

THE FINAL EVOLUTION OF ONeMg ELECTRON-DEGENERATE CORES

JORDI GUTIÉRREZ,¹ ENRIQUE GARCÍA-BERRO,² ICKO IBEN, JR.,³ JORDI ISERN,⁴
 JAVIER LABAY,¹ AND RAMON CANAL¹

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ABSTRACT

The final stages of the evolution of electron-degenerate ONeMg cores, resulting from carbon burning in “heavyweight” intermediate-mass stars ($8 M_{\odot} \lesssim M \lesssim 10 M_{\odot}$) and growing in mass either from carbon burning in a shell or from accretion of matter in a close binary system, are examined. When due account is taken of the Coulomb corrections, both in the equation of state and in the electron capture threshold energies, explosive NeO ignition takes place at densities high enough to ensure gravitational collapse to nuclear matter densities. It is shown that this result holds for two extreme assumptions concerning mixing in the presence of an overstable temperature gradient: no mixing (Ledoux criterion) and ordinary convective entropy mixing according to the Schwarzschild criterion (the latter delaying explosive ignition to still higher densities). Discrepancies among earlier calculations, due to omission of Coulomb corrections, are clarified with the use of the most recent electron capture rates on the relevant nuclides plus very finely zoned models.

Subject headings: nuclear reactions, nucleosynthesis, abundances — stars: evolution — stars: interiors — stars: supergiants — stars: white dwarfs

1. INTRODUCTION

The late stages of evolution of stars in the initial mass range $8 M_{\odot} \lesssim M \lesssim 12 M_{\odot}$ had remained almost unexplored until 15 years ago, in spite of the fact that this mass interval contains about one-half of the stars that are massive enough to ignite carbon nonexplosively. The pioneering work of Miyaji et al. (1980) concluded that all stars in the whole mass interval $8 M_{\odot} \lesssim M \lesssim 12 M_{\odot}$ would develop electron-degenerate ONeMg cores during C shell burning and capture electrons on ^{24}Mg and ^{24}Na first, and on ^{20}Ne and ^{20}F later, to explosively ignite Ne and O at central densities $\rho_c \gtrsim 2 \times 10^{10} \text{ g cm}^{-3}$. At these very high densities, the fast electron captures on the nuclear statistical equilibrium (NSE) material would quickly bring the Chandrasekhar mass below the actual mass of the core and thus induce gravitational collapse. Later, Woosley, Weaver, & Taam (1980) and Nomoto (1984) reduced the above range to $8 M_{\odot} \lesssim M \lesssim 10 M_{\odot}$ only: stars in the mass range $10 M_{\odot} \lesssim M \lesssim 12 M_{\odot}$ would nonexplosively burn Ne in flashes and then proceed through the O-burning and Si-burning stages, as more massive stars do. All of the preceding should apply not only to the cores of asymptotic giant branch (AGB) stars but also to ONeMg white dwarfs accreting material from a companion in a close binary system (Miyaji et al. 1980; Nomoto 1984, 1987).

Very recently, the evolution leading to the formation of electron-degenerate ONeMg cores has started to be systematically reinvestigated: Domínguez, Tornambé, & Isern (1994) have studied the formation of an ONeMg white dwarf through mass transfer in a close binary system, while García-Berro & Iben (1994) calculate the evolution of a

$10 M_{\odot}$ star from the H-burning main sequence up to exhaustion of C in the core (early “super”-asymptotic giant branch, or ESAGB phase). Clarification of the exact mass interval in which electron captures precede (and eventually trigger) explosive Ne ignition should be one important outcome of those studies.

Another very important point is whether the density at the ignition of explosive NeO burning is really high enough to ensure that fast electron captures on the NSE material do make the Chandrasekhar mass lower than the core mass before expansion induced by the thermonuclear energy release quenches them. This issue has been controversial in recent years, and its clarification is the main purpose of this paper. In the papers of Miyaji et al. (1980), convection was assumed to set in according to the Schwarzschild criterion: a convective core started to develop at the onset of the electron captures on ^{24}Mg . Convective heat transport contributed to keep the temperature below that of explosive Ne ignition ($\sim 2 \times 10^9 \text{ K}$) along core contraction until a central density $\rho_c \gtrsim 2 \times 10^{10} \text{ g cm}^{-3}$ was reached. That was questioned by Mochkovitch (1984), who argued that the steep gradient of the electron mole number, Y_e , produced by the same electron captures that create the superadiabatic temperature gradient, should stabilize the fluid against convective motions and a semiconvective region would form. Miyaji & Nomoto (1987) tested this situation by assuming semiconvective mixing to be negligible. They found that explosive NeO burning was ignited at a central density $\rho_c \simeq 9.5 \times 10^9 \text{ g cm}^{-3}$: heating by the electron captures and cooling by thermal neutrinos was then purely local.

Miyaji & Nomoto (1987) just assumed that the effects of burning propagation on the subsequent dynamics would be negligible, and they thus only took into account the electron captures on the NSE material: the fast decrease in Y_e very soon induced collapse. This was disputed by Isern, Canal, & Labay (1991), who argued that the outcome (either gravitational collapse to nuclear matter densities or thermonuclear explosion) depended on the competition between burning propagation and electron captures, and that for the current

¹ Departament d’Astronomia i Meteorologia, Universitat de Barcelona, Martí i Franquès, 1, 08028 Barcelona, Spain.

² Departament de Física Aplicada, Universitat Politècnica de Catalunya, c/Jordi Girona Salgado, s/n, Mòdul B-5, Campus Nord, 08034 Barcelona, Spain.

³ Astronomy and Physics Departments, University of Illinois, 1002 West Green Street, Urbana, IL 61801.

⁴ Centre d’Estudis Avançats de Blanes (CSIC), Camí de Sta. Bàrbara, s/n, 17300 Blanes (Girona), Spain.

estimates of the former (Woosley & Weaver 1986) thermonuclear explosion appeared most likely for $\rho_{\text{ign}} \lesssim 10^{10} \text{ g cm}^{-3}$. The critical density separating explosion from collapse in an ONeMg core is still being debated, but the most recent estimates (Timmes & Woosley 1992) place it at $\rho_{\text{ign}} \simeq 9 \times 10^9 \text{ g cm}^{-3}$. Therefore, an accurate determination of the explosive ignition density is crucial.

The three factors that determine the explosive NeO ignition density are the chemical composition of the core at the end of C burning, the electron capture rates on ^{24}Mg , ^{24}Na , ^{20}Ne , and ^{20}F , and the degree of mixing (chemical composition and entropy) in the semiconvective region.

Previous calculations (Miyaji & Nomoto 1987; Nomoto 1987; Canal, Isern, & Labay 1992) adopted a chemical composition: $X_{\text{O}} = 0.12$, $X_{\text{Ne}} = 0.76$, $X_{\text{Mg}} = 0.12$. Instead, the recent calculation of Domínguez et al. (1994) yields $X_{\text{O}} = 0.72$, $X_{\text{Ne}} = 0.25$, $X_{\text{Mg}} = 0.03$ (this is the composition that we adopt in our present calculations). On the other hand, Hashimoto, Iwamoto, & Nomoto (1993) use $X_{\text{O}} = 0.34$, $X_{\text{Ne}} = 0.49$, $X_{\text{Mg}} = 0.12$. As can be seen, the abundance of ^{24}Mg is roughly a factor of 4 lower, whereas the abundance of ^{16}O is a factor of 2 larger, when compared with the abundances adopted by Hashimoto et al. (1993). Domínguez et al. (1994) used the reaction rates of Caughlan et al. (1985). Their results are consistent with the abundances obtained by García-Berro & Iben (1994), who used the reaction rates of Caughlan & Fowler (1988).

The electron capture rates used in the calculations of Miyaji et al. (1980), Nomoto (1984, 1987), and Miyaji & Nomoto (1987) were based on the gross theory of β -decay. Later, new rates based on shell-model calculations have become available (Takahara et al. 1989; Oda et al. 1994). The rates of Takahara et al. (1989) have been used by Canal et al. (1992) and by Hashimoto et al. (1993). Those of Oda et al. (1994) are used here.

Coulomb corrections enter in the physical models in two ways: by lowering the pressure and the internal energy of the electron gas as compared with the perfect, degenerate equation of state (Salpeter & Zepolsky 1967) and by raising the thresholds for the electron captures on the relevant nuclei (Couch & Loumos 1974). We will see that inclusion of the latter significantly increases the explosive ignition density.

Finally, mixing in the semiconvective region has two opposite effects: by allowing electron captures to take place at energies much above threshold, it increases the local heating rate, while by homogenizing entropy over an extended region, it cools the center of the core much more efficiently than conduction alone. We will see that the second effect dominates, and thus mixing increases the explosive ignition density.

The present calculations elucidate the origin of previous discrepancies as to the exact explosive ignition density in the ONeMg cores of AGB stars in the $8 M_{\odot} \lesssim M \lesssim 10 M_{\odot}$ range and in mass-accreting white dwarfs with such composition. The conclusion is that, from all current evidence, those cores do collapse to nuclear matter densities.

2. MODELS, RESULTS, AND DISCUSSION

Our calculations start from a model of an ONeMg core of $1.375 M_{\odot}$, with central density $\rho_c = 1.0 \times 10^9 \text{ g cm}^{-3}$, central temperature $T_c = 2.0 \times 10^8 \text{ K}$, and chemical composition $X_{\text{O}} = 0.72$, $X_{\text{Ne}} = 0.25$, $X_{\text{Mg}} = 0.03$ (we do not consider here the possibility of some ^{12}C being left unburned in

a small central region, as found by Domínguez et al. 1994). The models are divided into 500 mass zones, which ensures good resolution of the chemical composition and entropy gradients produced by the electron captures (note that electron captures and decays are extremely sensitive to density). We have performed a series of numerical experiments to study the effect of an increasing number of shells on the ignition density. We have found that increasing from 100 to 500 the number of mass shells induces a change of roughly 10% in the ignition density. Increasing even further the number of shells does not produce any significant change in the ignition density. Since the evolution in previous calculations was very close to the current estimates of the critical ignition density separating explosion from collapse, it is really important to have a good zoning once captures set in. The evolution from the start of mass growth up to the quasi-dynamic contraction that precedes explosive NeO ignition is followed by means of an implicit, one-dimensional hydrocode. Core growth is calculated as in Canal et al. (1992).

The equation of state includes Coulomb corrections (Salpeter & Zepolsky 1967). The Coulomb interaction energy has, moreover, an effect on the thresholds for the electron captures (Couch & Loumos 1974): the change in the interaction energy that occurs when an ion changes its charge by one unit can increase significantly the threshold. The exact expression for computing the energy shift is

$$\Delta E_{\text{thr}} = \mu(Z+1) - \mu(Z),$$

where $\mu(Z)$ is the chemical potential of the nucleus with charge Z , which is given by

$$\mu(Z) = -kT \left(\frac{Z}{\bar{Z}} \right) \left\{ \Gamma \left[0.9 + c_1 \left(\frac{\bar{Z}}{Z} \right)^{1/3} + c_2 \left(\frac{\bar{Z}}{Z} \right)^{2/3} + \left[d_0 + d_1 \left(\frac{\bar{Z}}{Z} \right)^{1/3} + \dots \right] \right] \right\},$$

where $\Gamma = \bar{Z}^{5/3} e^2 / a_e kT = 2.275 \times 10^5 \bar{Z}^{5/3} (\rho Y_e)^{1/3} / T$ is the Coulomb coupling parameter, a_e is the inter-electronic distance, \bar{Z} is the average charge, and the four constants are $c_1 = 0.2843$, $c_2 = -0.054$, $d_0 = -9/16$, and $d_1 = 0.460$. The rest of the symbols have their usual meanings.

At the densities and temperatures, and for the ions involved here, the shift amounts to $\Delta E_{\text{thr}} \simeq 200\text{--}300 \text{ keV}$. This effect was not included in the calculations of Canal et al. (1992). As we will see, it plays a crucial role since its inclusion shifts the explosive ignition density from below to above the current estimates of the critical density separating explosive disruption of the core from gravitational collapse. This will be illustrated for the two types of calculations that we present here.

We use the electron capture rates of Oda et al. (1994), based on shell model calculations. They do not significantly differ from those of Takahara et al. (1989). The electron capture rates of Oda et al. (1994) are given as a function of the product ρY_e and temperature. This is the same as saying the Fermi energy, ϵ_F , and temperature. Since these rates do not include the effect of the Coulomb interactions on the threshold, the rates were computed introducing an effective Fermi energy $\epsilon_F^{\text{eff}} = \epsilon_F - \Delta E_{\text{thr}}$, which translates into a lower effective density and electron capture rate at the same temperature and Y_e .

There has been some debate on the dependence of the ignition density of Ne from the electron capture rates used:

Canal et al. (1992) stated that by adopting the rates of Takahara et al. (1989) the ignition density decreased significantly as compared with that obtained using the rates of Miyaji et al. (1980). In contrast, Hashimoto et al. (1993) found the difference negligible. Part of the reason for the discrepancy can be traced to the anomalously high values of the neutrino energies for the electron captures on ^{20}Ne near threshold in the published electron capture rates of Miyaji et al. (1980). This can be seen in Figure 2 of Canal et al. (1992), where the ratios of the neutrino energy-loss rates from Miyaji et al. (1980) to those from Takahara et al. (1989) are displayed as a function of $\log(\rho Y_e)$, for $\log T = 8.6$: the older rates are systematically higher at all densities, and there is a large peak (somewhat smoothed in the figure) around the threshold. On the other hand, as stated above, corrections to the threshold energies for electron captures arising from Coulomb interactions were not included in the calculations of Canal et al. (1992), and this further increased the differences. When Coulomb corrections are taken into account and some revised version of the neutrino energy-loss rates of Miyaji et al. (1980) is used, both sets of calculations (with the old and the new rates, respectively) do give very similar explosive Ne ignition densities, as in Hashimoto et al. (1993).

In a first set of calculations, we adopt the Ledoux criterion for stability against convective motions. In practice, this amounts to suppressing any mixing until explosive NeO ignition. The evolution of central temperature versus density is shown in Figure 1 (solid line): it is qualitatively similar to that found by Miyaji & Nomoto (1987), Canal et al. (1992), and Hashimoto et al. (1993). The evolution that results from omitting the Coulomb corrections to the elec-

tron capture thresholds is shown by the dotted line. The most obvious difference in our calculations when compared with previous ones is the amplitude of the first peak in temperature (corresponding to electron captures on ^{24}Mg). This is due to the smaller amount of ^{24}Mg resulting from the previous evolution of the CO core and, in particular, from the adopted carbon burning rates (see García-Berro & Iben 1994). The change introduced by the increase in the thresholds of the electron captures on ^{20}Ne is responsible for the shift of the ignition density from $8.55 \times 10^9 \text{ g cm}^{-3}$ to $9.74 \times 10^9 \text{ g cm}^{-3}$ (and thus across the current estimate of the critical density for gravitational collapse: $\approx 9 \times 10^9 \text{ g cm}^{-3}$).

In a second set of calculations, we adopt the Schwarzschild stability criterion. Then, a convective core develops from the start of electron captures on ^{24}Mg at the center. Standard mixing-length theory gives complete composition and entropy mixing on a timescale much shorter than that of core contraction. It is difficult to evaluate the amount of mixing across the convective region and thus to precisely calculate the explosive ignition density. Given the timescales for electron captures, which are shorter than the diffusion timescales (both for heat and for ion diffusion: see Mochkovitch 1984 and Hashimoto et al. 1993), mixing is most probably negligible. We have thus assumed complete entropy mixing but *local* abundances. The resulting evolution is shown in Figure 2, and the size of the convective region is shown in Figure 3. As can be seen in Figure 2, the ignition densities in both cases (i.e., taking into account Coulomb corrections and neglecting them) are much higher than when the Ledoux criterion is used ($\rho_{\text{ign}} \approx 2.1 \times 10^{10} \text{ g cm}^{-3}$). Here also, the solid line corresponds to the calculation with all Coulomb corrections properly included, and the dotted line to that with the threshold effects omitted. As in the preceding case, threshold increase results in a higher explosive ignition density, but now in both cases such density is much higher than the critical one. As in previous calculations (Miyaji et al. 1980; Nomoto 1987), convection

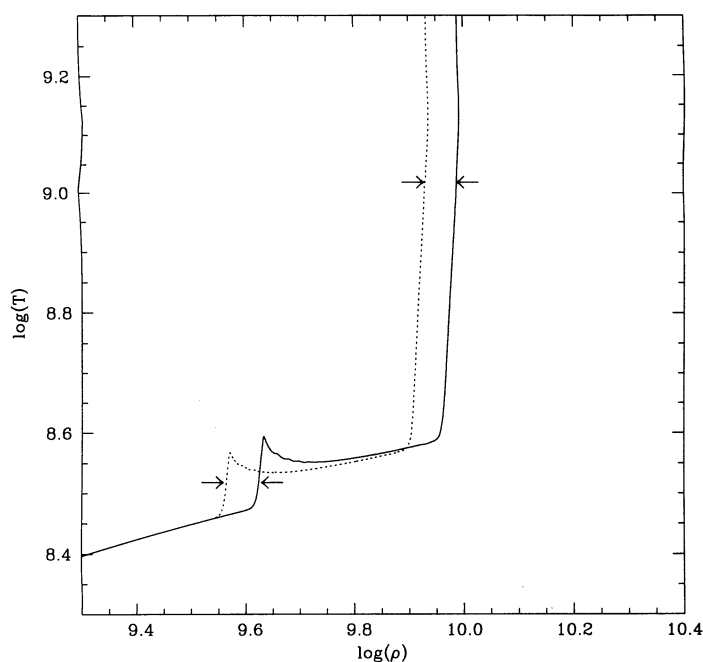


FIG. 1.—Evolution of the center of an ONeMg electron-degenerate core in the $\log \rho$ – $\log T$ plane as it grows in mass, either from a C-burning shell (AGB case) or by accretion from a close binary companion (white dwarf case), when the Ledoux stability criterion for stability against convection is adopted. The solid line corresponds to proper inclusion of all Coulomb corrections. The dotted line corresponds to omission of the effect of the Coulomb interaction energy on the electron capture thresholds (see text).

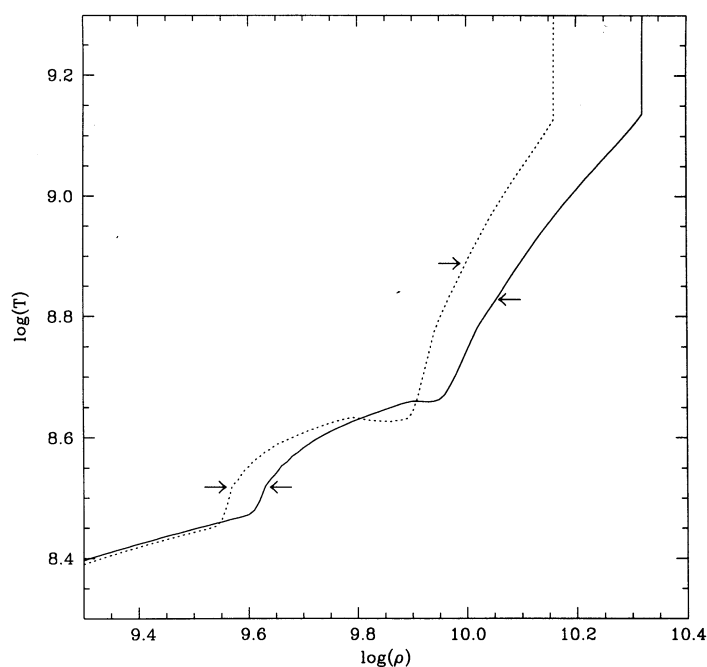


FIG. 2.—Same as Fig. 1, but adopting the Schwarzschild stability criterion

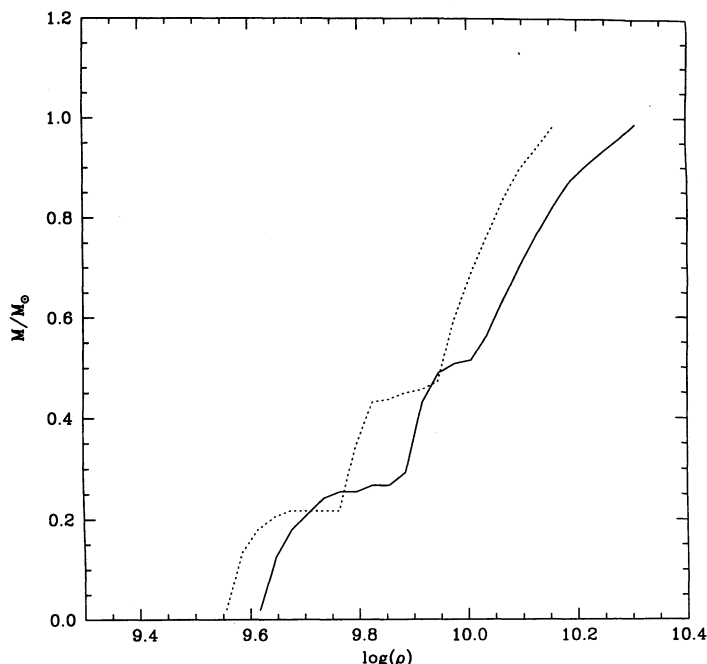


FIG. 3.—Size of the convective core when the Schwarzschild stability criterion is adopted. The solid line corresponds to proper inclusion of all Coulomb corrections. The dotted line corresponds to omission of the effect of the Coulomb interaction energy on the electron capture thresholds (see text).

increases the explosive ignition density as compared with the case in which convection is suppressed. The physical reason for this is the following: since we are assuming complete entropy mixing, the resulting temperature gradient is the adiabatic one, and since abundances are kept purely local, local heating is the same as in the preceding case, while the heat flow is much larger now (only conductive heat transport operates in the Ledoux case). This clearly results in higher ignition densities.

It was stated earlier (Canal et al. 1992) that some preliminary numerical experiments gave a *decrease* of the ignition densities for some prescriptions for the combined rates of heat transport and mixing of chemical composition across the semiconvective zone. This, however, has not been confirmed by more elaborate calculations.

In Table 1 we provide the timescales for electron capture ($\tau_\beta = -Y_e/\dot{Y}_e$) and core contraction ($\tau_\rho = \rho/\dot{\rho}$) for several relevant central densities (points marked in Figs. 1 and 2), and in Table 2 we compare our present results with those of all the preceding calculations of the evolution of ONeMg electron-degenerate cores up to explosive NeO ignition.

TABLE 1

TIMESCALES FOR ELECTRON CAPTURE AND CORE CONTRACTION

Case ^a	ρ_9	τ_β	τ_ρ	ρ	τ_β	τ_ρ
LCC	4.2	5.7×10^{11}	1.2×10^{11}	9.7	1.9×10^7	6.0×10^6
LNC	3.7	5.1×10^{11}	1.2×10^{11}	8.5	2.2×10^7	4.1×10^6
SCC	4.2	1.6×10^{12}	2.2×10^{11}	11.0	4.3×10^7	8.5×10^6
SNC	3.7	9.0×10^{11}	1.6×10^{11}	9.7	6.9×10^7	9.2×10^6

NOTE.—Timescales (in seconds) for electron capture (τ_β) and core contraction (τ_ρ) for several relevant densities (in g cm^{-3}) and for the four cases considered here.

^a LCC, Ledoux Criterion, Coulomb effects included; LNC, Ledoux criterion, Coulomb effects omitted; SCC, Schwarzschild criterion, Coulomb effects included; SNC, Schwarzschild criterion, Coulomb effects omitted.

TABLE 2

EXPLOSIVE IGNITION DENSITIES

Criterion	MNYS ^a	MN ^b	N ^c	CIL ^d	HIN ^e	This work
Ledoux	9	9.5	...	8.5	9.3	9.7
Schwarzschild	25	...	24	21.2

NOTE.—Explosive ignition densities (in units of 10^9 g cm^{-3}) from the present and previous calculations and for the two extreme hypotheses concerning mixing.

^a Miyaji et al. 1980.

^b Miyaji & Nomoto 1987.

^c Nomoto 1987.

^d Canal, Isern, & Labay 1992.

^e Hashimoto, Iwamoto, & Nomoto 1993.

Finally, in Figures 4 and 5 we show the evolution of Y_e for the four cases considered here. As can be seen from these figures, the final Y_e at the ignition density differs when different assumptions for the stability criterion against convection are adopted. When the Ledoux criterion is used, $Y_e \simeq 0.485$, whereas when the Schwarzschild criterion is chosen, $Y_e \simeq 0.472$. This will affect the dynamics of the ensuing collapse.

3. CONCLUSIONS

We have reexamined the evolution of ONeMg electron-degenerate cores experiencing mass growth either from a C-burning shell (the AGB case) or by accretion of material from a close binary companion (the white dwarf case). This has been done using the latest electron capture rates, properly taking into account Coulomb corrections (both to the equation of state and to the electron capture threshold energies), and by means of finely zoned models in which the quasi-dynamic nature of core contraction is adequately dealt with. The initial chemical composition has been taken from recent evolutionary calculations of the previous stages

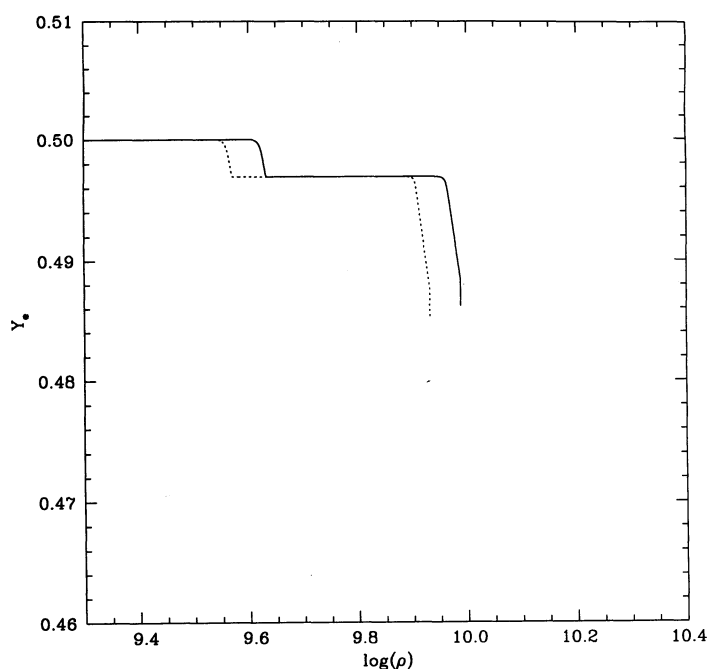


FIG. 4.—Evolution of the central Y_e when the Ledoux stability criterion is adopted. The solid line corresponds to proper inclusion of all Coulomb corrections. The dotted line corresponds to omission of the effect of the Coulomb interaction energy on the electron capture thresholds (see text).

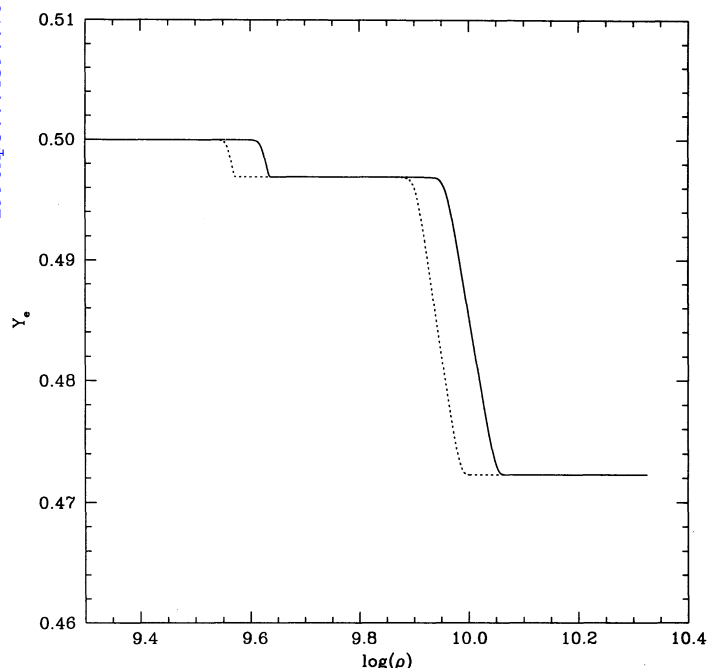


Fig. 5.—Same as Fig. 4, but adopting the Schwarzschild stability criterion

that use the most recent thermonuclear reaction rates. Two extreme assumptions, concerning chemical composition and entropy mixing in the semiconvective zone created by the electron captures, have been tested: no mixing at all (strict application of the Ledoux stability criterion), and ordinary convective mixing producing complete entropy homogeneity but keeping local chemical composition (the presence of convection being determined from the Schwarzschild stability criterion).

As in earlier calculations (Nomoto 1987; Miyaji & Nomoto 1987), the explosive ignition density is higher if convection is allowed to develop: it is $\rho_{\text{expl}} = 9.74 \times 10^9 \text{ g cm}^{-3}$ when the Ledoux criterion is used, and $\rho_{\text{expl}} = 2.12 \times 10^{10} \text{ g cm}^{-3}$ when the Schwarzschild criterion is adopted. The actual ignition density, if some mixing occurs in the semiconvective region, should be some intermediate value between these two extreme ones (most likely close to the lower value). Such a range of values lies above the best current estimates of the minimum ignition density for gravitational collapse ($\rho_{\text{crit}} \approx 9 \times 10^9 \text{ g cm}^{-3}$). Therefore, to the best present evidence, ONeMg cores that fail to ignite Ne prior to electron captures on Mg and Ne (stars in the $8 M_{\odot} \lesssim M \lesssim 10 M_{\odot}$ initial mass range?) do collapse to nuclear matter densities. A gravitational collapse supernova (with formation of a neutron star) should be the result in the AGB case, and nonexplosive collapse to a neutron star should be the result in the mass-accreting white dwarf case. Unless explosive NeO burning were found to propagate significantly faster than currently estimated, core disruption by a thermonuclear explosion would be possible only if the chemical composition differed from that adopted here either in having a much higher fraction of ^{24}Mg or in some ^{12}C being left unburned at the center (Dominguez et al. 1994).

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