (Soker 1994). We claim that we are lucky that the most massive planet in our solar system is *light* and at a *large* distance, since in many other systems the companions are likely to be closer to the primary star and more massive than Jupiter. Had Jupiter been more massive or much closer, say at ~ 3 AU from the Sun, Earth could not have had its nice, stable orbit around the Sun. At best, there would have been an asteroid belt at the location of Earth's orbit.

According to the scenario proposed here, a large fraction of horizontal-branch (HB) descendants of low-mass stars should rotate fast. Take the solar system, for example: The angular momentum of the Sun is $J \approx 2 \times 10^{48}$ g cm² s⁻¹, with its equatorial rotational velocity being $u_{\text{MS}} \approx 2$ km s⁻¹. The orbital angular momentum of Jupiter is ~ 100 times that of the Sun. Thus, if Jupiter deposits all its angular momentum in the Sun on the RGB and $\sim 10\%$ of this input angular momentum survives to the HB phase, then as the Sun shrinks to the HB the rotational velocity will be $u_{\text{HB}} \sim 50$ km s⁻¹. With no angular momentum deposition, the HB rotational velocity would be less than 1 km s⁻¹. Rotational velocities of $u_{\text{HB}} \sin i \sim 10$ –30 km s⁻¹ have been found for HB stars in globular clusters (Peterson 1985). Pinsonneault, Deliyannis, &

Demarque (1991) translated the rotational velocities to angular momenta on the HB and claim that much higher rotational velocity in the interior of main-sequence stars can explain the fast-rotating HB stars. It seems to us that the model of Pinsonneault et al. (1991) cannot explain the high HB rotational velocities, in light of the new results of Tomczyk, Schou, & Thompson (1995). Tomczyk et al. found that the Sun rotates as a solid body down to $r \approx 0.2 R_{\odot}$, and therefore Sun-like stars are unlikely to retain a large fraction of their angular momentum in their interiors. Based on the fast rotation velocities of many HB stars, Peterson et al. (1983), following a suggestion of L. Mertz, have discussed the possibility that a planet can spin up these stars. They cite the amount of required angular momentum to be about equal to the orbital angular momentum of Jupiter.

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