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CM DRACONIS AND YY GEMINORUM: AGREEMENT BETWEEN THEORY AND OBSERVATION

GILLES CHABRIER AND ISABELLE BARAFFE
Centre de Recherche Astrophysique de Lyon (UMR CNRS 142), Ecole Normale Supérieure, 69364 Lyon Cedex 07, France
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ABSTRACT

We present new evolutionary models which accurately reproduce the observed masses, radii and luminosities of the eclipsing binary systems CM Draconis and YY Geminorum. Our calculations include new equation of state and atmosphere models appropriate for low-mass stars. The models yield not only an accurate determination of the age, the helium abundance, and the heavy element content of these objects, but also their absolute magnitude and their distance.

The agreement between theory and observation assesses the validity of the present evolutionary models and the related mass-luminosity relation for M dwarfs, a cornerstone to derive accurate mass functions for low-mass stars from the observed luminosity functions.

Subject headings: binaries: eclipsing — stars: distances — stars: low-mass, brown dwarfs

1. INTRODUCTION

The correct modeling of the structure and the evolution of low-mass stars (LMSs) and very low mass stars (VLMSs) is of great importance not only for reliable astrophysical calibrations, but also for the identification of the nature of dark matter in spiral galaxies (Méra, Chabrier, & Schaeffer 1995). The tremendous improvement in IR technology and the developement of future ground-based and space-based IR surveys will produce a wealth of data on LMSs within the next few years. Accurate models will be necessary for a correct determination of the various parameters, in particular the *mass*, of these objects. More generally, a correct determination of the *mass function* of disk and halo low-mass stars from the observed luminosity functions (LFs) relies on the accuracy of the so-called *mass-luminosity* relationship for these objects.

Recently Baraffe et al. (1995) derived new evolutionary models for late M dwarfs, over a large metallicity range, which accurately reproduce the observed magnitude-color and color-color diagrams (Monet et al. (1992; Kirkpatrick et al. 1993, 1994). But the most stringent test for any model is certainly the *direct* comparison with the observed mass, when possible. The only LMS systems for which this parameter, among others, is accurately determined are the two eclipsing binary systems CM Draconis and YY Geminorum. Observations by Lacy (1977) and Leung & Schneider (1978) have determined the masses, radii, and luminosities of the aforementioned systems. Moreover, a correct determination of the helium and the heavy element abundance of CM Draconis is of considerable importance for cosmological purposes.

The mechanical structure of a star depends essentially on the equation of state (EOS) which, for LMSs, strongly departs from the perfect gas approximation. On the other hand, the surface gravity of these objects is log $g \approx 5$, whereas the effective temperature is below 4000 K. Under these conditions, CM Dra ($M \approx 0.2\ M_{\odot}$) and YY Gem ($M \approx 0.6\ M_{\odot}$) offer an excellent opportunity to test models of dense plasma EOS and cool and dense atmospheres. Moreover, if the opacities are well known, the envelope models allow the determination of the heavy element content of the star.

Tremendous progress has been made in the theory of LMSs within the past few years, in particular in the EOS and opacity calculations. This yielded significant improvement in the comparison between theory and observation. However, so far, all the models failed to reproduce the observed characteristics of CM Dra and YY Gem. Such a goal definitively represents the most stringent test to assess the validity of any theoretical model aimed at describing the mechanical and thermal properties of LMSs. In this Letter, we apply the recent calculations of Baraffe et al. (1995), based on new EOS, nuclear enhancement factors, and atmosphere models, to CM Dra and YY Gem. These calculations yield the determination of the age, the metallicity, and the helium abundance of these objects and suggest a new determination of the bolometric magnitude, and then the distance, of CM Dra.

2. THE MODEL

The first attempt to derive accurate models for CM Dra was conducted by Paczyński & Sienkiewicz (1984). The goal was to determine the helium content of this system, believed to belong to the halo stellar population (see Lacy 1977). Their first estimate was based on a simple polytropic calculation, with a polytrope $n = 3/2$, representative of fully adiabatic systems. Though crude, these simple calculations show very elegantly the dependence of the luminosity on the hydrogen content and the mean molecular weight, and thus on the chemical composition of the star. Although CM Dra is essentially convective, a substantial part of the energy is transported by radiation for more massive objects, like YY Gem, and in this case a simple polytropic relation is no longer accurate enough. A more elaborate model by the same authors was based on the so-called FGV EOS (Fontaine, Graboske, & Van Horn 1977), which takes into account nonideal effects between atoms and ions. However, this model could not reproduce the mass ratio of Dra A and Dra B within the observed error bars, for the correct effective temperatures. Their results lie 2 σ away from the observations. Though the FGV EOS represented a significant improvement over existing EOS for the internal structure of dense objects, these results clearly illustrate the *extreme* accuracy required for an EOS to reproduce the observed parameters within their reduced error bars. A further improvement in the EOS of low-mass stars and giant planets was achieved by Saumon and Chabrier (Chabrier

1990; Saumon & Chabrier 1991, 1992; Saumon, Chabrier, & Van Horn 1995, hereafter SCV95). These calculations include a detailed treatment of the interactions between ions, atoms, and electrons in dense plasmas. They give in particular a consistent treatment of *pressure* ionization and *pressure* dissociation of hydrogen, which is found to occur in the interiors of cooler objects like brown dwarfs and giant planets (Chabrier et al. 1992; Saumon et al. 1992). Moreover, the Saumon, Chabrier, & Van Horn (SCV) EOS allows various composition of the hydrogen/helium ratio in the mixture, whereas the FGV EOS was calculated for pure hydrogen, helium, and carbon and requires interpolation procedures for a mixture. A detailed comparison between the SCV EOS and other EOSs for dense objects, in particular with FGV, can be found in SCV95 and in Saumon (1994).

A correct theoretical determination of the effective temperature of a star requires (i) an accurate calculation of the energy distribution, which must reproduce the *observed spectrum*, and (ii) a proper treatment of the boundary conditions between atmosphere and interior models. As shown initially by Allard (1990), for the effective temperature range characteristic of M dwarfs, the energy distribution *strongly* departs from the blackbody distribution, essentially because of molecular absorption, stressing the need for detailed model atmosphere calculations. Our calculations include the recent nongray atmosphere models calculated by Allard & Hauschildt (1995, hereafter AH95) for temperatures and gravities characteristic of LMSs. Moreover, the grid of AH95 includes various metallicities, which allow a relatively precise determination of the overall abundance of heavy elements.

As shown by several authors (Dorman, Nelson, & Chau 1989; Allard 1990; Burrows et al. 1993; Baraffe et al. 1995), between 2500 K $\lesssim T_{\rm eff} \lesssim$ 4000 K, the convection zone penetrates deep into the optically thin outermost layers ($\tau \ll 1$), a direct consequence of H_2 molecular dissociation. In this case, the Eddington approximation is no longer valid, and an accurate determination of the boundary condition between the atmosphere and the internal structure is required along the evolutionary track.

At last, the mechanical and the thermal structure of the star depends also on the energy generation. For such low-mass objects, the nuclear reaction network is truncated after the PPI chain. The essential difficulty in the calculation of the nuclear energy production arises from the *screening factor*, i.e., the enhancement factor of the nuclear reaction rate between two particles as a result of the strongly correlated surrounding ions. Moreover, for the internal temperatures and densities characteristic of these objects, the electrons are only *partially* degenerate, i.e., $T/T_{\rm F} \sim 1$ (Baraffe & Chabrier 1995), where $T_{\rm F}$ is the Fermi energy of the electrons, so that, though weak, the electron gas polarizability is not negligible and must be taken into account for a correct calculation of the enhancement factors. We used the factors calculated by Chabrier (1995).

3. RESULTS

Figures 1 and 2 show the comparison between observations and theory for CM Dra (A and B) and YY Gem (A and B) in a mass–radius and a mass–effective temperature diagram, respectively. The basic calculations include the SCV EOS, the AH95 *nongray* atmosphere models, and the enhancement factors mentioned above. They correspond to an age τ = 10 Gyr and a solar helium initial mass fraction $Y = 0.275$. A mixing-length parameter $\alpha = l/H_P = 1$ was used throughout the calculations. Variations around these values will be considered below.

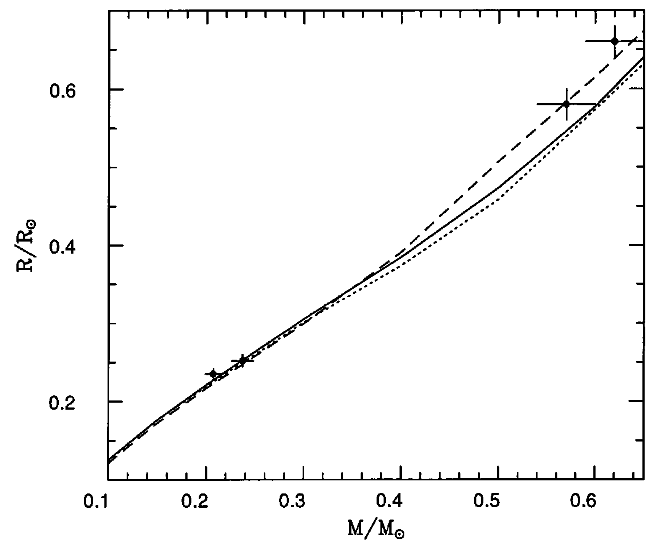

FIG. 1.—Mass-radius relationship for CM Dra and YY Gem. The points and the error bars are the observations (Lacy 1977; Leung & Schneider 1978). The results are for different metallicities, *Solid line*, [*M*/H] = 0; *dashed line*, [*M*/H] = −0.5; *dotted line*, [*M*/H] = 0 within the Eddington approximation. All calculations are for an age τ = 10 Gyr.

3.1. *CM Draconis*

Metallicity effect. —Although the agreement between theory and observation for the radius is obtained over a fairly large metallicity range, the effective temperature yields a metallicity [*M*/H] = 0 to ~−0.3, i.e., $Z \approx 0.01$–0.02 for CM Dra. Metallicities below this value are definitively ruled out, as clearly seen in Figure 2. This raises a problem for this system whose space velocity (u = +105; v = −118; w = −38 km s^{-1}) (Lacy 1977) strongly suggests a membership to the old disk

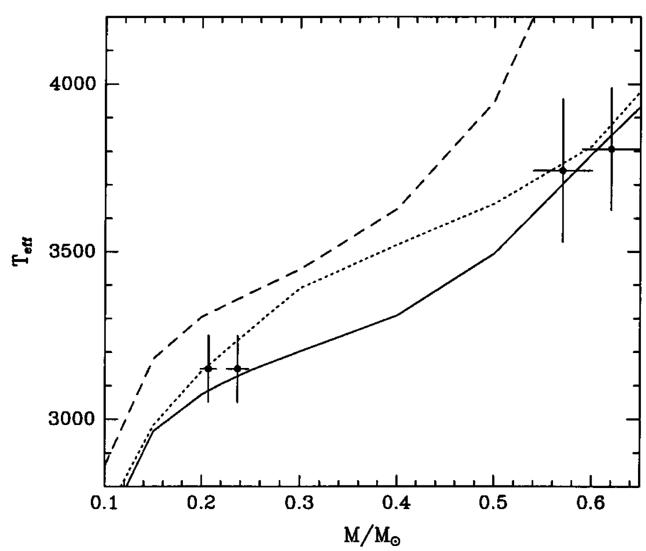

FIG. 2.—Mass–effective temperature relationship for CM Dra and YY Gem. Same legends as Fig. 1.

TABLE 1

CHARACTERISTIC PARAMETERS FOR CM DRACONIS AND YY GEMINORUM

A. CM DRACONIS

Object	$M/M_\odot$	[M/H]	τ (yr)	Y	$R/R_\odot$	T_{eff}	M_{bol}	M_v	d(pc)
Observations									
A	0.237 ± 0.011	...	...	...	0.252 ± 0.008	3150 ± 100	10.39 ± 0.11	12.77	14.49
B	0.207 ± 0.008	...	...	...	0.235 ± 0.007	3150 ± 100	10.54 ± 0.11	12.92	14.49
A (new distance)	0.237 ± 0.011	...	...	...	0.252 ± 0.008	3300 ± 100	10.16 ± 0.11	12.56	15.93
B (new distance)	0.207 ± 0.008	...	...	...	0.235 ± 0.007	3300 ± 100	10.31 ± 0.11	12.71	15.93
Theory									
A (model 1)	0.237	0	10^{10}	0.275	0.253	3128	10.33	13.18	...
(model 2)	0.237	−0.5	10^{10}	0.25	0.248	3366	10.05	12.07	...
B (model 1)	0.207	0	10^{10}	0.275	0.227	3086	10.63	13.58	...
(model 2)	0.207	−0.5	10^{10}	0.25	0.223	3318	10.35	12.47	...

B. YY GEMINORUM

Object	$M/M_\odot$	[M/H]	τ (yr)	Y	$R/R_\odot$	T_{eff}	M_{bol}	M_V	d(pc)
Observations									
A	0.62 ± 0.03	...	...	...	0.66 ± 0.02	3806 ± 180	7.54 ± 0.1	8.99	13.89
B	0.57 ± 0.03	...	...	...	0.58 ± 0.02	3742 ± 214	7.89 ± 0.1	8.99	13.89
Theory									
A (model 1)	0.62	0	10^{10}	0.275	0.6	3854	7.55	9.32	...
(model 2)	0.62	0	10^{8}	0.275	0.618	3898	7.44	9.13	...
B (model 1)	0.57	0	10^{10}	0.275	0.545	3696	7.95	9.97	...
(model 2)	0.57	0	10^{8}	0.275	0.565	3711	7.85	9.87	...

NOTES.—The "new distance" values for CM Dra correspond to the determination of a new parallax when fitting the observed color spectrum (Lacy 1977) with synthetic colors at [M/H] = −0.5 (Allard & Hauschildt 1995). The models numbered 2 are the most likely solutions (see text). The M_{bol} for YY Gem has been calculated from the observed luminosity (Leung & Schneider 1978), with the bolometric correction derived specifically for this system by Hoxie (1973).

population (see, e.g., Beers & Sommer-Larsen 1995), and then a metallicity [M/H] ≤ -0.5 (Leggett 1992). This paradigm will be examined below. Also shown on Figure 2 are calculations done within the Eddington approximation (for solar metallicity), with the most recent Rosseland opacities (Alexander & Ferguson 1994). As clearly shown, this approximation leads to effective temperatures significantly larger than the results obtained with accurate atmosphere models, and then to overluminous objects. This confirms the inadequacy of the diffusion approximation below ~4000 K and then the necessity to include a proper treatment of the atmosphere and the boundary conditions for the modeling of LMSs.

Age effect. —Stars with masses of 0.2 $M_\odot$ are found to reach the main sequence (MS) at $\tau \approx 5\ 10^8$ yr. Variations of the effective temperature and luminosity, and then of the radius, remain very small after this stage. As already mentioned, the calculations shown on Figures 1 and 2, which reproduce the observed radii and effective temperatures (and then the luminosities) within the error bars, correspond to an age $\tau = 10^{10}$ yr. This is consistent with a Population II stellar population, as suggested by the large space velocity, whereas younger ages are unlikely for such objects.

Helium abundance. —We find that the results depend *weakly* on the helium abundance. Calculations with $Y = 0.25$ and $Y = 0.275$, for a given metallicity, do not modify appreciably the results. A 10% decrease in Y yields ~0.6% decrease in T_{eff} ~ 3.6% decrease in L, and ~0.4% decrease in R for a 0.2 $M_\odot$ star. Therefore, the helium abundance is determined indirectly by the requirement of consistency with the age and the heavy element abundance.

Table 1 summarizes the results of the present analysis for CM Dra with [M/H] = 0 (models numbered 1). All models do reproduce the observations (mass, radius, and luminosity) within the error bars. However, as mentioned above, observations of CM Dra strongly suggest that it belongs to the *old disk*, whose typical metallicity is [M/H] ≤ -0.5. We found that, for this metallicity, the agreement between the observed (V, R, I, and J) colors (Lacy 1977) and the ones obtained from the synthetic spectra of AH95 is optimum for an effective temperature $T_{eff} = 3300$ K. The *observational* effective temperature has been determined with the Stefan-Boltzmann relation from the *absolute* bolometric magnitude. The latter requires the knowledge of the distance, and thus of the *parallax*. As acknowledged by Lacy himself, there is some uncertainty on the parallax. The characteristics displayed on the lower row for CM Dra A and B (models numbered 2), obtained with [M/H] = −0.5 and $T_{eff} = 3360$ and 3320 K, reproduce all the *direct* observational constraints within their error bars (though admitedly, the radius of CM Dra B is slightly underestimated). This yields a new determination of the parallax, $\pi = 0''.063$, about 1 σ away from the value adopted by Lacy. This corresponds to a ~10% larger distance $D' = 15.93$ pc, and absolute bolometric magnitudes $M_{bol} = 10.16$ and 10.31 ± 1%. In that case, *all* the characteristics of CM Dra (helium and heavy element abundance, age, space velocity) are consistent with an old disk stellar population.

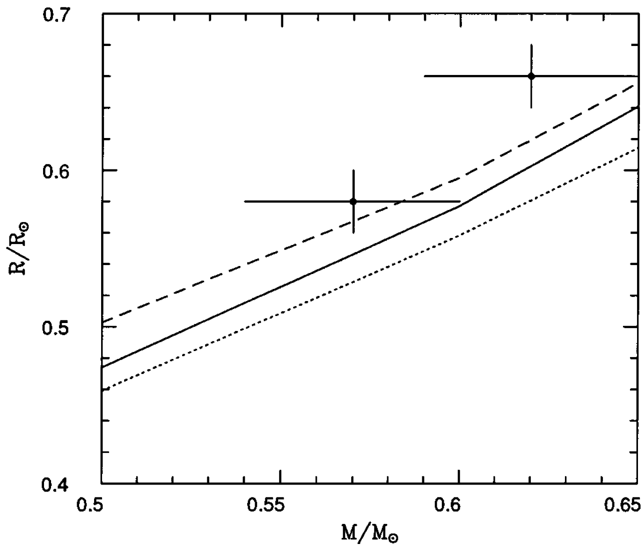

FIG. 3.—Age effect for YY Gem in a mass-radius diagram. *Dashed line*, $\tau = 10^8$ yr; *dotted line*, $\tau = 3 \times 10^8$ yr; *solid line*, $\tau = 10^{10}$ yr.

3.2. *YY Geminorum*

Metallicity effect. —Again, though agreement for the radius is found over a fairly large metallicity range (Fig. 1), the effective temperature clearly favors a solar-like metallicity ($[M/\mathrm{H}] \gtrsim -0.3$) (Fig. 2) and rules out metal-depleted models. As shown on Figure 1, the agreement for the radius is only marginal and can be improved, within the observational error bars, by decreasing the mixing length parameter to $\alpha = 0.5$. Indeed, although CM Dra is entirely convective, YY Gem develops a radiative core over ~70% in radius, for any metallicity, so that variations of the mixing-length parameter affect significantly the luminosity and the effective temperature. However, $\alpha = 0.5$ is unrealistically smaller than the solar value $\alpha \sim 1.5$ and thus is very unlikely.

Age effect. —The main sequence for a 0.6 $M_\odot$ star is reached after $\tau \approx 3 \times 10^8$ yr. Interestingly enough, an excellent agreement between theory and observations, inside *all* the error bars, is obtained for $\tau = 10^8$ yr (and $Y = 0.275$, $Z = Z_\odot$). Ages between this value and 10^{10} yr are clearly excluded, as shown on Figure 3. Similar results are obtained with $[M/\mathrm{H}] = -0.3$. This suggests that YY Gem either is on the late pre-MS contraction phase (model no. 2 in Table 1), or is a fairly old object (model no. 1 in Table 1). This latter possibility is clearly in conflict with a solar-like metallicity, as determined above. Moreover, YY Gem belongs to the Castor quartet, whose most massive star (Castor A) has an A3 spectral type. Since the MS lifetime for A3 stars is smaller than 10^9 yr, an age $\tau \geq 10^{10}$ yr for YY Gem (≡ Castor C) is certainly excluded. An age of 10^8 yr requires oversolar metallicities. Models with $[M/\mathrm{H}] = +0.5$ were found to predict much too low effective temperatures ($T_{\mathrm{eff}} \approx 3200$–3400 K). Then a likely metallicity for YY Gem is $[M/\mathrm{H}] \approx 0.1$–0.2.[1] The possible observable signature of young objects is the presence of Li in the spectrum. However, we find that, for solar-like metallicity, all the lithium is destroyed before 10^8 yr for the mass range of interest (Baraffe & Chabrier 1995).

Helium abundance. —In the case of YY Gem, the results are more sensitive to the helium abundance than for CM Dra. A 10% decrease in Y (0.275 to 0.25) yields ~1.7% decrease in T_{eff}, ~12% decrease in L, and ~2.5% decrease in R. Consistency between the helium abundance and the afore-determined age and metallicity suggests $Y \approx 0.30$.

4. CONCLUSION

In this Letter we have reported theoretical calculations which reproduce for the first time *all* the observed parameters of the eclipsing binaries CM Dra and YY Gem. These results assess the validity of the present models to describe the mechanical and thermal properties of LMSs and VLMSs and probe the mass-luminosity relationship within the corresponding mass range. These calculations show the necessity to use *very accurate* input physics, equation of state, nuclear enhancement factors, opacities, and a proper treatment of the atmosphere for a correct description of dense and cool objects. We determine the age, the helium abundance, and the metallicity of CM Dra and YY Gem. The most likely scenarios are that YY Gem is at the end of the pre–main-sequence contraction phase, with $[M/\mathrm{H}] \approx 0.1$–0.2 and $Y \approx 30\%$, and CM Dra is an old-disk Population II star, as suggested in particular by its spatial velocity, with a metallicity $[M/\mathrm{H}] \approx -0.5$ and a helium abundance $Y \approx 25\%$. Agreement between theory and observations of Lacy (1977) suggests a new determination of the distance, and absolute magnitudes, for this system.

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[1] Atmosphere models with these peculiar metallicities are not yet available, so precise evolutionary calculations could not be conducted.

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