Modelling Nearby Visual Binary Stars: η Cas

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Abstract: Quantitative tests of the theory of stellar interiors result from the confrontation of the observations of well-chosen objects with stellar models. The nearby visual binary systems are good candidates for such tests. We discuss here the case of η Cas with masses below the solar one (0.95 and 0.62 $M_\odot$ for component A and B respectively). A calibrated model of the whole system is slightly younger than the Sun, is helium deficient and shows the same mixing-length parameter as the Sun. We compare models calculated with the Saumon-Chabrier equation of state formalism, appropriate to the low-mass stars domain, to models obtained with the classical Eggleton, Faulkner and Flannery formalism. In this domain of mass the accuracy on the observational data does not yet allow to distinguish between these two formalisms.

1 Introduction

η Cassiopeiae (HD 4614) is a nearby visual binary at $\approx$ 6 pc on the north hemisphere. The luminosities of both stars, the metallicity and the effective temperature, $T_{eff,A}$, of η Cas A are determined with great accuracy thanks to high quality observations. $T_{eff,B}$ is determined through photometric calibrations and the orbital parameters yield an estimate of the individual stellar masses, M_A and M_B.

A "calibration" of the system as a whole permits to fix the different unknowns: the mixing-length parameter, α_{MLT}, for both stars, age, t, and helium abundance, Y.

Taking into account the present uncertainty on the treatment of the equation of state for low mass stars we use two different formalisms: the classical Eggleton, Faulkner & Flannery (1973) formalism (EFF) and the equation of state developed recently by Saumon and Chabrier (hereafter SC; Saumon & Chabrier, 1992; Saumon, Chabrier & Van Horn, 1995). The SC equation of state includes non-ideal effects and is appropriate for the modelling of very-low mass stars, brown dwarfs and giant planets.

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Observable	η Cas A	η Cas B	reference
Spectral type	G3 V	K7 V	Gliese&Jahreiss91
app. visual mag., m_v	3.45 $\pm$ 0.01	7.51 $\pm$ 0.01	Gliese&Jahreiss91
(R-I)	0.22	0.59	Gliese&Jahreiss91
$Teff$ (K)	6087 $\pm$ 60		Gray 94
metallicity $[\frac{Fe}{H}]$	-0.31 $\pm$ 0.05		Edvardsson et al. 94
Parallax, π(")	0.1684 $\pm$ 0.0031		Gliese&Jahreiss91
Period, P (yrs)	480 $\pm$ 10		Strand 69
semi-major axis, a(")	11.99$\pm$0.02		Strand 69

Table 1: Observational data for η Cas A and B.

2 Observational Data

The available observational data for η Cas are given in Table 1.

a) Luminosities. For both components they are derived from the apparent visual magnitude, m_v, and the parallax, π, using bolometric corrections of -0.21 $\pm$ 0.05 and -1.0 $\pm$ 0.2 for η Cas A and B respectively (Schmidt-Kaler, 1982). This yields: Log $\frac{L_A}{L_\odot}$ = 0.11 $\pm$ 0.05 and Log $\frac{L_B}{L_\odot}$ = -1.2 $\pm$ 0.1.

b) Effective temperatures. $T_{eff,A}$ is determined through detailed analysis using the method of line depth-ratio between vanadium and iron lines: $T_{eff,A}$ = 6087 $\pm$ 60 K (Gray et al. 1994). $T_{eff,B}$ is estimated with a $[T_{eff},(R-I)]$ photometric calibration, with a much lower accuracy: $T_{eff,B}$ = 4000 $\pm$ 150 K (Savanov, 1994).

c) Metallicity. From the metallicity of η Cas A (Table 1), we derive Z, the mass fraction of heavy elements, assuming Log $\frac{Z}{Z_\odot} \approx [\frac{Fe}{H}]$ and $Z_\odot$ = 0.0190 (Grevesse, 1991): $Z_{\eta Cas} \approx 0.009 \pm 0.001$. We assume that both stars have the same metallicity.

d) Stellar masses. According to Worley & Heintz (1983) the orbit is classified as reliable, which means "at least half of the orbit is defined"; long range photographic position observations are available. The total mass is derived from Kepler's law and the mass ratio $M_B/(M_A+M_B)$ from the photocentric orbit. It yields $M_A + M_B$ = 1.57 $\pm$ 0.11 $M_\odot$ and M_A = 0.95 $\pm$ 0.06 $M_\odot$ and M_B = 0.62 $\pm$ 0.05 $M_\odot$.

3 Stellar Models Calculations

The stellar models were computed with the CESAM code written by P. Morel with contributions from several members of the GDR 131 (Morel, 1993). We used the OPAL radiative opacities (Iglesias et al., 1992) complemented at low temperatures by Kurucz's (1991) data, with the solar mixture from Grevesse (1991). The effects of molecular opacities should be looked at but they are expected to be small in the range of mass considered in this work (Alexander & Fergusson

1994). The nuclear reactions rates are from Caughlan & Fowler (1988). The stellar convection is described according to the classical mixing-length theory (Böhm-Vitense, 1958). We used Eddington's T(τ)-law for the atmosphere which is suitable in the concerned temperature range (Allard & Hauschildt, 1995). Two series of models were calculated: reference models using the EFF equation of state and models using the more sophisticated SC equation of state. In solar models calculated with the EFF equation of state, an helium abundance, $Y = 0.286$, and a mixing-length parameter, $\alpha_{MLT} = 1.7$, are required to match the observed solar luminosity and radius at a solar age of 4.75 Gyrs.

4 Calibration of the System

Calibration is inspired from the method developed by Noels et al. (1991) for the α Centauri system. In the case of α Cen, both stars were supposed to have the same Z, Y, age and α_{MLT}, and were calibrated in the HR diagram taking into account the observed T_{eff} and L.

For η Cas the calibration method is slightly different because the metallicity is known with sufficient accuracy. Z is therefore an observable of the system which gives a rather strong calibration constraint, $0.008 \leq Z \leq 0.010$ and the determination of $M_A + M_B$ is more precise than the derived individual stellar masses. This gives a calibration constraint: $1.46\ M_\odot \leq M_A + M_B \leq 1.68 M_\odot$. So M_A and M_B will be considered as free parameters in the η Cas calibration.

We also assume for η Cas that both stars have same age and chemical composition but we allow different α_{MLT} values.
So, we have 6 unknows: M_A, M_B, t, Y, $\alpha_{MLT,A}$ and $\alpha_{MLT,B}$, for 6 observables: $T_{eff,A}$, $T_{eff,B}$, L_A, L_B, $M_A + M_B$ and Z.

We have calculated zero age main sequence models (ZAMS) for masses between 0.5 and 1.1 $M_\odot$ and evolutionary sequences for 1.0 $M_\odot$ with the EFF and SC equations of state, in order to estimate the age. The results are presented in Fig. 1 in which we also indicate the position and error box of the η Cas system.

We give in Table 2 the values of M_A, M_B, Y, $\alpha_{MLT,A}$, $\alpha_{MLT,B}$ and age ($t_{\eta Cas}$) which yield the observed T_{eff} and L. The errors presented in Table 2 for the calibrated values are obtained taking into account the observational error box on T_{eff} and L. To quantify the Z-dependence of the solution, we also present the results of a calibration made with a slightly different metallicity (Z = 0.010) and using the EFF equation of state.

5 Discussion and Conclusions

The results indicate that η Cas is helium-deficient relative to the Sun, $Y_{\eta Cas} \leq 0.27$. This leads to a relative helium to heavier elements enrichment $\Delta Y / \Delta Z \leq 5$. Because the location of low-mass stars models in the HR diagram is very weakly dependent on age, the age of the system only relies on the η Cas A model (see Fig. 1). The age we obtain for η Cas is in favor of an object slightly younger than the Sun. This is in agreement with the age derived using the chromospheric activity indicator CaII emission line (Hale 1994, Poveda et al., 1994). The difference between the age derived using SC and EFF is not very significant taking into account that $1.0 M_\odot$ is the limit of SC validity (Saumon et al., 1995).

Solutions were found adopting the solar α_{MLT} for both stars. This was also the case for α Cen (Fernandes & Neuforge, 95). This is in favour of small variations of α_{MLT} for a wide range

equation of state	SC	EFF	EFF
	Z=0.008	Z=0.008	Z=0.010
M_A	1.00 ± 0.05	1.00 ± 0.05	1.00 ± 0.05
M_B	0.55 ± 0.04	0.57 ± 0.03	0.58 ± 0.04
Y	0.25 ± 0.02	0.25 ± 0.02	0.25 ± 0.02
$\alpha_{MLT,A}$	1.7 ± 0.2	1.7 ± 0.2	1.7 ± 0.2
$\alpha_{MLT,B}$	1.7 ±(≥)1.	1.7 ±(≥)1.	1.7 ±(≥)1.
$t_{\eta Cas}$	3. ± 1	4. ± 1	5. ± 1.
$M_A + M_B$	1.55 ± 0.09 $M_\odot$	1.57 ± 0.08 $M_\odot$	1.58 ± 0.09 $M_\odot$

Table 2: Calibrated values of M_A, M_B, Y, $\alpha_{MLT,A}$, $\alpha_{MLT,B}$ and $t_{\eta Cas}$.

of low-mass stars ($0.5 \leq \frac{M}{M_\odot} \leq 1.0$); however as mass decreases, the sensitivity of stellar models with α_{MLT} decreases (note the large error bar for $\alpha_{MLT,B}$ in Table 2).
The change of the slope of SC-ZAMS for masses lower than about $0.7 M_\odot$ is probably due to "the inclusion of detailed hydrogen molecules and their partition functions (...)" as already discussed by Lebreton & Däppen (1988) concerning the MHD equation of state (Mihalas. Däppen & Hummer, 1988). A detailed study of the comparaisons of the SC equation of state with other equations of state by means of stellar models is in preparation (Lebreton, Fernandes. Chabrier, 1995).
In this work, possible tests of the equation of state could rely on the analysis of η Cas B stellar model. At $0.55 M_\odot$ the temperature difference between the two ZAMS models is ≈ 70 K and the luminosity difference is ≈ 0.1 dex (see Fig. 1). So that the present error in the HR diagram is still too wide to allow any test of the quality of the equation of state. So precise bolometric corrections and T_{eff} for η Cas B are also strongly needed. Improvements on $M_A + M_B$, $m_{v,A} - m_{v,B}$ and semi-major axis (may be) are expected from the Hipparcos results.

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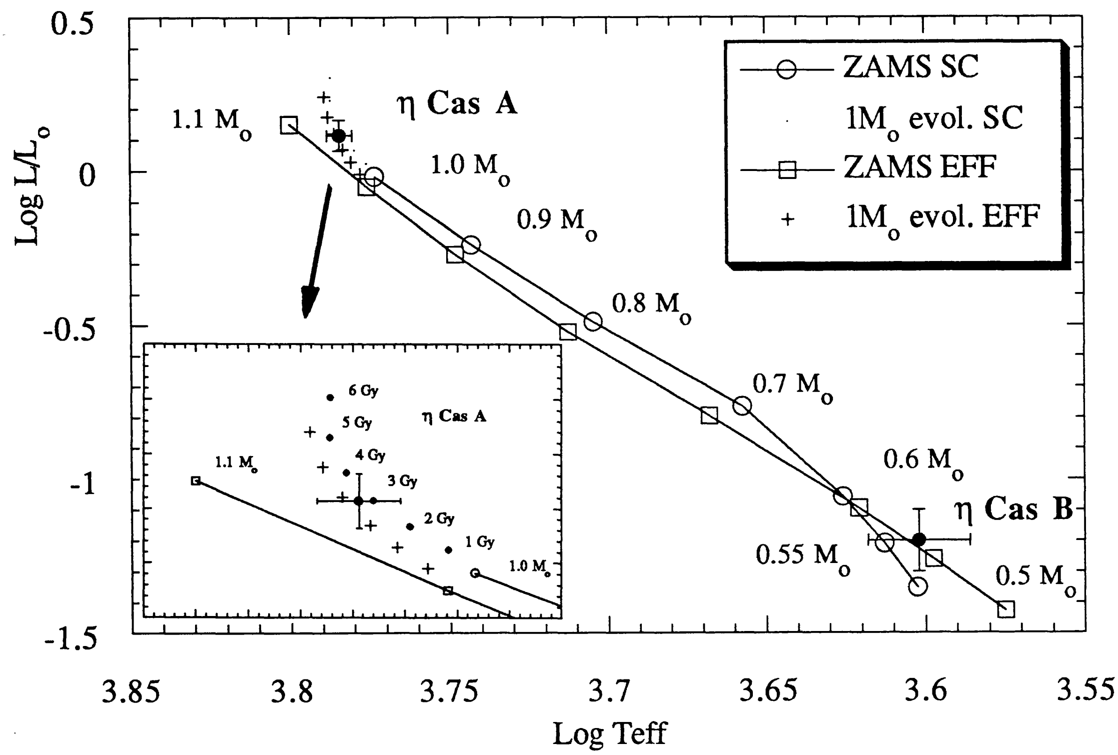

Figure 1: Calibration of the η Cas system in the HR diagram

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