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NONLINEAR EXCITATION OF GLOBAL MODES AND HEATING IN RANDOMLY DRIVEN CORONAL LOOPS

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ABSTRACT

We solve the nonlinear three-dimensional MHD equations for fully compressible, low-β, resistive plasma to model resonant Alfvén wave heating of a coronal loop. Alfvén waves are driven in the loop by a (pseudo)random time-dependent forcing with a bounded amplitude. We find that global modes are excited and resonantly heat the loop in the nonlinear regime in three dimensions. Resonant heating occurs in several narrow layers accompanied by high velocity and magnetic field shear. The narrow dissipation layers are affected by the self-consistent velocity shear and are carried around by the flow. Consequently, the topology of the perpendicular magnetic field and the ohmic heating regions differs significantly from the linear or single-frequency driver regimes, and the heating is spread more uniformly inside the loop. The heating rate varies significantly on a timescale of one to several global mode periods. We conclude that, in solar active regions, random field-line motions can excite global mode oscillations and resonantly heat the loops with a time-varying heating rate.

Subject headings: MHD — Sun: corona — Sun: magnetic fields — waves

1. INTRODUCTION

The exact mechanism that heats the corona to higher than 2×10^6 K is not well understood. However, it is well known from observations that the solar corona is highly structured and inhomogeneous and contains a large number of discrete magnetic loops, which contain the hot plasma, and there is observational evidence of MHD waves in the loops. The UV spectrum suggests nonthermal velocities of approximately 10–20 km s^{-1} (Cheng, Doschek, & Feldman 1979). Nonthermal soft X-ray line broadening corresponding to velocities of $\sim$50 km s^{-1} was observed above active regions with the X-ray polychromator (XRP) aboard the *Solar Maximum Mission* (*SMM*) (Acton et al. 1981). Recent study of soft X-ray lines from the XRP indicates nonthermal motions of 30–40 km s^{-1} above active regions. These motions have been suggested as a signature of MHD wave heating (Saba & Strong 1991 and references therein).

Resonant absorption of Alfvén waves appears to be one of the major candidates for the mechanism of coronal heating (Zirker 1993). Resonant absorption of Alfvén waves in coronal loops was first suggested by Ionson (1978) as a nonthermal heating mechanism of the corona, and since then has been studied by many authors in the linear regime (see Ofman, Davila, & Steinolfson 1995 and references therein). In the present Letter we present the first nonlinear three-dimensional MHD simulation of coronal loop heating with randomly excited Alfvén waves. According to our results, in an active region where many loops of various sizes are present, the heating due to random motions of the loop footpoints will be most effective at any given time in a number of distinct loops that match the global mode resonance condition, and these loops might appear brighter in X-ray images. This prediction is in qualitative agreement with the transient loop brightening observed by the *Yohkoh* Soft X-Ray Telescope (SXT) (Shimizu 1995) and is an encouraging result in support of the wave heating theory. A quantitative comparison of the global mode heating model with these observations is left for a future study.

The response of an inhomogeneous MHD cavity driven by random boundary motion was studied by Wright & Rickard (1995) in the linear regime. However, there are several important distinctions between the linear and the nonlinear theories of resonance absorption (see below). First, the enhanced dissipation may allow the predicted wave amplitudes and the observed turbulent velocities inferred from the nonthermal broadening of X-ray and EUV emission lines to be brought into agreement while still providing sufficient heating to balance coronal energy losses. Second, the transverse motion of the resonance layer due to the Kelvin-Helmholtz instability (KHI) vortices, which is strictly a nonlinear effect, allows the heat to be deposited over a much larger region of space. This is important for matching the observed emission measure with active region loops.

Recently, Ofman & Davila (1995, hereafter OD) and Ofman, Davila, & Steinolfson (1994) studied the nonlinear evolution of the resonant absorption of Alfvén waves and found KHI-type vortices. OD investigated quantitatively the parametric dependence of the heating rate and the velocity amplitudes on the resistivity and driver amplitude for both traveling and standing Alfvén waves via solution of the low-β three-dimensional MHD equations. They found that a coronal loop driven at the linear global mode frequency changes its density structure as a result of the self-consistent Lorentz force, which leads to a nonlinear shift of the global mode frequency of the loop and a subsequent drop in the heating rate. Thus, in the solar corona nonlinear effects are important in determining the global mode response of a given loop. They have also found that in the nonlinear regime the velocity amplitudes at the dissipation layer are lower than the values expected from the linear $S^{1/3}$ scaling, where S is the Lundquist number. In a related work, nonlinear saturation of Alfvén waves in the context of field-line resonances in the magnetosphere was considered by Rankin and coworkers (see Rankin et al. 1994, 1995 and references therein).

In the present Letter we present the first nonlinear results of excitation of a global mode and heating due to resonant

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absorption of Alfvén waves driven by temporally random forcing. We find that global mode resonant heating occurs in the nonlinear regime at a somewhat lower than linear frequency and that the heating rate varies significantly in time with a typical timescale of one to several global mode periods. These results may have implications for the observed coronal loop transient brightenings by the *Yohkoh* SXT (Shimizu 1995).

2. MHD EQUATIONS AND MODEL

For typical solar active region loop parameters the magnetic field strength $B = 100$ G, the temperature $T = 2 \times 10^6$ K, and the number density $n = 10^{10}$ cm^{-3}. The corresponding thermal/magnetic pressure ratio $\beta = 8\pi nkT/B^2 \approx 0.007$, therefore the low-$\beta$ approximation is justified. With the above parameters the plasma is in the collisional regime.

We solve the normalized low-β resistive MHD equations with the background density profile and the uniform background magnetic field as a simplified model of a solar coronal loop. This model and the method of solution are described in detail in OD. Here we summarized the main details of the model: the physical parameters are the Lundquist number $S = \tau_r/\tau_A$, where $\tau_r = 4\pi a^2/\eta c^2$ is the resistive timescale and $\tau_A = a/v_A$ is the Alfvén timescale. In the above definitions, a is the thickness of the loop, η is the resistivity, $v_A = B_0/[4\pi\rho_0(0)]^{1/2}$ is the Alfvén speed, B_0 is the magnitude of the background magnetic field in the loop, and $\rho_0(0)$ is the density at the center of the loop. The density, velocity, magnetic field, time, and space are made dimensionless by $\rho_0 \to \rho_0/\rho_0(0)$, $\boldsymbol{v} \to \boldsymbol{v}/v_A$, $\boldsymbol{B} \to \boldsymbol{B}/B_0$, $t \to t/\tau_A$, and $\boldsymbol{x} \to \boldsymbol{x}/a$.

The background magnetic field $\boldsymbol{B} = B_0\boldsymbol{e}_z$ is uniform; the initial density profile is given

$$\rho_0(x) = \rho_r + (1 - \rho_r)e^{-x^4}, \tag{1}$$

where $\rho_r = \rho_0(x \to \infty)/\rho_0(0)$. The slab of plasma has the dimensions $L_x \times L_y \times L_z$. The normalized resistive heating rate in the slab is calculated by integrating the ohmic dissipation per unit volume in the plasma,

$$H(t) = S^{-1} \int_V j^2\, d^3x, \tag{2}$$

where $j = j(x, y, z, t)$ is the magnitude of the current and V is the volume of the loop.

In the present study we use the following boundary conditions: x-direction: exponentially decaying, i.e., at $x = \pm x_{\max} = \pm L_x/2$ we assume that all the quantities depend on x as $e^{-\alpha|x|}$, where α is of the order of the fundamental wavenumber in the slab; y- and z-directions: periodic.

The maximal numerical value of the Lundquist number is limited by the resolution and numerical dissipation in the code and is small compared to the solar value. An undriven Alfvén wave trapped inside a resonant cavity will dissipate its energy within a small number of periods. In order to overcome this difficulty and obtain a standing Alfvén wave solution, we use periodic boundary conditions in the y- and z-directions and a driving force that is applied in a narrow region around $x = 0$. Because of the proximity of the dissipation layer to the driver, these boundary conditions allow us to investigate the nonlinear effects of standing Alfvén waves regardless of the small Lundquist number compared to the solar value. In our simulations we use $S = 10^3$ with 128^3 grid points, and the dissipation scale is well resolved (there are ~10 grid points across the dissipation layer).

The Alfvén waves in solar coronal loops are believed to be driven by the footpoint motions of the loops anchored in the high-β solar photosphere (e.g., Halberstadt & Goedbloed 1995). The footpoint motions are excited by the motion of the solar convection cells of the high-β photospheric gas, which is many orders of magnitude denser than the low-β coronal plasma. Thus, the response of the solar convective motions (the driver) to the coronal loop motions is negligible. In agreement with the above picture, we impose an external random driver in our model and do not consider the driver's response to the resulting dynamics.

Fast waves are driven by a forcing term in the x-component of the momentum equation. The fast waves are coupled to shear Alfvén waves and drive the global mode resonance. The forcing term is confined to a narrow region around $x = 0$ (i.e., the full width at half-maximum of the forcing-term magnitude is much less then the thickness of the loop), is periodic in the y- and z-directions, and is given by

$$\boldsymbol{F}_d = F_0(t)e^{-(x/W)^2} \cos(2\pi y/L_y + 2\pi z/L_z)\boldsymbol{e}_x, \tag{3}$$

where $F_0(t) = F_d(1 - 2\mathfrak{R})$, $\mathfrak{R} \in \{0, 1\}$ is a computer-generated random number, F_d is the driving-force amplitude, and W defines the full width at half-maximum of the driving force magnitude. In the present study we have used $W = 0.05$. The value of $\mathfrak{R}$ is changed every given time interval δt. In the present study $\delta t = 0.5 \gg \Delta t$, where Δt is the integration time step. Thus, δt determines the band of frequencies that contain most of the power of the driving spectrum.

The value of the normalized global mode frequency can be found from linear theory (Ionson 1978):

$$\omega_g = k_z \sqrt{\frac{2}{1 + \rho_r}}, \tag{4}$$

where k_z is the parallel wavenumber. The location of the ideal resonance is determined by $\omega^2 - k_z^2 v_A(x)^2 = 0$, where $v_A(x) = B_0[\rho(x)]^{1/2}$ is the normalized local Alfvén velocity. Recently, Ofman, Davila, & Steinolfson (1995) investigated the dependence of ω_g on both k_z and k_y. Nonlinear simulations of OD show that when the loop is driven with the linear global mode frequency, the heating rate drops mostly because of nonlinear changes in the background density. In the present study the global mode is not driven directly but emerges from the response of the loop to the random driver.

3. NUMERICAL RESULTS

Below we present the results of a simulation of a model coronal loop driven by a random forcing term (eq. [3]), with $F_d = 0.2$, $S = 10^3$, $L_x = 6.0$, $L_y = 10.47$, and $L_z = 31.42$. These dimensions correspond to $k_z = 0.2$, $k_y = 0.6$, and $\omega_g = 0.317$ in the linear simulations of Ofman, Davila, & Steinolfson (1995).

In Figure 1 we present the temporal evolution of the heating rate. It is evident that at $t \approx 50\ \tau_A$ the heating rate reaches its mean value of ~0.006. After the initial transition period the heating rate varies within 1 order of magnitude on a typical timescale of 10–70 τ_A. The shorter timescale corresponds to half the global mode timescale, and the longer timescale is due to the response of the loop to the lower frequency variations in the driving spectrum, i.e., the modulation of the global mode frequency amplitude of the driver. Effectively, when low frequencies dominate the driving power spectrum at some

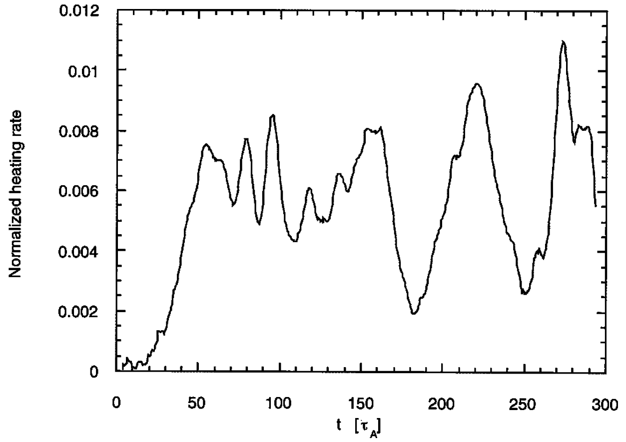

FIG. 1.—Temporal evolution of the heating rate. The fluctuation of the heating rate on a timescale of 10–70 τ_A is evident.

time interval, the heating rate does not drop immediately to zero, because of the trapped fast waves that continue to drive the Alfvén resonance at the global mode frequency in linear (Wright & Rickard 1995) and nonlinear regimes. In the solar corona the global mode frequency for each loop is determined by its length, geometry, and density structure (e.g., Steinolfson & Davila 1993; Ofman, Davila, & Steinolfson 1995). Thus, various loops heated by randomly driven Alfvén waves might vary in brightness considerably in X-ray images over timescales of one to several corresponding global mode periods.

In Figure 2 we present the power spectra of the driver and the response of the model coronal loop obtained by fast Fourier transform (FFT) of the variables at a given location in the loop. The power spectrum of the driver is shown in Figure 2*a*. It is evident that most of the power is contained in the frequency range $0 < \omega < 2$. The upper limit of the spectrum is determined by choosing the appropriate value of δt in the driver (see § 2). Thus, the linear global mode frequency is contained in the driving spectrum.

In Figure 2*b* we show the response of the plasma by Fourier transforming the $v_y(t, \boldsymbol{x}_0)$ component of the velocity and the $B_y(t, \boldsymbol{x}_0)$ component of the magnetic field, where $\boldsymbol{x}_0 = (0.874, 7.77, 23.3)$ is a given location inside the loop. It is apparent that the highest power in the spectra of v_y and B_y is at a single frequency, $\omega = 0.27 \pm 0.02$ (the uncertainty in ω is due to FFT of the final duration simulation), which is somewhat lower then the linear global mode frequency, $\omega_g = 0.317$. The response of the loop at other frequencies that are near the global mode frequency results in resonant absorption at additional, weaker resonant heating layers. Note that a small peak appears at double the global mode frequency in the power spectra of v_y and B_y. This peak is due to a higher harmonic of the global mode frequency that is generated by the nonlinear second-order terms in the Lorentz force (see Rankin et al. 1994; Ofman & Davila 1995).

4. SUMMARY AND DISCUSSION

We solve the nonlinear three-dimensional MHD equations for fully compressible, low-β, resistive plasma to model the resonant heating of a coronal loop. Standing Alfvén waves are excited in the loop by a (pseudo)random time-dependent forcing with a bounded amplitude. We find that global modes are excited by a random driver in a three-dimensional nonuniform slab of plasma in the nonlinear regime. Resonant heating occurs in several narrow layers inside the loop, accompanied by high velocity and magnetic field shear. The narrow dissipation layers are affected by the self-consistent velocity shear and are carried around by the flow (see also OD). The heating rate varies within 1 order of magnitude on a timescale of 10–70 τ_A. Consequently, the ohmic heating regions differ significantly from the linear or single-frequency driver regimes, and the heating is spread more uniformly inside the loop.

The amount of heating in any given loop depends on the amount of energy in the random driving spectrum, contained in a narrow frequency band, centered on the global mode frequency (or frequencies, if several global modes may be excited in a loop), and the wavelengths of the perturbations that can be excited in any given loop. The power spectrum due to random motion at any finite time interval might be different from the power spectrum at any other time interval. In an active region where many loops of various sizes are present, the heating due to random motions of the loop footpoints will be most effective at any given time in a number of distinct loops that match the global mode resonance condition, and

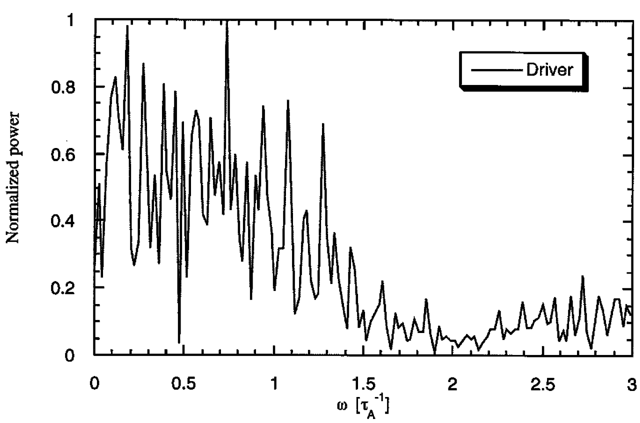

FIG. 2*a*

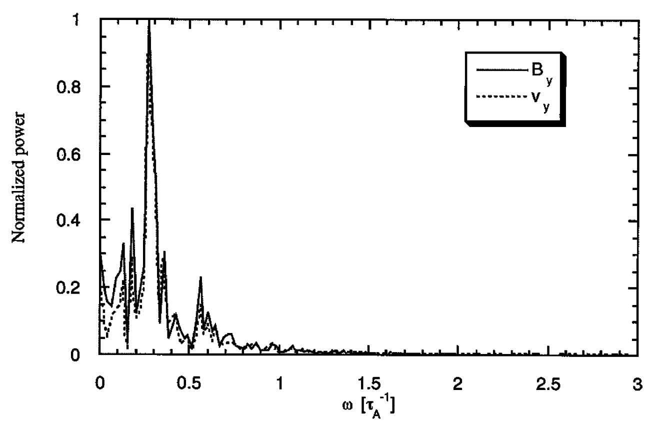

FIG. 2*b*

FIG. 2.—(*a*) Power spectrum of the imposed random driving term. Note that most of the power is contained in the $0 < \omega < 2$ frequency range, which includes the global mode frequency. (*b*) Power spectrum of the response: magnetic field component B_y (*solid line*) and velocity component v_y (*dashed line*) inside the coronal loop. The highest peak in the power spectrum is at $\omega = 0.27$. The peak that corresponds to the second harmonic of the global mode appears at $\omega = 0.56$. Note the low response to higher frequencies.

these loops might appear brighter in X-ray images. The loop brightness will vary as the power contained in each driving frequency evolves with time. The loops appear to act as narrowband filters that absorb energy contained in one or several global mode frequencies in the driver. This holds in the linear (Wright & Rickard 1995) and the nonlinear regime.

According to the results of our model, the heating rate of the loop due to randomly driven global modes is modulated on a longer timescale of several global mode periods. This timescale may be affected by the loop parameters, the effective dissipation, and losses inside the coronal loop. Our results are in qualitative agreement with transient loop brightenings, observed by the *Yohkoh* SXT (Shimizu 1995). Thus, we conclude that in solar active regions, random field-line motions can excite global mode oscillations that resonantly heat the loops with a time-varying heating rate.

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