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CAN TANGLED MAGNETIC FIELDS SUPPRESS THERMAL CONDUCTION IN CLUSTER COOLING FLOWS?

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ABSTRACT

We show that the standard scenario of a highly tangled magnetic field in cluster cooling flows is consistent with observations and theory only under very restrictive nonrealistic conditions. This result implies that plasma instabilities (wave pitch angle scattering) are the most likely cause of heat flux inhibition in cluster cooling flows.

Subject headings: cooling flows — intergalactic medium — magnetic fields — plasmas

1. INTRODUCTION

The existence of cooling flows is inferred from X-ray observations of a significant fraction of clusters of galaxies. It already has been argued in the first theoretical discussions of the observed phenomena that the presence of cool gas in the middle of the very hot intracluster medium implies that classical (à la Spitzer) heat conduction cannot play a significant role in the central parts of galaxy clusters (Binney & Cowie 1981). Binney & Cowie (1981) proposed two mechanisms that can, in principle, suppress the transport properties in the intracluster gas. The first mechanism involves the large-scale geometry of the magnetic field. Binney & Cowie conjectured, for example, that a magnetic field isolates M87 from the surrounding Virgo cluster and prevents the flow of heat from the cluster gas into the galaxy. The second mechanism involves the topology of the magnetic field on scales smaller than the mean free path of the electron. Binney & Cowie suggested that the field is tangled on distance scales that are about 100 times smaller than the typical mean free path of the electrons. The existence of a tangled magnetic field was ruled out by the above authors on the grounds that it requires unlikely values of the magnetic field. Regardless of Binney & Cowie's evaluation, this idea serves as the standard explanation for the suppression of conduction in cluster cooling flows (see, e.g., Fabian, Nulsen, & Canizares 1991; Fabian 1994).

The mechanism for suppression of thermal conduction by tangled fields is based on the reduction of transport processes in the direction perpendicular to magnetic field lines. It is known (Spitzer 1962) that, in the case of transport perpendicular to field lines, the particle gyroradius replaces the mean free path as the basic physical length scale that determines the transport. By analogy, one could assume that if the field is tangled and the coherence length of the field is smaller than the mean free path, then the coherence length might assume the role of the particle mean free path and hence be the factor in determining the suppression of the conduction. This would lead to isotropic suppression of conduction, provided the tangling is isotropic. Intuitively, under these assumptions, the ratio of the field coherence length to the particle mean free path yields the required suppression factor of the classical Spitzer value for the heat flux (Sarazin 1986). There has been some controversy with regard to the assumption that the magnetic field is both dynamically unimportant and able to suppress conduction at the very same time.

We note in passing that a field that consists of disconnected loops, the radius of which is smaller than the mean free path, is also a plausible solution for the required suppression of the transport. In this case, the mean thermal conduction along the temperature gradient may be suppressed to a value of the order of the heat conduction perpendicular to the magnetic field. A loop structure of the field complies with the underlying assumptions of the *homogeneous* cooling flow model. However, the observations require a multiphase model (Fabian et al. 1991; Fabian 1994). A transition from a *homogeneous* to a *multiphased* cooling flow induced by spontaneous weak fluctuations is very restricted (Mathews & Bregman 1978; Nulsen 1986; Malagoli, Rosner, & Bodo 1987; Balbus & Soker 1989; Loewenstein 1990; Balbus 1991). In this case, a nonlinear instability to a finite-size perturbation is required to provide the transition mechanism to the observed multiphased cooling flow.

Rosner & Tucker (1989) tried to set limits on the field strength and structure required to suppress conduction by a simply connected strongly fluctuating magnetic field. They concluded that, for this subset of tangled magnetic fields, the field cannot be dynamically unimportant and suppress conduction at the same time. In other words, they found that fields with these geometries can suppress conduction only when the magnetic field is in equipartition with the thermal gas pressure. In doing so, they verified the results of Binney & Cowie (1981).

In recent years it became clearer that the magnetic field in *noncooling flow* clusters of galaxies must indeed be somewhat tangled. The lack of inverse Compton X-ray emission from clusters of galaxies with radio halos allows one to set a lower limit on the strength of the intracluster magnetic field. The lower limit so found is typically $B \gtrsim 0.1\ \mu$G (Gursky & Schwarz 1977; Rephaeli & Gruber 1988). On the other hand, the Faraday rotation measure (FRM) of background radio sources caused by the general intracluster medium is not very large. Since the X-ray observations of clusters furnish values for the electron density of the thermal plasma in the cluster, the FRMs provide upper limits to the average component of the magnetic field along the line of sight. In general, these upper limits are incompatible with a relatively uniform field at the lowest limiting value set by the inverse Compton measurements. The upper limit due to FRM and the lower limit due to X-ray observations can only be reconciled if the field is tangled on a length scale that is much smaller than the path length through the cluster. Lawler & Dennison (1982) investigated the FRM in clusters of galaxies and concluded that the *inferred* field coherence length is about 10 kpc. More recently, Kim et al. (1990) reached similar conclusions on the basis of a larger sample of data.

The work of Kim et al. (1990) dealt mainly with noncooling flow clusters. Recently, Owen, Eilek, & Keel (1990), Ge (1991), Perley & Taylor (1991), and Ge & Owen (1993) performed FRMs in cooling flows. Their results have been collected by Godon et al. (1994). These FRMs are associated with the cooling flow region directly. The FRMs found in cooling flows are, in the most modest cases, about 2 orders of magnitude larger than those found by Kim et al. (1990) in noncooling flow clusters. The *measured* coherence length in clusters with cooling flows is of the order of 1–10 kpc, a value similar to the *inferred* scale in noncooling flow clusters (10 kpc). A 10 kpc value for the coherence length derived from the spatial variation of the FRM, combined with the value of the FRM, implies a magnetic field that is relatively ordered and nearly in equipartition with the gas. Hence, tangled magnetic fields (including closed loops) cannot be the factor responsible for the suppression of the conductivity inside the core of the cooling flow domain. Other interpretations of the same observational results discussed at the beginning of this paragraph were suggested by Bicknell, Cameron, & Gingold (1990) and Zuabi, Soker, & Regev (1995). These authors reach similar conclusions about the value of the coherence length and the magnitude of the field.

Another question in this context is: to what extent can a tangled magnetic field (simply connected) suppress the conduction in the outskirts of the cooling flow domain? We discuss this point and base our arguments in this paper only on the *observational constraint* set by the lack of a hard nonthermal X-ray tail. In other words, the magnetic field strength must be larger than 0.1 μG (Gursky & Schwarz 1977).

The next issue is whether there are any observational constraints on the field coherence length in noncooling flow clusters. The direct interpretation of the observations of Kim et al. (1990) implies that the coherence length in noncooling flow clusters is of the order of 10 kpc, i.e., is very similar to the value found in cooling flow clusters. If so, the following problem arises: why do the two different types of clusters possess approximately the same parameters?

The reply to this question can be given along one of the following three lines:

The cooling flow can be a transient phenomenon suppressed by temporal events (mergers, etc.) (Briel et al. 1991; Henry, Briel, & Nulsen 1993; Schindler & Böhringer 1993; Schindler & Müller 1993).

The appearance of cooling flow depends on other parameters than those discussed here, e.g., different metallicity (White et al. 1994; Fabian 1994; Fukazawa et al. 1994).

The similarity drawn between the coherence lengths measured in the two types of clusters is false.

In § 2 we show that the third possibility is viable yet unlikely.

In this work we address only tangled fields that do not contain loops. However, we point out that a pure loop structure inside the core of the cooling flow region ($r < 30$ kpc) is ruled out by the observations. Some comments about the effects of loops will, however, be made when such inferences are simple. We obtain a solution to the Boltzmann equation under the standard assumptions made about the structure of the field and the gas state. We then use the solution to show that the suppression of the conduction inferred from the measured FRM is insufficient (as can be inferred intuitively) to reconcile the theory and observations.

The structure of this paper is as follows: We show in § 2 that the FRM observations rule out the possibility of small-loop structure inside the cooling flow region. We further demonstrate that, although the observations do not rule out the tangling mechanism completely, it could occur only under very restrictive conditions. We then resort to theory, and in § 3 we solve the Boltzmann equation under the standard conjectures, which are supposed to provide the suppression of conduction. In § 4 we consider nonlocal transport effects and show again that the extreme tangling limit as traditionally envisioned is ruled out by the observations, as can be implied from Owen, Eilek, & Keel (1990), Ge (1991), Perley & Taylor (1991), and Ge & Owen (1993). In § 5 we provide a short discussion on possible cooling flow models.

2. FARADAY ROTATION OBSERVATIONS

Let us first discuss the assumptions made during the reduction of the FRM observations. Consider a cluster with a stochastic magnetic field. A common definition of the field coherence length l_b (Sarazin 1986) is that it is the length over which the field changes its direction by 90°. If this length scale is sufficiently smaller than the mean free path, heat conduction is suppressed (but see Tribble 1989, who does not require this restrictive condition). It is widely accepted (Sarazin 1986; Fabian et al. 1991) that this length scale can be obtained directly from measurements of the FRM. Crusius-Wätzel et al. (1990) and Tribble (1992) considered the interpretation of the FRM in clusters of galaxies. We follow Tribble (1992) and discuss the length scale that is obtained from the FRM in stochastic magnetic fields and show that it differs from the coherence length. To this end we assume that the ordered component of the magnetic field vanishes identically everywhere; hence,

$$\langle \boldsymbol{B} \rangle = 0 \, , \tag{1}$$

where the angle brackets denote an ensemble average. In this case, the ordered Faraday component of the FRM vanishes identically leaving only the contribution $\langle \boldsymbol{B} \rangle = 0$ of the stochastic component. Following Tribble (1992), we define the autocorrelation function of the FRM as

$$\xi(s) = \frac{810^2}{(1+z)^4} \left\langle \iint dz\, dz' n_e(z) n_e(z') B_z(\boldsymbol{x}, z) B_z(\boldsymbol{x}+s, z') \right\rangle , \tag{2}$$

where n_e is the electron number density in cgs units, dz is measured in kiloparsecs, the field is measured in microgauss, and z is the cluster redshift. The quantity s is the projected distance. As noted by Tribble (1992), the ensemble average can be moved into the integral to yield the integral over the correlation function of the field. Assuming $\langle \boldsymbol{B} \rangle = 0$ and $\nabla \cdot \boldsymbol{B} = 0$, one obtains (Batchelor 1953)

$$R_{zz} \equiv \langle B_z(\boldsymbol{x}, z) B_z(\boldsymbol{x} + s, z') \rangle = \frac{\langle B^2 \rangle}{3} \left[W(\delta r) + \frac{s^2}{2\delta r} \frac{dW}{d(\delta r)} \right], \quad (3)$$

where

$$s = \sqrt{(\delta r)^2 - (\delta z)^2} = \sqrt{(\delta r)^2 - (z - z')^2}\,, \quad (4)$$

and where W is the isotropic autocorrelation function. This function is unknown a priori and must be found from observations (or theory). The observations carried out by Kim et al. (1990), and previously by Lawler & Dennison (1982), correspond to $s = 0$ since the measurements were carried out only for a point source behind the cluster. The value of the correlation function $\xi(s = 0)$ can be written as

$$\xi(s = 0) = \frac{810^2}{(1 + z)^4} \iint dz' n_e(z') dz n_e(z) \frac{\langle B^2(z) \rangle}{3} W(z - z')\,. \quad (5)$$

We can write $n_e(z) n_e(z') = n_e(z' + \delta z) n_e(z')$ and obtain

$$\xi(0) = 2 \frac{810^2}{(1 + z)^4} \iint d(\delta z) dz \frac{1}{3} n_e^2(z) \langle B^2(z) \rangle W(\delta z) \times \left[1 + \frac{d \ln n_e(z)}{dz} \delta z \right]. \quad (6)$$

The density scale height is given by $l_\rho^{-1} = d \ln n_e(z)/dz$; hence, the term in brackets is $1 + \delta z / l_\rho$. Since $\delta z \ll l_\rho$, the second term can be neglected.

The integration should be carried out over the line of sight; thus,

$$\xi(0) = \frac{2}{3} \frac{810^2}{(1 + z)^4} \int_\Delta dz n_e^2(z) \langle B^2(z) \rangle \int_0^\infty W(\delta z) d(\delta z)\,, \quad (7)$$

where Δ is the total depth of the cluster and we assume that $W(\delta z)$ decays to zero sufficiently fast (but $\Delta \gg L_B$, where L_B is defined in the next paragraph) for large δz, so that the integral can formally be extended to infinity.

The correlation length L_B can be defined as (Batchelor 1953)

$$L_B = \int_0^\infty W(\delta z) d(\delta z)\,, \quad (8)$$

whereas the coherence length can be shown to be (Batchelor 1953)

$$l_b = \left. \frac{dW(\psi)}{d\psi} \right|_{W(\psi) = 1/2} \quad (9)$$

The simplest function W is a step function:

$$W(\psi = \chi / l_b) = \begin{cases} 1 & \text{for } \psi \le 1\,. \\ 0 & \text{for } \psi > 1\,. \end{cases} \quad (10)$$

In this case $L_B = l_b$. For a Gaussian function of the form $W(\psi = \chi / l_b) = \exp(-\psi^2)$ one finds $L_B = \pi^{1/2}/2 l_b$ (Tribble 1992). Of special interest are functions of the form

$$W(\psi) = (1 + \psi)^{-1-\epsilon}\ \epsilon > 0\,, \quad (11)$$

for which we find

$$L_B = \frac{1}{\epsilon} l_b\,. \quad (12)$$

Hence, unless $\epsilon \ll 1$, which is a special case, $L_B \approx l_b$. The requirement to suppress heat conduction by a stochastic magnetic field when the measured value of L_B is 10 kpc requires l_b to be 2 orders of magnitude smaller than L_B for a homogeneous cooling flow (Cowie & Binney 1981; Fabian 1994) and 4 orders of magnitude smaller for the transition to a multiphase cooling flow (Balbus 1991), i.e., $\epsilon \approx 10^{-2}$–10^{-4}. In summary, (1) either the stochastic magnetic field has a fast decaying isotropic autocorrelation function W and $L_B \approx l_b$ so that practically the field does not suppress the heat conduction; or (2) the stochastic magnetic field has a slowly decaying isotropic autocorrelation function and $\Delta \gg L_B \gg l_b$, in which case a suppression of the heat conduction is possible.

We are unable to rule out the second possibility only on the basis of the FRM observations in noncooling flow clusters. The situation is different in cooling flow clusters since here there is observational information for $s \neq 0$. The observations for $s \neq 0$ provide a direct measurement of the coherence length, which is found to be of the order of 1–10 kpc (Owen, Eilek, & Keel 1990; Ge 1991; Perley & Taylor 1991; Ge & Owen 1993). Here we note that since this method of interpretation does not depend on the topology of the field, a loop structure in which the radius of the loop is smaller than the mean free path is not possible in the core of the cooling flow. However, if, as predicted by Soker & Sarazin (1990) in the standard cooling flow model, the field is dragged in from the cluster to the cooling flow region, it is expected to be in equipartition with the gas in this domain; hence, the loops would reconnect. The loops would have a general tendency to reconnect for the following reason: consider a circular loop that is dragged into the cooling flow from the cooling radius. As the loop is dragged in, it becomes, according to Soker & Sarazin's (1990) solutions, more and more elliptic, with the major axis pointing toward the center of the cooling flow. Once the ellipticity exceeds some critical value, the loop annihilates or is broken into smaller loops. Thus, the observational results, which unfortunately are available only in the cores of the cooling flows and not far away, cannot rule out the suppression of conduction by a highly tangled magnetic field or a closed loop structure. In the next sections we turn to theory and consider the effects of a simply connected field structure.

We stress that the FRM observations in cooling flow and noncooling flow clusters do not contradict Tribble's (1989) conclusion that the heat flux could be significantly reduced by a mildly tangled field. As a matter of fact, FRM observations support the idea of suppression by this mechanism. However, Tribble's (1989) model differs from the cooling flow model in that it is a priori multiphased. To the best of our knowledge, the details of this model have not yet been worked out. Furthermore, it should be noted that in the situation envisioned by Tribble (1989) the scales involved in the problem are just the spatial temperature scale height and the field correlation length. Under such conditions, heuristic arguments (Sarazin 1986) suggest that the conductivity should be reduced by only a factor of 3.

We note that other models interpreting the FRM in cooling flow clusters exist. These models (Bicknell et al. 1990; Zuabi et al. 1995) invoke magnetic lifting or mixing due to the Kelvin-Helmholtz instability to explain the FRM observations. In both types of models the typical length scale of the magnetic field is similar to that resulting from the Owen, Eilek, & Keel (1990), Ge (1991), Perley & Taylor (1991), and Ge & Owen (1993) interpretation. Hence, the coherence length of the field in this domain is larger than the mean free path; thus, the field geometry in this region does not allow for heat flux inhibition. In short, neither of the interpretations solves the problem of the transport coefficient.

3. SOLUTION OF THE BOLTZMANN DRIFT KINETIC EQUATION

The transport coefficients are obtained from an approximate solution of the Boltzmann equation. The hydrodynamic equations with classical transport á la Spitzer constitute such an approximate solution. Thus, to derive the effective transport coefficients which are due to a tangled magnetic field we need to solve the Boltzmann equation. Here we solve the steady state Boltzmann equation. The Boltzmann equation considered in this section is written in the drift approximation, and its validity to the ICM environment is discussed in Appendix A. Under the assumption of steady state, equation (A9) reduces to

$$(\boldsymbol{v}_{\parallel} + \boldsymbol{v}_{\mathscr{C}D} + \boldsymbol{v}_{\mathscr{F}D}) \cdot \nabla \bar{f} + \left\{ \left(\frac{v_{\parallel} B}{\Omega_L} \right) \boldsymbol{b} \cdot \nabla \left[\frac{v_{\parallel} \boldsymbol{b} \cdot (\nabla \times \boldsymbol{b})}{B} \right] \right\} \times \mu \frac{\partial \bar{f}}{\partial \mu} = C(f, f) . \quad (13)$$

The meaning and the physical interpretation of various terms is provided in Appendix A. Here we note that $\bar{f}$ is the guiding center distribution function, $\boldsymbol{v}_{\parallel}$ is the particle velocity parallel to the field lines, and $\boldsymbol{b}$ is a unit vector in the direction of the magnetic field. The other terms on the left-hand side are associated with the deviation which is due to various drift effects from motion parallel to the field lines. Note that the term $C(\bar{f}, \bar{f}) \approx O[\Lambda^2 = (r_L/l_b)^2]$ was neglected.

The terms that describe the deviation from the motion parallel to the field lines contain the self-consistent DC electric field $\boldsymbol{E}_{\parallel} = \gamma_{\parallel} \boldsymbol{bb} \cdot \nabla T$, where $\gamma_{\parallel}$ is the thermoelectric constant parallel to field lines. The field $\boldsymbol{E}_{\parallel}$ is given by the condition that the current vanishes along field lines and does not contribute to the drift. The thermoelectric constant perpendicular to field lines is of the order of the thermal conduction perpendicular the field and hence may be neglected. Thus, drift which is due to the self-consistent DC electric field is negligible, and we may set $E = 0$ in the drift terms. Collective plasma instabilities that might enhance the role of the electric drift are beyond the scope of this paper. However, collective plasma oscillations leave $\gamma_{\parallel} \gg \gamma_{\perp}$ but require $T_e \gg T_i$ (Tsytovich 1972).

Let us now make the standard assumptions of the cooling flow model, namely, that the magnetic field is arbitrary and dynamically unimportant, while the density and temperature depend on the radial distance only. Consider the order of various terms in equation (13) *assuming that* $\nabla \bar{f}$ is a slowly varying function in space. The order of the respective terms is

$$\left(\frac{v_{\text{th}}}{l_H^{\parallel}} : \frac{v_{\text{th}} r_L}{l_b^2} : \frac{v_{\text{th}} r_L}{l_b^2} \right) : \frac{v_{\text{th}} r_L}{l_b^2} : \frac{v_{\text{th}}}{\lambda_e} , \quad (14)$$

where $v_{\text{th}} = (2kT/m)^{1/2}$ and $l_H^{\parallel}$ is the shortest scale height along the magnetic field lines, namely, $l_H^{\parallel} = \min \{l_\rho^{\parallel}, l_T^{\parallel}, l_u^{\parallel}\}$. In the previous expression k is the Boltzmann constant and T is the gas temperature. The above relation with $\Lambda = r_L/l_b$ and $\sigma = l_H^{\parallel}/l_b$ is easily written in the following form:

$$1 : \Lambda\sigma : \Lambda\sigma : \Lambda\sigma : \frac{l_H^{\parallel}}{\lambda_e} . \quad (15)$$

The solution of equation (13) is very sensitive to the magnitude of $l_H^{\parallel}$. This quantity is first estimated on physical grounds and is later checked for consistency.

Previous discussions of the suppression of heat conduction by magnetic fields (Sarazin 1986; Fabian et al. 1991) assumed that the distance scale for changes in the direction of the field replaces the role of a mean free path. This assumption was supposed to lead to a situation in which transport in space is determined by l_b^2/λ_e, provided that $l_b/\lambda_e \approx 10^{-2} \ll 1$. This situation is realized once $l_H^{\parallel} = l_b$, namely, $\sigma = 1$. Thus, the curvature of the field lines acts as an effective scatterer of the electrons (as a result of a change in the direction of the field). This case is considered in Appendix B, in which it is shown that under these conditions the distribution function deviates from the thermal (Maxwellian) one and the saturated heat flux is much larger than the Spitzer heat flux (see § 4). From the above estimates of the magnitude of the various terms in the drift kinetic equation and an estimate based on the solution found in Appendix B, it is clearly seen that *the coherence length of the field does not replace the role of the mean free path, even if it is considerably smaller than the mean free path.* This conclusion is also applicable for (steady state) particle trapping since in this case $l_H^{\parallel}$ is just the size of the trap (Stix 1992).

The most conservative assumption, which is also mathematically more justified, is that the magnitude of $l_H^{\parallel}$ is $l_H^{\parallel} \equiv l_H^S/\cos \alpha$, where $\cos \alpha = \boldsymbol{b} \cdot \hat{\boldsymbol{r}}$, $\hat{\boldsymbol{r}}$ is the unit vector in the radial direction, and l_H^S is the shortest scale height variation in space. If $\cos \alpha = 0$, then $l_H^{\parallel}/\lambda_e \to \infty$ and the Chapman-Enskog solution of the drift kinetic equation is valid. If $\cos \alpha = 1$, then $l_H^{\parallel}/\lambda_e = l_H^S/\lambda_e = K_n^{-1}$, where K_n is defined as "the Knudsen number in space." The Chapman-Enskog expansion is valid if and only if $K_n < 10^{-2}$ (Max 1981) and not, as is often thought, if $K_n < 1$. The requirement on K_n is a result of the fact that the collisional mean free path depends on the inverse square of the electron velocity.

In a cooling flow environment the Knudsen number at the outer parts is somewhat larger than that required for the validity of the Chapman-Enskog expansion. As a first-order approximation we consider a situation in which $\lambda_e/l_H \le \lambda_e/l_H^S = K_n < 10^{-2} \ll 1$. This allows the Chapman-Enskog expansion along a single coordinate. This coordinate is the affine field length $l_{\parallel}$ and is obtained from the definition $\partial/\partial l_{\parallel} = \boldsymbol{B} \cdot \nabla/B$. The Spitzer-Härm expression for the conductivity along the field lines is obtained under these conditions (Hinton & Hazeltine 1976). This justifies the underlying physical assumptions made by Tribble (1989).

Tribble (1989) derived his effective transport formula by assuming that each field line has a different temperature, namely, an implicit flux-tube structure. Thus, as noted by Tribble (1989), one can implement the results of the thermal evaporation theory to study whether a flux tube with an arbitrary structure can cool. In order to obtain a flux tube in which one of its sides is colder than the other, its length must be larger than the Field (1965) length. The Field length in the cluster environment is $\lambda_F \approx 1$ Mpc. Winding a 1 Mpc flux tube inside a volume of 100–200 kpc^3 does not present a real difficulty, and the coherence length of the tube can easily be 10 kpc. Thus, such tubes can in principle account for the X-ray observations with an a priori multiphased model. Such a model has not yet been developed in detail (Fabian 1994) and is beyond the scope of this paper. We, however, note that recently Tao (1995) has addressed the same problem as did Tribble and included the effect of the magnetic fields on the dynamics in a self-consistent manner. The results of Tao for the suppression factor differ from those of Tribble, yet the magnitude of the suppression factor is not sufficient to explain the observations.

According to the expected field structure in cooling flows (Soker & Sarazin 1990; Pistinner & Shaviv 1995a), the component of the Gaussian curvature in the radial direction, $\boldsymbol{\kappa} \cdot \hat{\boldsymbol{r}} = (\hat{\boldsymbol{r}} \times \boldsymbol{b}) \cdot (\nabla \times \boldsymbol{b})$, is proportional to the density scale height and depends on the variation of the tangential component in the azimuthal and poloidal angles. Even if the temperature is a function of the angles, the suppression is not enough unless the field is dynamically important. This is easily seen if we average the heat flux vector over the surface of an arbitrary sphere, the center of which is at the center of the cooling flow. This argument shows that the suppression of the conductivity only in the direction perpendicular to the field is not sufficient to yield the observed macroscopic suppression. In other words, *suppression along the field lines is also required.* We note again that for a simply connected field structure, suppression resulting from magnetic field effects is not expected to be more than a factor of 3 if $l_H^S \gg l_B \approx \lambda_e$.

Consider now the case $l_B \gg \lambda_e$. To find the degree of suppression in this case, one has to average the heat flux over space. Let

$$\boldsymbol{q} = \kappa_{\rm SH}\, \boldsymbol{b} \frac{\partial T}{\partial l_{\parallel}} = \kappa_{\rm SH}\, \boldsymbol{b}(\boldsymbol{b} \cdot \nabla T) \tag{16}$$

be the heat flux vector along field lines. Here $\kappa_{\rm SH}$ is the Spitzer-Härm conductivity. Let $\boldsymbol{b} = \langle \boldsymbol{b} \rangle + \delta \boldsymbol{b}$, with the obvious meaning. We now average the heat flux vector on a scale that is much larger than l_b. This presents no difficulty since we can take the limits of the averaging volume to infinity effectively. The result is

$$\begin{aligned}\langle \boldsymbol{q} \rangle &= \kappa_{\rm SH}(\langle \boldsymbol{b} \rangle \langle \boldsymbol{b} \rangle \cdot \nabla \langle T \rangle + \langle \delta \boldsymbol{b} \rangle\} \boldsymbol{b} \cdot \nabla T \\ &\quad + \langle \boldsymbol{b} \rangle \cdot \langle \delta \boldsymbol{b} \nabla T \rangle + \langle (\delta \boldsymbol{b} \delta \boldsymbol{b}) \rangle \cdot \langle \nabla T \rangle) \\ &= \kappa_{\rm SH}(\langle \boldsymbol{b} \rangle \langle \boldsymbol{b} \rangle \cdot \nabla \langle T \rangle + \langle (\delta \boldsymbol{b} \delta \boldsymbol{b}) \rangle \cdot \langle \nabla T \rangle)\,. \end{aligned} \tag{17}$$

The heat flux depends now on two terms: the first depends only on the mean field, and the second on the fluctuations. For suppression of the heat flux the two terms must be suppressed. Rosner & Tucker (1989) considered the special case in which the first term is very small and found that in this case the field in not dynamically unimportant.

We now discuss the second term. Assume a turbulent isotopic magnetic field so that one can write $\langle \boldsymbol{q} \rangle = \kappa_{\rm SH}/3W(\lambda_e)\langle \partial T/\partial r \rangle$. The mean free path, at least at the edge of the cooling radius, is $\lambda_e \leq 20$ kpc $\approx 2 \times l_b \approx 2 \times L_B$. Hence, if $W(\lambda_e/l_b) \cong W(2) \ll 1$, one could reduce $\langle \boldsymbol{q} \rangle$. However, the assumption of an isotropic turbulent magnetic field is inconsistent with the finding of Soker & Sarazin (1990) for the magnetic field, namely, a solution that varies slowly in the radial direction.

While this paper was being refereed, it was brought to our attention that Tao (1995) approached the heat flux inhibition problem differently. Assuming Spitzer transport along field lines and using a self-consistent MHD model, Tao (1995) sets an upper limit on the maximal stretching of magnetic field lines that are embedded in a fully turbulent incompressible flow. His final conclusion for the ICM is that a modest upper limit on the amount of conduction suppression is about 100. Naturally, in such a flow the solution obtained by Soker & Sarazin (1990) is not valid.

4. NONLOCAL TRANSPORT ALONG FIELD LINES

We turn now to study a situation that is more applicable to clusters of galaxies, namely, a situation in which $K_n > 10^{-2}$.

Let us introduce a new variable $\zeta = v_{\parallel}/v$, the pitch angle of the gyrating particle, where $v = (v_{\parallel}^2 + v_{\perp}^2)^{1/2}$. The electron drift kinetic equation now reads

$$(\zeta v \boldsymbol{b} + \boldsymbol{v}_{\mathscr{C}D} + \boldsymbol{v}_{\mathscr{F}D}) \cdot \nabla \bar{f} + \left\{\left(\frac{\zeta v B}{\Omega_L}\right) \boldsymbol{b} \cdot \nabla \left[\frac{\zeta v \boldsymbol{b} \cdot (\nabla \times \boldsymbol{b})}{B}\right]\right\} \times \mu \frac{\partial \bar{f}}{\partial \mu} = C(\bar{f}, \bar{f})\,. \tag{18}$$

Next, we write the collision integral in this new set of coordinates. The electron-electron collision term does not present a problem since it depends only on v. The electron-ion collision integral depends on ζ. The explicit dependence can be found in Hinton & Hazeltine (1976) and in Stix (1992). If we further assume that the Mach number for the case under consideration is $< 10^{-2}$, the drift kinetic equation becomes

$$(\zeta v \boldsymbol{b} + \boldsymbol{v}_{\mathscr{C}D} + \boldsymbol{v}_{\mathscr{F}D}) \cdot \nabla \bar{f} + \left\{\left(\frac{\zeta v B}{\Omega_L}\right) \boldsymbol{b} \cdot \nabla \left[\frac{\zeta v \boldsymbol{b} \cdot (\nabla \times \boldsymbol{b})}{B}\right]\right\} \mu \frac{\partial \bar{f}}{\partial \mu} = \frac{D_{\perp}}{v^3} \frac{\partial}{\partial \zeta} (1 - \zeta^2) \frac{\partial \bar{f}}{\partial \zeta} + \frac{D_{\perp}}{v^2} \frac{\partial \bar{f}}{\partial v} + \frac{D_{\parallel}}{v^2} \frac{\partial}{\partial v} \frac{1}{v} \frac{\partial \bar{f}}{\partial v}\,, \tag{19}$$

where

$$D_{\perp} = \frac{v_T^4}{\lambda_e}\,; \quad D_{\parallel} = \frac{v_T^6}{\lambda_e}\,. \tag{20}$$

We expand in the small parameter A and ignore terms of $O(\Lambda^2)$ in this approximation since the drift kinetic equation is correct to only that order. The first two equations in the hierarchy read

$$\zeta v \frac{\partial \bar{f}^o}{\partial l_{\parallel}} = \frac{D_{\perp}}{v^3} \frac{\partial}{\partial \zeta} (1 - \zeta^2) \frac{\partial \bar{f}^o}{\partial \zeta} + \frac{D_{\perp}}{v^2} \frac{\partial \bar{f}^o}{\partial v} + \frac{D_{\parallel}}{v^2} \frac{\partial}{\partial v} \frac{1}{v} \frac{\partial \bar{f}^o}{\partial v}\,, \tag{21}$$

and

$$\begin{aligned} &\zeta v \frac{\partial \bar{f}^1}{\partial l_{\parallel}} + (\boldsymbol{v}_{\mathscr{C}D} + \boldsymbol{v}_{\mathscr{F}D}) \cdot \nabla \bar{f}^o + \left\{\left(\frac{\zeta v B}{\Omega_L}\right) \boldsymbol{b} \cdot \nabla \left[\frac{\zeta v \boldsymbol{b} \cdot (\nabla \times \boldsymbol{b})}{B}\right]\right\} \mu \frac{\partial \bar{f}^o}{\partial \mu} \\ &= \frac{D_{\perp}}{v^3} \frac{\partial}{\partial \zeta} (1 - \zeta^2) \frac{\partial \bar{f}^1}{\partial \zeta} + \frac{D_{\perp}}{v^2} \frac{\partial \bar{f}^1}{\partial v} + \frac{D_{\parallel}}{v^2} \frac{\partial}{\partial v} \frac{1}{v} \frac{\partial \bar{f}^1}{\partial v}\,. \end{aligned} \tag{22}$$

Consider the first equation of this hierarchy. This equation is just the steady state Boltzmann equation in one dimension, with the cosine of the pitch angle ζ replacing the cosine angle between the observer and the electron beam.

A theory of nonlocal solutions to equation (21) has been developed by Luciani & Mora (1985). Its main advantage is the fact that it reproduces very well the results of numerical computations and laboratory experiments (Luciani, Mora, & Pellet 1985). We apply this theory to our case.

The distribution function found by Luciani & Mora (1985) is given by

$$\bar{f} = \int_{-\infty}^{\infty} dl'_{\parallel} \frac{v^{-3}}{2\lambda'_{\rm eff}} \exp\left(-\frac{v^{-3}|l_{\parallel} - l'_{\parallel}|}{\lambda'_{\rm eff}}\right) f'_{\rm MB}\,, \quad (23)$$

where $\lambda_{\rm eff} \approx 32\lambda_e$. The value of $\lambda_{\rm eff}$ is somewhat ambiguous since different theoretical approximations provide different results. However, the results do not differ by a large numerical factor. The heat flux vector parallel to field lines resulting from this theory is

$$q_{\parallel} = \int_{-\infty}^{\infty} dl'_{\parallel} \frac{1}{2\lambda'_d} \exp\left(-\frac{|l_{\parallel} - l'_{\parallel}|}{\lambda'_d}\right) q_{\parallel}^{\rm SH}\,, \quad (24)$$

where $q_{\parallel}^{\rm SH}$ is the Spitzer-Härm heat flux parallel to field lines and λ_d is a delocalization length obtained from a numerical fit to the exact expression. The result is $\lambda_d \approx 5.5$–$32\lambda_e$, depending on the physical system. It is readily verified that the exponent appearing in the integrand can be approximated with a delta function if the path length of the field is much larger than the effective mean free path. Thus, a tangled field leads to a situation in which the heat flux vector is the Spitzer vector along field lines, in agreement with Tribble's (1989) underlying physical assumption.

If one makes now the unfit assumption that the coherence length replaces the role of the mean free path, then the theory yields a saturated heat flux expression and a non-Maxwellian distribution function. However, it is unlikely that under normal cluster conditions (a moderately tangled magnetic field) this nonlocal distribution would affect the energy distribution significantly. Furthermore, it will most likely become unstable (Levinson & Eichler 1992).

One may suspect that the modification of the heat flux affects the energy balance significantly. The integral nature of this formula requires that we take the derivative of the heat flux prior to the calculation of its contribution to the total energy balance. Toward that goal, we note that (i) because of the divergence-free condition of the magnetic field and (ii) since the conduction perpendicular to field line can be practically ignored, we have

$$\frac{\partial q_{\parallel}}{\partial l_{\parallel}} \approx \nabla \cdot \boldsymbol{q}\,. \quad (25)$$

Substituting now equation (24) in equation (25) and taking the exponent in the limit of strong tangling, namely, $(l_{\parallel} - l'_{\parallel})/\lambda'_d \rightarrow 0$, so that the exponent is equal to unity, one verifies after some algebra that

$$\frac{\partial q_{\parallel}}{\partial l_{\parallel}} \approx \kappa^{\rm SH} \frac{\Delta T}{\lambda_{\rm eff}^2} \approx \left(\frac{L_T}{\lambda_{\rm eff}}\right)^2 \kappa^{\rm SH} \frac{\Delta T}{L_T^2}\,, \quad (26)$$

where L_T is the radial temperature scale height and ΔT the temperature difference between two points in space along the radial distance. The suppression factor in the divergence of the heat flux is in this case the inverse of the effective Knudsen number. Thus, the effect of conduction on the energy balance is not suppressed; on the contrary, it is enhanced. One can estimate the Knudsen number from existing steady state cooling flow models that neglect conduction (White & Sarazin 1987) and find that it is always larger than or equal to unity. Therefore, the assumption that the coherence length replaces the role of the mean free path leads to an enhancement of the effect of heat conduction on the energy balance. This conclusion is valid as long as the modified distribution does not change the rate of cooling dramatically.

5. DISCUSSION

We have shown that the coherence length does not replace the role of the mean free path in determining the transport coefficients. We have also shown that the standard assumption of a radially slowly varying temperature gradient and a simply connected tangled magnetic field with a coherence length much shorter than the mean free path is not sufficient to provide the suppression of conduction in cluster cooling flows needed to reconcile theory and observation.

Using recent observations, we have been able to rule out the suppression of conduction by small-scale loops in the core of the cooling flow. Such a structure would very quickly become unstable and lead to reconnection in a spherical symmetric model. As pointed out previously, the crucial region where the suppression of heat conduction is required is the outskirts of the cooling flow domain. The dragged-in field lines are expected to straighten out (Soker & Sarazin 1990), and the field to be amplified, provided the field's own turbulence does not react back on the flow. These observations cannot rule out the possibility that a field with a closed-loop structure suppresses conduction in the outer regions of the cooling flow. However, it is difficult in this case to explain the transition to a multiphase flow by weak fluctuations.

This situation allows for conduction to be suppressed by small-scale loops. These loops would isolate the cooling flow region from the rest of the cluster, and once dragged in, would reconnect to give rise to the structure that is observed by the direct FRMs in cooling flows. This very special situation appears to us implausible.

There is a theoretical possibility to model the cluster by assuming that the entire cluster is dominated by the flux-tube structure. Such a model does not violate the FRM observational constraints and seems to account qualitatively for the X-ray observation. However, further study is required to explore the dynamic stability of such a structure.

The possibility that nonlocal kinematic factors may have significant effects is considered by Pistinner & Shaviv (1995b). Collective plasma oscillations are considered by Pistinner et al. (1995). Further study is needed to distinguish whether the nonlocal effects and the collective effects play a role in the cluster environment. Here we note that the answer to this question depends on the relative magnitude of the effective mean free path.

It has become unequivocally clear that the magnetic field is an inherent part even in the standard spherical symmetrical cooling flow model and induces the transition from a homogeneous cooling flow or a static configuration to a multiphased cooling flow (Loewenstein 1990; Balbus 1991; Pistinner & Shaviv 1995b). However, the results of Soker &

Sarazin (1990) and Pistinner & Shaviv (1995a) show that once an homogeneous flow is activated, the entire cooling flow region will most likely become turbulent, although Soker & Sarazin (1990) provide an argument why this back-reaction is not expected to be significant. Turbulence fails to react back on the flow if the turbulent component of the velocity and the field are parallel.

The recent FRMs and the results of X-ray observation clearly imply that the *standard* Chapman-Enskog transport is not valid in the *cluster environment* in general and in the outskirts of cooling flows in particular. The Chapman-Enskog expansion becomes valid *within the cooling flow*, implying that the Spitzer conductivity should be used, unless plasma turbulence becomes important.

All discussions on the suppression of conduction in the cluster environment considered hitherto consider the suppression by a single factor alone. However, we should not ignore the possibility of a combined effect from several factors playing together.

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APPENDIX A

THE BOLTZMANN DRIFT KINETIC EQUATION

In this appendix we discuss the applicability of the drift kinetic equation to describe the ICM. We start with the Boltzmann equation, the standard form of which reads

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \frac{1}{m}\left(\mathscr{F} + \frac{q}{c}\,\boldsymbol{v} \times \boldsymbol{B}\right) \cdot \frac{\partial f}{\partial \boldsymbol{v}} = C(f, f)\,, \tag{A1}$$

where f is the single-particle distribution function, q is its charge, and m is its mass. The external force is given by $\mathscr{F} = q\boldsymbol{E} + m\boldsymbol{g}$, where $\boldsymbol{E}$ is the external electric field and $\boldsymbol{g}$ is the gravity acceleration vector; the external magnetic field is denoted by $\boldsymbol{B}$; and t, $\boldsymbol{r}$, and $\boldsymbol{v}$ represent the time and the phase space coordinates, respectively. Finally, $C(f, f)$ is the binary collision operator written in some form under the Stosszahlansatz assumption.

We assume that the magnetic field is a simply connected vector field; in other words, there are no loops in the field. Under this assumption, the Boltzmann equation can be written in a different set of coordinates. Let $\boldsymbol{b}$ be a unit vector in the direction of the magnetic field. We define a set of unit vectors $\boldsymbol{e}_i$ by means of the following relations:

$$\boldsymbol{b} = \frac{\boldsymbol{B}}{B}\,; \qquad \boldsymbol{b} = \boldsymbol{e}_2 \times \boldsymbol{e}_3\,; \qquad \boldsymbol{e}_2 = \boldsymbol{e}_3 \times \boldsymbol{b}\,; \qquad \boldsymbol{e}_3 = \boldsymbol{b} \times \boldsymbol{e}_2\,. \tag{A2}$$

These equations define a new basis in the configuration space. This is just a formal procedure since the vectors $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$ will not appear in the final expressions. The corresponding velocity coordinates are decomposed according to

$$\boldsymbol{v} = \boldsymbol{v}_{\parallel} + \boldsymbol{v}_{\perp} = v_{\parallel}\boldsymbol{b} + v_{\perp}(\boldsymbol{e}_2 \cos\xi - \boldsymbol{e}_3 \sin\xi)\,, \tag{A3}$$

where

$$\xi = -\tan^{-1}\left(\frac{\boldsymbol{v} \cdot \boldsymbol{e}_2}{\boldsymbol{v} \cdot \boldsymbol{e}_3}\right). \tag{A4}$$

An extensive review of the subject can be found in Hinton & Hazeltine (1976). For purposes of convenience we have changed the notation slightly. Since we are dealing with a case in which gravity is important, the local gravitational acceleration is included. The expressions derived by Hinton & Hazeltine (1976) can be recovered from those given by us by replacing $\mathscr{F}$ with $q\boldsymbol{E}$ (which is equivalent to setting $\boldsymbol{g} = 0$). To verify that no overlooked subtle points are involved, we repeated their derivation and found this transformation rule justified. The derivation is straightforward but laborious.

It is more convenient to change the independent phase space variables again and use the angle ξ (eq. [A4]), the total energy of the particle ϵ, and the approximate adiabatic invariant μ. These quantities are given by

$$\epsilon = \frac{1}{2}\,(v_{\parallel}^2 + v_{\perp}^2) + \frac{q}{m}\,\phi^E + \frac{1}{m}\,\phi^G = \frac{1}{2}\,v^2 + \frac{1}{m}\,\phi^{\rm tot}\,, \qquad \mu = \frac{v_{\perp}^2}{2B}\,, \tag{A5}$$

where

$$-\nabla\phi^G = m\boldsymbol{g}\,, \qquad -\nabla\phi^E = \boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t}\,, \qquad \boldsymbol{A} = \nabla \times \boldsymbol{B}\,. \tag{A6}$$

In this set of coordinates the Boltzmann equation reads

$$\frac{\partial f}{\partial t} + (\boldsymbol{v}_{\parallel} + \boldsymbol{v}_{\perp}) \cdot \nabla f + \frac{\partial f}{\partial \mu}\frac{d\mu}{dt} + \frac{\partial f}{\partial \epsilon}\frac{d\epsilon}{dt} + \frac{\partial f}{\partial \xi}\frac{d\xi}{dt} = C(f, f)\,, \tag{A7}$$

where $d/dt = \partial/\partial t + \boldsymbol{v} \cdot \nabla$.

The expressions for the temporal derivatives of μ, ξ, and ϵ are readily obtained from the particle equation of motion. In the next step we note that $d\xi/dt \approx \Omega_L + O(\Lambda = r_L/l_b)$, where $\Omega_L = qB/mc$ is the gyrofrequency and r_L is the gyroradius. The lack of the hard X-ray tail combined with FRM in clusters of galaxies implies that $l_B \gg r_L \Rightarrow \Lambda \ll 1$. This is a necessary and sufficient condition for averaging the Boltzmann equation over the gyroangle (for more details, see Hazeltine 1973). The averaging procedure is performed with respect to the quantity ξ, which is the fastest varying quantity in the problem. Let

$$\bar{f} = \frac{1}{2\pi}\int_o^{2\pi} d\xi f\,; \qquad \tilde{f} = f - \bar{f}\,. \tag{A8}$$

With this definition Hazeltine (1973) obtains

$$\frac{\partial \bar{f}}{\partial t} + (\boldsymbol{v}_\parallel + \boldsymbol{v}_{\mathscr{C}D} + \boldsymbol{v}_{\mathscr{F}D})\cdot\nabla\bar{f} + \left\{\frac{v_\parallel}{\Omega_L}\nabla\cdot\left(\frac{\partial \boldsymbol{b}}{\partial t}\times\boldsymbol{b}\right) - \left(\frac{\boldsymbol{b}}{B}\cdot\frac{\partial \boldsymbol{A}}{\partial t}\right)\boldsymbol{b}\cdot(\nabla\times\boldsymbol{b}) + \left(\frac{v_\parallel B}{\Omega_L}\right)\boldsymbol{b}\cdot\nabla\left[\frac{v_\parallel \boldsymbol{b}\cdot(\nabla\times\boldsymbol{b})}{B}\right]\right\}\mu\frac{\partial\bar{f}}{\partial\mu}$$
$$-\left[v_\parallel\frac{\partial v_\parallel}{\partial t} - \frac{q}{mc}(\boldsymbol{v}_\parallel + \boldsymbol{v}_{\mathscr{C}D} + \boldsymbol{v}_{\mathscr{F}D})\cdot\frac{\partial \boldsymbol{A}}{\partial t}\right]\frac{\partial\bar{f}}{\partial\epsilon} = C(\bar{f}, \bar{f}) - \overline{C(\tilde{f}, \tilde{f})}\,, \tag{A9}$$

where

$$\boldsymbol{v}_{\mathscr{F}D} = \frac{c}{q}\frac{\mathscr{F}\times\boldsymbol{b}}{B} + \frac{\boldsymbol{b}}{\Omega_L}\times\left(\mu\nabla B + v_\parallel^2\boldsymbol{\kappa} + v_\parallel\frac{\partial \boldsymbol{b}}{\partial t}\right)\,; \qquad \boldsymbol{v}_{\mathscr{C}D} = \frac{v_\perp^2}{\Omega_L}\boldsymbol{b}\cdot(\nabla\times\boldsymbol{b})\boldsymbol{b}\,; \qquad \boldsymbol{\kappa} = (\boldsymbol{b}\cdot\nabla)\boldsymbol{b}\,. \tag{A10}$$

Note that $\boldsymbol{\kappa}$ is the Gaussian curvature of the field lines. Other terms in the definition of $\boldsymbol{v}_{\mathscr{F}D}$ and $\boldsymbol{v}_{\mathscr{C}D}$ have a simple physical interpretation. The first term in the equation for $\boldsymbol{v}_{\mathscr{F}D}$ is the drift term resulting from the interaction of external forces with the magnetic field. The terms in parentheses from right to left are the gradient drift term and the curvature drift. The last term and the second term in parentheses comprise the acceleration drift term. The velocity $\boldsymbol{v}_{\mathscr{C}D}$ is the velocity along the field lines resulting from the curvature of the field lines. All these terms emerge as a result of the distortion of the gyro-orbit. Thus, the function f is the phase space distribution function of the guiding centers.

APPENDIX B

FAST VARIATION OF QUANTITIES ALONG FIELD LINES

There are two known ways by which the suppression of conduction by a *steady state* magnetic field is thought to occur. The first mechanism requires that two conditions be fulfilled:

1. The field lines are sufficiently long so that the path length along field lines is long enough.
2. The field lines are tangled so that the effective distance traveled by the particle in space is short.

This mechanism was considered in some detail in § 3. The suppression factor is clearly not l_b/λ_e but a function of this variable. The numerical value of l_b/λ_e has to be determined from MHD considerations or observations.

In this appendix we discuss the second possibility, i.e., a mechanism by which the suppression of conduction and the closure relation in deriving hydrodynamic equations are postulated to depend linearly on l_b/λ_e, and the particles bounce as a result of the change in the direction of the field lines. In this model it is generally assumed that the important length scale that the determines the transport parallel to field lines is l_b, just as r_l is the length scale in transport perpendicular to the field lines. We further assume that $r_L \ll l_b$, so that equation (13) holds. To check the validity of the *assumption* that the particles bounce because of the magnetic field lines, we estimate the order of various terms in equation (13) as

$$\left(\frac{v_{\rm th}}{l_b} : \frac{v_{\rm th} r_L}{l_b^2} : \frac{v_{\rm th} r_L}{l_b^2}\right) : \frac{v_{\rm th} r_L}{l_b^2} : \frac{v_{\rm th}}{\lambda_e}\,. \tag{B1}$$

Thus,

$$1 : \Lambda = \frac{r_L}{l_b} : \Lambda : \Lambda : \frac{l_b}{\lambda_e}\,. \tag{B2}$$

The traditional mechanisms for the suppression of conduction in cooling flows require that l_b/λ_e be of the order of 10^{-2} for the homogeneous case (Cowie & Binney 1981) and of the order of 10^{-4} for the multiphase case (Balbus 1991).

The density, temperature, and magnetic field can be estimated from the observations and from them λ_e and r_L can be evaluated (to within an order of magnitude). The requirement for the suppression of the conductivity provides a constraint on l_b, namely, $\eta = l_b/\lambda_e \ll 10^{-4}$ (Balbus 1991). Hence, we can expand f in the form

$$\bar{f} = \bar{f}^o + \eta\bar{f}^1 + \eta^2\bar{f}^2 + \cdots\,; \tag{B3}$$

here $\bar{f}^o$ is the solution for a collisionless plasma, which we will later assume to be a Maxwellian. We now substitute equation

(B3) into equation (13) and equate coefficients of η. The first two equations of this expansion read

$$\boldsymbol{v}_{\parallel} \cdot \nabla \bar{f}^{o} = 0 \,, \tag{B4}$$

$$\boldsymbol{v}_{\parallel} \cdot \nabla \bar{f}^{1} + (\boldsymbol{v}_{\mathscr{C}D} + \boldsymbol{v}_{\mathscr{F}D}) \cdot \nabla \bar{f}^{o} + \left\{ \left(\frac{v_{\parallel} B}{\Omega_L} \right) \boldsymbol{b} \cdot \nabla \left[\frac{v_{\parallel} \boldsymbol{b} \cdot (\nabla \times \boldsymbol{b})}{B} \right] \right\} \mu \frac{\partial \bar{f}^{o}}{\partial \mu} = C(\bar{f}^{o}, \bar{f}^{o}) \,. \tag{B5}$$

Note that if we now make the assumption (which, unlike the case of the Chapman-Enskog expansion, cannot be rigorously justified here and, in fact, according to § 4 is not appropriate here) that $\bar{f}^{o}$ is Maxwellian, then $C(\bar{f}^{o}, \bar{f}^{o}) = 0$. We stress again that the solution $\bar{f}$ of equation (13) is a function of the magnetic field geometry (special coordinates), the bulk velocity, the phase space velocity, all thermodynamic variables, *and the gravitational potential.* In other words, $\bar{f}^{o} = \bar{f}(r, v, \epsilon, \mu)$, where $\epsilon = \epsilon(\boldsymbol{g})$ (see eq. [17]). The assumption that $\bar{f}(r, v, \epsilon, \mu)$ is Maxwellian means that the phase space dependence and the dependence on the thermodynamic variables is determined. In general, the thermodynamic variables and the gravitational potential vary in space—in particular, along field lines. From equation (B4), we see that $\bar{f}^{o}$ = constant along field lines, namely, the thermodynamic variables and the gravitational potential vary along the field lines in such a way that the guiding center velocity distribution is spatially invariant. In the case of hydrostatic equilibrium, this is possible only if the gas is isothermal. If we allow the bulk velocities to vary along field lines, we obtain an average outflow instead of inflow. Since the assumptions made in the derivation of the above equation (i.e., a fast variation *along* field lines and Maxwellian distribution function) are inconsistent with the solution of the equation, we conclude that the assumptions are contradictory. Therefore, if heat flux inhibition is caused by the tangled magnetic field, the degree of suppression depends on the correlation function of the magnetic field and not on $\eta = l_b/\lambda_e$.

The contribution of $\bar{f}^{1}$ is at most of order η. This is essentially the same result that was obtained by Stix (1992), neglecting terms of order Λ in comparison to terms of order η. If we now use the solution $\bar{f}^{o}$ = constant and assume that $\bar{f}^{o}$ is a Maxwellian, equation (B5) reduces to

$$\boldsymbol{v}_{\parallel} \cdot \nabla \bar{f}^{1} + \boldsymbol{v}_{\mathscr{F}D} \cdot \nabla \bar{f}^{o} = -\left(\frac{m^2 c v_{\parallel} v_{\perp}^2}{2kTq} \right) \boldsymbol{b} \cdot \nabla \left[\frac{v_{\parallel} \boldsymbol{b} \cdot (\nabla \times \boldsymbol{b})}{B} \bar{f}^{o} \right] = 0 \,. \tag{B6}$$

The left-hand side of equation (B6) vanishes because it contains terms that are proportional to the current parallel to the field lines. Let us see if $\bar{f}^{1}$ can vary appreciably along field lines so that it could accumulate an appreciable contribution to $\bar{f}$. In order to perform this task, we introduce the following derivative:

$$\frac{\partial}{\partial l_{\parallel}} = \boldsymbol{b} \cdot \nabla \,. \tag{B7}$$

Using this relation and equation (A10) in equation (B6), one readily obtains

$$\bar{f}^{1} = \int_0^{l_{\parallel}} \frac{dl_{\parallel}}{v_{\parallel}} \frac{\boldsymbol{b}}{\Omega_L} \cdot (\boldsymbol{g} \times \nabla \bar{f}^{o}) + \int_0^{l_{\parallel}} \frac{dl_{\parallel}}{v_{\parallel}} \frac{\boldsymbol{b}}{\Omega_L} \cdot [(\mu \nabla B + v_{\parallel}^2 \boldsymbol{\kappa}) \times \nabla \bar{f}^{o}] \approx \int_0^{l_{\parallel}} \frac{dl_{\parallel}}{v_{\parallel}} \frac{\boldsymbol{b}}{\Omega_L} [(\mu \nabla B + v_{\parallel}^2 \boldsymbol{\kappa}) \times \nabla \bar{f}^{o}] \,. \tag{B8}$$

The second approximation holds, provided, as one expects, $\boldsymbol{g}$ and $\nabla \bar{f}^{o}$ are parallel. Thus, the correction to $\bar{f}^{o}$ is at most of the order of $r_L/\lambda_e \approx 10^{-16}$. We conclude that the standard picture of conduction suppression, in which the coherence length of the field is much smaller than the mean free path in a manner that allows fast variation of quantities along the field lines, does not comply with the standard cooling flow model.

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