

DISCOVERY OF STRONG VIBRATIONALLY EXCITED WATER MASERS AT 658 GHz
TOWARD EVOLVED STARSKARL M. MENTEN¹ AND KEN YOUNG²*Received 1995 May 24; accepted 1995 July 2*

ABSTRACT

We report the astronomical discovery of the $1_{10} \rightarrow 1_{01}$ rotational transition within the $\nu_2 = 1$ vibrationally excited state of water vapor (H_2O). Using the 10.4 m telescope of the Caltech Submillimeter Observatory, we detect strong maser emission in this line, which has a frequency near 658 GHz, toward a diverse sample of oxygen-rich red giant and supergiant stars. In circumstellar envelopes these 658 GHz H_2O masers appear to be as common as SiO masers and H_2O masers in other transitions, while we fail to detect 658 GHz H_2O emission toward the W49 N and W51 N star-forming regions. For all of the 11 stars detected, the luminosity in the 658 GHz H_2O transition is comparable to or higher than the luminosity of any other known SiO or H_2O maser line.

Subject headings: circumstellar matter — masers — stars: AGB and post-AGB — stars: mass loss — stars: variables: other

1. INTRODUCTION

Water is one of the most abundant molecules in the envelopes of oxygen-rich cool giant and supergiant stars (Tsuji 1964) and its numerous rotational transitions, which occur at submillimeter and far-infrared wavelengths, dominate the radiative cooling of these regions (Goldreich & Scoville 1976; Justtanont, Skinner, & Tielens 1994). Observational information on circumstellar H_2O is difficult to obtain since water absorption in the Earth's atmosphere effectively prohibits ground-based studies of most rotational lines. All of the centimeter and (sub)millimeter wavelength H_2O transitions detected so far are from levels at relatively high energies above the ground state and show maser action, which complicates their interpretation, but, given their high brightness temperatures, allows interferometric studies with high spatial resolution. In particular, radio interferometry has shown that the most widely observed transition, the 22 GHz $6_{16} \rightarrow 5_{23}$ line, arises toward Mira and semi-regular variables from regions with radii ranging from several stellar radii (r_*) to several $10 r_*$ (see, e.g., Reid & Menten 1990; Bowers & Johnston 1994). So far no interferometric data have been published for any of the recently detected (sub)millimeter H_2O maser transitions from within the vibrational ground state (Menten, Melnick, & Phillips 1990a; Menten et al. 1990b; Cernicharo et al. 1990; Melnick et al. 1993), which may arise from similar parts of the envelope as the 22 GHz line or somewhat closer to the central star (Menten & Melnick 1991; Yates, Cohen, & Hills 1994).

Water located even closer to the star has been studied toward the Mira variable R Leo by infrared absorption measurements of high excitation lines from various vibrational states (Hinkle & Barnes 1979). At millimeter wavelengths, circumstellar emission from two pure rotational lines from within the $\nu_2 = 1$ vibrational state, the first excited state of the bending mode, has been detected by Menten & Melnick (1989). Both, the $\nu_2 = 1, 4_{40} \rightarrow 5_{33}$ and $5_{50} \rightarrow 6_{43}$ transitions at 96 and 233 GHz, respectively, show weak emission with flux

densities around a few Jy toward the M-type supergiant VY CMa. The 233 GHz line is also detected toward the nearby semiregular variable W Hya. The presence of narrow spectral features indicates maser action in both lines, which are at energies of more than 3000 K above the ground state. Observations of another rotational line from within the $\nu_2 = 1$ state toward VY CMa were reported by Feldman et al. (1993).

In this Letter we describe the first astronomical observations of the 658 GHz $J_{K_a K_c} = 1_{10} \rightarrow 1_{01}$ rotational transition within the $\nu_2 = 1$ vibrationally excited bending mode of H_2O toward oxygen-rich giant and supergiant stars. The 1_{01} level, the lowest rotational level of ortho- H_2O in the $\nu_2 = 1$ state, is at an energy of 2329 K above the 0_{00} (para) level of the vibrational ground state. This energy is intermediate between that of the $\nu = 1$ and 2 states of SiO, which are around 1800 and 3500 K, respectively, and whose lower rotational transitions exhibit strong maser action toward numerous evolved stars (see, e.g., Benson et al. 1990). Our observations show that circumstellar maser emission in the 658 GHz H_2O transition appears to be as widespread as maser emission in these SiO and other H_2O lines. Moreover, for the stars in our sample the luminosity of the 658 GHz water line is comparable to or higher than that of any other known maser line.

2. OBSERVATIONS

The submillimeter observations were made on 1995 March 18 using the 10.4 m telescope of the Caltech Submillimeter Observatory (CSO) on Mauna Kea, Hawaii, under very good weather conditions. The liquid-helium-cooled SIS mixer receiver described by Kooi et al. (1994) was used in double-sideband (DSB) mode, with the lower sideband (LSB) centered at the frequency of the $\nu_2 = 1, 1_{10} \rightarrow 1_{01}$ transition of H_2O . The rest frequency of this line is 658.00655 GHz and has an uncertainty ≈ 0.1 MHz, corresponding to a radial velocity uncertainty of less than 0.05 km s^{-1} (Helminger, Messer, & De Lucia 1983; Pearson et al. 1991). The upper sideband (USB) was at a 3 GHz higher frequency, so that the $J = 6 \rightarrow 5$ transition of ^{13}CO at 661.0673 GHz (Lovas 1992; Graf et al. 1990) was within the observed band. Weak emission in the latter line is detected toward some of our program sources. To

¹ Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge MA 02138; menten@cfa.harvard.edu.

² Caltech Submillimeter Observatory, P.O. Box 4339, Hilo, HI 96720; rtm@tacos.caltech.edu.

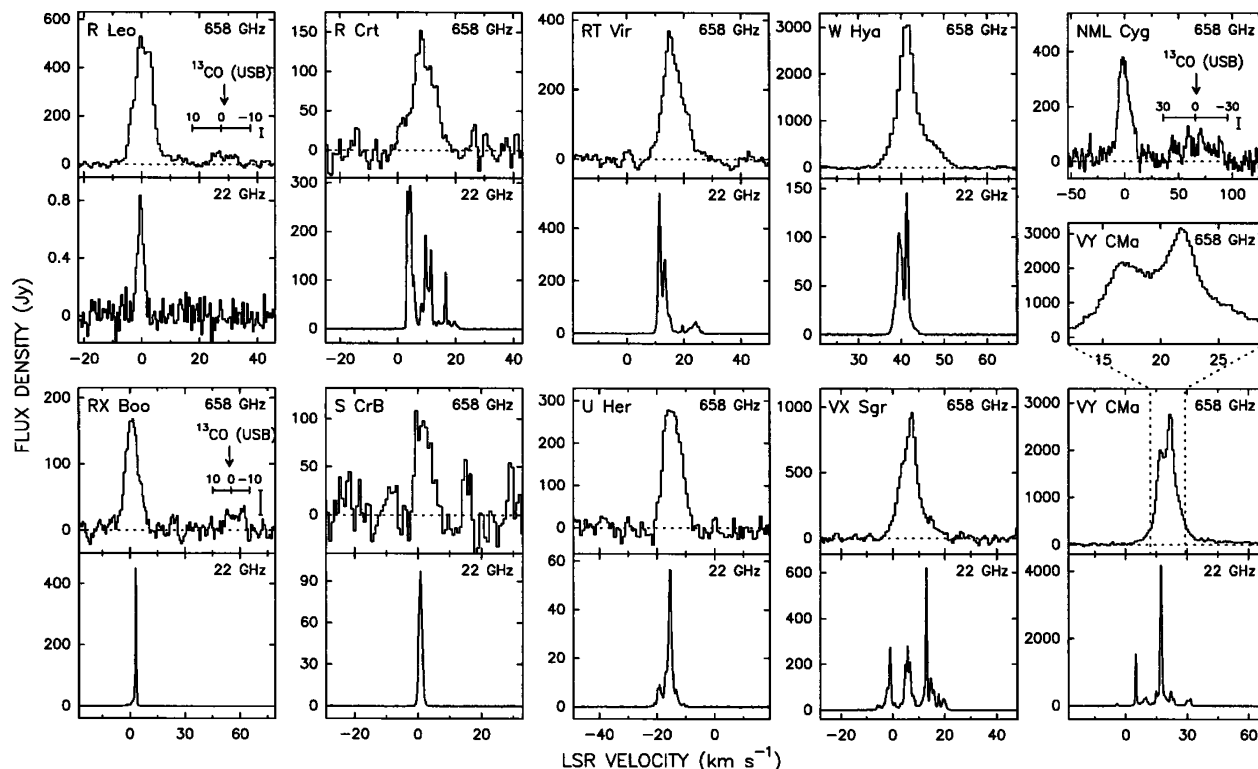


FIG. 1.—Observed spectra of the 658 GHz $\nu_2 = 1, 1_{10} \rightarrow 1_{01}$ and 22 GHz $6_{16} \rightarrow 5_{23}$ transitions of H_2O toward oxygen-rich evolved stars. The 658 GHz spectra have been smoothed to a channel spacing of 0.78 km s^{-1} . For the 22 GHz spectra the channel spacing is 0.17 km s^{-1} , except for the R Leo spectrum, for which it is 0.49 km s^{-1} . The horizontal dotted lines indicate the zero levels of the 658 GHz spectra. No 22 GHz spectrum was obtained for NML Cyg. For VY CMa, two 658 GHz spectra are shown in the final column. The wider band spectrum was taken with the 500 MHz bandwidth AOS and has a channel spacing of 0.78 km s^{-1} . The narrower spectrum was taken with the 50 MHz AOS, has a channel spacing of 0.22 km s^{-1} , and covers, as indicated by the dotted lines, only part of the velocity range showing emission. Very low level emission in the wideband 658 GHz spectrum at velocities $\geq 35 \text{ km s}^{-1}$ probably is from the $^{13}\text{CO } J = 6 \rightarrow 5$ transition in the image sideband (USB). For R Leo, RX Boo, and NML Cyg, the displayed velocity range also contains weak image sideband $^{13}\text{CO } J = 6 \rightarrow 5$ emission. For these stars, an insert shows the velocity scale appropriate for USB emission at the rest frequency of the ^{13}CO line. The arrow indicates the stellar velocity in this velocity frame and the vertical bar corresponds to an intensity of 0.5 K in main-beam brightness temperature units.

prevent blending of H_2O emission from the LSB with ^{13}CO emission from the USB, in most cases the local oscillator frequency was chosen slightly ($\approx 20 \text{ MHz}$) below the frequency of the H_2O line, which caused both lines, if present in a given source, to cover separate velocity intervals. The latter procedure also allowed unambiguous identification of both lines. A beam-switching observing mode was employed with the secondary mirror wobbling at a rate of $\approx 1 \text{ Hz}$ between the source position and, alternately, two off-source positions displaced in azimuth by $-60''$ and $+60''$. This observing mode resulted in very flat spectral baselines. Calibration was initially achieved with the chopper-wheel method. At the frequencies in question, the atmospheric opacity in the LSB is different from the USB value, resulting in a miscalibration, which was corrected by employing the two-step process described by Menten et al. (1990b). The resulting corrected antenna temperatures were converted into Jy units by assuming an aperture efficiency of 29%, which was determined from observations of Mars. The USB ^{13}CO data are quoted in main-beam brightness temperature units using a main-beam efficiency of 0.41% (Fig. 1). The telescope pointing is estimated to be accurate to within $3''$ and the beam width is $11''$ (FWHM). We estimate that the absolute calibration of our spectra is accurate to within 30%. The spectra were simultaneously analysed with two 1024 channel acousto-optic spectrometers (AOS) with bandwidths of 500 and 50 MHz. Their effective velocity resolutions were 0.88 and 0.090 km s^{-1} , respectively.

To compare our submillimeter H_2O data with spectra of the most widely studied water maser line, the $6_{16} \rightarrow 5_{23}$ transition at 22.23508 GHz , we observed the latter line toward most of our target sources with the 100 m telescope operated by the Max-Planck-Institut für Radioastronomie near Effelsberg, Germany. These observations were made on 1995 April 8–10, about three weeks after the submillimeter observations. A total-power observing mode was used. The beamwidth was $43''$ (FWHM). The observations were made in rather poor weather and we estimate that the flux density scale, which was determined by using NGC 7027 as a calibrator source, is accurate within 20%. The spectrometer was a 1024 channel autocorrelator with 12.5 MHz bandwidth, yielding a velocity resolution of 0.17 km s^{-1} .

3. RESULTS AND DISCUSSION

We detected strong emission in the $\nu_2 = 1, 1_{10} \rightarrow 1_{01}$ transition toward 10 out of the 11 oxygen-rich stars observed (Fig. 1 and Table 1). Because of the high intensities it is clear that we are observing maser emission. More quantitatively, we can exclude thermal emission for the following reasons: Vibrational excitation almost certainly only plays a role in the innermost part of the circumstellar envelope (Justtanont et al. 1994). Assuming that the 658 GHz line arises from a region of radius $10 r_*$ (or $5 \times 10^{14} \text{ cm}$ for a “typical” Mira or semi-regular variable), that within this region the H_2O level popu-

TABLE 1
RESULTS OF 658 GHz H₂O $\nu_2 = 1, 1_{10} \rightarrow 1_{01}$ OBSERVATIONS

SOURCE ^a	α_{1950}	δ_{1950}	TYPE	PHASE ^b	S_{peak}^c (Jy)	v_{LSR}^c (km s ⁻¹)	$\int S dv^d$ (Jy km s ⁻¹)	$v_{\text{LSR-range}}$ (km s ⁻¹)	L_{658}^e ($\times 10^{43} \text{s}^{-1}$)	L_{658}/L_{22}^f
U Ori	05 ^h 52 ^m 50 ^s .9	20°10'06''	M	0.37	< 60	39
VY CMa	07 20 54.7	-25 40 12	Irr	...	2760	22	27800(200)	≈ [0, 40]	3060	3.0
R Leo	09 44 52.2	11 39 40	M	0.97	532	0	3790(50)	[-5, 6]	6.0	2000
R CrI	10 58 06.0	-18 03 22	SRb	...	152	8	1250(80)	[-1, 19]	6.5	1.1
RT Vir	13 00 05.7	05 27 22	SRb	...	366	15	2810(60)	≈ [8, 25]	36	2.2
W Hya	13 46 12.0	-28 07 09	SRa	0.81	3020	42	17300(200)	[35, 50]	10	57
RX Boo	14 21 56.7	25 55 47	SRb	...	171	1	1470(50)	[-7, 10]	4.5	3.5
S CrB	15 19 21.4	31 32 47	M	0.39	100	1	560(80)	[-2, 7]	4.8	4.4
U Her	16 23 34.7	19 00 17	M	0.81	275	-15	2100(70)	[-21, -8]	19	18.0
VX Sgr	18 05 03.0	-22 13 55	Irr	0.59	953	7	7460(130)	[-4, 21]	1300	3.6
W49 N	19 07 49.8	09 01 17	(SFR)	...	< 60	10	< 0.12
W51 IRS2	19 21 22.3	14 25 15	(SFR)	...	< 70	60	< 0.50
χ Cyg	19 48 38.5	32 47 10	S	0.73	< 60	10
NML Cyg	20 44 33.8	39 55 57	Irr	...	377	-1	4930(150)	≈ [-14, 12]	74	...

^a References for stellar positions can be found in Table 1 of Menten & Melnick 1991, except for VX Sgr and χ Cyg, whose positions were taken from Wright et al. 1990.

^b Phases listed correspond to the date of the 658 GHz observations and are given for those stars for which period and date of last maximum were available from the AAVSO (Mattei 1995).

^c S_{peak} and v_{LSR} are peak flux density and LSR velocity of the strongest emission observed in the 658 GHz H₂O line with the 500 MHz AOS (0.78 km s⁻¹ channel spacing). In cases where an upper limit is given for S_{peak} , this is 3 times the rms noise and the listed v_{LSR} value is the center of the observed velocity range.

^d Flux densities integrated over the whole v_{LSR} range showing emission, as listed in the following column, are given by $\int S dv$ together with 1 σ errors, which are derived from the rms noise in the spectra and do not include the $\approx 30\%$ absolute calibration uncertainty.

^e The photon luminosities in the 658 GHz line, L_{658} , were calculated assuming isotropic emission and distances listed in Table 1 of Menten & Melnick 1991. The distance for VX Sgr was assumed to be 1.7 kpc (Humphreys et al. 1972).

^f Ratio of the observed photon luminosities in the 658 and 22 GHz lines. We did not obtain 22 GHz data for U Ori, NML Cyg, and the star-forming regions (SFRs) W49N and W51 IRS2. The upper limits in L_{658}/L_{22} for the latter two sources were calculated as described in the text.

lations are thermalized at an ‘‘average’’ excitation temperature of 1000 K, and that the 658 GHz line is optically thick (all of which are *extremely* optimistic assumptions), one predicts to observe with the CSO a main-beam brightness temperature (T_{MB}) of 3 K toward a star at a distance of 100 pc. This is much smaller than $T_{\text{MB}} = 39$ K observed toward W Hya, which is at that distance. In Figure 1 we compare our $\nu_2 = 1, 1_{10} \rightarrow 1_{01}$ spectra with spectra of the 22 GHz $6_{16} \rightarrow 5_{23}$ transition of H₂O. While in most cases the emission in both lines covers similar velocity intervals, the 658 GHz spectra look quite different from the 22 GHz spectra. Spectra in the latter line are dominated by many narrow features with widths $\lesssim 1$ km s⁻¹, while the 658 GHz spectra show contiguous, broad, emission with asymmetric line shapes centered at the stellar velocity. These differences are not due to the different velocity resolutions and appear most clearly in the case of the supergiants VX Sgr and VY CMa (see Fig. 1), which have particularly complex 22 GHz spectra. The photon luminosities observed in the 658 GHz line, L_{658} , range from 5×10^{43} to 3×10^{46} s⁻¹ (Table 1). Although no clear relationship between stellar luminosity (L_*) or mass-loss rate (\dot{M}) and maser luminosity is apparent in our data, we note that the highest maser luminosities are observed toward the luminous ($L_* > 10^5 L_\odot$), high mass-loss ($\dot{M} > 10^4 M_\odot \text{yr}^{-1}$) supergiants VY CMa and VX Sgr. In all cases the photon luminosity of the 658 GHz line exceeds that of the 22 GHz line and other submillimeter H₂O maser lines (Menten & Melnick 1991; Yates et al. 1995).

We did not detect the 658 GHz line toward the Mira variable U Ori, which usually shows 22 GHz maser emission (Benson et al. 1990) and toward which the 321 GHz $10_{29} \rightarrow 9_{36}$ water maser line was detected by Menten & Melnick (1991). We also failed to detect 658 GHz emission toward the S-type star χ Cyg, which, while showing strong SiO maser emission,

has not been detected in any H₂O maser transition (Benson et al. 1990). Finally, the 658 GHz H₂O line was not detected toward the star-forming regions W49 N and W51 IRS2 (also known as W51 d or W51 N), both of which show strong emission in the 22 GHz and most of the known (sub)millimeter water maser lines. Since we do not have recent 22 GHz spectra of W49 N and W51 IRS2 available, we calculate the upper limits on the L_{658}/L_{22} luminosity ratio quoted for these sources in Table 1 by, first, assuming isotropic 22 GHz luminosities of $L_{22} = 4 \times 10^{48}$ and 9×10^{46} s⁻¹, respectively (Menten et al. 1990a). Second, we calculated conservative upper limits in L_{658} from the listed 3 σ upper limits in S_{peak} , assuming that the 658 GHz line, if present, would cover similar velocity intervals as other submillimeter H₂O lines in the sources in question, namely 130 km s⁻¹ (W49 N) and 20 km s⁻¹ (W51 IRS2) (see Melnick et al. 1993). The resulting upper limits on L_{658}/L_{22} are significantly lower than the values obtained for the evolved stars.

Vibrationally excited H₂O masers probably are located in the same parts of circumstellar envelopes as SiO masers, i.e., within a few stellar radii of the central star (Diamond et al. 1994; Greenhill et al. 1995). This is strongly suggested by the fact that the 96 GHz $\nu_2 = \text{H}_2\text{O}$ line and several isotopic SiO maser lines observed toward VY CMa have identical line shapes (Menten & Melnick 1989, 1991). To compare the observational properties of 658 GHz H₂O and SiO masers, we use, in the absence of contemporaneous measurements, the SiO data taken by Martínez, Bujarrabal, & Alcolea (1988). These authors monitored the strong SiO $v = 1$ and 2, $J = 1 \rightarrow 0$ maser transitions toward, among others, VY CMa, R Leo, R CrI, RT Vir, W Hya, U Her, and VX Sgr over several years. For all stars the 658 GHz photon luminosity exceeds the

maximum photon luminosity measured in the strongest SiO line during the monitored time interval by factors of a few.

Generally, 43 GHz $J = 1 \rightarrow 0$ SiO and, as mentioned above, 22 GHz H₂O spectra show a larger number of narrow velocity components than the 658 GHz H₂O spectra. This may be explained if saturation sets in for 658 GHz H₂O masers at lower brightness temperatures (T_s) than for SiO and 22 GHz H₂O masers, so that all the observed 658 GHz masers are heavily saturated. Indeed, assuming equal beaming for SiO and 658 GHz H₂O masers, we find, using equation (9) of Reid & Moran (1981), the ratio of the saturation temperatures of the 658 GHz H₂O and the 43 GHz $J = 1 \rightarrow 0$ SiO masers, $T_s^{658}/T_s^{43} \approx 8 \times 10^{-3} \Gamma^{658}/\Gamma^{43}$. The population redistribution rates, Γ , depend on the pump mechanism, which, as discussed below, is poorly known in the case of the 658 GHz H₂O line. However, for radiative infrared pumps one might expect $\Gamma^{658} \approx \Gamma^{43}$. The observed rather extreme differences between 658 and 22 GHz H₂O spectra may be expected if stellar 22 GHz H₂O masers are mostly unsaturated, which is indeed indicated by observational evidence (Reid & Menten 1990).

Recent models of SiO masers, advocating pump mechanisms dominated by either collisional or radiative processes, suggest molecular hydrogen densities around 10^9 – 10^{10} cm⁻³ and temperatures $\gtrsim 1000$ K for the masing region (Lockett & Elitzur 1992; Bujarrabal 1994). Alcolea & Menten (1993) performed numerical calculations which showed that inversion in the 96 GHz $4_{40} \rightarrow 5_{33}$ and 233 GHz $5_{50} \rightarrow 6_{43}$ $\nu_2 = \text{H}_2\text{O}$ transitions can be achieved under similar conditions. Moreover, their calculations predict maser action in other $\nu_2 = 1$ transitions whose upper level quantum numbers are of the form $J_{K_a=J, K_c=0 \text{ or } 1}$. Although the 1_{10} upper level of the 658 GHz transition is one of these “transposed” backbone levels, the

calculations of Alcolea and Menten do not predict strong maser action in the latter line. The 658 GHz line is also not mentioned by Deguchi & Nguyen-Q-Rieu (1990) who studied circumstellar H₂O excitation including vibrational excitation and predict line fluxes for numerous submillimeter and far-infrared transitions. We note that, at present, all efforts to model vibrational excitation of H₂O in circumstellar envelopes are seriously hampered by the lack of collisional excitation rates for transitions between the ground state and vibrationally excited states and also for collisions within the excited states.

Given the strength of the 658 GHz line, it is clear that rotational emission from vibrationally excited water provides a powerful new means for detailed studies of the innermost, hot regions of circumstellar envelopes just outside the photosphere, which at (sub)millimeter wavelengths, so far, have only been probed by vibrationally excited SiO maser emission. Of particular interest will be interferometric studies with spatial resolutions $\lesssim 0''.1$, which will be attained with instruments such as the Submillimeter Array currently under construction by the Smithsonian Astrophysical Observatory and the Millimeter Array planned by the National Radio Astronomy Observatory.

We would like to thank Tom Phillips, the director of the CSO, for granting us observing time, Eric Herbst for comments on H₂O spectroscopy, and Mark Reid for helpful discussions. We are grateful to Javier Alcolea for assisting in the Effelsberg observations and useful discussions and Janet Mattei for providing data from the AAVSO International Database. Work at the Caltech Submillimeter Observatory is funded by the National Science Foundation under contract AST 93-13929.

REFERENCES

- Alcolea, J., & Menten, K. M. 1993, in *Astrophysical Masers*, ed. A. W. Clegg & G. E. Nedoluha (Berlin: Springer), 399
- Benson, P. J., et al. 1990, *ApJS*, 74, 911
- Bowers, P. F., & Johnston, K. J. 1994, *ApJS*, 92, 189
- Bujarrabal, V. 1994, *A&A*, 285, 953
- Cernicharo, J., Thum, C., Hein, H., John, D., Garcia, P., & Mattiocco, F. 1990, *A&A*, 231, L15
- Deguchi, S., & Nguyen-Q-Rieu. 1990, *ApJ*, 360, L27
- Diamond, P. J., Kemball, A. J., Junor, W., Zensus, A., Benson, J., & Dhawan, V. 1994, *ApJ*, 430, L61
- Feldman, P. A., Matthews, H. E., Amano, T., Scappini, F., & Lees, R. M. 1993, in *Astrophysical Masers*, ed. A. W. Clegg & G. E. Nedoluha (Berlin: Springer), 65
- Goldreich, P., & Scoville, N. 1976, *ApJ*, 205, 144
- Greenhill, L. J., Colomer, F., Moran, J. M., Backer, D. C., Danchi, W. C., & Bester, M. 1995, *ApJ*, 449, 365
- Graf, U. U., Genzel, R., Harris, A. I., Hills, R. E., & Russell, A. P. G. 1990, *ApJ*, 358, L49
- Helminger, P., Messer, J. K., & De Lucia, F. C. 1983, *Appl. Phys. Letters*, 42, 309
- Hinkle, K. H., & Barnes, T. G. 1979, *ApJ*, 227, 923
- Humphreys, R. M., Strecker, D. W., & Ney, E. P. 1972, *ApJ*, 172, 75
- Justanont, K., Skinner, C. J., & Tielens, A. G. G. M. 1994, *ApJ*, 435, 852
- Kooi, J. W., Walker, C. K., LeDuc, H. G., Hunter, T. R., Benford, D. J., & Phillips, T. G. 1994, *Int. J. Infrared Millimeter Waves*, 15, 477
- Lockett, P., & Elitzur, M. 1992, *ApJ*, 399, 704
- Lovas, F. J. 1992, *J. Phys. Chem. Ref. Data*, 21, 181
- Martínez, A., Bujarrabal, V., & Alcolea, J. 1988, *A&AS*, 74, 273
- Mattei, J. A. 1995, Observations from the AAVSO International Database, private communication
- Melnick, G. J., Menten, K. M., Phillips, T. G., & Hunter, T. 1993, *ApJ*, 416, L37
- Menten, K. M., & Melnick, G. J. 1989, *ApJ*, 342, L91
- . 1991, *ApJ*, 377, 647
- Menten, K. M., Melnick, G. J., & Phillips, T. G. 1990a, *ApJ*, 350, L41
- Menten, K. M., Melnick, G. J., Phillips, T. G., & Neufeld, D. A. 1990b, *ApJ*, 363, L27
- Pearson, J. C., Anderson, T., Herbst, E., De Lucia, F. C., & Helminger, P. 1991, *ApJ*, 379, L41
- Reid, M. J., & Menten, K. M. 1990, *ApJ*, 360, L51
- Reid, M. J., & Moran, J. M. 1981, *ARA&A*, 19, 231
- Tsuji, T. 1964, *Ann. Tokyo Astron. Obs.*, 9, 1
- Wright, M. C. H., Carlstrom, J. E., Plambeck, R. L., & Welch, W. J. 1990, *AJ*, 99, 1299
- Yates, J. A., Cohen, R. J., & Hills, R. E. 1995, *MNRAS*, 273, 529