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MAGNETIC EFFECTS ON OSCILLATIONS IN roAp STARS

W. A. DZIEMBOWSKI[1] AND PHILIP R. GOODE[2]
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ABSTRACT

We calculate the effect of a simple dipole magnetic field on high-order *p*-mode oscillations. The stellar models, oscillation modes, and range of field intensity were selected to correspond to the data on roAp stars. We did not account for the field in the static models. Some of the modes we calculate exhibit a strong driving due to the κ-mechanism acting in the hydrogen ionization zone. This driving is only somewhat smaller than the radiative damping occurring beneath. We argue that the situation is likely to reverse after needed improvements are made in model calculations.

The effect of the field is very significant. At KG photospheric intensity, the mode frequencies are shifted by about 10–20 μHz from their nonmagnetic values. Such shifts are comparable to the small separations. Damping rates due to Alfvénic wave losses are in the 2–10 μHz range and are comparable to nonadiabatic damping rates.

Surface amplitudes significantly depart from pure, single spherical harmonic dependence, which severely complicates mode identification and observational determination of large separations. Thus, taking into account the effects of the magnetic field is a prerequisite to any meaningful roAp star asteroseismology and to understanding mode selection in these objects.

Subject headings: stars: magnetic fields — stars: oscillations — stars: peculiar

1. INTRODUCTION

The unexpected discovery of high-order *p* modes in Ap stars by Kurtz (1982) has provoked considerable interest. Several theoretical papers have been devoted to explaining his discovery. However, they left unanswered the most basic questions: the nature of the driving mechanism and the connection between the oscillations and the magnetic field. In the latter case, observational evidence leaves no doubt that such a link exists. First, the type of mode observed in roAp stars is unique to these objects. We stress that there are significant differences between these oscillations and the high-order *p*-modes excited in the Sun. In particular, the roAp star oscillations are characterized by much larger amplitudes and much greater mode stability. Second, in roAp stars the pulsations have the same symmetry as the magnetic field.

It is fair to say that essentially all of our knowledge of roAp stars reflects only direct inference from observation. The number of roAp objects discovered has expanded considerably since Kurtz's original discovery. There are now 27 such objects known (Kurtz & Martinez 1994). Further, we have learned during this time that the modes cannot be represented by a single Legendre polynomial (Kurtz 1992), and that there is evidence for cyclic changes in oscillation periods which have a timescale of years (Kurtz et al. 1994). These are new observational facts which require interpretation.

Our first goal is to explain the role of the magnetic field in the excitation of these oscillations and its part in mode selection. Our observational clues here are that only a few modes are observed in individual objects, and all of these modes are symmetric about the magnetic axis. The role of the magnetic field may be indirect, for instance, through its effect on elemental diffusion. Or, it may be direct as in Alfvénic wave losses as first noted by Roberts & Soward (1983). The effect of such losses was investigated by Spruit & Bogdan (1992) in the context of *p*-mode absorption by sunspots. We will assess the significance of these Alfvénic losses in roAp stars by comparing them with the nonadiabatic damping rates.

Our second goal is to examine the effect of the magnetic field on mode frequencies and the angular structure of the surface amplitudes. Understanding these, in our view, is a prerequisite to asteroseismic use of roAp star data.

2. MODELS

We do not have good models of roAp stars. Even if we neglect the dynamical effect of the magnetic field, we should account for the stratification of the chemical elements. All of these stars are chemically peculiar which is generally considered to be evidence for the action of elemental diffusion and in particular, gravitational settling of helium. Diffusion is not taken into account in the stellar evolution code that we use. Thus our models have chemically homogenous envelopes. For the two models we consider, we adopt the standard Population I composition ($X = 0.7$ and $Z = 0.02$). Parameters of the two models are given in Table 1. These model surface parameters are well within the range of those for roAp stars (Kurtz 1990).

One purpose in selecting these two models was to illuminate the possible driving mechanism. The form of the pulsation (size and stability of amplitudes) is similar to that in opacity-driven pulsators in the main-sequence band. Thus the opacity mechanism seems to be the most likely cause. Our nonadiabatic pulsation calculations support this conjuncture. We noted that, in the two selected models as well as in other standard models of main-sequence stars with similar effective temperatures, it is only the H ionization zone which exerts a significant driving effect for the high-order ($n > 20$) *p*-modes detected in roAp stars. The detected modes have periods in the 6–12 minute range. An example of such a mode ($n = 24$) from model 2 is given in Figure 1. For this mode, we emphasize that there is no

[1] Copernicus Astronomical Center, ul. Bartycka 18, 00-716 Warszawa, Poland. Also Physics Department, New Jersey Institute of Technology.

[2] Physics Department, New Jersey Institute of Technology, 323 Martin Luther King Boulevard, Newark, NJ 07102. Also Copernicus Astronomical Center, Warsaw, Poland.

TABLE 1

roAp Model Parameters

Model	$M/M_{\odot}$	$R/R_{\odot}$	$\log T_{\rm eff}$	$\log L/L_{\odot}$	X_c
1	1.8	1.529	3.9235	1.014	0.70
2	2.0	2.138	3.9240	1.307	0.38

driving in the He II ionization zone. Rather, all the driving comes from the hydrogen ionization zone. The figure also shows that the driving is not quite strong enough to compensate for the damping which ultimately renders the mode stable. This is in sharp contrast to the situation of the lower frequency mode ($n = 7$) shown in Figure 1 for which the hydrogen ionization zone plays a negligible role in the mode's stability, while the He II ionization zone provides all of the driving. This mode is unstable, and it has a frequency like those detected in δ Scuti stars. This sharp contrast between the work integrals in these two modes has two causes. The lack of any significant driving in the hydrogen ionization zone for the $n = 7$ mode is due to the fact that the mode period is much longer than the thermal timescale. On the other hand, for the $n = 24$ mode, the lack of driving in the He II ionization zone is related to the oscillatory behavior of the eigenfunction in that zone which implies a dominance of the radiative losses over the opacity effect driving.

One can easily imagine that if He settles in roAp stars, as is plausible, then the mode owing its driving to the H ionization zone becomes unstable while the one owing to the He ionization zone becomes stable. If this were true, then there is an indirect role for the field. That is, the field slows rotation and thereby renders meridional circulation insignificant which in turn enables gravitational settling to occur.

D. Kurtz & J. Matthews (1995, private communication) raised an objection to this proposal pointing out to us that invoking a similar argument coupling slow rotation and settling, we should expect excitation of high-order p-modes in Am stars, which are nonmagnetic, slowly rotating chemically peculiar stars, at corresponding $T_{\rm eff}$ values. However, short-period oscillations in Am stars have not been detected. This objection must be kept in mind, but we would like to stress that these stars would not be the first ones in which oscillations predicted by linear stability calculations have not been detected. Thus, we regard the opacity mechanism acting in the H ionization zone as the most plausible cause of roAp star oscillations.

The driving by the hydrogen ionization zone begins to be appreciable in model 2 at $n \sim 20$ for low l modes. Of course, our interest in modes of low degree is motivated by the observational data. Also in model 1, we begin to see the driving effect of the hydrogen ionization zone at similar n values. However, in this latter model, the maximal driving occurs above the acoustic cut-off frequency. Because of uncertainties in modes close to the critical cutoff, we avoided approaching them. In fact, the maximal driving occurs for the untrapped modes (modes with frequency above the acoustic cutoff). But there, we find large damping due to the acoustic losses.

Our models were calculated with the use of the Eddington approximation for radiative transfer which is inadequate in the atmosphere and, in particular, results in an underestimate of the minimum temperature and, consequently, the critical acoustic frequency (Weiss 1986). We leave for future work both a proper accounting of the effect of diffusion and an improvement in the treatment of atmospheric layers in our models.

The behavior of the work integral for the $n = 24$ mode demonstrates that the nonadiabatic effect on the mode is important in the very outermost regions. More or less in the same layers, magnetic effects are important as is clear from Figure 1b. In detail, Figure 1b shows that the layer in which the magnetic field is strongly coupled to the oscillations ($\beta \sim 1$) encompasses only about the outer 1% of the star by radius.

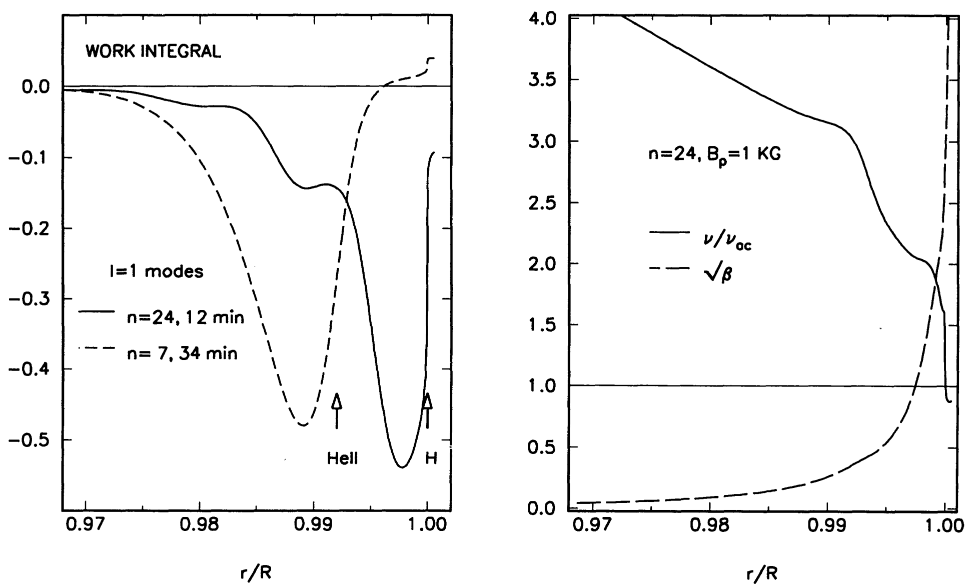

FIG. 1.—(*left*) Cumulative work integral (in arbitrary units), calculated from the center to the surface, for selected p-modes in Ap star model 2. A local decrease of the work integral implies local dissipation and an increase implies local driving. The total integral determines the mode stability where values greater than zero imply mode instability. (*right*) The $(\beta)^{1/2}$ is the ratio of the Alfvén speed to the adiabatic speed of sound. $\nu/\nu_{\rm ac}$ is the ratio of the frequency to the acoustic cut-off frequency for an isothermal atmosphere. Thus acoustic propagation occurs if $\nu \gtrsim \nu_{\rm ac}$. The magnetic field decouples from acoustic oscillations if $\beta \ll 1$.

Although it should be expected that both magnetic and nonadiabatic effects are globally small, they are locally large and cannot be treated by standard perturbation techniques. It is known, for instance, that the application of the so-called quasi-adiabatic treatment of the nonadiabatic effect leads to completely unreliable results. In spite of these caveats, we will adopt an unjustified superposition approach so that we separately treat magnetic and nonadiabatic effects and compare them. We think that this separation will nonetheless allow us to assess the role of the two effects. In this first work, we deal with the magnetic field in an adiabatic approximation which is difficult enough.

Even in the layers where β is very small, care must be taken because of the singular nature of the perturbation. No matter how weak the field is, it introduces Alfvénic waves. As first noted by Roberts & Soward (1983), these modes introduce a new means of energy dissipation, and therefore are important in mode stability considerations.

3. FORMALISM

We assume that the magnetic field is curl-free and purely dipolar in nature,

$$\boldsymbol{B} = B_p\left(\frac{R}{r}\right)^3\left(\cos\theta \boldsymbol{e}_r + \frac{\sin\theta}{2}\boldsymbol{e}_\theta\right), \tag{1}$$

where $\boldsymbol{e}$ and $\boldsymbol{e}_\theta$ are unit vectors in the spherical coordinate system. We consider only the case of adiabatic oscillations that are symmetric about the magnetic axis. Thus, the displacement for an oscillation may be represented in the following form:

$$\xi = r[y(r, \theta)\boldsymbol{e}_r + z(r, \theta)\boldsymbol{e}_\theta] \exp(i\omega t) . \tag{2}$$

Our aim is to determine eigenfrequencies, ω, as well as corresponding eigenfunctions y and z. Below we outline how we do it. Some additional details are given in the Appendix.

Magnetic corrections to p-mode frequencies in an Ap star model have already been calculated by Shibahashi & Takata (1993). We believe, however, that the approximation they adopt cannot be justified and introduces an uncertainty which is impossible to assess. The authors acknowledge the uncertainty in their treatment. They applied a standard perturbation treatment, ignoring the fact that the magnetic perturbation cannot be treated as small in the outer layers and that the perturbation is singular (raising the differential order of the system of equations).

In our treatment of the magnetic boundary layer, we use a local plane-parallel approximation at each latitude, ignoring all angular derivatives of the perturbed quantities. The latter approximation is a consequence of focusing our interest on modes of low angular degree. The bottom of this layer, $r = r_{\text{fit}}$, was determined by the condition $\beta \ll 1$, for which the typical r_{fit} ranged from 0.98 to 0.99. At $r < r_{\text{fit}}$, a full decoupling between Alfvénic and acoustic oscillations was assumed,

$$\xi = \xi_A + \xi_p . \tag{3}$$

The Alfvénic modes are described by the WKB approximation, and following the suggestion of Roberts & Soward (1983), an ingoing wave solution was adopted. The ingoing Alfvénic waves are assumed to be dissipated well before they would have reached $r = 0$. The ξ_p obey the ordinary, nonmagnetic equations of stellar pulsation. The boundary layer solution, at $r = r_{\text{fit}}$, yields for each trial ω, the complex function $\mathscr{F}$, where

$$\left(\frac{\delta p}{p}\right)_p = \mathscr{F}(\theta)y_p , \tag{4}$$

where δp denotes a Lagrangian pressure perturbation, which with use of the continuity equation and the adiabaticity condition (eqs. [A2] and [A3] of the Appendix) may be expressed in terms of y and z. The determination of the boundary layer solution for the $\mathscr{F}(\theta)$ is presented in detail in the Appendix. Equation (4) may be regarded as an outer boundary condition imposed on the solution of the oscillation equations satisfying the inner boundary conditions. Additional boundary conditions imposed on the solutions at $r = r_{\text{fit}}$ follow from the requirement that the perturbed gravitational potential is described by a decreasing solution of the Laplace equation, which is a consequence of negligible mass of the magnetic layer.

Since $\mathscr{F}$ depends on θ, the solution cannot be described by a lone Legendre polynomial. We consider instead a truncated expansion

$$y_p = \sum_k D_k y_{p,k}(r)P_k(\cos\theta) , \tag{5}$$

of consecutive Legendre polynomials of the same parity, where the $y_{p,k}$ are solutions for degree k of the equation for oscillations satisfying the inner boundary condition at $r = 0$. The parity selection arises because $\mathscr{F}(\theta)$ is symmetric about the equator. We note that Kurtz (1992) finds that in the case of HR 3831 the data require expansion of the surface amplitude in terms of both odd and even Legendre polynomials. The implication of his finding is that the magnetic field cannot be a pure dipole.

Using this expansion and the corresponding one for $\delta p/p$ in equation (4), we obtain the following homogeneous system of equations for the D's with ω being a complex eigenvalue,

$$\sum_k \left[\delta_{l,k}\left(\frac{\delta p}{p}\right)_{p,k} + F_{l,k}y_{p,k}\right]D_k = 0 , \tag{6}$$

where

$$F_{l,k} = \frac{1}{2l+1}\int_0^{\pi/2} P_l \mathscr{F} P_k \sin\theta\, d\theta . \tag{7}$$

The frequency ω occurring implicitly in $\mathscr{F}$, y, and $\delta p/p$ is the eigenvalue. Although the system is adiabatic, it is, nonetheless, complex because of the wave losses. The imaginary part of the eigenfrequency, $\Im(\omega)$, yields the amplitude damping rate.

In most of the cases considered, an accuracy of 0.01 μHz was achieved with five to nine terms included. However, in some cases of interest close resonances cause problems. We will discuss such a case in the next section.

For the remainder of the paper, we use cyclic frequencies, ν, instead of angular frequencies, ω, since the former are favored by observers.

4. SOLUTIONS

In model 2, we considered modes for which the nonmagnetic mode identifiers are in the ranges $l = 0$–3 and $n = 21$–24. These are the modes which exhibit a significant driving effect from the hydrogen ionization zone. We varied B_p from 0.5 to 1 KG—a typical range for roAp stars. Figures 2 and 4 show the real frequency shifts due to such fields for four selected modes. The changes of these shifts with increasing field are complicated and strongly affected by an interaction with adjacent modes of the same parity. One may see that the size of the magnetic shifts is comparable to the frequency separation in the nonmagnetic limit. To measure the degree of the interaction, we calculated contributions to the total kinetic energy of the mode from various terms of the expansion in equation

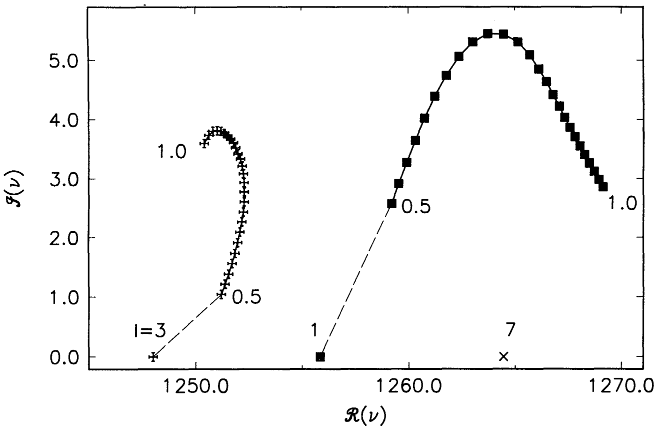

FIG. 2.—The real [$\mathscr{R}(\nu)$] and imaginary [$\mathfrak{I}(\nu)$] parts of oscillation frequencies (in μHz) in the presence of the dipole magnetic field having B_p = 0.5–1.0 KG for the modes which in the nonmagnetic limit are $l = 1$, $n = 22$ and $l = 3$, $n = 21$. The nonmagnetic frequencies for l = 1, 3, and 7 mode are shown. Frequencies of the l = 5 and 9 are outside the range.

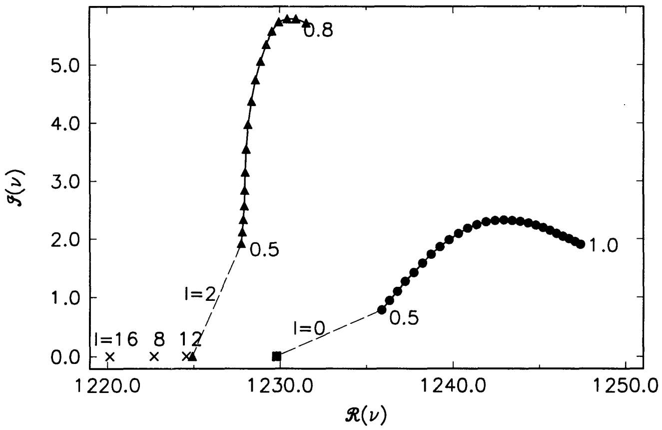

FIG. 4.—The same as Fig. 2, but for modes which are, in their nonmagnetic limit, $l = 0$, $n = 22$ and $l = 2$, $n = 21$. Positions of nearly resonant modes of the same parity are shown in the nonmagnetic limit. The calculation for $l = 2$ was terminated at B_p = 0.8 KG because the truncation failed.

(5). These contributions denoted E_k, for the four modes selected are shown in Figures 3 and 5.

For the case of the odd parity modes illustrated in Figures 2 and 3, we observe a strong interaction between modes beginning with $B_p \approx 0.65$ KG. The $l = 1$ mode has a strong $k = 7$ component, which is a consequence of a near-resonance. At a somewhat stronger field, modes of $l = 1$ and 3 undergo an avoided crossing. In spite of a significant contamination from the higher degree components, the truncation works quite well in the range of field intensity we consider.

Turning now to the case of even parity modes illustrated in Figures 4 and 5, we call attention to a complicated situation arising for the case of the $l = 2$ mode. The complications originate because of the proximity of modes with l = 8, 12, and 16. We had to terminate the calculations at B_p = 0.8 KG because in extending the series in equation (5) up to $k = 28$, we could not achieve the requisite stability in the calculated eigenfrequencies. It is important to realize that one cannot continue the expansion to too large a k-value because in our treatment of the magnetic layer, we assume that the derivatives of the perturbed quantities with respect to θ may be ignored. This approximation is not valid if the expansion involves Legendre polynomials of high degree.

The situation for other modes in the n = 21–24 range in model 2 is very similar to that shown in Figures 2–5, except that the problem concerning the $l = 2$ mode appears at a somewhat weaker field for the higher n's. In model 1, we also considered modes with l = 0–3 and obtained similar results, although we did not encounter the aforementioned difficulties with $l = 2$ modes. In Figure 6, we plot frequency shifts due to a 1 KG field for model 2 modes with l = 0–1 in a wide frequency range. Through most of this range, until about 2300 μHz, the

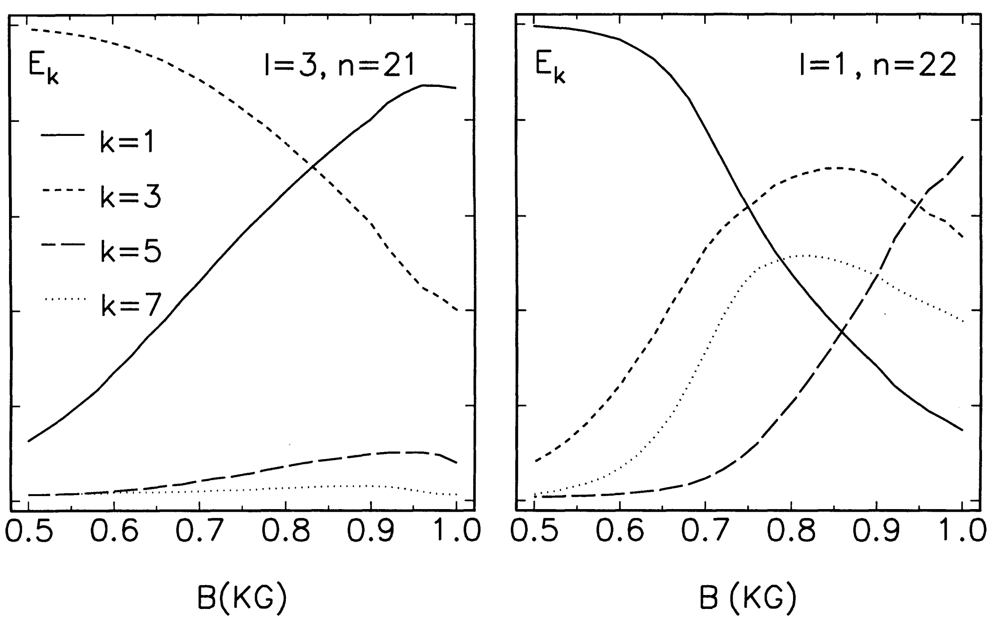

FIG. 3.—The contribution to mode energy from the components of various Legendre polynomial degrees, k, for the same two modes presented in Fig. 2. The expansion given in eq. (5) was truncated at k = 19 and the total energy was normalized to unity. The components are shown as a function of magnetic field strength.

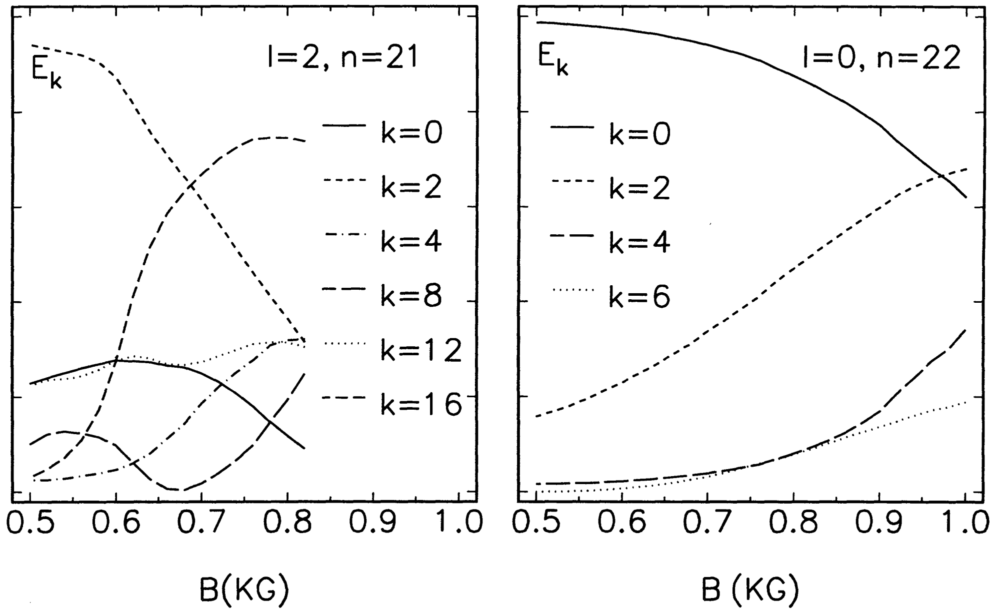

FIG. 5.—Counterpart of Fig. 3 for the modes considered in Fig. 4. The expansion given in eq. (5) was truncated at $k = 28$. For the $l = 2$ case, there are exceptionally large contributions from k values of 12 and 16.

energy of these modes is clearly dominated by the component due to the nominal l value. Evidently, the real mode mixing occurs relatively close to the acoustic cut-off frequency, which in model 1 is about 1500 μHz and in model 2 is about 2700 μHz.

It is not surprising that the effect of the magnetic field increases with mode frequency. To understand it, let us go back to Figure 1 and observe in the right panel that for $n = 24$, the magnetic layer ($\beta \gtrsim 1$) encompasses a part of the acoustic propagation zone ($\nu \gtrsim \nu_{ac}$). For lower frequency modes, this common part shrinks and it disappears at a frequency about half that of the $n = 24$ mode. Only perturbations occurring in the propagation zone change the frequency in a significant way. Perturbation of the outer boundary condition imposed in the evanescent zone has only a very small effect on the frequencies. The modes detected in roAp stars are of high order and are among those most strongly affected by KG magnetic fields. Furthermore, it is only for these modes that we see any significant driving in the hydrogen ionization zone.

One should note in Figure 6 that there is a smooth change in the frequency shift with ν_0 demonstrating that we should not expect appreciable changes in the large separations ($\nu_{l,n} - \nu_{l,n-1}$) due to the magnetic field. We should emphasize, however, that even for relatively weak fields, the flux variations

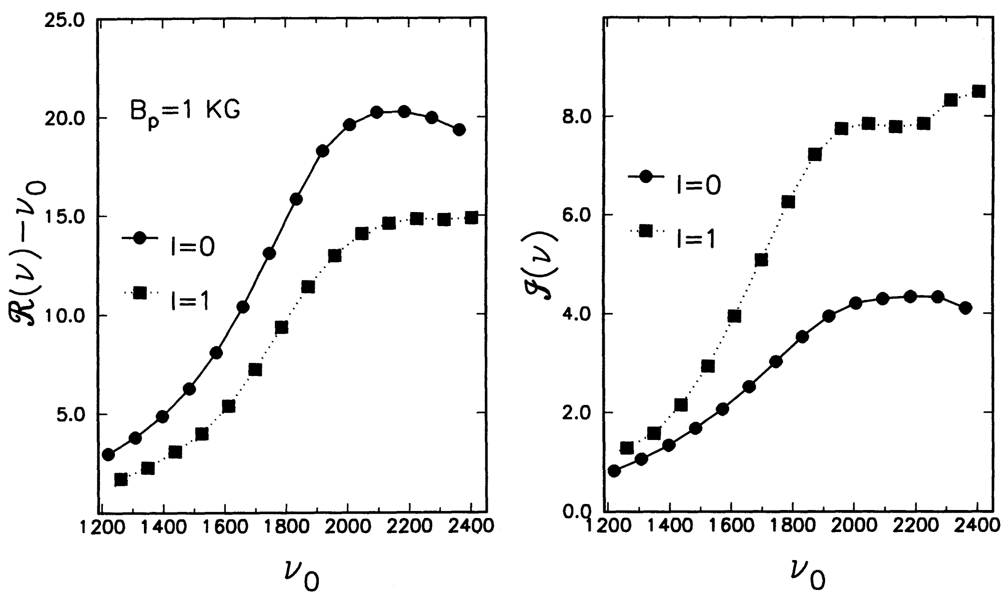

FIG. 6.—Real and imaginary frequency shifts, from their unperturbed values, due to a $B_p = 1$ KG. These shifts are plotted against the unperturbed frequency, ν_0.

over the surface are not described by a single spherical harmonic. This leads to the real possibility of confusing the large separations. We will discuss this point further in § 6.

5. SMALL SEPARATIONS

In the absence of magnetic fields, the so-called small separations probe the deep interior (Christensen-Dalsgaard 1988; Ulrich 1988). The most commonly used definition of the small separations is

$$S_{l,n} = \nu_{l,n+1} - \nu_{l+2,n} . \quad (7)$$

In the asymptotic limit ($\nu \to \infty$), there are simple relations between the $S_{0,n}$ and the $S_{1,n}$. Here it is useful to consider them separately, as well as considering one additional small splitting,

$$s_n = \frac{\nu_{0,n} + \nu_{0,n+1}}{2} - \nu_{1,n} . \quad (8)$$

Figure 7 shows that even for a weak magnetic field the small separations are significantly perturbed. Therefore, in roAp stars the small separations certainly cannot be regarded as a probe of the internal structure.

This conclusion may sound disappointing. On the other hand, our results point out the possibility of probing the structure of subsurface magnetic fields in Ap stars, which is very important. For the sake of simplicity we assumed that the field is purely dipolar. Therefore, its vital radial structure was prescribed. Here we emphasize that the assumption lacks theoretical justification and that it should be observationally tested.

6. SURFACE AMPLITUDES OF THE MODES

Oscillations of roAp stars are observed as fluctuations in intensity. It may not be too inaccurate to assume that the bolometric intensity has the same angular dependence as the relative pressure amplitude. This assumption would certainly be true in the case of high-order *p* modes in the nonmagnetic limit. The pressure amplitude is directly obtained from our solution of the oscillation equations. In Figure 8 (*top*), these amplitudes are shown for the modes previously considered for model 2. The values refer to the lowest field considered (0.5 KG) for which all of these modes maintain their nonmagnetic identity insofar as the contribution to mode energy is concerned. For the $l = 2$, $n = 21$ mode, the dominant contaminant is from the P_{12} component.

Our models are not sophisticated enough to allow us to make detailed comparisons with observations like those of Kurtz (1992). We have, however, demonstrated that one must expect a strong contamination of the surface amplitudes from high-order modes. This contamination may well be the critical clue needed to explain the sizable harmonic amplitudes found in Ap stars at relatively low amplitudes of the fundamental frequency implying that the pulsations are more nonlinear than their observed amplitudes may suggest.

In Figure 8, we also show the contribution to the luminosity after averaging over the disk. Naturally only low-*k* components survive. Still, the contributions are not pure. Note, for instance, that the $l = 3$ case shown may be observed as a dipole mode. Other calculations reveal that modes with higher nominal l may contain sizable contamination from $l = 0$ or 1 components. Therefore, they may be detectable. Imagine, for instance, a mode with a nominal l value of 7 which has a 10%–20% contamination by $l = 1$—which would not be uncommon for the magnetic fields and modes we have been discussing. Such a mode would be identified by observers as $l = 1$ because the amplitude reduction of $P_7(\cos\theta)$ is about 3×10^{-3}, while for $P_1(\cos\theta)$ it is 0.7. This misidentification could lead to real problems in determining large splittings.

7. THE ROLE OF THE MAGNETIC FIELD IN DAMPING OSCILLATIONS

How significant is the effect of the magnetic field on mode stability? This could be assessed by comparing the imaginary parts of the ν values arising from the field to those arising from

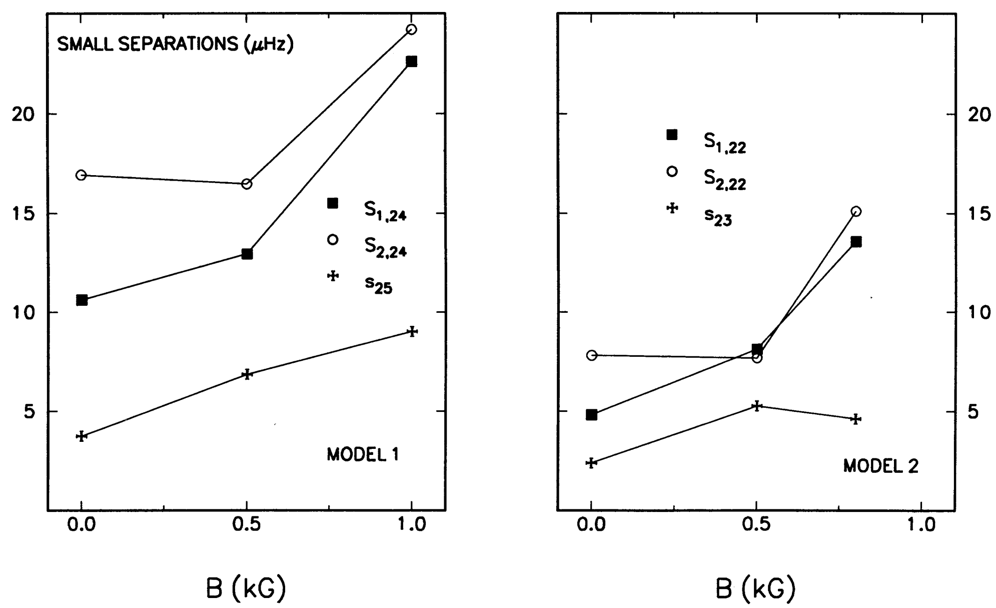

FIG. 7.—The small separations defined in eqs. (7) and (8) are shown in the absence of a magnetic field and at two finite values of the field. For model 2, the higher value of the field is 0.8 KG which was selected because of the problems at 1 KG for the $l = 2$ modes.

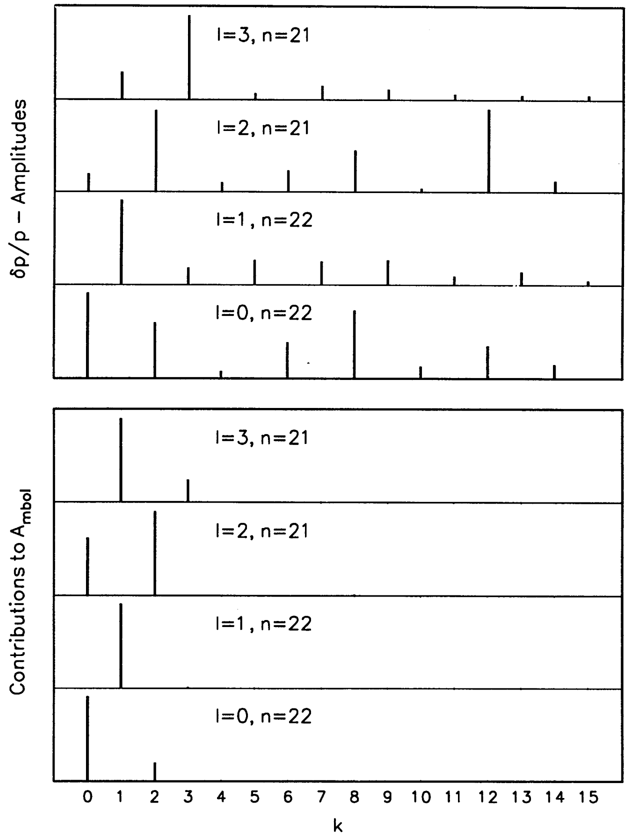

FIG. 8.—(*top*) Legendre polynomial expansion for the relative pressure amplitude at the photosphere for four modes of model 2 at $B_p = 0.5$ KG. (*bottom*) Contribution to bolometric amplitude variation calculated under the assumption that the bolometric intensity follows the angular dependence of the relative pressure amplitude. The results are averaged over random orientations of the axis of the magnetic field.

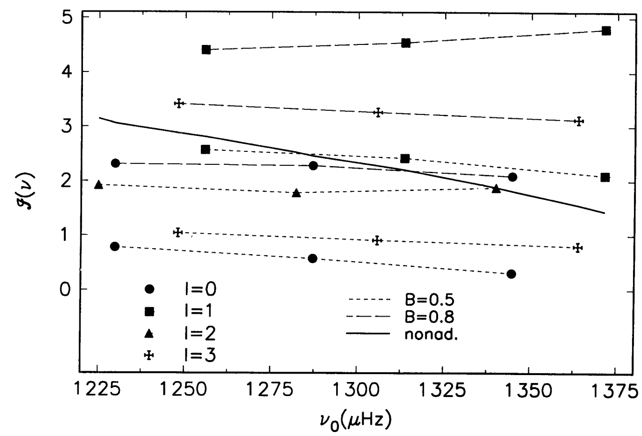

FIG. 9.—The imaginary part of the frequency for selected modes of model 1 at selected values of the magnetic field compared with the same quantity due to nonadiabatic effects.

the interaction with the radiative field obtained from a standard nonadiabatic oscillation code. As we have pointed out in § 2, though such calculations reveal significant hydrogen ionization zone driving in some high-frequency modes, they do not show net extinction [$\Im(\nu) < 0$]. The driving effect, therefore, manifests itself only in a depression of the value in Figure 9. Actually, the maximum driving effect occurs somewhat above the upper value of the range of frequencies in the figure and there the minimum value of $\Im(\nu)$ is about 0.8 μHz. These values are compared to those due to the magnetic field effect at $B_p = 0.5$ and 0.8 KG. We should look with some reservation at this comparison because nonadiabatic and magnetic effects should be treated simultaneously. Two observations, however, can be made from these results. First, the damping due to Alfvén losses is very significant. And in contrast to the damping due to nonadiabatic effects, those due to Alfvén losses are strongly l dependent. This implies that we should expect value of l to be important in the selection of unstable modes. We see that $l = 0$ modes are the least affected by the Alfvénic losses. We can see in Figure 9 that this difference is quite robust. This seems to be in sharp contrast to the observational finding that the $l = 1$ modes dominate in roAp starlight variations. We should point out, however, that in model 1 we found the $l = 3$, 5, and 7 modes to be less damped than those with $l = 0$, and that for these modes the surface luminosity amplitude variation is dominated by the $l = 1$ component.

8. CONCLUSIONS

We have shown that magnetic fields of kilogauss intensity, typical for roAp stars, significantly affect the properties of p-mode oscillations. The typical change in frequencies induced by such fields range up to 20 μHz. Alfvénic wave dissipation implies damping rates in the range of 2–10 μHz, and as such are comparable to damping rates due to nonadiabatic effects. We have also seen that the intensity distribution of individual eigenmodes over the surface can no longer be described by a single spherical harmonic. The departures from mode purity are quite large even for situations in which mode energies differ little from their pure p-mode antecedents in the nonmagnetic limit.

We have argued that the most probable driving mechanism of roAp oscillations is the κ mechanism operating the hydrogen ionization zone. In standard stellar models, we did not find modes which are actually unstable, but we have ones that are very close to instability. That is, driving in the hydrogen ionization zone nearly compensates the radiative damping beneath. We argued that departures from simple, standard models such as gravitational settling of helium or improved treatment of atmospheric layers could easily render some modes unstable. Enhancement of the driving effect must also be large enough to compensate the aforementioned Alfvénic wave losses.

In spite of improvements in models, our treatment of the stellar oscillations is still too crude to enable us to compare model predictions with observational data. What we have actually demonstrated is the need to account for magnetic field effects in the description of roAp star oscillations. In fact, we regard this as a prerequisite to any meaningful effort in the asteroseismology of these stars. The crucial improvement in treating oscillations is our inclusion of nonadiabatic and magnetic effects. However, our simple Eddington description of radiative transfer in the outer layers is certainly inadequate. Furthermore, to understand the alignment of the modes with the field, we need to consider modes which are not symmetric about the magnetic axis.

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APPENDIX

BOUNDARY LAYER APPROXIMATION

Our treatment of the magnetic boundary layer may be regarded as a special case of that due to Campbell and Papaloizou (1986). Our specialization is for low-degree oscillations symmetric about the magnetic axis. Thus, the system of equations we solve is reduced to fourth order in r. Terms involving the horizontal wave vector are absent. The only important difference concerns the outer boundary for which they used a polytropic model. We, on the other hand, use an infinite isothermal atmosphere. The main advantage of our approach, however, lies in a rigorous treatment of the eigenvalue problem for the whole model as it has been described in § 3. We feel that the method of Campbell & Papaloizou, consisting of solving separate eigenvalue problems at each θ, does not provide a useful approximation for roAp star models.

We present here an outline of the method leading to the determination of $\mathscr{F}(\theta)$ given in equation (4) which is our crucial equation in determining the eigenfrequencies. We begin with the equations for linear, adiabatic oscillations in the presence of a curl-free magnetic field,

$$\omega^2 \xi = \frac{1}{\rho} \nabla p_1 + g e_r \frac{\rho_1}{\rho} + \frac{\boldsymbol{B} \times (\nabla \times \boldsymbol{B}_1)}{4\pi\rho} , \tag{A1}$$

$$\delta\rho = -\rho \nabla \cdot \xi , \tag{A2}$$

$$\delta p = c^2 \delta\rho , \tag{A3}$$

where $c^2 = \Gamma p/\rho$ and Γ is the adiabatic exponent and

$$\boldsymbol{B}_1 = \nabla \times (\xi \times \boldsymbol{B}) , \tag{A4}$$

where $\boldsymbol{B}$ and ξ are given by equations (1) and (2). The subscript " 1 " denotes an Eulerian perturbation of the respective quantity, " δ " denotes a Lagrangian perturbation, and g is the gravitational acceleration.

In our boundary layer approximation, we ignore derivatives with respect to θ, assume $r \approx R$, and $|(r/a)(da)/(dr)| \gg 1$, where a is y, z, or a thermodynamical parameter in the unperturbed model. With these approximations, we get from equation (A4)

$$\boldsymbol{B} \times (\nabla \times \boldsymbol{B}_1) = \frac{B_p^2}{R \sin\theta} \frac{d^2 h}{dx^2} \left(-\frac{\sin\theta}{2} \boldsymbol{e}_r + \cos\theta \boldsymbol{e}_\theta \right) , \tag{A5}$$

where

$$h = \frac{y}{2} \sin^2\theta - z \sin\theta \cos\theta , \tag{A6}$$

and $x = r/R$. Except for the special cases of the pole and the equator which will be considered below, equations (A1)–(A6) lead to

$$\frac{d^2 y}{dx^2} + \left(\frac{d \ln \Gamma}{dx} - V \right) \frac{dy}{dx} + \left(\frac{\omega r}{c} \right)^2 \left(y + \frac{\tan\theta}{2} z \right) = 0 \tag{A7}$$

and

$$\frac{d^2 h}{dx^2} - \frac{1}{\beta} \left(\frac{\omega r}{c} \right)^2 \frac{\tan\theta}{2} z = 0 , \tag{A8}$$

where

$$\beta = \frac{B_p^2}{4\pi p \Gamma} , \tag{A9}$$

and $V = g r \rho / p$. At the magnetic pole ($\theta = 0$), the oscillations are decoupled from the magnetic field, and we use equation (A7) with $z = 0$. At the equator ($\theta = \pi/2$), we have $h = y/2$ and the system again reduces to a second-order equation,

$$\frac{d^2 y}{dx^2} + \frac{4}{4+\beta} \left[\left(\frac{d \ln \Gamma}{dx} - V \right) \frac{dy}{dx} + \left(\frac{\omega r}{c} \right)^2 y \right] = 0 . \tag{A10}$$

We apply the outer boundary condition in the atmosphere at very low optical depths where we may crudely regard the atmosphere as being isothermal. We require there that $\beta \gg 1$. The solution for h should be such that it can be continuously fitted to the vacuum solution, which implies that

$$\frac{dh}{dx} = 0 . \tag{A11}$$

The exact form of the boundary condition does not matter as long as $|dh/dx| \ll V$. Equation (A7) with V and Γ being constant and

$h = 0$ has an exponential solution which yields

$$\frac{dy}{dx} = \lambda y , \tag{A12}$$

where

$$\lambda = \frac{V}{2}\left[1 - \sqrt{1 - \frac{\omega^2}{\omega_{ac}^2}\left(1 + \frac{\tan^2\theta}{4}\right)}\right], \tag{A13}$$

and $\omega_{ac} = Vc/2r$. The choice of the branch of the square root is such that for real values, the solution describes trapped modes and for imaginary values it describes outgoing waves. Note that near the equator, trapping never occurs because of the behavior of the tangent.

In the opposite limit ($\beta \ll 1$), an asymptotic decomposition of the system occurs beneath $r = r_{fit}$, as already discussed in the papers of Roberts and Soward (1983) and Campbell & Papaloizou (1986). One solution is the one describing p-modes in the absence of the magnetic field. It corresponds to the choice $z \to 0$. Then, from equation (A6) we have

$$h_p = \frac{y_p}{2}\sin^2\theta . \tag{A14}$$

The other solution is rapidly oscillating and it describes Alfvénic wave. Equation (A8), where we may neglect y, yields

$$h_A \propto \exp(i\Phi_A) , \tag{A15}$$

with

$$\frac{d\Phi_{Al}}{dx} = \frac{\omega r}{c\sqrt{\beta}\cos\theta} . \tag{A16}$$

Following Roberts & Soward's suggestion we selected the branch describing the ingoing wave. For this solution equation (A7) yields

$$y_A = \frac{\beta}{2} h_A . \tag{A17}$$

For intermediate values of θ, we have two sets of solutions for y and h satisfying the outer boundary condition. Thus, at $r = r_{fit}$ write

$$C_1 y_1 + C_2 y_2 = -\frac{\beta}{2} h_A + y_p \tag{A18}$$

and

$$C_1 h_1 + C_2 h_2 = h_A + \frac{\sin^2\theta}{2} y_p . \tag{A19}$$

There are obvious analogous equations for the derivatives of y and h. The resulting system of four equations involves five unknown quantities, C_1, C_2, y_p, y'_p, h_A. It allows us to express, in particular, y'_p in terms of y_p for each θ and trial eigenvalue. From the relation $\delta p/p = -\Gamma dy/dx$, we then obtain $\mathscr{F}(\theta)$ in equation (4).

The system of equations (A6)–(A8) was solved by the Runga-Kutta method with an adjustable step. Note that near the equator, oscillatory behavior occurs much earlier than near the pole. The step size is adjusted so that $\mathscr{F}(\theta)$ behaves smoothly. In standard calculations, we used 64 gridpoints for the colatitude, but the results were insensitive to this choice as long as more than 20 steps were employed.

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