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ASSESSING THE ENVIRONMENTAL IMPACT ON PSR B1620−26 IN M4

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ABSTRACT

The $\ddot{P}$ of the binary millisecond pulsar PSR B1620−26 in the Galactic globular cluster M4 (Backer 1992; Backer, Sallmen, & Foster 1993; Thorsett, Arzoumanian, & Taylor 1993), indicates the pulsar is a member of a hierarchical triple. The tertiary may have a mass, of from $\sim 10^{-3}\ M_\odot$ to $1\ M_\odot$, and orbits the inner binary with a semimajor axis of between 10 and 50 AU. The observed spin period derivatives constrain the mass, m_2, semimajor axis, a_2, eccentricity, e_2, and angle between the line of sight and the semimajor axis, ω_2, of the tertiary. We consider the expected values of some of the observable variables for different values of m_2, a_2, and e_2, and we show that a nonzero e_2 permits a surprisingly large range of values for m_2, a_2. In particular, the apparent mean motion provides a poor measure of the tertiary orbital period when $e_2 \sim 0.3$. We consider perturbations of the inner binary orbital parameters, in particular, the inner binary orbital period, P_1. Measurements of higher time derivatives of the spin period, and time derivatives of the orbital elements of the inner binary, will soon provide very strong constraints on the orbital parameters of the system. We also discuss scenarios for formation and subsequent evolution of planetary and stellar triples in M4, and the implications for PSR B1620−26. If the tertiary is substellar, the system must have spent a large fraction of its lifetime outside the core of M4 and may have survived one or more close encounters with a field star. If the tertiary is of stellar mass, the system is likely to be younger than inferred from its characteristic age and to have undergone multiple encounters with field stars. The confirmation of PSR B1620−26 as a hierarchical triple pulsar would provide fascinating insight into pulsar formation and stellar dynamics in globular clusters. A planetary mass tertiary would offer strong evidence for planet formation being common in solar-type stars, even those of low metallicity.

Subject headings: globular clusters: individual (M4) — planetary systems — pulsars: individual (PSR B1620−26) — stars: low-mass, brown dwarfs

1. INTRODUCTION

NGC 6121, better known as Messier 4, is a very mediocre example of a Galactic globular cluster. M4 has an estimated mass of some $10^5\ M_\odot$, a central density, ρ_0, of some 1–3 × $10^4\ M_\odot\ \mathrm{pc}^{-3}$, a core radius, r_0, of ~0.5 pc, and a projected one-dimensional velocity dispersion of $\sim 5\ \mathrm{km\ s^{-1}}$ in its core. M4 has an estimated metallicity of ~0.05, relative to solar metallicity and is thought to have formed ~15 billion years ago (Richer & Fahlman 1984; Djorgovski 1993; Trager, Djorgovski, & King 1993). There is a negligible amount of gas in M4, and it is believed that no star formation has taken place in the cluster since its formation.

At a distance of ~2 kpc, M4 is one of the closest globular clusters to the Sun, and in 1987 a millisecond pulsar, PSR B1620−26, was discovered near the center of the cluster (Lyne et al. 1987). The pulsar's projected position is near the edge of the cluster core, with a most probable position at a radius of 1–1.5r_0 (Goss, Kulkurni, & Lyne 1988). PSR B1620−26 has a spin period of $P \approx 11$ ms. Observations have revealed it to be a member of a binary, with an orbital period $P_1 \approx 191$ day and an eccentricity of $e_1 = 0.025$ (McKenna & Lyne 1988; Thorsett et al. 1993; Taylor, Manchester, & Lyne 1993). Continuous monitoring of the system has revealed an anomalous second time derivative of the spin period, $\ddot{P} = -2.3 \times 10^{-27}\ \mathrm{s\,s^{-2}}$ (Backer 1992; Backer et al. 1993; Thorsett et al. 1993). With an observed spin period time derivative, $\dot{P}$, of $8.2 \times 10^{-19}\ \mathrm{s\,s^{-1}}$, and expected intrinsic $\ddot{P} = \dot{P}^2/P = 6.1 \times 10^{-35}\ \mathrm{s\,s^{-2}}$, the observed $\ddot{P}$ is several orders of magnitude too large and of the wrong sign. The pulsar has a characteristic age $\tau_c = P/2\dot{P} = 2.2 \times 10^8$ yr, which is similar to that of other cluster pulsars and which suggests the intrinsic time derivative of the spin period should be of the same order as the one observed. The timescale for the spin period time derivative to reverse sign, $\tau_p = \dot{P}/\ddot{P}$, is only ~10 yr if $\ddot{P}$ remains constant.

We assume here, without further discussion, that the $\ddot{P}$ observed is due to an external gravitational jerk from a bound companion in a stable hierarchical orbit. Alternative scenarios have been discussed elsewhere (Phinney 1993; Backer et al. 1993; Sigurdsson 1993; Michel 1994) and do not seem to provide a good explanation for the current observations. We also assume the secondary is a white dwarf of mass $\sim 0.3\ M_\odot$ and that the pulsar was spun up by accretion during a mass transfer phase when the secondary evolved off the main sequence sometime in the last 1–2 × 10^9 yr. This implies that the secondary was originally a main-sequence star of mass 0.7–0.8 $M_\odot$, most probably in a tighter orbit about the pulsar than the current orbit. Such a system most likely formed during a binary-single or binary-binary exchange (Rappaport, Putney, & Verburt 1990; Sigurdsson 1993). We refer to other stars in the globular clusters as "field stars" when discussing the dynamical effects of individual cluster stars on the triple.

2. DYNAMICAL INFLUENCES

A pulsar can be considered to have some true spin period P_0, with instantaneous observed spin period, $P = P_0(1 + \boldsymbol{v} \cdot \boldsymbol{n})$, where $\boldsymbol{v}$ is the pulsar velocity relative to the observer, and $\boldsymbol{n}$ is the unit vector along the line of sight to the pulsar

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(Blandford, Romani, & Applegate 1987; for detailed discussion, see Phinney 1992, 1993). As pulsars spin, they emit energy and spin down, and thus there is an intrinsic time derivative of the pulsar spin period in the pulsar rest frame, $\dot{P}_0$. It is useful to define the true spin-down age, $\tau_0 = P_0/2\dot{P}_0$. The observed time derivative of the spin period, $\dot{P}$, is then given by

$$\frac{\dot{P}}{P} = \frac{\dot{P}_0}{P_0} + \frac{\boldsymbol{a} \cdot \boldsymbol{n}}{c} + \frac{v_t^2}{cR}, \qquad (2.1)$$

where v_t is the transverse component of the pulsar velocity as measured by the observer, $\boldsymbol{a}$ is the acceleration of the pulsar; c is the speed of light, as usual, and R is the distance to the pulsar from the observer. For pulsars in globular clusters, v_t is small and cR is large, even for a cluster as nearby is M4 and allowing for the cluster velocity relative to the solar barycentric velocity, so the third term above is more than an order of magnitude smaller than the observed $\dot{P}/P$. For higher period derivatives, only the second term contributes significantly, as $1/R\, dR/dt$ is small and the transverse velocity does not vary rapidly, which gives

$$\frac{1}{P}\frac{d^n P}{dt^n} = \frac{1}{c}\frac{d^{m-1}\boldsymbol{a}}{dt^{n-1}} \cdot \boldsymbol{n}. \qquad (2.2)$$

In particular, the $\ddot{P}$ is dominated by the jerk, $\dot{\boldsymbol{a}}$, $\dddot{P}$ is dominated by the jounce, $\ddot{\boldsymbol{a}}$, and $d\dddot{P}/dt$ is dominated by $\dddot{\boldsymbol{a}}$, which we provisionally refer to as the jolt.

2.1. *Projected Jerks, Jounces, and Jolts*

Consider a hierarchical triple, with a primary mass m_p (here implicitly taken to be the pulsar with canonical mass 1.4 $M_\odot$), secondary of mass m_1 ($\approx 0.3\ M_\odot$), in orbit with period P_1, semimajor axis a_1, and eccentricity e_1. Orbiting about the inner binary is a tertiary of mass m_2, orbiting with period P_2, with a semimajor axis a_2 relative to the center of mass of the inner binary, and eccentricity e_2. In general, the orbital plane of the inner binary will be inclined to the line of sight by some angle i_1, and the orbital plane of the tertiary relative to the center of mass of the inner binary will be inclined, relative to the line of sight, by $i_2 \neq i_1$. As the projected properties of the system scale uniformly with inclination, we will explicitly ignore factors of $\sin i_1$, $\sin i_2$, and factor them into the masses of the secondary and tertiary for calculations. We define ω_1 and ω_2 as the angles between the line of sight to the system center of mass and the longitude of periastron of the inner and outer orbits, respectively (note this is 180° off the definition in Sigurdsson 1993).

We classify our solutions as "stellar mass tertiaries" for $m_2 > 0.1\ M_\odot$ and "planet mass tertiaries" for $m_2 \leq 0.1\ M_\odot$. The stellar mass tertiaries have $a_2 \sim$ 40–45 AU, and high expected eccentricity, while the planet mass tertiaries have $a_2 \sim$ 10–40 AU and eccentricity $\sim$0.3–0.5. The constraint imposed by $\ddot{f}$ requires $\omega_2 \sim 180°$ for the stellar mass tertiaries.

Consider the orbit of the tertiary about the center of mass of the inner binary. To first approximation we treat the inner binary as a point mass at its center of mass. We will want to consider the derivatives of the observed pulsar spin frequency, $f = 1/P$, $\dot{f}/f = \dot{f}_0/f_0 + \boldsymbol{a} \cdot \boldsymbol{n}/c$, $\ddot{f}/f = \dot{\boldsymbol{a}} \cdot \boldsymbol{n}/c$, etc. Following Danby (1988, chap. 6), define

$$X = a_2(\cos E - e_2), \quad Y = a_2\sqrt{1 - e_2^2}\sin E, \qquad (2.3)$$

where E is the eccentric anomaly, $r = a_2(1 - e\cos E)$. Differentiating and using $dE/dt = n_2 a_2/r$, where, $n_2 = 2\pi/P_2$ is the tertiary mean motion, and $dr/dt = -e_2\dot{X}$, we get

$$\dot{X} = -\frac{n_2 a_2^2}{r}\sin E,$$

$$\dot{Y} = \frac{n_2 a_2^2}{r}\sqrt{1 - e_2^2}\cos E,$$

$$\ddot{X} = -n_2^2 a_2 \frac{a_2^2}{r^2}\left(\cos E - e_2 \sin^2 E \frac{a_2}{r}\right),$$

$$\ddot{Y} = -n_2^2 a_2 \sqrt{1 - e_2^2}\frac{a_2^2}{r^2}\left(\sin E - e_2 \sin E \cos E \frac{a_2}{r}\right). \qquad (2.4)$$

Then

$$\boldsymbol{a} \cdot \boldsymbol{n} = a_X \sin\omega_2 + a_Y \cos\omega_2$$

$$\dot{\boldsymbol{a}} \cdot \boldsymbol{n} = \dot{a}_X \sin\omega_2 + \dot{a}_Y \cos\omega_2$$

$$\ddot{\boldsymbol{a}} \cdot \boldsymbol{n} = \ddot{a}_X \sin\omega_2 + \ddot{a}_Y \cos\omega_2, \qquad (2.5)$$

where $a_X = Gm_2 X/r^3$, $a_Y = GM_2 Y/r^3$, and

$$\dot{a}_X = \frac{Gm_2}{r^3}\left(\dot{X} + \frac{3e_2 X\dot{X}}{r}\right),$$

$$\dot{a}_Y = \frac{Gm_2}{r^3}\left(\dot{Y} + \frac{3e_2 Y\dot{X}}{r}\right),$$

$$\ddot{a}_X = \frac{Gm_2}{r^3}\left\{\ddot{X} + 3e_2\left[\frac{X}{r}\ddot{X} + \frac{2\dot{X}}{r}\left(\dot{X} + \frac{2e_2 X\dot{X}}{r}\right)\right]\right\}$$

$$\ddot{a}_Y = \frac{Gm_2}{r^3}\left\{\ddot{Y} + 3e_2\left[\frac{Y}{r}\ddot{X} + \frac{2\dot{X}}{r}\left(\dot{Y} + \frac{2e_2 Y\dot{X}}{r}\right)\right]\right\}. \qquad (2.6)$$

For $e_2 \neq 0$, there is no unique solution for m_2, a_2, e_2, given $\dot{f}$, $\ddot{f}$, $\dddot{f}$. The equations for $\ddot{a}_X$, $\ddot{a}_Y$ are readily solved numerically (see Figs. 1 and 2). One peculiarity of the solution is apparent by inspection. For circular orbits, $(-\dddot{f}/\dot{f})^{1/2} = n_2$; this is not true in general for eccentric orbits.

Defining the apparent mean motion $n_a = (-\dddot{f}/\dot{f})^{1/2}$, we find $n_a \neq n_2$ for most values of e_2 when observed near apastron ($E \approx 180°$). Most solutions for the tertiary require it to be at, or just past, apastron. For PSR B1620−26, $n_a = 1.2 \times 10^{-9}\ \mathrm{s^{-1}}$, naively assuming $P_2 = 2\pi/n_a$, this implies $P_2 \sim 160$ yr. As can be seen in Figures 1 and 3, for $e_2 \sim \frac{1}{3}$ the apparent mean motion can be large at $E \sim 180$, for a large fraction of the orbital period. For orbital eccentricities near 0.3, the observed $\dddot{f}$ provides a very poor measure of the true orbital period when the tertiary is near apastron.

Assuming $\dot{P}_0 \sim \dot{P}$, the orbital geometry requires the tertiary be near apastron, and $\omega_2 \lesssim 180°$. This is significant, as we may expect planet mass tertiaries to have $e_2 \sim$ 0.3–0.7 (Sigurdsson 1993). *The orbital period inferred from the apparent mean motion may be significantly larger than the true orbital period.*

For highly eccentric orbits, n_a may be large, which leads to an inferred orbital period significantly smaller than the true orbital period (see Fig. 2). For stellar mass tertiaries, the small $\dot{P}$ requires the semimajor axis to be uncomfortably closely aligned to the line of sight, however, we are not selecting from the prior distribution of ω_2. Rather, we have to accept the system as observed and that the observations have simply pro-

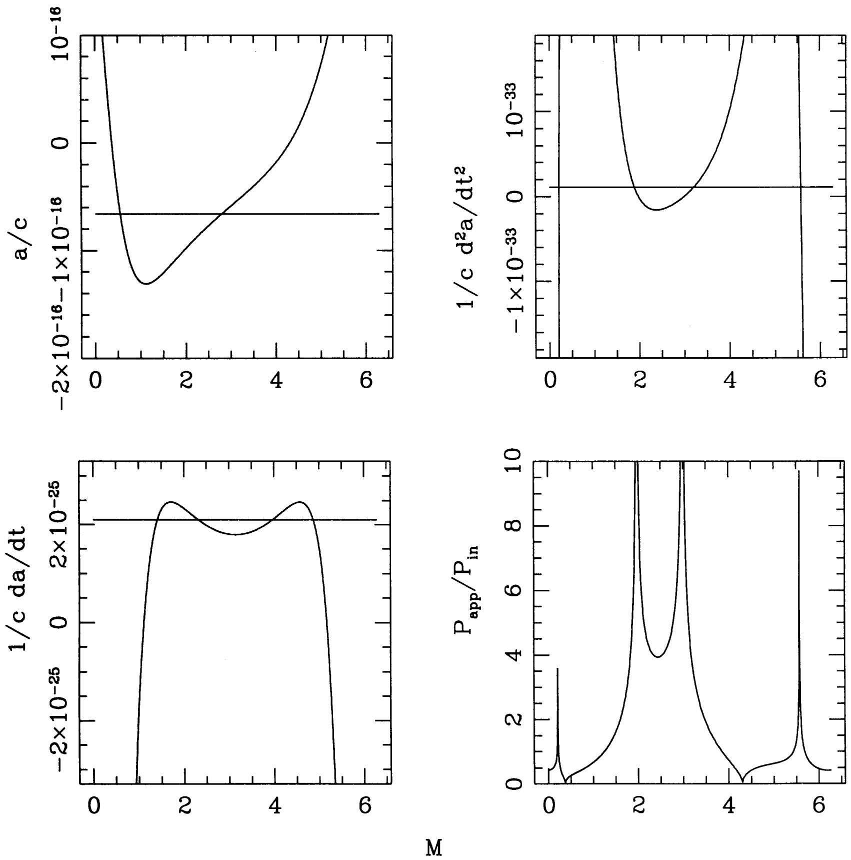

FIG. 1.—The projected jerk, jounce, and jolt for a planet mass tertiary, vs the mean anomaly, M. An M of unity is equivalent to a period of ~ 6 yr, which is comparable to the length of observations. The planet was mass $1.2 \times 10^{-3}\, M_\odot$ in a 13 AU ($P_2 = 36$ yr) orbit of eccentricity $e_2 = 0.3$. The angle between the line of sight and the semimajor axis, $\omega_2 = 140°$. This represents a "minimum mass" solution for the current observations. The horizontal lines in the first three panels represent the observed $\dot{f}/f$, $\ddot{f}/f$ and $\dddot{f}/f$ from Thorsett et al. (1993). The fourth panel shows the apparent orbital period normalized to the true orbital period, as inferred from the observed jounce. With a true orbital period of 36 yr, compared to an apparent orbital period of 160 yr, we expect the tertiary to be rounding apastron at this time.

vided a constraint on the solution, given the existence of the system. For planet mass solutions, the constraint on ω_2 is not so severe.

What distinguishes the planet mass solution from the stellar mass tertiary is that the current $\ddot{f}$ cannot be sustained for more than a decade. Within a few years the tertiary must approach periastron, if the tertiary is low mass. The predicted $d\dddot{f}/dt \sim 3 \times 10^{-40}$ s s^{-4} should be observable in the near future, which gives an estimate of the jolt, $\dddot{a} \sim 10^{-31}$ cm s^{-5}.

2.2. *Orbital Variations*

Rasio (1994a, b) considers the effects of perturbations by the tertiary on the orbital elements of the inner binary. In particular, he finds that orbital precession should be detectable with a little more timing and that the short-term variation of e_1 is below that detectable by current timing. Here we consider perturbations of the semimajor axis and period of the inner binary.

2.2.1. *Semimajor Axis and Inclination of Inner Binary*

There are two sources of variation in the inner binary apparent semimajor axis, a_{1p}. There may be intrinsic variation in he true semimajor axis, $\dot{a}_1(t)$, and there may be a variation in the inclination, which would cause a variation in the apparent semimajor axis, $a_p = a_1 \sin i_1$, and $\dot{a}_p = a_1 \cos i_1\, di_1/dt$. The total variation in the apparent semimajor axis is then

$$\dot{a}_{1p}(t) = \dot{a}_1(t) \sin i_1 + a_1 \cos i_1 \frac{di_1}{dt}\,. \tag{2.7}$$

The observable inner binary period, P_1, is only a function of the true variation in a_1,

$$\dot{P}_1 = \frac{dP_1}{da_1}\frac{da_1}{dt}\,. \tag{2.8}$$

P_1 is directly measurable by timing the interval between either the near or far turnaround points of the binary orbit, which is invariant under inclination variation. Ths $\dot{P}_1$ and $\dot{a}_{1p}$ are independent observables and the $\dot{a}_1$ and di_1/dt may be measured.

The inclination variation has been estimated by Thorsett (1995) and, using an equivalent formalism, by Rasio (1994a). We can write

$$\frac{di_1}{dt} = \frac{3}{2}\frac{a_1^3}{r_2^3}\frac{m_2}{m_p + m_1} n_1 F(i_1, i_2)\,, \tag{2.9}$$

where n_1 is the mean motion of the inner binary, r_2 is the separation between the tertiary and the inner binary center of

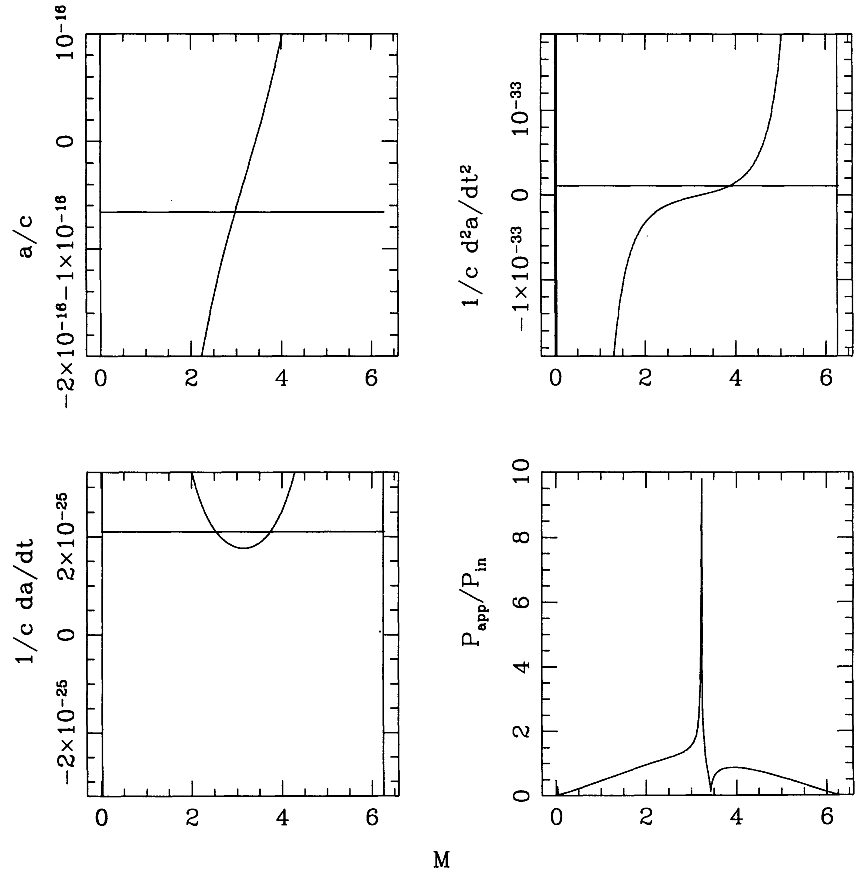

FIG. 2.—Same as Fig. 1, but for a "high-mass" solution of $m_2 = 0.5\ M_\odot$, $e_2 = 0.9$, $a_2 = 42$ AU ($P_2 = 184$ yr) A change in mean anomaly M of 0.2 corresponds to an interval of ~ 6 yr. In this case, we require the apparent orbital period to be a little less than the true orbital period, and the tertiary has probably recently passed apastron.

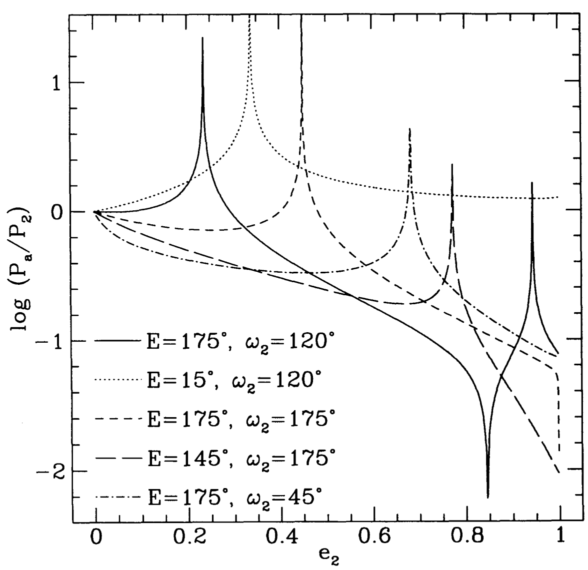

FIG. 3.—The apparent orbital period normalized to the true orbital period, as inferred from the observed acceleration and jounce, as a function of eccentricity for fixed tertiary eccentric anomaly, E, and angle to line of sight, ω_2. Note for $P_{app}/P_2 > 1$ we expect $e_2 \sim 0.3$, whereas for $e_2 \gtrsim 0.5$, most angles give $P_{app}/P_2 \lesssim 1$.

mass, and F is a function of the inclination and orbital phase of the binaries. Assuming the tertiary is near apastron, $r_2 \approx a_2(1 + e_2)$, as implied by the solution for m_2, a_2. F is proportional to sin $(i_1 - i_2)$ and cos i_1, with expected values of $F \sim 0.1$. Hence we find $\dot{a}_p = 0.7F$ cm s^{-1} for a stellar mass tertiary, and $\dot{a}_p = 0.16F$ cm s^{-1} for a planet mass tertiary.

There is no secular perturbation of a_1 over P_2, but on timescales t, $P_2 \gg t \gg P_1$, there are significant perturbations to a_1. Note the approximation used in Rasio (1994a) is for a fixed tertiary position; using this approximation there can be no change in a_1. In reality, the tertiary is not fixed, and there is energy transfer between the inner and outer binary on timescales less than P_2. Physically, the inner binary semimajor axis shrinks through the tidal shock of periastron passage of the tertiary. If the tertiary orbit was parabolic, a_1 would slowly increase after periastron, which would lower the speed of the receding tertiary. For a bound orbit the periastron passages are periodic, and the postperiastron relaxation of a_1 must assume a symmetric form, with a decrease in a_1 near periastron and an increase in a_1 near apastron. In general, the relaxation is not monotonic. Rather there is an initial "overcorrection" in a_1 and a rebound.

We estimate the variation in a_1, from $d(t) = r_1(t) - r_1(t = 0)$, where r_1 is the separation of the primary and secondary, using the method developed by Bailyn (1987). We define a time-

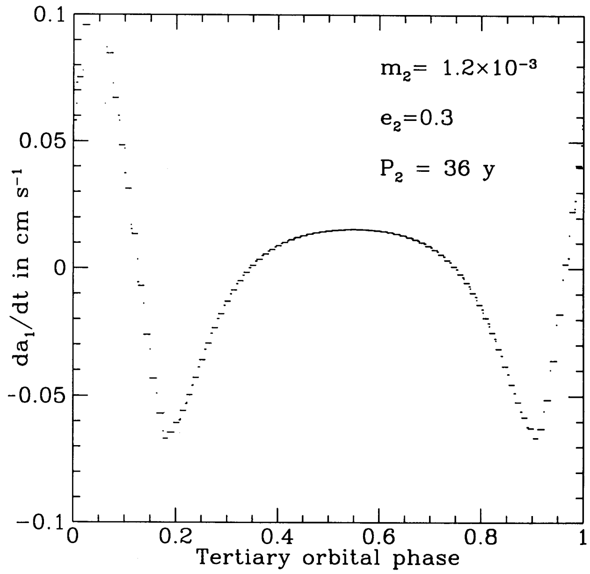

FIG. 4.—The instantaneous running average $\dot{a}_1(t)$ vs. the orbital phase $(M/2\pi)$ of the tertiary for the planet mass tertiary above. An orbit inclination $i_1 = 60°$ was assumed and the change in a_1 was smoothed on a timescale $\tau = P_1$. Variations in $d(t) = a_1(t) - a_1(0)$ on timescales shorter than P_1 were considerably larger than the smoothed variations, with strong perturbation to a_1 near periastron. The eccentricity fluctuations implied by the short-term variations in $d(t)$ were $\delta e_1 \sim 10^{-7}$ per P_1.

averaged $\dot{d}_\tau(t) = [d(t) - d(t-\tau)]/\tau$. In practice, what is observed is the time delay of the pulsar signal as the pulsar return to some reference orbital phase of the inner orbit. It is not possible to determine $a_1(t)$ from instantaneous measurements. The quantity $\dot{d}_\tau$ is best compared to a discrete variation

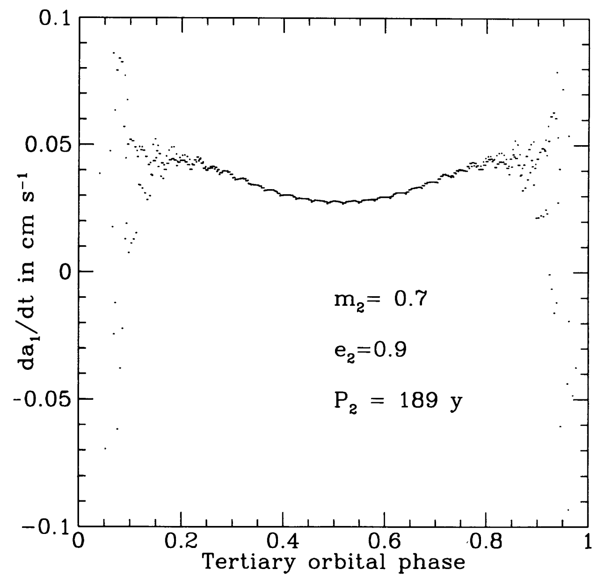

FIG. 5.—Same as Fig. 4, but for a stellar mass solution, $m_2 = 0.7\ M_\odot$, $\epsilon_2 = 0.9$, $a_2 = 44$ AU ($P_2 = 189$ yr.) Note that $\dot{a}_1 > 0$ away from periastron (phase = 0.1). The model shows that the inner binary semimajor axis is sharply reduced by the strong perturbations at periastron and relaxes to a larger value for most of the rest of the orbit, quite unlike the behavior seen for a planet mass tertiary. It is conceivable that this is an artifact of the particular solution method used here. The eccentricity fluctuations implied by the short-term variations in $d(t)$ were $\delta e_1 \sim 10^{-4}$ per P_2.

in a_1 over the interval. Then doing a running average of the solution for d_τ produces a fit for $a_1(t)$. The variations in $d(t)$ on timescales shorter than P_1 are effective smoothed over, and the $\dot{d}(t)$ measures the change in the amplitude of the envelope of the short-term fluctuations in r_1.

The solution assumes the mean motion of inner binary is approximately constant, which is a fair approximation for the weak perturbations involved here. Bailyn's original solution assumed circular orbits. We have extended the solution for the case where the outer orbit is eccentric, and we approximate the initial inner orbit as circular. The orbits are assumed to be co-planar, with appropriately lower masses. That is, the forces normal to the plane of the inner binary are neglected. The solution should provide a good approximation to the true dynamics over a large range in inclination, and the relative inclination cannot be very large, as the triple would then be unstable. Retrograde tertiary orbits and nonzero inclination may change the magnitude of the perturbation by a factor of order 2.

We solve for

$$\ddot{d}(t) = -n_1^2 d(t) + \sum_{j=0,3} c_j \sin^j [n_1 t - v_2(t)] \qquad (2.10)$$

The values c_j are coefficients calculated numerically, and v_2 is the true anomaly of the tertiary. We integrated $d(t)$ numerically for many orbits, with initial conditions that the tertiary is at some arbitrary initial orbit phase, and the line joining the tertiary to the center of mass of the inner binary is orthogonal to the line between the stars of the inner binary. One must be careful to discard the particular solution $d(t) \propto t^2$, which violates the boundary conditions implicit in the solution. The solution for the first couple of outer orbits is discarded, as it is susceptible to transients due to the artifical initial conditions. Later orbits show no such spurious excursions in d, and solutions for later orbits are robust to variations in the initial phase of the outer companion.

For the stellar mass tertiary, $\dot{d}_\tau \approx 0.03$ cm s^{-1}, with the tertiary anywhere near apastron. For the planet mass tertiary, the situation is more complicated. The relaxation timescale for the inner binary ($\sim$few times P_1) is not much shorter than P_2, and -0.02 cm s$^{-1} \lesssim \dot{a}_1 \lesssim 0.02$ cm s^{-1} changes sign within a few years of apastron passage. Near apastron, the observed $\dot{P}_1$ may be of either sign and, averaged over several periods, will likely be small (see Figs. 4 and 5).

Thorsett et al. (1993) quote an upper limit on $|\dot{P}_1| \leq 10^{-8}$, which corresponds to $\dot{a}_1 \leq \pm 0.005$ cm s^{-1}. If we assume the approximation used here is valid, the observational limits would tentatively rule out the stellar mass tertiary, based on the constraints on $\dot{P}_1$. The total semimajor axis variation should be dominated by the variation in inclination, $\dot{a}_{1p} \sim \pm 0.02F/0.12$ cm s^{-1} for a planet mass tertiary; $\dot{a}_{1p} \sim \pm 0.08F/0.12$ cm s^{-1} for a stellar mass tertiary. No currently favored formation scenario predicts a coplanar inner and outer binary, and, barring an a priori improbably high inner binary inclination, the two cases may be distinguished from observations of $\dot{a}_{1p}$. It is likely that $\dot{a}_{1p}$ will be measured to a theoretically interesting level in the very near future (S. Thorsett, private communication).

The change in e_1 inferred from the calculation of $d(t)$, on the timescale integrated over, is consistent with the amplitude calculated by Rasio (1994a, b).

2.2.2. *Eccentricity of Inner Binary*

The orbital eccentricity of the inner binary, e_1 is anomalously large (Phinney 1992; Sigurdsson 1993). As noted by Rasio (1994a, b), the observed e_1 is naturally induced by secular perturbations from a stellar mass tertiary. A planet mass tertiary induces an eccentricity through secular perturbations that are 2 orders of magnitude smaller. Sigurdsson (1993) conjectured that perturbations of the tertiary by field stars would preclude the long-period saturation of the growth in e_1 and lead to long-term random walk increase in e_1. Rasio (1994b) argues the timescale for the eccentricity growth is too long for it to be effective during the pulsar lifetime.

The saturation in the growth of e_1 can be understood to be a consequence of the fixed ratios of the mean motions of the inner binary and tertiary. If the orbital phase of the tertiary is perturbed by an external force, the long-period phase relation between the inner and outer orbits is destroyed and the eccentricity perturbation may continue to grow.

Perturbations by field stars induce secular perturbations in the orbital phase of the tertiary. The perturbations in orbital phase due to passing field stars decline like $(a_2/a_f)^3$, where a_f is the distance of closest approach by the field star. We may expect $O(100)$ distant $(4 \ll a_f/a_2 \ll 100)$ perturbations for every close approach. The timescale for approaches of $a_f(\propto a_f^2)$ is only 10^5 yr in the core of M4 for $a_f \sim 100$ AU, so it is conceivable that e_1 could have random walked to $O(10^2)$ times the maximum value predicted by secular perturbation theory.

Possibly the e_1 observed was induced by a close passage by a field star to the inner binary. There is a probability of ~ 0.5 of the tertiary surviving such an encounter, and such a close encounter is expected to occur every few times 10^8–10^9 yr, depending on whether the system has remained in the core of the cluster during its lifetime, the encounter rate is ~ 2 orders of magnitude lower at the half-mass radius then in the core. That is, a close encounter is probable on timescales $\gtrsim \tau_c$.

An interesting, albeit speculative, alternative is possible if the system was originally a quadruple system containing two planet mass objects in a hierarchical orbit about the inner binary. Such a configuration is possible if the system was formed during an exchange process as discussed in Sigurdsson (1993) and if two planets were captured into orbit about the inner binary. In general, such a configuration would not be stable and should lead to the eventual ejection of one of the planets. The most probable ejection process is for the orbit of one of the planets to evolve to cross the orbit of the secondary. The increase in a_1 as the secondary evolved off the main sequence is likely to trigger such an instability. A sling-shot ejection of a planet mass by the secondary changes the energy and angular momentum of the secondary by a small amount. If the planet ejected had mass of order $10^{-3}\ M_\odot$, the fractional change in angular momentum is $\sim 10^{-2}$, and there should be a change in the secondary eccentricity $\Delta e_1 \sim 10^{-2}$. Unfortunately, since it is impossible to rule out such a process by observation, it must simply be considered an illustrative example of a process by which the observed e_1 can be induced by one of the many complex dynamical processes possible in crowded stellar environments.

Given the numerous possible sources of perturbation on the inner binary, one might conclude that the peculiarity of e_1 is how small it is, not how large it is. While the secular perturbation of a stellar mass tertiary on the inner binary is sufficient to induce the observed inner binary eccentricity, it is premature to conclude that it is the necessary mechanism.

3. EVOLUTION AND THE ENVIRONMENT

If PSR B1620−16 is a hierarchical triple, the long-term stability of the system to encounters by field stars must be a concern. The stability of the triple is determined both by the "hardness" of the system and the rate of encounters with field stars. Whether the system is "hard" (encounters tend to increase the binding energy) or "soft" (encounters tend to decrease the binding energy) depends on both a_2 and m_2, and also on the encounter velocity and mass of any field star (see, e.g., Sigurdsson 1992). The rate of encounters depends on the local field star density and is several orders of magnitude smaller outside the cluster core than in the core.

To model the dynamical impact of the field stars, we evolved an ensemble of hierarchical trinaries of the appropriate orbital parameters in a multimass model of M4, a total of 100 representative systems of each type. The calculation for stellar mass tertiaries provides an independent estimate of the life expectancy of such a triple system, as calculated by Rasio, McMillan, & Hut (1995).

The cluster model was an explicit realization of the distributon function of a 10 mass–model fit to the observed structural parameters of M4, the mass distribution representing a discrete realization of an evolved Salpeter zero-age mass function. The triples were assumed to form by a binary exchange in the core of the cluster, with the neutron star a member of a tight neutron star–heavy white dwarf binary, and the current secondary assumed to have been a turnoff mass main-sequence star. The fourth star was assigned either a planetary mass of $\sim 10^{-3}\ M_\odot$ or a stellar mass between 0.5–0.8 $M_\odot$ drawn randomly from the stellar mass function. In the encounter, it is assumed that the heavy white dwarf is ejected and a hierarchical triple is formed, with the turnoff mass star and neutron star forming the inner binary and the fourth star (planet) providing the tertiary. The triple is assumed to recoil from the core with a velocity drawn from a distribution characteristic of the exchange process. The mean recoil is somewhat less for a stellar mass tertiary at fixed exchange energy, scaling as $1/(m_p + m_1 + m_2)$. The resultant motion of the triple was then integrated explicitly in the model cluster potential. We treated the triple as a point mass, and assumed that it moved according to

$$\ddot{\boldsymbol{r}} = \nabla\Psi(r) + \boldsymbol{a}_{\rm dyf} + \boldsymbol{a}_{\rm diff}\ , \tag{3.1}$$

where $\nabla\Psi(r)$ is the potential gradient due to the mass interior to r, $\boldsymbol{a}_{\rm dyf}$ is the dynamical friction experienced by the binary, and $\boldsymbol{a}_{\rm diff}$ is the effective acceleration due to scattering by individual stars in the cluster. We calculated $\boldsymbol{a}_{\rm dyf}$ and $\boldsymbol{a}_{\rm diff}$ explicitly at each step, using the second-order diffusion coefficients derived from the distribution function (Sigurdsson & Phinney 1995).

At each step of the integration, the probability of a field star encountering the binary on a trajectory with pericenter $a_f \lesssim 4a_2(1 + e_2)$ was calculated by integrating over the cross section for encounters and hence we derived the encounter rate as a function of the local distribution function and the triple's velocity. The probability for encounters was compared to a random number drawn at each integration step, and we deemed an encounter to have occurred if the latter was smaller than the instantaneous encounter probability. If an encounter was deemed to have occurred, initial orbital parameters for the field star and triple were drawn from the underlying distribution and the encounter integrated explicitly as a three-body encounter, with the inner binary of the triple treated as a point mass (this was necessary because of the large difference in inner and outer orbital periods). The encounter was integrated until resolved. If the triple survived the encounter, it was placed back in the cluster potential with an appropriate new velocity, and the integration through the cluster continued. Each of the triples with planet mass tertiary was integrated in the cluster for a total of $f_r \times 2 \times 10^9$ yr. The triples with a stellar mass tertiary were integrated for $f_r \times 10^9$ yr, where $f_r \in [0, 1)$ is a random number drawn uniformly on the interval.

During each encounter integration, the separation between the inner binary and both the tertiary and field star was monitored. If the tertiary–inner binary separation became less than $\sim 3a_1$, the triple was no longer hierarchical and would become unstable. If the field star approached within $\lesssim 3a_1$, the orbital elements of the inner binary would be strongly perturbed, and the encounter was flagged as having lead to a "collision" and was terminated.

For both sets of runs, 21 of the initial triples were ejected from the cluster, mostly by the initial recoil and in a few cases by diffusion of marginally bound orbits across the tidal radius of the cluster. These runs were discarded, although in practice some might have become bound to the cluster again through dynamical friction before being completely stripped by the Galactic tidal field. In no case could they have returned to the cluster core in the lifetime of the system.

In the case of triples with stellar mass tertiary, 58 of the remaining 79 systems ended up on orbits inside the core after $\leq 10^9$ yr. Only 31 of the surviving triples with planet mass tertiaries ended up in the core after $\leq 2 \times 10^9$ yr, with 34 of the triples on orbits extending between 1 and 5 core radii. As PSR B1620−26 is most probably located outside the core of M4, this suggests that either the tertiary is low mass or that the system formed less than 5×10^8 yr ago.

3.1. *Planet Mass Tertiaries*

Of the 79 runs bound to the cluster, 17 underwent encounters which strongly perturbed the orbital element of the planet, rendering the triple unstable. For 1 run the planet was directly stripped from the inner binary by an encounter with a field star. In no case was the planet exchanged by an encounter with a field star, as expected given the low planet mass. For the 18 cases where the planet was ejected, 16 took place in the core, the remaining two between 1 and $1.5r_0$. Thus, of the 61 surviving planetary triples, only 15 were inside a core radius after $f_r \times 2 \times 10^9$ yr, while 32 were between 1 and $5r_0$. Of the 17 planetary triples that became unstable, seven had $a_2 > 20$ AU at the time of the encounter that destroyed them. Encounter probabilities are not sensitive to the exact tertiary mass for $m_2 \ll m_p + m_1$; this shows that triples with $a_2 \gtrsim 20$ AU and brown dwarf mass tertiaries ($m_2 \sim 0.001\ M_\odot$) are unlikely to survive for timescales of order τ_c. All runs with $a_2 \leq 20$ AU where the planet was ejected were from low recoil initial encounters where the triple trajectory was brought rapidly back to the core through dynamical friction and the planet stripped.

Thirty-two of 79 candidate triples with planet mass tertiaries survived the interaction with cluster environment for up to 2×10^9 yr, and were at 1–$5r_0$ at the end of the run, compared to 15 who survived and reached the core, and 15 who remained at radii $\geq 5r_0$. A planet mass tertiary can thus comfortably survive for timescales $\gtrsim 10^9$ yr, provided the triple is formed with modest recoil in the cluster core in the range naturally expected for exchange formation scenarios. A triple with planet mass tertiary is more likely to be observed outside the cluster core than not, as is the case with PSR B1620−26.

3.2. *Stellar Mass Tertiaries*

Of the 79 bound stellar mass tertiaries, two underwent exchanges with field stars where the field star and tertiary formed a separate binary. For 6 runs, the triple underwent an exchange between the tertiary and a field star, without the inner binary being perturbed significantly during the encounter. For 15 of the surviving runs, encounters with field star either stripped the tertiary from the triple or increased a_2 to such a large value that subsequent encounters would have rapidly completed the stripping of the tertiary. A run was terminated if the tertiary semimajor axis exceeded 12 times its initial value after an encounter with a field star.

For 11 of the remaining 56 runs, an encounter with a field star lead to a close approach between the inner binary and either the field star or tertiary, which was close enough to strongly perturb the orbital elements of the inner binary. For only 20 of the remaining 45 runs was the outer semimajor axis still in the range inferred for PSR B1620−26, with a stellar mass teritary, the other 25 runs having undergone (typically multiple) encounters during which a_2 increased due to perturbations by the field star.

Of the 28 surviving triples with $a_2 < 100$ AU at the end of the run, only 10 were outside the core at the end of their run, while 18 were inside the core. Of the 18 stars inside the core, only one had $f_r > 0.5$, which leads us to the conclusion that triples with stellar mass tertiary return to the core through dynamical friction on a timescale of less than 10^9 yr, and once in the core, the inner binary survives undisturbed for less than 5×10^8 yr. Rasio et al. (1995) find the lifetime of such stellar triples in the core to be of order 10^8 yr, which is consistent with the results above.

The characteristic age of the pulsar is only 2.2×10^8 yr, but that is likely to be contaminated by the acceleration of the tertiary. It is possible that the true spin-down age of the pulsar is less than τ_c; that is, that the intrinsic $\dot{P}$ is larger than that observed and $\boldsymbol{a} \cdot \boldsymbol{n} < 0$, but it is more likely that $\dot{P}_0 < \dot{P}$ and the true age of the pulsar is somewhat larger than τ_c. The time at which the tertiary was formed is $\geq \tau_0$, if the current tertiary was captured at the same time as the current secondary. If the secondary had already left the main sequence at the time of the formation of the triple, accretion onto the primary could have commenced immediately and been completed on a timescale $\ll \tau_c$. This is necessary in order for the tertiary to be of stellar mass. A more leisurely spin-up and a long characteristic age required that the system should have survived field perturbations longer than is probable and that the system should have returned to the cluster core through dynamical friction. Thus, a stellar mass tertiary implies the pulsar must be young or that the tertiary was acquired after spin-up (Rasio 1994a).

The perturbation of the inner binary eccentricity by field star encounters make solutions with black hole tertiaries, $m_2 \gg 1\ M_\odot$, unlikely. While low-mass black holes may be present in globular clusters (Kulkarni, Hut, & McMillan 1993; Sigurdsson & Hernquist 1993), and would be likely to become triple members by exchange, this is unlikely to be the case for PSR B1620−26. A triple containing a black hole, $m_2 \gg M_\odot$, has a very short dynamical friction timescale and is unlikely to be observed outside the core, as PSR B1620−26 is. Such triples have very large cross section for resonant interaction with field stars (Sigurdsson & Hernquist 1993), which would perturb the inner binary until $e_1 \sim 0.7$ on a timescale $\ll \tau_e$.

A problem is presented by the magnitude of the orbital angular momentum of a triple with a stellar mass tertiary. If the triple is conjectured to have formed during a binary-binary encounter, a_2 is determined by the total angular momentum available in the encounter center-of-mass frame. Typically, the star ejected during the encounter carries away a small fraction of the angular momentum. The inner binary also has comparably low angular momentum. It is likely that the binary that originally contained the pulsar had a semimajor axis smaller than 1 AU. This is expected both from evolutionary arguments and from the requirement that an inner binary be formed with a_1 small enough that mass transfer can take place as the secondary evolves off the main sequence. The bulk of the angular momentum must thus have resided in the other binary involved. If we assume a favorable prograde encounter, the impact parameter of the binary-binary encounter could have been, at most, twice the semimajor axis of the wider binary. This implies the wider binary must have had a semimajor axis of $\gtrsim 10$ AU. With a projected core dispersion of 5 km s^{-1}, mean encounter velocities in the core of M4 are ~ 10 km s^{-1}. At such encounter velocities, binaries with semimajor axis $\gtrsim 10$ AU tend to be disrupted by encounters with field stars. It is unlikely that a binary of large enough angular momentum to directly form our triple system could survive long enough in the core of M4 to undergo the initial binary-binary encounter. Any triple system formed by exchange with a hard core binary is likely to have $a_2 \lesssim 30$ AU, simply from the limited total angular momentum available during encounters. Rasio et al. (1995) reach similar conclusions. Their Figure 3 shows the most probable $a_2 \gtrsim 50$ AU, but they require a wide binary originally containing the current tertiary to have recently entered the core from the cluster halo.

It is still possible for a triple with stellar mass tertiary to have formed, as it could have formed through a prompt encounter between a wide binary just entering the core after spending most of the cluster lifetime in the halo of the cluster. This scenario has difficulty explaining the current position of the pulsar triple outside the core of M4, as recoil during wide binary-binary exchanges is insufficient to eject such a triple from the core. Diffusion out of the core is unlikely for such a high-mass object. The encounter would have had to have occurred recently and most likely outside the cluster core. That is, in this scenario, the inner binary formed through the exchange of a neutron star–heavy white dwarf binary with a turnoff mass main-sequence star, the resulting binary being ejected from the core, and the pulsar being spun up outside the cluster core. The current tertiary is then formed through a binary-binary exchange, as proposed by Rasio (1994a), but the exchange took place *outside* the core of M4, where the necessary wide binaries may survive for long enough timescales. Alternatively, the triplet could have formed with a_2 smaller than currently observed, and encounters with field stars could have lead to a subsequent increase in a_2, in which case the triple is likely to be disrupted on its next return to the core. Both scenarios though require accepting a lower *prior* probability for observing the system in its current state.

4. CONCLUSION

PSR B1620−26 is turning out to be one of the most fascinating binary pulsar known. There is strong indication that the pulsar is a member of a hierarchical triple system, and a few more years of timing are certain to resolve both whether there is a tertiary and the mass of any tertiary present. It is distinctly possible that the tertiary is a jovian planet, in which case it is likely the tertiary was exchanged into the system during a binary-single star encounter (Sigurdsson 1993). If the tertiary is of stellar mass, it will be conclusive proof that the pulsar was involved in a binary-binary exchange, which will provide further support for theories of pulsar recycling involving binary interactions in globular clusters and hence an indirect constraint on the binary fraction in globular clusters.

Dynamical constraints strongly favor the extremes of the permitted mass range for the tertiary. A planet in the tightest orbit permitted can remain bound to the system for timescales comparable to the pulsar characteristic age, τ_c, despite being formally "soft" and vulnerable to stripping by field stars. A stellar mass tertiary provides a marginally "hard" outer binary and thus may remain bound, but such systems are vulnerable to perturbations of the inner binary on timescales comparable to the τ_c. The intermediate-mass solutions—although attractive, as they would provide a detection of a brown dwarf—produce systems that are "soft" and thus vulnerable to stripping, have high interaction rates with field stars, comparable to stellar mass triples, and thus are dynamically not favored.

Either a planet or a stellar mass tertiary requires that the pulsar has undergone an exchange. In the former case, the tertiary was captured at the same time as the secondary. In the latter case, the tertiary may have been captured during a second exchange. Both scenarios require a binary to have spent significant fraction of the cluster lifetime outside the cluster core. For a planet mass tertiary, the whole triple must have spent the last $O(10^9)$ yr in the cluster halo, the pulsar only now returning to the core. For a stellar mass tertiary, the pulsar likely spent the last 10^8–10^9 yr outside the core, and the tertiary spent most of the cluster lifetime in the halo. In either case, the most probable origin for the neutron star was as a member of a binary with a heavy white dwarf, similar to PSR 1713+0747 (Camilo, Foster, & Wolszczan 1994).

Future observations should determine the mass and orbital parameters of the tertiary with the next few years at most. In particular, observations of higher spin period derivatives, starting with the jolt induced fourth derivative, should be observed within 5 yr if the tertiary is planetary mass. Observations of the orbital period derivative may provide a stronger discriminant on a shorter timescale, combined with a measurement of the apparent change in semimajor axis, a direct estimate of the tertiary inclination and mass may be possible in the near future.

If the tertiary is stellar mass, it is most likely that the pulsar is young. Extracting the true spin-down age of the pulsar from $\dot{P}$ would provide an important data point for estimates of the pulsar birthrate in globular clusters. As M4 is the nearest globular cluster, a young PSR B1620−26 implies a steep pulsar luminosity function and a proportionately high pulsar birthrate. A candidate star has been detected near the location of the pulsar (Bailyn et al. 1994) although an association with the pulsar is not certain.

If the tertiary is of planetary mass, the pulsar is naturally expected to have a relatively long spin-down age, which implies a comfortably lower pulsar birthrate. The inferred presence of planet mass companions among low-metallity main-sequence stars (assuming the exchange hypothesis is correct) would imply that planet formation is common. Indeed, it would strongly imply that jovian planets form around low-metallicity main-sequence stars, particularly globular cluster stars. Knowing that jovians form around low-metallicity protostars would provide additional constraints on models for planet formation.

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