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ON THE DESTRUCTION AND OVERMERGING OF DARK HALOS IN DISSIPATIONLESS N-BODY SIMULATIONS

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ABSTRACT

N-body simulations of collisionless dark matter have failed to produce galaxy halos or substructure within the dense environments of clusters. We investigate the "overmerging" problem using analytic results and new simulations designed to calculate destruction times of halos owing to numerical and physical dynamical effects. Current numerical resolution is sufficient to suppress mass loss from two-body relaxation and particle-halo heating. Substructure in these simulations can still be destroyed by the combined action of large-force softening together with tidal heating by the cluster and encounters with other dissolving halos. In the limit of infinite numerical resolution, whether or not individual halos or substructure can survive depends sensitively on their inner density profiles. Singular isothermal halos should always survive at some level, although the computational cost of resolving halo cores becomes very large. However, if halos form with large core radii, then the overmerging problem will always exist within dissipationless *N*-body simulations. In this case a dissipational component can increase the halos central density, enabling galaxy halos to survive.

Subject headings: dark matter — galaxies: formation — galaxies: halos — galaxies: interactions — large-scale structure of universe — methods: numerical

1. INTRODUCTION

Understanding how structure evolves in the universe is a fundamental problem in cosmology. Cold dark matter–dominated universes have been extensively investigated with some success at reproducing observations from galactic to cluster scales (e.g., Davis et al. 1985; White et al. 1987). Structure formation within these models proceeds in a bottom-up fashion, with small overdensities in the mass distribution collapsing first and subsequently merging hierarchically to form larger objects. Galaxies form from gas that has cooled within the deep potential wells provided by the dark matter halos and the accretion of gas-rich subclumps that are part of the merging hierarchy (White & Rees 1978). The dominant force that drives structure formation is gravity, hence numerical simulations have proved a useful tool to study the nonlinear growth of structure formation over a wide range of dynamical scales.

Several authors have attempted to study the formation of single rich galaxy clusters within a cosmological context in order to obtain very high spatial and mass resolution (Carlberg & Dubinski 1991; Summers 1993; Carlberg 1994; Evrard, Summers, & Davis 1994). Their aim was to resove individual halos in clusters at the present epoch. However, even with mass and length resolution $\sim 10^9\ M_\odot$ and ~ 15 kpc, respectively, no halos were found to survive within a distance of at least half the virial radius from the cluster center. The so-called "overmerging" problem, or the inability to resolve substructure within dense environments, was first noted by White et al. (1987) and Frenk et al. (1988). This problem is most serious when comparing cosmological models with the observations. In order to create artificial galaxy catalogs it is customary to identify galaxies with overdense peaks in the simulated mass distribution; i.e., peaks with one-dimensional velocity dispersion ~ 150 km s^{-1} could be identified with L_* galaxies (the Milky Way is close to an L_* spiral galaxy). However, the cluster-sized objects that actually form lack significant substructure or smaller peaks, leading investigators to artificially populate dense regions in the dark matter distribution with "galaxies" (Davis et al. 1985; Gelb & Bertschinger 1994).

Clusters in the real universe contain many galaxies that have retained their dark halos (at least within their optical radii). When the overmerging problem was first noticed, it was suggested that cooling gas would increase the central density of the halos, helping them stay intact within the cluster. Preliminary results from smoothed particle hydrodynamic simulations show that galaxy-like clumps of gas and dark matter do survive to the present epoch (Katz, Hernquist, & Weinberg 1992; Katz & White 1993; Summers 1993; Evrard et al. 1994; Navarro, Frenk, & White 1995). These techniques are potentially very valuable but are presently limited by their low resolution. Summers, Davis, & Evrard (1995) discuss different methods for identifying galaxy tracers within dissipationless simulations. They argue that the dynamics of the galaxy tracers identified in Carlberg (1994) is different from the dynamics of the clumps that form after a gaseous component is included.

In this paper we discuss the mechanisms that cause dark halos to dissolve in dense environments within a Hubble time. Artificial numerical effects include two-body relaxation of particles within the softened dark halos and the heating of halos by artificially massive *N*-body particles. Two physical heating effects will always be present, even in the limit of infinite numerical resolution: tidal heating of halos on eccentric orbits and impulsive heating from rapid encounters between halos. We shall use numerical simulations to follow the evolution of halos within a cluster environment in order to isolate these effects and test analytic estimates of the dissolution timescales. Our aim is to determine what erases substructure in present simulations and to determine whether or not future simulations at higher resolution will be able to resolve halos within dense environments.

2. DISRUPTION MECHANISMS

2.1. *Numerical Effects*

2.1.1. *Relaxation*

A galaxy halo with internal velocity dispersion σ_h at a distance R_c from a cluster with velocity dispersion σ_c will have a tidally limited mass

$$m_h = 7.8 \times 10^{11}\, M_\odot \left(\frac{\sigma_h}{150 \text{ km s}^{-1}}\right)^3 \times \left(\frac{R_c}{500 \text{ kpc}}\right)\left(\frac{1000 \text{ km s}^{-1}}{\sigma_c}\right). \quad (1)$$

Here we have assumed isothermal cluster and halo potentials so that the tidal radius of a halo is simply $r_t = R_c\, \sigma_h/\sigma_c$. Typical particle masses in cosmological simulations are between 10^9 $M_\odot$ and 10^{10} $M_\odot$; therefore the number of particles, N_p, within an L_* halo at $R_c = 500$ kpc is in the range 50–500. The evaporation timescale from two-body encounters is of order $300 t_r$, where the median relaxation timescale for a halo with half-mass radius $r_h \approx r_t/3$ can be written $t_r = 0.14 N_p/\ln(0.4 N_p)[r_h^3(G m_h)]^{1/2}$ (Spitzer & Hart 1971), i.e., for isothermal potentials

$$t_{\text{evap}} = 3 \text{ Gyr}\left[\frac{N_p}{\ln(0.4 N_p)}\right]\left(\frac{R_c}{500 \text{ kpc}}\right)\left(\frac{1000 \text{ km s}^{-1}}{\sigma_c}\right). \quad (2)$$

Clearly, t_{evap} is larger than a Hubble time for moderately large N_p.

Evaporation is accelerated in the presence of a tidal field (see Chernoff & Weinberg 1990). To test the evaporation timescale of a halo orbiting within a cluster, we constructed equilibrium halos with truncated isothermal density profiles. Our aim is to simulate both the evaporation rate from halos that form in current simulations and those which might form given much better length resolution. Current cosmological simulations have force resolutions limited by gravitational softening $\sim$5–20 kpc, and produce halos with correspondingly soft, low-density cores with a size comparable to the force resolution. Within the next few years a resolution of $\sim$1 kpc will be obtained and the structure of individual halos should be better resolved.

To test the ideas presented above, we evolved a series of model halos constructed with a total mass of 2.5×10^{11} $M_\odot$ within a outer radius of 30 kpc. Such a halo might surround an L_* galaxy 300 kpc from the center of a rich cluster. The halos were placed on circular orbits at 600 kpc in an isothermal cluster potential with a one-dimensional velocity dispersion of 1000 km s^{-1}. This distance was chosen so that the halo's outer radius was well inside the tidal radius imposed by the cluster, in order to minimize the effects of tidal heating. Figure 1 shows the fraction of mass remaining within a fixed distance of 30 kpc from the center of each halo over a period of 100 Gyr. We evolved the particle distributions using the TREECODE (Barnes & Hut 1986; Hernquist 1987). The number of particles, n, spline force softening s (Hernquist & Katz 1989), and core radius, r_c, of each model are also indicated in Figure 1. Throughout this paper we use the criteria that at twice the internal velocity dispersion of the system, a particle takes at least three steps across the softening length.

We find that the initial rate of mass loss is consistent with the standard evaporation formula above. However, the rate of mass loss is not linear with time, but rapidly slows down as the

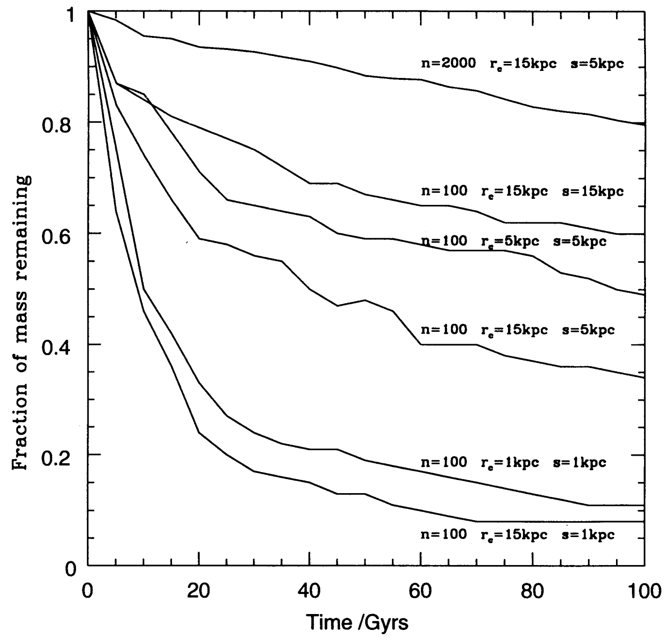

FIG. 1.—Mass-loss rates from dark halos owing to evaporation. The halos are constructed using the indicated model parameters for the core radius, r_c, and the softening length s, and particle number n. Each halo was placed on a circular orbit at 600 kpc from the center of an isothermal potential for 100 Gyr.

physical size of the halo approaches the softening or core size. Hence, halos with large softening and as few as 20 particles can survive many Hubble times. The rate of evaporation increases as the softening, hence core size, is reduced, i.e., individual encounters between particles can transfer more energy at the same impact parameter. Halos with the same softening will evaporate faster if the physical core is large since the binding energy is correspondingly lower. From these tests we conclude that relaxation effects are not important at driving mass loss from halos within current simulations.

2.1.2. *Particle–Halo Heating*

Carlberg (1994) explained the halo disruption in his 10^6 particle cluster simulation as a consequence of heating by massive N-body particles, particles much more massive than any viable dark matter candidate. We can make an analytic estimate for t_{ph} using the impulsive approximation, since the typical encounter timescale is smaller than the galaxy's internal timescale, i.e., $r_h/\sigma_h > 2b/\pi\sigma_c$, where b is the impact parameter. This calculation was originally performed for the disruption of open clusters by giant molecular clouds (Spitzer 1958). As a cluster particle passes by or through a halo it tidally distorts the system and increases its internal kinetic energy. Following Binney & Tremaine (1987), we equate the disruption timescale as the time for the halo's binding energy, E_{bind}, to change by order of itself owing to impulsive energy input:

$$t_d \approx \left(\frac{\Delta E}{E_{\text{bind}}}\right) = \left(\frac{0.03\sigma_c}{G}\right)\left(\frac{m_h}{r_h^3}\right)\left(\frac{r_p^2}{m_p^2 n_p}\right), \quad (3)$$

where m_p is the perturber's mass and r_p is the perturber's half-mass radius, which we equate to the gravitational softening length. Here, we have assumed that the galaxy's mean square radius is similar to the half-mass radius r_h. Therefore, we derive

$$t_{\text{ph}} \approx 65 \text{ Gyr}\left(\frac{\sigma_c}{1000 \text{ km s}^{-1}}\right)\left(\frac{r_p}{10 \text{ kpc}}\right)^2\left(\frac{10^9\, M_\odot}{m_p}\right) \quad (4)$$

by substituting the tidal radius of the halo for the half-mass radius and noting that the number density of perturbers within an isothermal potential, $n_p = \sigma_c^2/(2\pi m_p R_c^2 G)$.

The analogous analytic estimate for globular cluster dissolution by black holes in the halo of the Milky Way was tested using N-body simulations by Moore (1993) and found to be accurate within a factor of ~ 2. We therefore conclude that present dissipationless cluster simulations have obtained sufficient resolution to avoid this problem. For example, Carlberg's (1994) cluster simulations have $m_p = 4 \times 10^9\ M_\odot$ and $r_p \approx s = 20$ kpc, and Summers et al. (1995) have $m_p = 10^9\ M_\odot$ and $r_p \approx s = 10$ kpc, yielding $t_{\rm ph}$'s of about 65 and 23 Gyr, respectively.

2.2. *Physical Effects*

2.2.1. *Tidal Heating*

At a certain distance from the halo, particles will escape and become bound to the cluster potential. Our truncated softened isothermal halos will have somewhat smaller tidal radii than those given by the above formula for true isothermal potentials but are more realistic halo potentials. We test the analytic formula by placing model halos on circular orbits at different radii within a cluster and following their evolution with the TREECODE as in § 2.1.1. We define the tidal destruction radius to be the distance from the cluster center at which the halos lose 50% of their mass over 5 Gyr (i.e., typical halos completely disrupt at this cluster-centric distance over a Hubble time).

We evolve several model halos for 10 Gyr, varying the force softening and core radius. The initial number of particles is 10,000 so as to minimize relaxation effects. Our model halo with a core radius $r_c = s = 1$ kpc survives to $R_c = 200$ kpc. Halos with $r_c = 15$ kpc and $s = 5$ kpc will survive to $R_c = 300$ kpc. Halos with $r_c = s = 15$ kpc can survive at $R_c = 450$ kpc.

Halos that survive several orbits have tidal radii close to the value given by equation (1) using R_c set equal to their pericentric distance. Hence, in the limit of infinite resolution, singular isothermal halos will be tidal stripped to a radius where their outer density is approximately equal to the density of the cluster at pericenter.

Halos become very unstable when the tidally imposed radius approaches a value smaller than 2 or 3 times the core radius. Furthermore, halos with a physical core will be more unstable with a larger softening. Hence, in the absence of all other disruption mechanisms, tidal disruption from the cluster potential is sufficient to erase all L_* halos within 500 kpc for a Plummer force softening of 10 kpc. Note that a Plummer softening equivalent to our spline softening would be 30% smaller. Hence, simulations with 20 kpc Plummer softening will dissolve L_* halos at about 1 Mpc within a Coma-sized cluster! Any additional source of heating will drive this survival radius even farther outward.

Weinberg (1994) has stressed the importance of resonant orbit coupling between stellar dynamical systems orbiting within a tidal field. The orbital time of a halo at half the clusters virial radius is of order 10^9 yr, a timescale similar to the internal orbital period within an L_* halo at its half-mass radius. Halos on eccentric orbits within a cluster will lose mass as a consequence of heating by the tidal field, even though the "tidal shock" varies quite slowly with time. We test the importance of this effect numerically by placing the above model halos in eccentric orbits within the same cluster potential. Typically over a Hubble time, between 10% and 20% of the mass is lost owing to the slow tidal shocks over a period of about 10 Gyr. Halos with larger cores lose more mass than more tightly bound systems, but tidal shocking plays a relatively minor role in erasing halos.

2.2.2. *Halo-Halo Heating*

An additional heating term arises from impulsive encounters between halos within the cluster. The rate of energy input via impulsive heating is $\propto m_p^2 n_p = f m_p \rho_c$, where f is the fraction of the cluster mass contained within the perturbers. Therefore, the relative importance of heating from cluster particles versus individual halos is $f m_h/m_p$. Halos are typically between 10 and 100 times as massive as the particles in current simulations so we expect that the heating rate from halo-halo encounters will be about an order of magnitude more important than that from particle-halo encounters.

Performing the same calculation for estimating $t_{\rm ph}$, but replacing the softened particle perturbers with halos we find

$$t_{\rm hh} \approx 3.5 \frac{1}{f}\,{\rm Gyr}\left(\frac{R_c}{1\ {\rm Mpc}}\right)\left(\frac{100\ {\rm km\ s^{-1}}}{\sigma_h}\right). \quad (5)$$

Assuming tidally truncated isothermal halos (see § 2.1.1), f varies linearly with R_c (at the cluster virial radius $f_{\rm vir} \sim 0.5$) and we can write equation (5) as

$$t_{\rm hh} \approx 5.3 \frac{1}{f_{\rm vir}}\,{\rm Gyr}\left(\frac{\sigma_c}{1000\ {\rm km\ s^{-1}}}\right)\left(\frac{100\ {\rm km\ s^{-1}}}{\sigma_h}\right). \quad (6)$$

The scaling of this formula shows that $t_{\rm hh}$ is independent of position within the cluster. However, note that this derivation breaks down when the perturber mass is much larger than the halo mass and when the impact parameter is so large that use of the impulse approximation breaks down, i.e., when the encounter occurs beyond about 75 kpc from the halo. Hence, at large distances from the cluster center, very few encounters occur within this impact parameter and the heating rate falls to zero. Furthermore, in deriving this timescale we have ignored the effect of the tidal field in estimating the binding energy of the halo. As we demonstrate below, halos closer to the center have lower binding energies and hence will be easier to disrupt. Thus we would expect halos to survive at the edge of clusters and to be disrupted with increasing ease toward the cluster center.

We test this analytic estimate by constructing a cluster of galaxies similar to the Coma Cluster. Within the virial radius (r_v/kpc $= v_c$/km s^{-1}), galaxies are drawn from a Schechter luminosity function (Schechter 1976) with $\alpha = -1.25$ and $M_* = -19.7$ ($H_0 = 100$ km s^{-1} Mpc^{-1} and $\Omega = 1$), including all galaxies brighter than $2.8 \times 10^8\ L_\odot$. Each galaxy is modeled by an isothermal potential that is tidally limited using equation (1) with R_c equal to its pericentric distance. Velocity dispersions are assigned using the Faber-Jackson relation (1976); our minimum luminosity corresponds to a halo with one-dimensional velocity dispersion $\sigma_h = 50$ km s^{-1}. For $\sigma_c = 1000$ km s^{-1} the cluster mass within the virial radius of 1.5 Mpc is $6.9 \times 10^{14}\ M_\odot$, hence for a mass-to-light ratio of 250, the total cluster luminosity within this radius is $2.8 \times 10^{12}\ L_\odot$. This normalization yields approximately 950 galaxies brighter than our minimum luminosity and 30 brighter than L_*. At 500 kpc from the cluster center, about 25% of the total cluster mass remains associated with the cluster galaxies.

Our high-resolution model halos, constructed as in § 2.2.1, are placed on circular orbits at 300 kpc and 600 kpc and feel the potential field of all the cluster members and the analytic potential of the remaining cluster background. The 950 perturbing cluster galaxies orbit on eccentric orbits within the cluster. Halos lose up to half of their mass over a Hubble time; we multiply the initial perturbers mass by 0.75. Since the impulsive energy input is proportional to the square of the perturber mass, this is a conservative means of accounting for the gradual mass loss from the other cluster halos. The typical encounter velocity is equal to $\sqrt{2}\,\sigma_c \sim 1500$ km s^{-1}; hence for all values of the softening we use 10,000 time steps over a Hubble time. In a single time step a perturbing galaxy moves ~ 1 kpc which would be too large for the smallest softening we adopt; however, the encounters are never penetrating and occur beyond the tidal radius of the high-resolution halo.

Our treatment of the cluster environment contains all of the important dynamical effects. Its major shortcoming is that we assume that the cluster is in place at the beginning of the simulation.

Figure 2 shows the results from several numerical calculations. We find that the survival of a given halo depends sensitively on the ratio of the core radius to the tidal radius. As the core radius approaches the tidal radius the halos disrupt completely in a relatively short timescale. Our estimate of the halo-halo disruption timescale from equation (5) is in relatively good agreement with the numerical results for halos with small core radii.

The curves show a stochastic evolution that leads to a dissolution timescale of halos with fixed physical properties that varies by a factor ~ 2. The heating from a single encounter depends upon the square of the perturber mass; hence given a Schechter luminosity function, most of the heating is due to galaxies with luminosities $\gtrsim L_*/2$. This explains why the evolution is fairly chaotic since the number of massive perturbers is relatively small. Typically, the evolution of a halo is driven by between 5 and 10 large impulsive encounters.

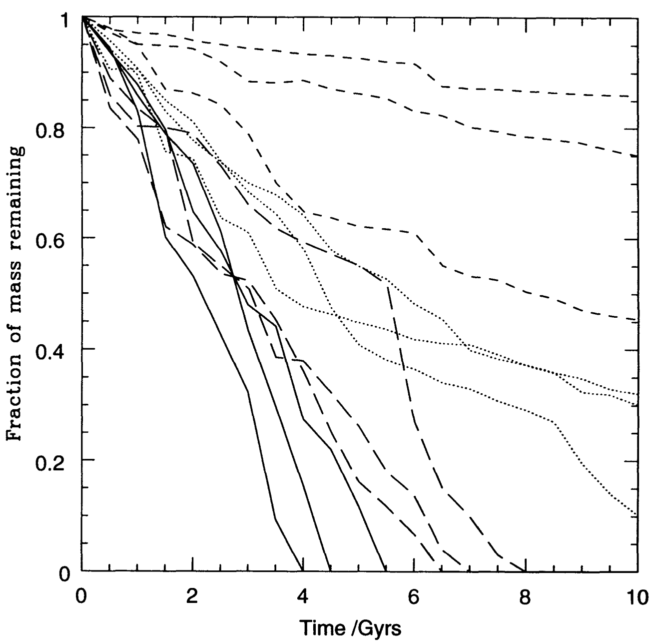

FIG. 2.—Mass-loss rates from dark halos owing to halo-halo heating. For each model, circular orbits in different directions were followed to demonstrate the stochastic nature of halo-halo heating. The dotted curves show models with $r = s = 1$ kpc in orbit at 300 kpc. The solid curves show models with $r_c = s = 5$ kpc at 300 kpc. The short dashed curves show models with $r = s = 5$ kpc at 600 kpc. The long dashed curves show models with $r = s = 15$ kpc at 600 kpc.

3. DISCUSSION AND CONCLUSIONS

Current dissipationless N-body simulations have achieved a resolution sufficient to resolve large halos within cluster environments. However, little substructure and no galaxy halos were found in these cluster simulations. We find that two-body evaporation is unimportant for halos with more than 30 particles. A second artificial numerical effect, particle-halo heating, does not pose a problem within numerical simulations as long as the particle mass is kept below $10^{10}\ M_\odot$. However, present cosmological simulations have softening lengths of order 5–20 kpc leading to halos with large low-density cores. As the limiting tidal radius approaches the halo core radius the dissolution of the halo occurs very rapidly. Hence, the overmerging problem within large overdense regions is due to the large force softening combined with tidal heating from the mean field of the cluster and encounters with other dissolving halos.

In the limit of infinite numerical resolution, the survival of individual halos within dense environments depends critically on their inner structure. If halos form with singular isothermal density profiles, then they can always be resolved at some level. Alternatively, including a gaseous component that dissipates energy can increase the central density of halos, effectively decreasing their disruption timescale under halo-halo collisions. Recent results on the structure of dark matter halos give conflicting results (Moore 1995). The highest resolution simulations of Carlberg (1994), Warren et al. (1992), and Crone, Evrard, & Richstone (1994) give very steep inner density profiles. Furthermore, after correcting for force softening, Crone et al. show that profiles are singular and fall steeper than r^{-2} on all scales. These results disagree with those of Katz & White (1993) and Navarro et al. (1994), who find that the density profiles of halos fall with radius as r^{-1} over a large central region. If the former is true, then it will be possible to identify halos within dissipationless simulations, although the halos themselves would be inconsistent with some observations of galaxy rotation curves (Moore 1994). If the simulated halos have shallow inner density profiles as some observations indicate, or as the latter authors find, then the overmerging problem can be resolved only by including a gaseous component.

If dissipationless dark matter forms singular isothermal halos, then in order to resolve galaxies in a cluster environment, force softenings less than a kpc must be adopted. In this case the computational cost becomes enormous if sufficient numbers of particles are used to resolve high-density regions and to minimize relaxation effects. Including a dissipational component effectively increases the density of galaxy-sized halos with far fewer particles and with only a relatively small increase in the computations run time. Therefore, increasing the resolution in collisionless simulations will serve mainly as a consistency check for identifying galaxy tracers in dense environments.

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