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A DYNAMO INTERPRETATION OF STELLAR ACTIVITY CYCLES

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ABSTRACT

Twenty-five-year records of Ca II H and K chromospheric emission fluxes measured in single lower main-sequence stars at Mount Wilson Observatory reveal surface magnetic activity cycles in one-third of the sample of roughly 100 stars. For those stars with cycles, we compare the ratio of the observed periods of the cycle of magnetic activity and axial rotation, P_{cyc}/P_{rot}, to predictions available from stellar dynamo theory. Theoretical considerations suggest that the ratio is the observational equivalent of the stellar dynamo number, D.

The stellar sample is comprised of two groups separated by their mean level of activity, $\langle R'_{HK}\rangle$, and rotation, P_{rot}: one group has high levels of average activity and fast rotation, while the other group has relatively low levels of activity and slower rotation. Both groups also occupy different regions on the diagram of X-ray flux versus stellar dynamo number. For the older group of stars (ages $\gtrsim 2$ Gyr) which includes the current Sun, we find a statistically significant inverse relation between the intensity of the cycle, $c = \Delta R'_{HK}/\langle R'_{HK}\rangle$, and the stellar dynamo number, empirically determined to be $D \sim (P_{cyc}/P_{rot})^{1.35^{+0.65}_{-0.35}}$.

Subject headings: MHD — stars: activity — stars: magnetic fields — stars: rotation

1. INTRODUCTION

Extended records of observations of chromospheric Ca II H and K emission fluxes in lower main-sequence stars reveal variations similar to the 11 yr solar cycle of surface magnetic activity (Wilson 1978; Baliunas et al. 1995). Such records permit the Sun's activity cycle to be studied in the broader context of other stars with different properties which are thought to influence the period and amplitude of the cycle. Patterns in the observed quantities that characterize stellar cycles reinforce the notion of a general theory underlying and unifying solar and stellar activity variations. We examine regularities apparent in the records of chromospheric activity, namely, the period of stellar cycle, P_{cyc}, and the period of stellar rotation, $P_{rot} \sim \Omega^{-1}$, where Ω is the rotation velocity, in the framework of one theory of magnetic activity cycles, i.e., the dynamo theory (e.g., see the recent review by Rosner & Weiss 1992).

The results from dynamo theory can be compared to observable quantities in the stellar activity records in several different ways. Early empirical attempts (e.g., see reviews by Baliunas & Vaughan 1985; Belvedere 1985) looked for the dependence of P_{cyc} on mass (estimated from $B-V$ photometric index) and age (estimated from P_{rot} or the time-averaged level of activity, $\langle R'_{HK}\rangle$). No clear trends were discerned, perhaps because the interpretation of hydrodynamical results in terms of stellar properties such as mass and age is complex.

Several theoretical interpretations have also been proposed which involve different parameterizations. First, P_{rot} can be normalized by the convective overturn time, τ_c (Noyes et al. 1984a). Second, P_{cyc} can be normalized by the characteristic turbulent magnetic diffusion timescale, τ_D (Tuominen, Rüdiger, & Brandenburg 1988). In the mixing-length theory of convection, the two parameters τ_D and τ_c are roughly equivalent, differing only by factors which depend on the stellar radius and convection zone depth (e.g., Noyes, Weiss, & Vaughan 1984b; Saar & Baliunas 1992; Kim & Demarque 1996).

Finally, in a recent study, Soon, Baliunas, & Zhang (1993) commented on P_{cyc}/P_{rot} as a third parameterization of measurable quantities in the stellar records of Ca II chromospheric activity. They proposed to interpret this ratio in terms of a dynamo number, D, the most important parameter controlling magnetic field generation in the mean-field stellar dynamo models. Soon et al. (1993) suggested the following observational equivalent of the dynamo number:

$$\frac{P_{cyc}}{P_{rot}} \sim D^{1/2} . \quad (1)$$

In fact, stellar dynamo theory yields the following general result:

$$\frac{P_{cyc}}{P_{rot}} \sim D^{\iota} , \quad (2)$$

where ι is a positive constant of order of unity, and its exact value depends on the regime of dynamo generation. On a logarithmic scale (i.e., a power-law relation), the difference between equations (1) and (2) is negligible. We will synthesize below results from dynamo theory in order to support relation (2).

We emphasize that the proposed parameterization, namely, P_{cyc}/P_{rot}, is *observationally* based. That observable quantity, P_{cyc}/P_{rot}, directly connects the observational results to stellar dynamo theory without relying on the uncertainty of the estimates of the convective overturn time or diffusion time associated with the mixing length theory of convection.

2. DETERMINATION OF OBSERVABLE QUANTITIES

Both P_{rot} and P_{cyc} come from periodogram analysis of Ca II chromospheric activity records of lower main-sequence stars obtained at the Mount Wilson Observatory (MWO) (Horne & Baliunas 1986; Baliunas et al. 1995). The

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Ca II chromospheric emission fluxes in about 100 stars are monitored at the 60 inch (1.5 m) telescope equipped with a spectrophotometer. The relative Ca II emission, S, is the ratio of fluxes in two narrow 0.1 nm passbands centered at H (396.8 nm) and K (393.3 nm) to two 2 nm passbands in the nearby photospheric continuum centered at 390.1 nm and 400.1 nm, respectively (Vaughan, Preston, & Wilson 1978). Since stars in our sample range from spectral type F to K, we have converted the S index to R'_{HK} (which is the ratio of the net chromospheric emission flux corrected for the contribution from photospheric Ca II emission to the bolometric luminosity) in order to compare the activity levels in stars of differing $B-V$ colors (i.e., masses) (Middlekoop 1982; Noyes et al. 1984a). The net chromospheric emission is identified as nonthermal heating and, in the case of the spatially resolved Sun, it is observed to be related to the photospheric magnetic flux, on roughly a one-to-one basis (e.g., Skumanich, Smythe, & Frazier 1975; Schrijver et al. 1989). That net chromospheric emission has a partial contribution by the dissipation of acoustic waves generated by the turbulence in the convective zone. In addition, detailed energy-balance models in magnetically active regions (i.e., plages), suggest chromospheric heating by the dissipation of slow mode magnetoacoustic waves in the low chromosphere and by Alfvén waves in the higher chromosphere (e.g., Ulmschneider & Stein 1982). Therefore, both empirical and theoretical studies justify the use of disk-integrated stellar Ca II measurements to obtain information on surface magnetic inhomogeneities.

In the following, we discuss only those 34 stars displaying cyclic chromospheric activity, about one-third of Wilson's sample. (Discussion of other [e.g., those with no apparent cycles] stars will be deferred.) The sunspot record since the 18th century reveals a range in P_{cyc} from 9 to 13 yr. Based on that known variation of the length of the sunspot cycle, it is difficult to assign formal uncertainties to the detection of P_{cyc}. The values of P_{cyc} are determined from the stellar records covering one to three activity cycles (about 25 yr), and have a spread of roughly 1 yr (Baliunas et al. 1995). Five stars in the sample have two significant and independent periods. The physical origin of multiple periodicities in the records of solar and stellar surface activity is not well understood (e.g., Hoyng 1990). The mean stellar rotation periods, P_{rot}, used in this analysis also have a range of $\pm 25\%$ from the mean.

In addition to P_{cyc} and P_{rot}, two other quantities can be derived from the 25 yr MWO stellar Ca II chromospheric emission records: (i) the time-averaged level of activity, $m = \langle R'_{HK} \rangle$ and (ii) the amplitude of the activity cycle, $\delta = \Delta R'_{HK}$. Identifying the two observational quantities m and δ (or the ratio $c = \Delta R'_{HK}/\langle R'_{HK} \rangle$) with the magnetic (i.e., plage) area filling factor, f, and the large-scale magnetic field strength, $\boldsymbol{B}$, would require a detailed two-dimensional stellar surface imaging model (e.g., Vogt, Penrod, & Hatzes 1987; Piskunov, Tuominen, & Vilhu 1990).

In the framework of this paper, one needs to justify that

$$\boldsymbol{B} \sim f\boldsymbol{b}' , \tag{3}$$

where $\boldsymbol{b}'$ is a fluctuating component of magnetic field. Such a statement is implicit in dynamo theory (see below).

3. DYNAMO NUMBER FOR THE STELLAR DYNAMO

According to the concept of the $\alpha\Omega$-dynamo, the large-scale stellar magnetic field is generated by the simultaneous action of two mechanisms. The first one—dimensional rotation—produces a toroidal magnetic field from an initial poloidal field; however, this mechanism alone cannot produce self-excitation of the magnetic field. A poloidal field supported only by differential rotation will decay due to dissipative forces. An additional mechanism—mean helicity—is necessary to drive self-excitation. Helicity creates a new poloidal field from the toroidal one generated by the differential rotation. Since the chain of self-excitation is connected by the action of two mechanisms, the intensity of the corresponding dynamo depends on the product of mean helicity and differential rotation shear. In a dimensionless form, this product is known as the dynamo number, D.

For an $\alpha\Omega$-dynamo in a spherical convective shell, with a thickness h and the outer radius R, the dynamo number is

$$D = \frac{\alpha^* R^3}{\beta} \frac{1}{r} \frac{\partial \Omega^*}{\partial r} , \tag{4}$$

where * denotes the amplitudes of the corresponding parameters, length is measured in units of h and time in units of diffusion time R^2/β, where β is the coefficient of turbulent diffusivity.

Suppose that we are investigating a sample of stars in which the only varying parameter is the amplitude of angular velocity, Ω^*. In that case,

$$D \sim \alpha^* \Omega^* . \tag{5}$$

(In the following, the superscript * will be omitted for convenience). Let us discuss now how D depends on α and Ω. The quantity α (in units of velocity) describes the helicity of the flow, which is a component of the turbulent velocity and a measure of the mirror-symmetry violation. In a simple model of turbulent motion described as a collection of random curls, helicity is proportional to the mean number of the right-hand curls minus a mean value of the left-hand curls. This is the origin of a well-known upper limit of the mean helicity. In a stratified medium, the mean helicity is created by the action of the Coriolis force and it is proportional to Ω. Then, provided that Ω is not too large, helicity is

$$\alpha \sim \Omega . \tag{6}$$

If the rate of angular rotation is very high, a constraint is needed for the upper limit of helicity, i.e.,

$$\alpha = \text{const} . \tag{7}$$

Relation (6) holds when the rotation period of a star is longer than a turnover time of convective cell while (7) is valid in the opposite situation. The latter possibility usually applies to stars and the former to the galactic dynamo. Thus,

$$D \sim \Omega \tag{8}$$

seems to be a reasonable estimate for solar and stellar dynamos.

However, there is yet no definitive, direct measurement of helicity in astrophysical or laboratory settings (see, however, Pevtsov, Canfield, & Metcalf 1994). Thus, we cannot rule out estimate (6), at least for the activity cycles of slowly rotating stars. In that case,

$$D \sim \Omega^2 . \tag{9}$$

4. PERIOD OF STELLAR ACTIVITY

Calculation of the period (or length) of the stellar activity cycle is a complex problem. Consider the case of the Parker (1955) kinematic migratory dynamo; the kinematic regime is valid as long as the magnetic field is not sufficiently strong to affect the velocity field. For a kinematic dynamo model, the magnetic field is proportional to

$$\boldsymbol{B} \sim e^{\gamma t} , \tag{10}$$

where γ is a complex growth rate, $\gamma = \Gamma + i\omega$. The value Γ corresponds to the mean growth rate of, say, magnetic energy, and $\omega^{-1} \sim P_{\rm cyc}$.

The Parker migratory dynamo considers the specific case of the $\alpha\Omega$-dynamo model (Steenbeck, Krause, & Rädler 1966): the convective shell must be relatively thin with respect to the stellar radius, yet sufficiently large so that the diffusion of magnetic field across the shell is negligible. Equations of the Parker migratory dynamo can be formulated as follows (with $[t, \theta]$ denote the coordinates of time and colatitude, respectively)

$$\frac{\partial A_t}{\partial t} = \alpha B_t + \frac{\partial^2 A_t}{\partial \theta^2} ,$$

$$\frac{\partial B_t}{\partial t} = D \frac{\partial A_t}{\partial \theta} + \frac{\partial^2 B_t}{\partial \theta^2} , \tag{11}$$

where A_t and B_t are toroidal components of magnetic potential and magnetic field, respectively. Those simplified equations describe only the latitudinal distribution of large-scale stellar magnetic field. In this case, the magnetic field is smoothly distributed in the radial direction with a spatial scaling of the order $k_r^{-1} \sim h$, where h is the thickness of the convective shell.

Solution of equations (11) can be obtained in the form of a harmonic wave

$$A_t = |D|^{-2/3} a(\xi) B_0 ,$$

$$B_t = b(\xi) B_0 , \tag{12}$$

where $a = a_1 \exp(i\xi)$, $b = b_1 \exp(i\xi)$ are dimensionless functions, B_0 is a unit of magnetic field, which is arbitrary in the kinematic approach, and

$$\xi = (\omega - i\gamma)t + k\theta . \tag{13}$$

Using appropriate scalings,

$$\gamma = |D|^{2/3}\Gamma , \quad \omega = |D|^{2/3}\tilde{\omega} , \quad k = |D|^{1/3}K , \tag{14}$$

we can remove D from equations (11). More precisely, the scalings (14) are adopted to scale all terms in equations (11) to the same order in D. Canceling this common multiplier, we obtain the following equations in the case of $D < 0$:

$$(\Gamma + i\tilde{\omega})a_1 = b_1 - K^2 a_1 , \tag{15}$$

$$(\Gamma + i\tilde{\omega})b_1 = -iKa_1 - K^2 b_1 . \tag{16}$$

These equations do not contain D. Solving equations (15) and (16) using the scalings (14), we find the proper dependence between dynamo number and dynamo wave properties.

The scaling for k in equation (14) means that the spatial scale of the kinematic dynamo wave is of order $|D|^{-1/3}$ and hence

$$P_{\rm cyc} \sim D^{-2/3} . \tag{17}$$

A similar scaling can also be obtained for a nonlinear version of equation (11) adopting $\alpha = \alpha(B)$. Recent numerical results of Jennings & Weiss (1991) in the nonlinear dynamo generation regime give $P_{\rm cyc} \sim D^{-0.61}$, which is consistent with estimate (17). However, it is unclear whether the agreement is fortuitous or a physical connection can be established from the agreement.

We combine equations (8) and (17) to obtain

$$\frac{P_{\rm cyc}}{P_{\rm rot}} \sim D^{1/3} . \tag{18}$$

Let us now consider the degree to which the estimate given in equation (18) depends upon the model.

Estimate (17) is based on an Ansatz that the radial magnetic field scale is determined by convective shell thickness and the latitudinal magnetic field scale could be extracted from the dynamo equations, for example, by adopting the scale that maximizes the growth rate of the magnetic field. That scaling was suggested by Parker (1971a) in a slightly different context and has been discussed by Kleeorin, Ruzmaikin, & Sokoloff (1983) for the stellar dynamo.

The original relationship (1) between the dynamo number and $P_{\rm cyc}/P_{\rm rot}$ suggested by Soon et al. (1993) can be obtained with a slightly different thought experiment. Assume the radial magnetic field size, k_r^{-1}, maximizes the magnetic field growth rate while the latitudinal field size is given as a fraction of the distance between the pole and equator. Those assumptions yield $k_r \sim D^{-1/2}$ and $P_{\rm cyc} \sim D^{-1/2}$, so that

$$\frac{P_{\rm cyc}}{P_{\rm rot}} \sim D^{1/2} . \tag{19}$$

This scaling has been suggested by Noyes et al. (1984b). According to computer simulations developed by Moss, Tuominen, & Brandenburg (1990), equation (19) corresponds to a dynamo in a turbulent sphere.

Rüdiger & Brandenburg (1995; see also Brandenburg et al. 1994) suggested $P_{\rm cyc} \sim D^{-1/6}$, hence

$$\frac{P_{\rm cyc}}{P_{\rm rot}} \sim D^{5/6} \tag{20}$$

for a dynamo model with anisotropic helicity. Dynamo models by Rüdiger et al. (1994), who considered both the α-quenching and magnetic diffusivity quenching, give predictions of $P_{\rm cyc} \sim D^k$, where k varies from values of about -0.4 to about -0.1, with the possibility of k being positive, depending on the magnetic quenching mechanism and the geometry of model.

Those dynamo models assume some simple parameterization of the back-reaction of magnetic field on the dynamo, e.g., α-quenching or quenching of the differential rotation. Jennings (1993) demonstrated that if the back reaction is modeled in the form of a Navier-Stokes equation for the mean velocity field, then $P_{\rm cyc} \sim D^0$ or $P_{\rm cyc}$ even increases with D. The latter prediction agrees qualitatively with the one-dimensional dynamo results of Rüdiger et al. (1994) who considered only α-quenching.

In summary,

$$a = \frac{P_{\rm cyc}}{P_{\rm rot}} \sim D^{\iota} , \qquad (21)$$

where ι is a positive constant $\gtrsim \frac{1}{3}$.

The observational results can test the theoretical predictions discussed above. Figure 1 shows a positive trend between log $(P_{\rm cyc}/P_{\rm rot})$ and $d \sim \log(1/P_{\rm rot})$ for all stars showing cycles. Slopes between $0.5 \lesssim \iota \lesssim 1$ are allowed by the data (the 95% confidence interval), with the most probable value of $\iota \sim 0.74$. The probability, based on 39 measurements of $P_{\rm cyc}$ (34 stars, five with two periods), that there is no correlation between that two quantities is 1.7×10^{-6} (from a correlation coefficient r of 0.68). Similar results are obtained if we exclude the five "double-period" stars in Figure 1.

None of the theoretical estimates of ι is contradicted by the observational results in Figure 1. The value $\iota = \frac{5}{6}$ (Rüdiger & Brandenburg 1995; Rüdiger et al. 1994) is closest to the mean slope of the observations. On the other hand, if the estimate (6) for helicity should prevail for the older stars, then the observations seem to favor the results (18) from the kinematic (linear) model or the nonlinear dynamo results of Jennings & Weiss (1991; see also Tuominen et al. 1988).

Even though the comparison between theoretical predictions and observational results based on the ratio $P_{\rm cyc}/P_{\rm rot}$ is encouraging, some caution is warranted. First, even though the comparison uses robust measurables of the stellar record, the a priori assumption that surface activity is cyclic with periods determined accurately from the power spectra analysis may be biased. Indeed, nonperiodic variations are also observed in the stellar records (Baliunas et al. 1995). Second, Jennings & Weiss (1991) caution that although correlations among those observed quantities may be statistically significant in some cases, a diagnostic for the quenching mechanism by comparing, e.g., observed $P_{\rm cyc}$ versus Ω or $P_{\rm cyc}$ versus Rossby number ($Ro = P_{\rm rot}/\tau_c$) relations with one-dimensional dynamo models is not yet available (see also discussion in Noyes et al. 1984b; Soon et al. 1993). Third, the idealization of the theoretical results is complex.

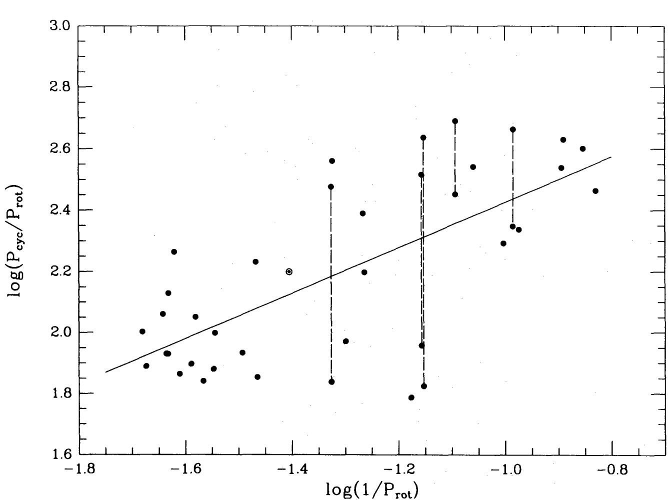

FIG. 1.—The quantity log $(P_{\rm cyc}/P_{\rm rot})$ vs. log $(1/P_{\rm rot})$ [$\sim$log (D)] for the lower main-sequence stars with cyclic activity based on 25 yr records of Ca II fluxes. The dotted circle denotes the Sun. The solid line is the least-squares fit using all points (including both periods of the five "double-period" stars). The dashed vertical lines connect the two *independent* periods detected in each "double-period" star.

5. INTENSITY OF THE STELLAR CYCLE

According to the dynamo theory, correlation between dynamo number and intensity of stellar magnetic field should not be as pronounced as correlation between stellar period and dynamo number. While $P_{\rm cyc}$ is expected to scale $\sim D^k$, the equilibrium value of the stellar magnetic field should be determined by the value in equipartition with, say the kinetic energy of turbulence, which is not directly related to D. However, the spatial structure of magnetic field depends on D. As a result, a correlation between intensity of stellar magnetic field and dynamo number may exist under certain conditions, for example, for highly ordered surface magnetic fields (i.e., smooth activity cycles—with highly significant periods; Baliunas et al. 1995). In order to examine the intensity of the stellar magnetic field, we shall consider independently the two groups of the sample that are known to be separated by age (Vaughan & Preston 1980).

5.1. *Results for Old and Young Stars*

Vaughan & Preston (1980) divided a similar but much larger sample of stars within 25 pc of the Sun into two subsets, based on the average level of activity or rotation, or both, i.e., age. They also noted a dearth of stars with intermediate Ca II emission which is quite visible in the range $0.5 \lesssim B-V \lesssim 1.0$.

Another factor that separates the two stellar subgroups is the observed negative correlation between long-term (year-to-year; specifically from 1984 to present) chromospheric activity (measured by Ca II emission) and photospheric variability (in the Strömgren photometric b and y passbands, centered at 472 and 551 nm, respectively) for the young stars and the switch to positive correlation for the older stars (Radick, Lockwood, & Baliunas 1990).

The old and young stars are also distinct when another measure of magnetic heating is used, e.g., the X-ray flux, F_X, measured by the *ROSAT* satellite (Hempelmann, Schmitt, & Stępień 1995). Figure 2 shows the separation between old and young stars when log F_X is plotted against the empirical log (D). The X-ray flux might be considered a more direct indicator of magnetic heating than the Ca II emission, which must be corrected in nontrivial ways for nonmagnetic contributions to the flux.

Several competing explanations exist for the Vaughan-Preston gap. Hartmann et al. (1984) interpreted the break in chromospheric activity as a fluctuation in the local stellar birthrate on timescales of a few times 10^8 yr, so that the lack of stars in the gap area results from a slowly varying star formation rate. That explanation does not involve the dynamo. Another suggestion is that the activity of the young stellar group is not associated with a regenerative dynamo but instead is connected with a primordial magnetic field (see, e.g., Vainshtein & Rosner 1991).

Differences in the physical mechanisms (i.e., in terms of changes in the pattern of convection) of stellar dynamo operating at fast or slow rotation rates (hence ages) for a given spectral type have been suggested by Knobloch, Rosner, & Weiss (1981). Results from dynamo theory (Kleeorin et al. 1983) are able to suggest the existence of the

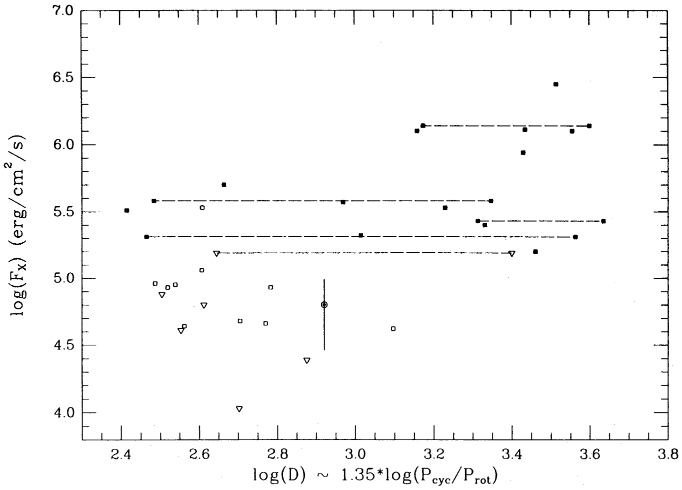

FIG. 2.—The *ROSAT* X-ray flux of lower main-sequence stars as a function of log $(D) \sim 1.35$ log (P_{cyc}/P_{rot}). The filled squares denote stars with a high average level of activity and fast rotation (which are presumably young) and the open squares denote stars with a low average level of activity and slow rotation (which are presumably old). The inverted triangles indicate the upper limit of nondetection of F_X in old stars. The vertical line through the Sun's symbol shows the range of the variability of F_X over an activity cycle (Schrijver 1983; Rutten & Schrijver 1987). The range of solar variability describes the physical uncertainty of F_X.

gap (see, e.g., Durney, Mihalas, & Robinson 1981; Soon et al. 1993), although the interpretation of the Vaughan-Preston gap as a break in the dynamo characteristic is far from settled.

The relationship between $c = \Delta R'_{HK}/\langle R'_{HK}\rangle$ and P_{cyc}/P_{rot} $(\sim D)$ for the two groups of stars is shown in Figure 3. A statistically significant inverse trend is apparent but only for the older stellar group. The slope of the relation determined by a linear least-squares fit for the old stars is -0.68 on the log-log scale, with a 95% confidence level of 0.20. The correlation coefficient, r, is -0.83 (based on the sample size of 17 and excluding the "double-period" old star HD 100180) which has a probability of 3.4×10^{-5} that no correlation exists between these two quantities.

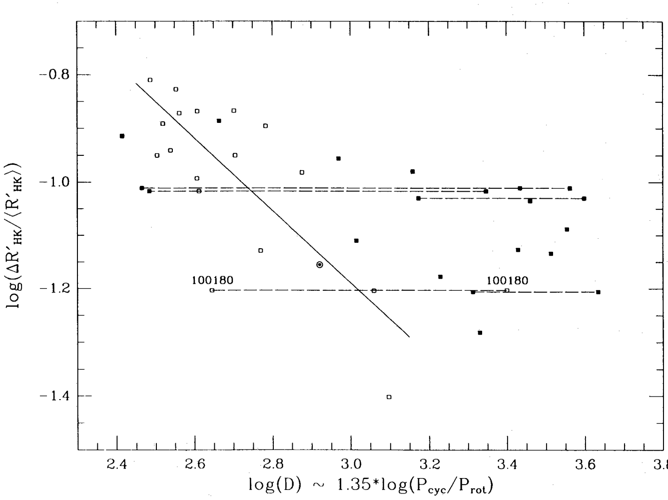

FIG. 3.—The quantity log $(\Delta R'_{HK}/\langle R'_{HK}\rangle)$ vs. log (D) for the lower main-sequence stars with cyclic activity. The symbols are the same as in Fig. 2. The solid line is the least-squares fit for only the older stars (*open squares;* excluding the "double-period" star HD 100180). The dashed horizontal lines connect the two *independent* periods detected in the five "double-period" stars.

5.2. *Idealizations from Dynamo Theory*

We examine next the relation between a and m, a and δ, and a and c. According to dynamo theory, the magnetic field, $\boldsymbol{H}$, generated by the dynamo mechanism can be decomposed as follows:

$$\boldsymbol{H} = \boldsymbol{B} + \boldsymbol{b}' + \boldsymbol{b} . \quad (22)$$

Here $\boldsymbol{B}$ is the mean (large-scale) magnetic field component, described by the $\alpha\Omega$-dynamo equations; $\boldsymbol{b}'$ is a small-scale component associated with the large-scale field. This component does not appear directly in the $\alpha\Omega$-dynamo equations; however, it is considered under the mean-field formalism.

The small-scale magnetic field component, $\boldsymbol{b}$, is generated by a dynamo mechanism independently of large-scale differential rotation and mean helicity. This type of small-scale magnetic field probably exists on the Sun (see e.g., Stenflo 1990). The intensity of the field. $\boldsymbol{b}$, does not depend on dynamo number D.

The large-scale component, $\boldsymbol{B}$, can be further decomposed as follows

$$\boldsymbol{B} = \boldsymbol{B}_t + \boldsymbol{B}_p , \quad (23)$$

where $\boldsymbol{B}_t$ and $\boldsymbol{B}_p$ are the toroidal and poloidal components, respectively. According to the standard dynamo concept, the toroidal magnetic field is in equipartition with the energy of convective motions (see, however, the recent criticism by Vainshtein & Cattaneo 1992). Thus

$$B_t \sim D^0 . \quad (24)$$

In the stellar convective shell, the magnetic Reynolds number, Re_m, is very large, which allows determination of the dominant term in equation (22):

$$\langle (b + b')^2 \rangle \sim Re_m \langle B \rangle^2 . \quad (25)$$

The result in relation (25) suggests that the small-scale magnetic field, not the large-scale, is in equipartition with the small-scale velocity field.

If that picture were correct, dynamo theory would not be so relevant to observational study of solar and stellar magnetic activity because $\boldsymbol{B}$ would be negligible (see, however, Vainshtein, Parker, & Rosner 1993). However, recent numerical simulations by Brandenburg (1994; see also Brandenburg et al. 1995) indicate that

$$\langle (b + b')^2 \rangle \sim F \langle B \rangle^2 , \quad (26)$$

where the dependence of F is much weaker than that predicted by equation (25), e.g., $F \sim \ln (Re_m)$. The result implies that the small-scale magnetic field, though less intense than the large-scale field, cannot prevent the generation of large-scale field. Nevertheless, the situation remains unclear. In interpreting stellar cycles in terms of the dynamo theory, here, we accept the optimistic viewpoint of the regenerative solar and stellar dynamos (Vainshtein et al. 1993).

Let us now estimate the strength of the poloidal magnetic field, B_p. This field is connected with a latitudinal derivative of the toroidal component from equations (12) and (14)

$$B_p \sim kA_p \sim D^{-1/3} . \quad (27)$$

Both toroidal and poloidal magnetic fields are generated deep inside the stellar convective shell. Toroidal field is transported to the stellar surface by some additional physical mechanism, i.e., that leading to the formation of surface magnetic features (e.g., Zwaan 1987). On the other hand, the poloidal magnetic field can penetrate through the stellar surface, even without the help of a local dynamo process, but is not connected with surface features. It is yet unclear which component, B_p or B_t, is mainly responsible for the net stellar Ca II chromospheric emission flux. The net flux is presumably formed by the small-scale fluctuating field $\boldsymbol{b}'$ which is proportional to $\boldsymbol{B}$. Strictly speaking, one needs to distinguish between the components b'_p and b'_t associated with B_p and B_t, respectively. We assume here that the contribution from B_p to the net flux is not negligible, which seems to be supported by observations (Livingston 1994). A differentiation of the contribution of B_t and B_p into a net flux being shifted in time could produce complicated time behavior resulting in a multiperiodic or erratic variability activity consistent with observations (Baliunas et al. 1995).

Thus, we expect

$$m \sim \sqrt{C_1 + C_2 + C_3 D^{-2/3}}\,, \qquad (28)$$

$$\delta \sim \sqrt{C'_1 + C'_3 D^{-2/3}}\,, \qquad (29)$$

and

$$c \sim \sqrt{\frac{C'_1 + C'_3 D^{-2/3}}{C_1 + C_2 + C_3 D^{-2/3}}}\,. \qquad (30)$$

Here C_1, C_2, and C_3 denote contributions of b, B_t, and B_p, respectively, and the prime allows for the possibility of different constants in m and δ. We have considered different magnetic field components and have developed arguments based on magnetic energies rather than magnetic field strengths.

Let us now consider the theoretical interpretation of Figure 3. The scatter of the trend in the young stars could be intrinsic to that group. As noted above, the spatial structure of the magnetic fields for the young stars is expected to be less orderly than in the older stars since the dynamo number is higher in young stars (see, e.g., Parker 1971b; Belvedere, Pidatella, & Proctor 1990; Jennings 1991). For dynamos operating in a highly critical regime, there is no reason to expect a priori the intensity of the magnetic activity to have a well-defined relation with D (i.e., terms C_1, C_2 may dominate over C_3 in relation [30] at such regime).

Estimates provided by relation (30) are for the simplest case of the Parker migratory dynamo with α-quenching. The dependence of c on D (the $-\frac{2}{3}$ index) derived in relation (30) results from the dependence between $P_{\rm cyc}$ and D in relation (17). A specific prediction for the contribution of each term in relation (30) is difficult. However, the relation for the older stars in Figure 3 may suggest that C'_3 (B_p field contribution) dominates the numerator (δ) and $C_1 + C_2$ ($b + B_t$ fields contributions) dominate the denominator (m).

6. DISCUSSION

$P_{\rm cyc}/P_{\rm rot}$ provides a useful comparison between the observations of cyclic stellar activity variations and results of dynamo theory. It remains to be seen if an observable like $P_{\rm cyc}$ may be generalized beyond strictly periodic variations to include, e.g., those low and quiescent state of variability resembling the solar activity during the Maunder minimum interval (see e.g., Sokoloff & Nesme-Ribes 1994; Soon, Baliunas, & Zhang 1994). If so, the observable ratio $P_{\rm cyc}/P_{\rm rot}$ would indeed be a robust measure of the underlying magnetic field generator and provide a closer connection to theoretical study of stellar dynamos.

The observed intensity of the stellar cycle may also be linked to the individual components of the large-scale and small-scale magnetic fields. A relation can be established for the older stars which suggests a decreasing intensity of surface flux as D increases. That inverse relation appears to be counterintuitive, which may indicate the presence of dynamical coupling between the subsurface magnetic fields and fluid motions. If the physical significance of the relation can be validated, a common nonlinear mechanism (yet to be identified) may indeed be realizable, at least for the older stars.

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