THE ASTROPHYSICAL JOURNAL, 450:851–861, 1995 September 10

STUDIES OF HOT B SUBDWARFS. XI. A REDETERMINATION OF THE DENSITY OF HOT, HYDROGEN-RICH SUBDWARFS IN THE GALACTIC PLANE[1]

B. VILLENEUVE,[2] F. WESEMAEL, AND G. FONTAINE
Département de Physique, Université de Montréal, C.P. 6128, Succursale Centre Ville, Montréal, Québec H3C 3J7, Canada; villeneu, wesemael, fontaine@astro.umontreal.ca
Received 1994 December 16; accepted 1995 March 21

ABSTRACT

We have reanalyzed the statistical properties of the sample of hydrogen-rich hot subdwarfs identified in the Kitt Peak–Downes survey in order to derive an improved value of the density D_0 of these stars in the plane of the Galaxy. Our analysis is based on photoelectric Strömgren photometry for 22 of these stars, all brighter than $V \sim 15.0$. The value of the average absolute magnitude of the KPD objects we redetermine is at least 1 mag brighter than that derived in the initial investigation. Three methods are then considered to derive D_0, under the assumption of an exponential distribution of stars in the direction perpendicular to the Galactic plane: fits to the number of stars as a function of apparent brightness and as a function of vertical distance above the plane, as well as the $1/V'_m$ method. We find that the distribution of hot, hydrogen-rich subdwarf stars can be satisfactorily reproduced with a midplane space density $D_0 = 3.8 \pm 1.7 \times 10^{-7}$ pc^{-3}, a factor of ~ 5 lower than initial estimates, but in agreement with the recent determination of Villeneuve et al. based on the Palomar-Green sample at high galactic latitudes.

Subject headings: stars: early-type — stars: statistics — subdwarfs

1. INTRODUCTION

Villeneuve et al. (1995, hereafter Paper I) have recently provided a new determination of the space density and distribution scale height of the hot, hydrogen-rich subdwarfs. Their analysis is based on a sample of 209 stars culled from the Palomar-Green survey, for which Strömgren photometry had previously been secured. Their final values for these parameters are, respectively, $D_0 = 3 \pm 1 \times 10^{-7}$ pc^{-3} and $z_e = 450 \pm 150$ pc. The space density D_0 is smaller than the values obtained previously (Heber et al. 1984; Downes 1986; Green, Schmidt, & Liebert 1986; Heber 1986; Green & Liebert 1987; Reid et al. 1988; Moehler et al. 1990; Bixler, Bowyer, & Laget 1991; Theissen et al. 1993), which were all in the range 1–4 $\times 10^{-6}$ pc^{-3}. Similarly, the exponential scale height z_e derived in Paper I is at the high end of those found previously, most recently by Moehler et al. (1990), Bixler et al. (1991), Saffer (1991), and Theissen et al. (1993). While earlier values of z_e argued convincingly for the hydrogen-rich subdwarfs belonging to an old disk population, the latest determination of z_e suggests that there might be, as well, a small admixture of stars belonging to the thick disk envisioned by Gilmore & Reid (1983).

Paper I includes a detailed rediscussion of most of the earlier determinations. Among those, that of Downes (1986) plays a distinct role: it is the only determination which does not make use of a sample of stars obtained at high galactic latitudes. This is in contrast, for example, to the Slettebak & Brundage (1971) objective prism survey or to the Palomar-Green (PG) survey, to name but two; the Kitt Peak–Downes (KPD) survey was conducted exclusively in the plane of the Galaxy. Unfortunately, this is where interstellar extinction becomes a significant source of uncertainty in the interpretation of the properties of Downes's statistically complete subsample of objects. Nevertheless, the galactic-plane survey has a strategic importance, since it provides us with the opportunity to determine the midplane space density of hydrogen-rich subdwarfs in a manner which is essentially independent of the vertical scale height of their distribution. It was pointed out in Paper I that the value of D_0 originally derived by Downes (1986), 1.5–2 $\times 10^{-6}$ pc^{-3}, could be reconciled with the lower value derived in Paper I when the large difference in absolute magnitude derived for the hydrogen-rich subdwarfs was taken into account. While the analysis of Paper I gave $\langle M_V \rangle \sim 3.7$, Downes's values were $\langle M_V \rangle \sim 5.0$–5.3, depending on the assumed intrinsic color. However, Paper I offered no specific suggestion as to the origin of this discrepancy.

In light of the importance of Downes's survey highlighted above, we reconsider here the problem of determining the space density of hot, hydrogen-rich subdwarfs in the plane on the basis of the KPD data. Our analysis, which can be considered a follow-up to Paper I, is similar to that of the PG stars presented there and hinges on photoelectric Strömgren photometry secured for a number of KPD stars by Wesemael et al. (1992). These data are first analyzed to derive approximate atmospheric parameters; three independent methods are then used to derive the midplane density of these objects in the Galaxy.

2. THE SAMPLE OF HYDROGEN-RICH SUBDWARFS IN THE PLANE

The KPD colorimetric survey covers 1144 deg^2 divided into 52 fields, all located within 12° of the Galactic plane. The mean completeness limit of the survey is $B_{pg} \sim 15.3$, a result derived on the basis of the V'/V'_m test[3] applied to the white dwarf

[1] Based on observations obtained at the Kitt Peak National Observatory, National Optical Astronomy Observatories, which is operated by the Association of Universities for Research in Astronomy under contract with the National Science Foundation.

[2] Permanent address: Collège André Grasset, 1001 Crémazie Est, Montréal, Québec H2M 1M3, Canada.

[3] The use of primed volume elements here simply indicates that all volumes are weighted by the barometric density distribution (see Paper I).

sample. Downes isolates 31 of the 40 hydrogen-rich subdwarfs discovered in the survey as being within that completeness limit and forming a complete statistical sample. Wesemael et al. (1992) obtained Strömgren photometry for 22 of the 25 subdwarf candidates brighter than $V \sim 15$. For the three stars left out (KPD 2109+4401, KPD 2118+3841, and KPD 2237+4924), it is a simple matter to relate the missing Strömgren photometry to the Johnson photometry secured by Downes (1986).

Figure 1*a* displays the observed correlation between the Strömgren *y*-magnitude and the Johnson *V*-magnitude of Downes for the 22 objects for which these data are available, while Figure 1*b* shows a similar correlation between Strömgren *b* and *B*. These relations can be expressed in the form

$$y = 1.024V - 0.324 , \quad (1a)$$

and

$$b = 0.944B + 0.888 . \quad (1b)$$

Similarly, Figure 2*a* displays the correlation between the Strömgren $(b - y)$ index and the Johnson $(B - V)$, while Figure 2*b* shows a similar correlation between $(u - b)$ and $(U - B)$. These relations take the form

$$(b - y) = 0.579(B - V) + 0.056 , \quad (2a)$$

and

$$(u - b) = 1.490(U - B) + 1.404 . \quad (2b)$$

The use of these transformations permits a smooth transition from the Johnson to the Strömgren system for the three unobserved stars. The KPD subsample we consider here thus consists of 25 stars with $V \leq 15.0$, or $y \leq 15.04$ (see eq. [1a]).

At the bright end, we use truncated cones (introduced in Paper I) to compute the different integrals and volumes required in § 4, in order to avoid problems related to a possible incompleteness of the KPD survey at that end. The survey is thus considered to be statistically complete between Strömgren *y*-magnitudes 13.0 and 15.04.

3. DETERMINATION OF PHYSICAL PARAMETERS

3.1. *Effective Temperatures and Surface Gravities*

Estimates of the effective temperature were obtained on the basis of the calibrated relation between the reddening-free color index Q' and the effective temperature. This relation is described by Bergeron et al. (1984) and Wesemael et al. (1992), while the sensitivity of the index to the transformation to a calibrated system is discussed in that last paper, as well as by Saffer et al. (1994). Furthermore, care was taken, in Paper I, to treat those objects which appeared to be composites in a slightly different manner in order to avoid the contamination of the colors due to the companion. Here the presence of extinction on the line of sight is such a significant problem that we have neglected this distinction altogether. The importance of this omission will be addressed below (§ 3.2).

Our gravity assignments follow the procedures developed in Paper I in that we used four different methods to assign surface gravities to individual objects in our sample, and complete sequences of calculations were carried out for each method. There are, however, small changes in the details of the sequences: The C_1 and C_2 sequences consist of assigning constant surface gravities, $\log g = 5.25$ and 5.5, respectively, to all program stars. A third sequence, termed BBL, uses the best-fitting, least-square linear relation given by Bixler et al. (1991) between $\log g$ and $\log T_{\rm eff}$ ($\log g = -11.70 + 3.87 \log T_{\rm eff}$). The last sequence, termed GS, mimics the Greenstein & Sargent (1974) method for assigning $\log g$ values ($\log g\theta^4 = 2.3$). Note that the BBL method is equivalent, in practice, to adopting a slightly modified Greenstein & Sargent–like relation of the

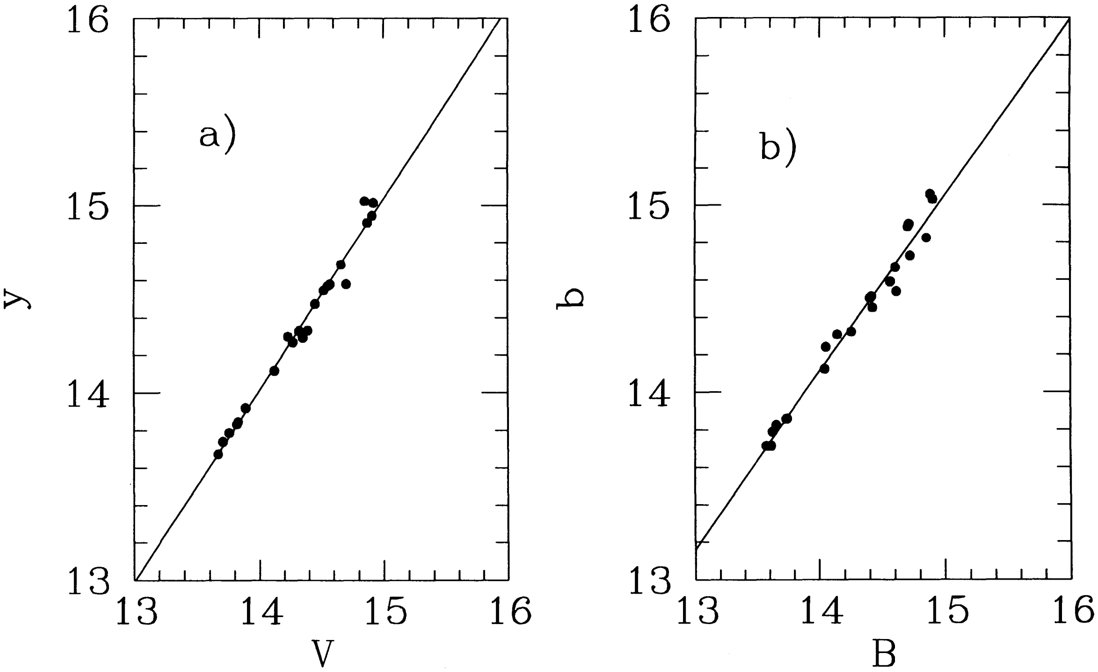

FIG. 1.—Photometric properties of our sample of 22 objects brighter than $y = 15.04$. (*a*) Strömgren *y*-magnitude as a function of Johnson *V*-magnitude of Downes (1986). The straight line displays the linear regression of eq. (1a). (*b*) Strömgren *b*-magnitude as a function of Johnson *B*-magnitude of Downes (1986). The straight line displays the linear regression of eq. (1b).

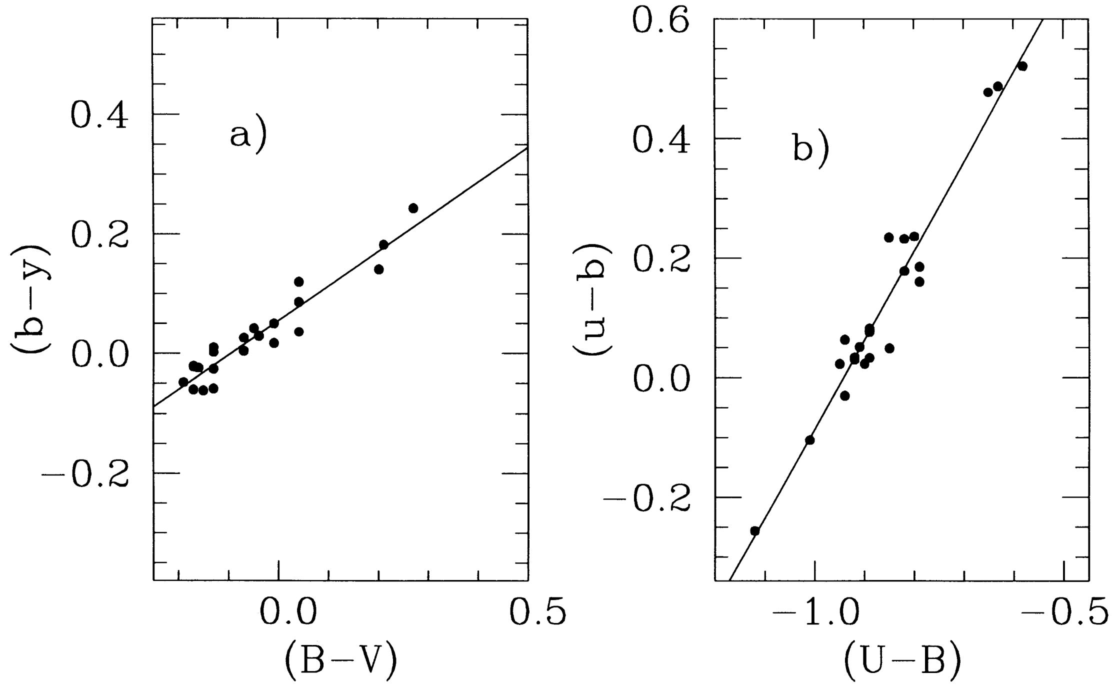

FIG. 2.—Photometric properties of our sample of 22 objects brighter than $y = 15.04$. (*a*) Strömgren $(b - y)$ color as a function of Johnson $(B - V)$ color of Downes (1986). The straight line displays the linear regression of eq. (2a). (*b*) Strömgren $(u - b)$ color as a function of Johnson $(U - B)$ color of Downes (1986). The straight line displays the linear regression of eq. (2b).

form $\log g\theta^{3.87} = 2.63$. A brief discussion of the features associated with the C_1 and GS sequences can be found in Paper I. The sequences C_2 and BBL are new to this analysis.

It should be noted that our reanalysis of the KPD sample is part of a comprehensive effort aimed at carrying out statistical analyses of large samples of hydrogen-rich subdwarfs through analyses of their photometric properties. As already discussed in Paper I, it would thus have been an unreasonable drain of telescope time to attempt to secure spectra for the 209 stars analyzed there. Here, in the same spirit, no spectra have been secured for this reinvestigation, even though the much smaller size of the KPD sample (one-tenth that of Paper I) makes the procurement of these data much easier. So much so, in fact that we have become aware, after this analysis was completed, that the KPD sample had recently been almost completely reobserved by Saffer & Liebert (1995). We will return to this important development in § 5.

3.2. *Distance Determinations and the Problem of Reddening*

Individual distances, d, are calculated from

$$d = 3.241 \times 10^{-19}\left(\frac{4\pi GMH_{5500}}{gf_{5500}}\right)^{1/2} \text{pc} , \quad (3)$$

where H_{5500} and f_{5500} are, respectively, the Eddington flux at 5500 Å computed for a given $T_{\rm eff}$ and $\log g$ from models similar to those of Wesemael et al. (1980), and the flux detected at the top of the Earth atmosphere at the same wavelength. Other symbols have their usual meaning. As in Paper I, we adopt $M = 0.5\ M_\odot$ (Moehler et al. 1990), and the flux is obtained from the Heber et al. (1984) relation, whereby $m_y = 0.000$ corresponds to 3.60×10^{-9} ergs cm^{-2} s^{-1} Å^{-1}. However, the m_y magnitude used here must be corrected for interstellar extinction.

This is achieved by first determining the unreddened $(b - y)_0$ color index by combining the equation defining Q',

$$Q' \equiv (u - b) - 1.56(b - y) = (u - b)_0 - 1.56(b - y)_0 , \quad (4a)$$

with a relationship between $(u - b)_0$ and $(b - y)_0$ derived on the basis of the stellar atmosphere models used by Wesemael et al. (1992; e.g., Fig. 9), which we express in the form

$$(u - b)_0 = 10.485(b - y)_0 + 1.258 . \quad (4b)$$

We thus obtain

$$(b - y)_0 = 0.112(Q' - 1.258) . \quad (4c)$$

The color excess E_{b-y} can now be estimated, and the extinction follows from the relation $A_y = 5.06E_{(b-y)}$, based on the average reddening curves of Seaton (1979). The flux f_{5500}, corrected for extinction, then follows, while H_{5500} is obtained from the effective temperature $T_{\rm eff}(Q')$. All ingredients are now in hand to estimate d from equation (3). Absolute magnitudes follow from the usual expression

$$M_y = m_y - A_y + 5 - 5 \log d , \quad (5)$$

with d expressed in parsecs, while the height of each star above the galactic plane is simply $z = d \sin b$, where b is the galactic latitude of the object. A summary of the average absolute magnitudes characterizing our sample of objects, derived with the two temperature calibrations and the four distinct gravity assignments, is presented in Table 1.

We derive effective temperatures on the basis of Q', and this procedure implicitly assumes that the observed colors are uncontaminated. The impact on d of our ignoring possible contamination of the observed y-magnitude by a cool main-sequence companion will be felt differently for our different sequences of $\log g$ assignments. To illustrate this, we recall that

TABLE 1

MEAN M_y OF THE SAMPLE

Sequence	Calibration	$\langle M_y \rangle$
C_1	Schulz	3.48
C_2	Schulz	4.11
BBL	Schulz	4.35
GS	Schulz	3.78
C_1	Olson	3.33
C_2	Olson	3.95
BBL	Olson	4.54
GS	Olson	3.98

our estimate of H_{5500} derives from a determination of $T_{\rm eff}(Q')$. Let us assume, furthermore, that the main effect associated with the presence of a companion will be to redden $(b-y)$, while leaving $(u-b)$ essentially unchanged. The value of Q' for such a composite system will thus decrease (see eq. [4a]), and a higher effective temperature would be assigned to the subdwarf than would otherwise have been the case had uncontaminated colors been used. For the first two sequences, the effect of this nominally higher $T_{\rm eff}$ is partially offset by the higher f_{5500} associated with the contaminated y-magnitude. While this is also true for the last two sequences, the higher $T_{\rm eff}$ also leads, for those, to the assignment of a larger log g through the imposed relationship between g and θ. It then follows from equation (3) that, in these two cases, the final distance should be smaller than that assigned had the presence of a companion been properly accounted for.

The importance of this effect, which should affect only a small fraction of the sample (only three KPD stars are candidates for composites, according to Allard et al. 1994), is easily appreciated when one examines its impact on the absolute magnitude. To this end, we recalculated the average M_y for the 209 stars in the sample of Paper I; effective temperatures were determined directly from the Q' index, thus *ignoring the possibility* that a certain number of these stars might be composites. The Schulz (1978) calibration was used, and both the C_1 and GS log g sequences were considered. By contrast, we also determined M_y for a subsample of 154 stars located, in a $(u-b)$ versus $(b-y)$ diagram, far from the region of contamination from composites identified by Allard et al. (1994). The difference between the mean values of M_y for these two samples were less than 0.1 dex, smaller by a factor of 3–4 than the standard deviation of the M_y distribution. Thus, whatever the true fraction of contaminated hydrogen-rich subdwarfs and the level of contamination they suffer, a maximal supplementary error of 10% may affect some of the distance determinations.

3.3. *The Absolute Magnitude of Hydrogen-rich Subdwarfs*

There are alternative methods to determine distances to individual objects. Thus, Downes (1986) assumes instead a constant value of $(B-V)_0$ for all stars and determines E_{B-V}, and A_V, for each object. Then, through the use of published relations between color excess and distance for specific galactic directions, d follows, while M_V is obtained through equation (5).

Following this approach for the 31 hydrogen-rich subdwarfs forming the KPD complete sample, Downes found a mean $\langle M_V \rangle$ of 5.0 ± 1.4 (5.3 ± 1.4) on the basis of $(B-V)_0 = -0.35$ (-0.30). Here, on the basis of the various methods used to assign individual log g values, and the various temperature calibrations used, we obtain instead a mean value of $\langle M_y \rangle = 3.94$, with a standard deviation of 0.41 (see Table 1). Our result is in accordance with the mean values representative of these stars in Paper I—which are in the range 3.32–3.93—as well as with several estimates of M_V derived recently, and already summarized in Paper I: $\langle M_V \rangle = 3.14$, $\sigma = 0.76$, based on the 26 sdB stars with reliable M_V in Table A3 of Greenstein & Sargent (1974); $\langle M_V \rangle = 3.85$, $\sigma = 0.60$, for the eight objects of Heber et al. (1984); $\langle M_V \rangle = 4.41$, $\sigma = 0.31$, for the seven new, single objects analyzed by Heber (1986); $\langle M_V \rangle = 3.53$, $\sigma = 0.52$, for the 37 objects studied by Moehler et al. (1990); $\langle M_V \rangle = 3.8 \pm 0.8$ for the 11 objects in the complete sample considered by Theissen et al. (1993); and a range of 4.3–4.5 derived by Liebert, Saffer, & Green (1994) for three B subdwarfs in the old, metal-rich galactic cluster NGC 6791.

Downes's generally fainter magnitudes must originate either with his choice of a constant $(B-V)_0$ value or with the extinction versus distance relations which are at the core of his calculation of $\langle M_V \rangle$. The first of these options can be readily checked by calculating a mean value of $(B-V)_0$ for the sample of Wesemael et al. (1992), which can be obtained by assuming that—for the PG stars—the observed $(b-y)$ value is very nearly $(b-y)_0$, and by further using equation (2a) to convert individual $(b-y)_0$ indices to $(B-V)_0$. The resulting mean is $\langle (B-V)_0 \rangle = -0.325$, with a standard deviation of 0.025. This result confirms the adequacy of Downes's choice, which consisted of simplifying the problem by choosing the intrinsic $(B-V)_0$ color as a constant and ignoring its dependence on $(U-B)_0$.

This agreement suggests that one must consider the extinction versus distance relations considered by Downes (1986) as the source of the discrepancy. These relations, extracted from three distinct sources (FitzGerald 1968; Lucke 1978; Nandy et al. 1978), tend to be generally noisy, and it is unfortunately not possible for us to reproduce exactly the way individual distances were determined on the basis of these plots. However, while the *determination of distances and absolute magnitudes* on the basis of these graphs appears fraught with risks, it remains of some importance for our later discussion (§ 4) to investigate to what extent the extinction and distances we derive on the basis of our own methods are compatible with those extracted from these graphs. For this, we use the following two sources of information on the spatial variation of interstellar extinction: the data of FitzGerald (1968), one of the three sets used by Downes (1986), and the more recent compilation of Neckel & Klare (1980). Both present graphs of extinction in the plane as a function of distance for numerous galactic longitudes. We define a match with the graphical data as acceptable if the extinction we determine falls within a certain magnitude range of that suggested by either of the two sources (FitzGerald or Neckel & Klare) at the mean distance derived for the object. The latter is simply defined as the mean of the distances obtained with each of our four methods to assign log g. The acceptable range is fixed by the general uncertainty inherent to our distance determination, even in the absence of significant extinction. From the analysis of Paper I, we estimate that the uncertainty on d and z is of the order of 50%, a value consistent with that derived by Moehler et al. (1990). This figure translates into an acceptable uncertainty of $^{+1.5}_{-0.9}$ on A_y. If, on the other hand, we adopt a smaller uncertainty on d of the order of 25%, the acceptable uncertainty range on A_y decreases to $\sim \pm 0.54$.

The combination of estimated extinction and distance we derive can be matched with the graphical data for 21 of the 25 stars in our sample if the typical uncertainty on d is set at 25%, while we secure a match for 23 out of 25 objects for an allowed uncertainty on the distance of 50%. Among the four problematic stars, one (KPD 2053+5632) is over-reddened, in the sense that our value of A_y is ~ 1.3 mag larger, at our estimated mean distance to the object, than the largest value suggested by the graphs; this star, however, is not among the composite candidates identified in the KPD survey by Allard et al. (1994). For the other three objects (KPD 0025+5402, KPD 0550+1922, and KPD 2040+3955), the extinction we derive is 1.0–1.6 mag smaller than the smallest value estimated on the basis of the graphs.

Table 2 presents the physical parameters determined for the original sample of 25 stars isolated by Downes. Strömgren y-magnitudes are from Wesemael et al. (1992), except for the three stars for which equation (1a) was used to transform the KPD Johnson V-magnitude to the Strömgren system. The extinction A_y is that derived here on the basis of our method. Individual temperatures follow from Q' and the Schulz (1978)

TABLE 2

Parameters of Downes's Complete Sample

KPD Name	y	A_y	$T_{\rm eff}$	log g[a]	d (pc)	M_y	KPD Name	y	A_y	$T_{\rm eff}$	log g[a]	d (pc)	M_y
0025+5402.......	13.92	0.3	25352	5.25	939	3.76	1943+4058.......	14.91	0.5	28174	5.25	1444	3.57
				5.50	704	4.38					5.50	1082	4.19
				5.34	843	3.99					5.52	1057	4.24
				5.11	1108	3.40					5.29	1379	3.67
0054+5406.......	14.12	0.7	30901	5.25	1000	3.39	1946+4340.......	14.30	0.4	32748	5.25	1328	3.27
				5.50	750	4.02					5.50	996	3.89
				5.68	612	4.46					5.77	726	4.58
				5.45	795	3.89					5.55	939	4.02
0311+4801[b]......	14.33	0.7	50099	5.25	1578	2.61	2022+2033[b]......	13.67	0.7	19999	5.25	559	4.25
				5.50	1184	3.23					5.50	419	4.87
				6.49	379	5.71					4.94	794	3.49
				6.29	477	5.21					4.69	1060	2.86
0422+5421.......	14.68	1.3	22905	5.25	775	3.96	2024+5303[c]......	14.58	1.2	35997	5.25	1158	3.08
				5.50	581	4.58					5.50	868	3.70
				5.17	847	3.76					5.93	528	4.79
				4.93	1120	3.16					5.72	678	4.24
0550+1922.......	14.57	0.3	27124	5.25	1311	3.63	2040+3955.......	14.47	0.8	26570	5.25	1009	3.67
				5.50	983	4.26					5.50	757	4.30
				5.46	1033	4.15					5.42	828	4.10
				5.22	1351	3.57					5.19	1084	3.52
0629−0016.......	15.01	0.7	26766	5.25	1338	3.66	2053+5632.......	14.29	1.9	26502	5.25	563	3.68
				5.50	1003	4.28					5.50	423	4.30
				5.43	1082	4.12					5.42	464	4.10
				5.20	1416	3.53					5.18	608	3.51
0640+1412.......	15.02	0.9	32528	5.25	1473	3.28	2109+4401[d]......	13.37	0.4	33976	5.25	906	3.19
				5.50	1105	3.91					5.50	679	3.82
				5.76	817	4.56					5.84	461	4.66
				5.54	1056	4.01					5.61	595	4.11
0716+0258.......	14.95	0.4	27566	5.25	1535	3.61	2118+3841[d]......	13.88	0.3	31334	5.25	1080	3.36
				5.50	1151	4.23					5.50	810	3.99
				5.48	1172	4.19					5.70	644	4.49
				5.25	1532	3.61					5.47	834	3.92
0721−0026[b]......	13.85	0.5	22601	5.25	761	3.98	2215+5037[c]......	13.74	0.5	28608	5.25	855	3.54
				5.50	571	4.61					5.50	641	4.17
				5.15	854	3.73					5.55	607	4.28
				4.91	1130	3.12					5.32	792	3.71
1901+1607.......	14.27	0.9	27812	5.25	899	3.59	2237+4924[d]......	14.95	1.3	25589	5.25	969	3.74
				5.50	674	4.22					5.50	727	4.37
				5.50	675	4.21					5.36	854	4.01
				5.27	882	3.63					5.12	1122	3.42
1905+1144.......	14.54	1.7	35301	5.25	899	3.12	2254+5444[c]......	14.33	1.4	41652	5.25	1051	2.83
				5.50	674	3.74					5.50	788	3.46
				5.90	425	4.74					6.18	361	5.15
				5.68	547	4.20					5.97	459	4.63
1924+2932.......	13.79	0.7	34294	5.25	942	3.17	2319+5014.......	14.58	0.6	24175	5.25	1042	3.85
				5.50	707	3.80					5.50	781	4.47
				5.85	472	4.68					5.26	1026	3.88
				5.63	608	4.13					5.02	1352	3.28
1930+2752.......	13.83	0.9	33235	5.25	887	3.24							
				5.50	666	3.86							
				5.80	472	4.61							
				5.58	609	4.05							

[a] Listed in the following order: C_1, C_2, BBL, and GS.
[b] Not a hydrogen-rich subdwarf according to Saffer & Liebert 1995 (see § 5). Derived parameters may thus be especially inaccurate.
[c] Probable composite, according to Allard et al. 1994.
[d] Not observed on the Strömgren system by Wesemael et al. 1992.

transformation (see Wesemael et al. 1992 and Paper I). The four different log g assignments (in the order C_1, C_2, BBL, and GS), distances, and M_y follow for each star.

4. DETERMINATION OF THE SPACE DENSITY

4.1. *General Considerations*

We use three methods, each with distinctive characteristics, to determine the space density based on the complete statistical sample defined in § 2. Our first approach—clearly the most direct one—is to consider the observed $N(y)$ distribution of program stars and compare it with predicted model distributions calculated with assumed values of the exponential scale height, z_e, and of the midplane density, D_0. The values of these parameters can be adjusted in the model to match as closely as possible the observed $N(y)$ variation. The optimal values of D_0 and z_e obtained depend, in this instance, on the constant value of M_y which is assumed to characterize the population of objects.

The next two techniques are already described in detail in Paper I. One of these is Schmidt's (1975) technique, which consists of summing the $1/V'_m$ for each star of a complete sample, where V'_m is the maximal density-weighted volume accessible to each object in the survey. One advantage of the KPD sample is its relative insensitivity to the assumed scale height of the distribution. This feature allows a test for completeness (which requires us to assume a value of z_e), as well as a determination of the space density, even if the exponential scale height cannot be determined accurately. Finally, the third method we use considers fits to the observed $N(z)$ distribution of stars; values of the exponential scale height and of the space density in the plane are, once again, adjusted in the model stellar distribution, characterized by a unique value of the absolute magnitude, to force a match to the observed $N(z)$ variation.

In contrast to the work presented in § 3.2, where we circumvented—through the use of equation (3)—the need for an extinction versus distance relation to determine M_y values, such a relation is needed here to determine D_0. This is because the evaluation of the predicted $N(y)$ or $N(z)$ distributions requires that we calculate the distance at which a star of given absolute and apparent magnitudes would be located in each field of the survey. This information cannot be obtained without referring to an extinction versus distance relation *for each field*, which we extract from the Neckel & Klare (1980) data. Similarly, the $1/V'_m$ summing technique requires that the volume accessible to each object in the survey be calculated. This datum again requires a knowledge of the extinction versus distance relation for each field.

4.2. *Fitting the Observed* N(y) *Distribution*

To obtain a first value of the space density of hydrogen-rich subdwarfs, as well as to provide a connection to the value derived earlier by Downes (1986), we first attempt to fit, with a χ^2 technique, the observed $N(y)$ distribution to those predicted within the exponential model, treating the density D_0 and the scale height z_e as adjustable parameters. The number of stars of absolute magnitude M_y (assumed identical for all stars) in a bin centered on y_i in a survey of n fields, each characterized by a solid angle ω_j and a central galactic latitude b_j is, for a given choice of D_0 and z_e,

$$N(y_i) = \sum_{j=1}^{n} \frac{D_0 \omega_j}{\sin^3 b_j} \int_{z_{i-1}}^{\min[z_{i+1}, z_{\max}(j)]} z^2 \exp(-z/z_e) dz . \quad (6)$$

In the above, the height above the plane, z_k, of a star of assumed absolute magnitude M_y and of apparent magnitude y_k in a given field, characterized by its galactic latitude b_j and its extinction $A_{y,j}(d)$, is simply

$$z_k = d_k \sin b_j = 10^{[y_k - A_{y,j}(d_k) - M_y + 5]/5} \sin b_j . \quad (7)$$

The maximal height at which field j contributes, $z_{\max}(j)$, is determined by the limiting magnitude of the field and by the absolute magnitude of the stars within it. Furthermore, note that each individual distance d_k associated with a particular apparent magnitude depends on the extinction in the field, which is itself also dependent on distance. Thus, before performing any calculation, we need the $A_y(d)$ relation for each of the 52 fields. FitzGerald (1968) presents 74 graphs of color excess versus distance based on photoelectric data for more than 7000 stars in order to cover the 360° of galactic longitude at low latitudes. Neckel & Klare (1980) used photoelectric data for more than 11,000 stars and generate 325 different graphs of A_V versus d to cover the entire galactic plane at low galactic latitudes. On the basis of both these studies, the general behavior of the extinction can be modeled as a straight line of constant slope for distances smaller than ~1 kpc, followed by a constant value which correspond to that reached at 1 kpc. Because this description is adequate for most of our 52 fields, we adopt it for all of them. The calculations were repeated for the two different values of the extinction at 1 kpc obtained from the compilations of FitzGerald and Nickel & Klare.

An iterative technique is used to compute the individual distances d_k. As a first estimate of A_y, we use the 1 kpc value. If, in this first pass, the computed distance is larger than 1 kpc, the procedure is stopped. If it is less than 1 kpc, this distance is probably underestimated, because the true extinction is most likely smaller than the value initially adopted. A second estimate of A_y is then made, using the previous distance estimate and the linear relationship between A_y and distance. This second distance estimate is greater than the first and is itself overestimated because the true extinction is larger than that used. The procedure is repeated, and converges quickly (four or five steps); it is stopped when the difference between the last two distance estimates is smaller than 10 pc. The final distance is the mean of the last two values.

The general χ^2 for our problem is

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{N(y_i) - N(y_i, D_0, z_e)}{\sigma_i}\right]^2 , \quad (8)$$

where N represents the number of bins $N(y_i)$ is the observed number of stars in bin y_i, $N(y_i, D_0, z_e)$ is the number of stars predicted for a given D_0 and z_e, and σ_i is the error on $N(y_i)$. The latter is estimated under the assumption that the observed $N(y_i)$ can be described by a Poisson distribution; thus $\sigma_i = [N(y_i)]^{1/2}$.

Our fits assume that all the objects in the survey area can be characterized by a constant absolute magnitude M_y. Rather than adopting the average absolute magnitude characteristic of one or another of the C_1, C_2, BBL, or GS sequences, we have elected to perform this first set of fits with absolute magnitudes in the range from 3.5 to 5.0 in steps of 0.5. This wide range permits us to generate results for M_y values as faint as those adopted by Downes in his earlier calculations of D_0, which none of our four sequences would otherwise reach. Table 3 gives the optimal values of the scale height z_e and the midplane density for each choice of absolute magnitude, as well as for the

TABLE 3

TWO-DIMENSIONAL FIT TO $N(y)$

$\langle M_y \rangle$	Extinction	z_e (pc)	D_0 (pc^{-3})
3.5......	Neckel & Klare	150	2.8×10^{-7}
4.0......	Neckel & Klare	275	3.6×10^{-7}
4.5......	Neckel & Klare	300	5.6×10^{-7}
5.0......	Neckel & Klare	175	9.8×10^{-7}
3.5......	FitzGerald	125	3.0×10^{-7}
4.0......	FitzGerald	225	3.8×10^{-7}
4.5......	FitzGerald	375	5.4×10^{-7}
5.0......	FitzGerald	175	9.8×10^{-7}

two sources of extinction data described above. Figure 3 displays the four different distributions obtained with the Neckel & Klare extinction data as continuous lines, while the observed distributions appear as histograms. The results of Table 3 do not appear to depend critically on the adopted extinction data, but they are sensitive to the assumed M_y value, as had been argued in Paper I. Larger midplane densities are naturally associated with fainter absolute magnitudes, since all stars are then located closer and are thus more concentrated. In other words, to account for the same number of observed stars in an apparent magnitude bin, a higher density is required for fainter M_y because the volume associated with the bin is smaller than it is for a brighter object. The six density determinations associated with the three brightest absolute magnitudes are consistent with the results of Paper I, while those obtained for $\langle M_y \rangle = 5.0$ are similar to those encountered in Downes's original analysis, which yielded $D_0 = 1.5 \times 10^{-6}$ pc^{-3}.

The values of z_e obtained for the $\langle M_y \rangle = 5.0$ cases are quite consistent with those adopted by Downes (1986). In general, however, the values of the exponential scale height are not consistent with those obtained in Paper I. This is not really a surprise since, as mentioned earlier, the Galactic plane KPD sample is not really suited to a determination of z_e. This is best seen in Figure 4, which displays the confidence levels associated with our fits in the (D_0, z_e) plane. The confidence levels around the best fit tend to form banana-shaped curves, open-ended along the vertical axis. This morphology simply indicates that the density is relatively well constrained, i.e., that different values of z_e can be used, and similar values of D_0 obtained nevertheless; similar contour plots were obtained in Paper I while fitting subsamples of the PG survey limited to low z. This result is exactly that expected from a sample of stars concentrated to the Galactic plane: because the mean z of the sample is lower than the scale height, no significant vertical density decrease can be detected, and any z_e sufficiently larger than $\langle z \rangle$ is appropriate to describe the sample.

4.3. *The V'/V'_m Test and its Associated Density*

The V'/V'_m test is often used as a tool to determine the optimal value of the exponential scale height z_e for a complete sample. When the test is performed for different values of z_e, that which gives—within the uncertainty—a value of $\langle V'/V'_m \rangle$ near 0.5 is defined as the optimal exponential scale height. The density then follows by summing the individual $1/V'_m$ over the entire survey.

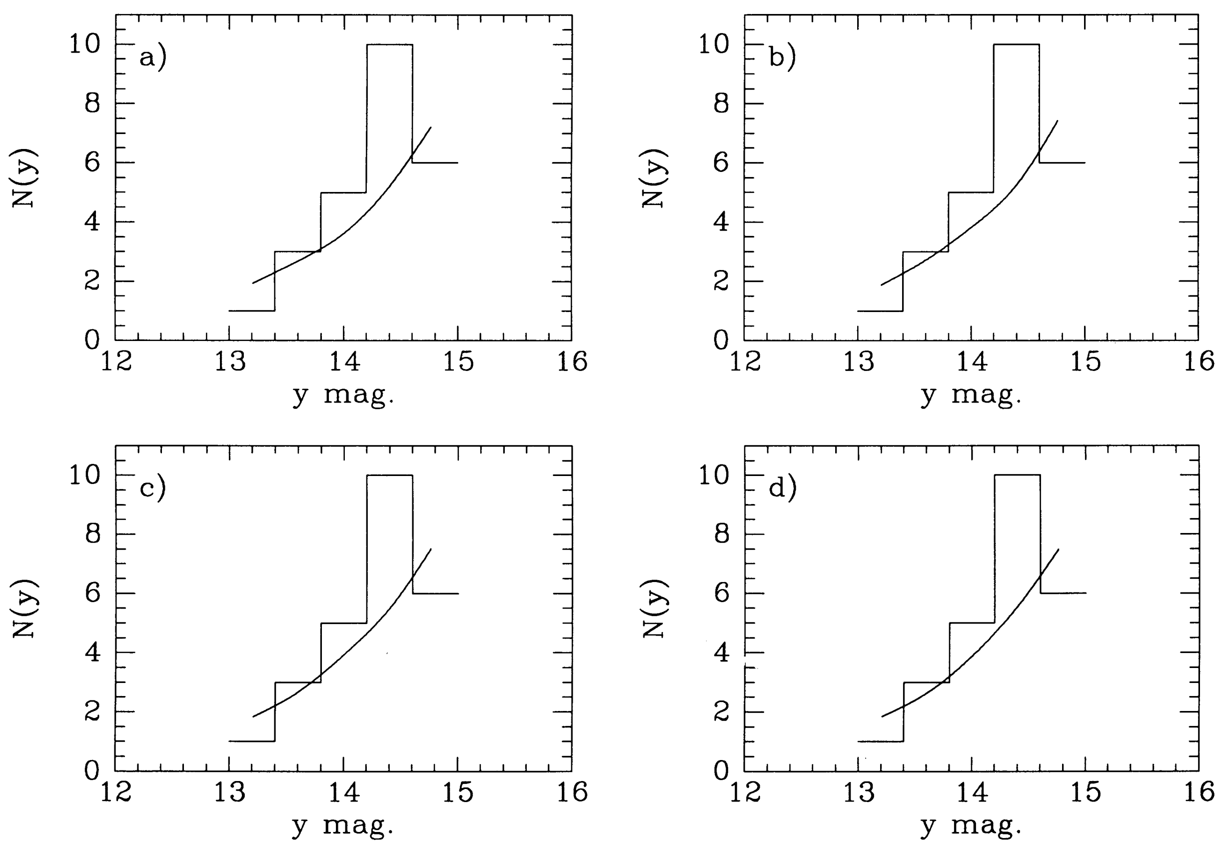

FIG. 3.—Observed $N(y)$ distribution (*histogram*), together with our optimal fit within the exponential model (*solid line*). Extinction data are from Neckel & Klare (1980): (*a*) for $M_y = 3.5$; (*b*) for $M_y = 4.0$; (*c*) for $M_y = 4.5$; (*d*) for $M_y = 5.0$.

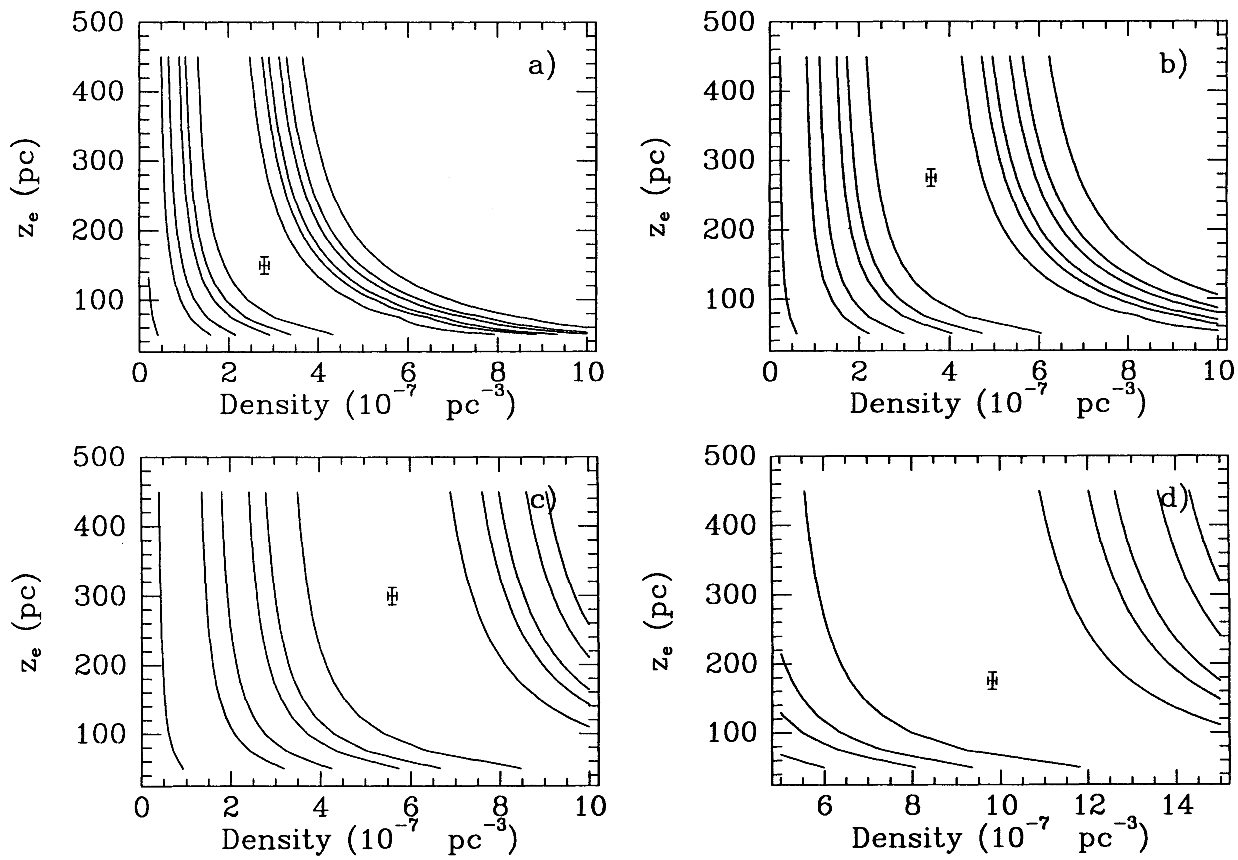

FIG. 4.—(*a*) Contours of equal confidence level associated with the fit to the $N(y)$ distribution. Extinction data are from Neckel & Klare (1980). The contours shown are for 68.3%, 90%, 95.4%, 99%, 99.73%, and 99.99% confidence levels. The cross indicates the set of D_0 and z_e associated with the minimal deviation. These values are given in Table 3: (*a*) for $M_y = 3.5$; (*b*) for $M_y = 4.0$; (*c*) for $M_y = 4.5$; (*d*) for $M_y = 5.0$.

The evaluation of V' requires the distance at which a star characterized by given apparent and absolute magnitudes should be in each individual field. This distance differs from field to field, according to the characteristics of the extinction in that direction. Similarly, the maximal distance at which a star can be detected depends both on its absolute magnitude, and on the magnitude limit of the field, as well as on the extinction which characterizes the different fields. We again follow the procedure described in § 4.2 to determine distances within each field of the survey, with due account taken of the extinction.

Note that, in contrast to the procedure involving fits to the observed $N(y)$ distribution discussed in § 4.2, the $1/V'_m$ summing technique uses the *individual* absolute magnitudes determined for each star in § 3.2. Tables 4 and 5 show the results obtained with each of the four gravity assignment techniques, with effective temperatures based on both the Schulz (1978) and the Olson (1974) calibrations, and for the extinction data of Neckel & Klare (1980). We used two different z_e values to carry out the V'/V'_m test, as well as to compute its associated space density.

For all sequences, the $\langle V'/V'_m \rangle = 0.5$ requirement is met within the uncertainty of 0.06 [given by $(12N)^{-1/2}$, where $N = 25$; Green 1980]. This again shows the relative unimportance of the chosen value of z_e in analyses of the KPD sample. As expected, the lower scale heights produce the largest densities by lowering the extent of the maximal density-weighted volume. In agreement with the results of § 4.2, the value of the midplane density D_0 is sensitive to the mean absolute magnitude which characterizes each sequence. Furthermore, the

TABLE 4

V'/V'_m TEST:
SCHULZ CALIBRATION AND NECKEL & KLARE EXTINCTION

Sequence	$\langle \log g \rangle$	z_e (pc)	$\langle V'/V'_m \rangle$	$\Sigma 1/V'_m$
C_1	5.25	500	0.46	2.2×10^{-7}
		250	0.47	2.6×10^{-7}
C_2	5.50	500	0.48	4.2×10^{-7}
		250	0.49	4.8×10^{-7}
BBL......	5.60	500	0.48	5.5×10^{-7}
		250	0.49	6.3×10^{-7}
GS	5.37	500	0.47	3.2×10^{-7}
		250	0.48	3.7×10^{-7}

TABLE 5

V'/V'_m TEST:
OLSON CALIBRATION AND NECKEL & KLARE EXTINCTION

Sequence	$\langle \log g \rangle$	z_e (pc)	$\langle V'/V'_m \rangle$	$\Sigma 1/V'_m$
C_1	5.25	500	0.45	1.9×10^{-7}
		250	0.46	2.3×10^{-7}
C_2	5.50	500	0.48	3.6×10^{-7}
		250	0.49	4.2×10^{-7}
BBL......	5.74	500	0.48	6.7×10^{-7}
		250	0.49	7.5×10^{-7}
GS	5.51	500	0.48	4.0×10^{-7}
		250	0.49	4.6×10^{-7}

values derived here are comparable to those of § 4.2. One should note that none of the current sequences reach the very faint absolute magnitudes explored previously within the $N(y)$ fits.

4.4. *Fitting the Observed* $N(z)$

We now attempt to fit, with a χ^2 technique, the observed $N(z)$ distribution to those predicted within the exponential model, treating the density D_0 and the scale height z_e as adjustable parameters (additional details are given in § 5.3 of Paper I). As in §§ 4.2 and 4.3, the calculated distance to a given star in each of the fields takes into account the extinction particular to that field. The maximal z at which each field contributes is determined by the limiting magnitude of the field and by the absolute magnitude of the stars within it. As in § 4.3, calculations are carried out for each sequence in turn (C_1, C_2, GS, and BBL), but here we assume that the theoretical sequence can be characterized by the average absolute magnitude value appropriate to it and summarized in Table 1.

Figures 5 and 6 display our results. The optimal $N(z)$ distributions are displayed as continuous lines, while the observed distributions are represented by histograms. Table 6 presents the derived values of z_e and D_0, all for the Schulz (1978) temperature calibration and the Neckel & Klare extinction data, and this for the now customary four sequences of log g assignments. Because the KPD sample is restricted to the Galactic plane, the vertical distribution of stars within it is essentially uniform. Thus, the method of fitting the observed $N(z)$ distribution is, for all practical purposes, incapable of providing useful constraints on z_e. From a practical point of view, this is reflected in our fits by a systematic tendency for our procedure to opt for large values of the scale height, $z_e \gtrsim 1$ kpc, since increasingly larger scale heights lead to increasingly more homogeneous distributions. Despite these limitations, the densities derived from this method of analysis remain well constrained, as shown in Figure 6, and are in agreement with those derived in §§ 4.2 and 4.3.

TABLE 6

TWO-DIMENSIONAL FIT TO $N(z)$: SCHULZ CALIBRATION AND NECKEL & KLARE EXTINCTION

Sequence	z_e (pc)	D_0 (pc^{-3})
C_1	$\gtrsim$1000	1.4×10^{-7}
C_2	$\gtrsim$1000	2.8×10^{-7}
BBL......	$\gtrsim$1000	3.2×10^{-7}
GS	$\gtrsim$1000	2.2×10^{-7}

5. CONCLUDING REMARKS

On the basis of the statistically complete sample of hydrogen-rich subdwarfs isolated by Downes (1986), we have redetermined the midplane density of these stars. We have

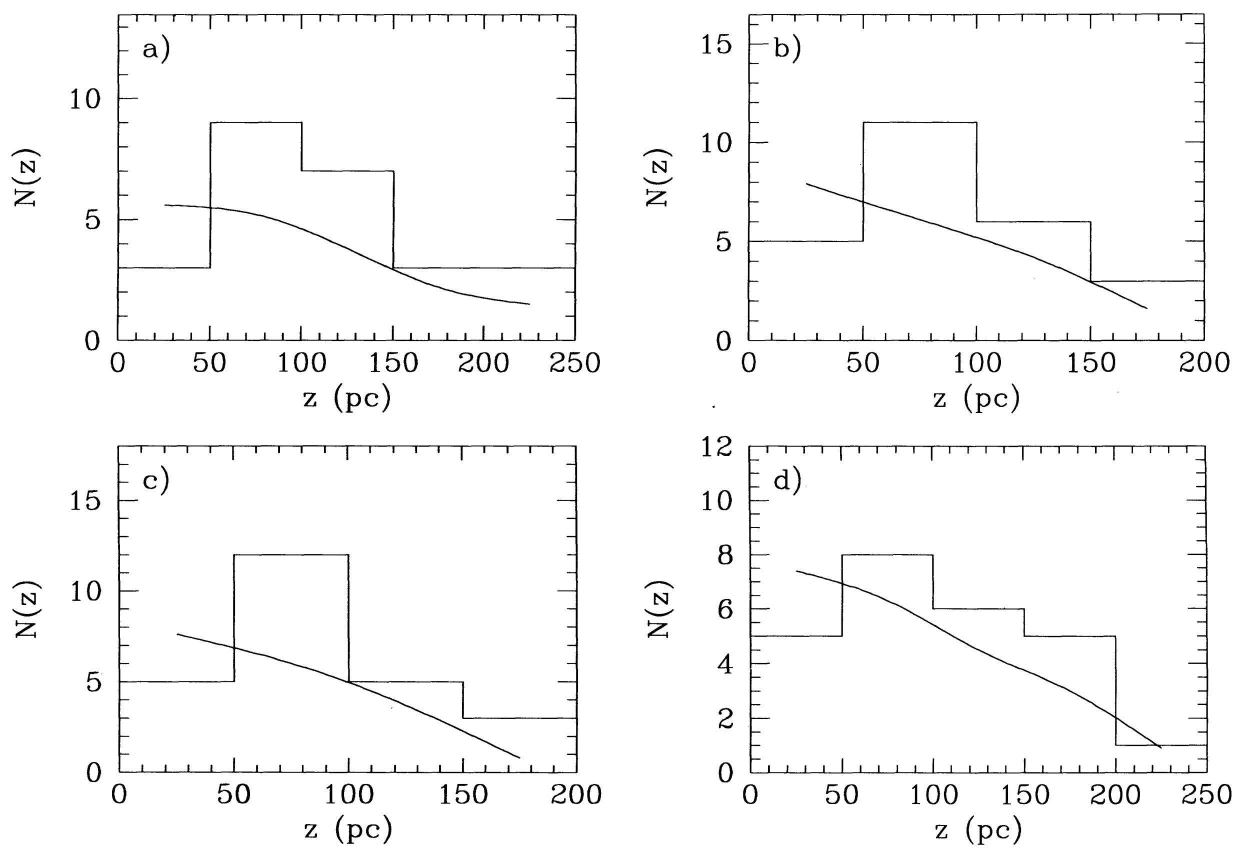

FIG. 5.—(*a*) Observed $N(z)$ distribution (*histogram*), together with our optimal fit within the exponential model (*solid line*). Extinction data are from Neckel & Klare (1980), and effective temperatures are derived from Q', on the basis of the Schulz (1978) calibration. (*a*) For the C_1 sequence, where surface gravities are taken to be a constant, log $g = 5.25$; (*b*) for the C_2 sequence, where surface gravities are taken to be a constant, log $g = 5.50$; (*c*) for the BBL sequence, where surface gravities are assigned on the basis of log $g = -11.70 + 3.87 \log T_{\rm eff}$; (*d*) for the GS sequence, where surface gravities are assigned on the basis of log $g\theta^4 = 2.3$.

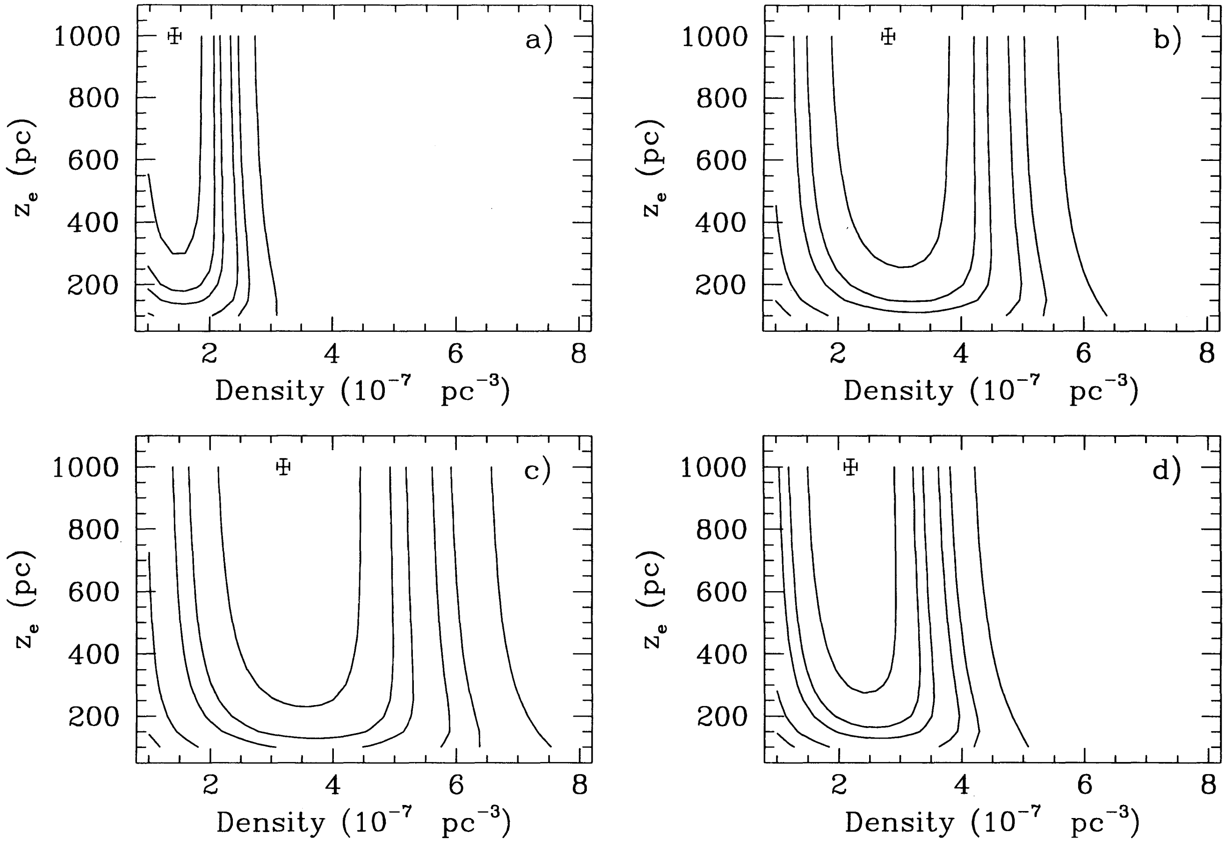

FIG. 6.—Contours of equal confidence level associated with the fit to the $N(z)$ distribution. Extinction data are from Neckel & Klare (1980), and the Schulz (1980) calibration is used. The contours shown are for 68.3%, 90%, 95.4%, 99%, 99.73%, and 99.99% confidence levels. The cross indicates the set of D_0 and z_e associated with the minimal deviation. These values are given in Table 6: (*a*) for the C_1 sequence; (*b*) for the C_2 sequence; (*c*) for the BBL sequence; (*d*) for the GS sequence.

assumed that the vertical distribution of the objects is correctly described by an exponential dependence (see the discussion of Paper I) and have used a subsample of stars brighter than $V \sim 15.0$. By simply averaging the results obtained from the $1/V'_m$ technique (Tables 4 and 5) and from our fits to the observed $N(z)$ distribution (Table 6), which take into account two temperature calibrations as well as several approximate ways to assign surface gravities to the sample, we find a mean value of the midplane space density of $3.8 \pm 1.7 \times 10^{-7}$ pc^{-3}. This value is in good agreement with that derived in Paper I ($D_0 = 3 \pm 1 \times 10^{-7}$ pc^{-3}) in a statistical analysis of the PG sample. However, it is roughly 4–5 times smaller than that initially derived by Downes (1986) on the basis of essentially the same sample. As was already suggested in Paper I, this discrepancy can be traced back to the absolute magnitudes assigned to the program objects. It is now likely that we have found the specific origin of these differing assignments: the distances derived by Downes (1986) for his program objects rely heavily on diagrams of extinction versus distance in the Galactic plane (FitzGerald 1968; Lucke 1978; Nandy et al. 1978), diagrams which—in retrospect—may have been too noisy for the task of determining individual absolute magnitudes. Our estimates of M_y, in contrast, are independent of these diagrams, since they rely on estimates of $T_{\rm eff}$ based on the reddening-independent, temperature-sensitive index Q'.

This being said, however, *we must still rely on diagrams of extinction versus distance in the Galactic plane to derive space densities with any of the three methods explored here*: as was pointed out earlier fits to both the $N(y)$ and the $N(z)$ distributions require the calculation of a predicted distribution of stars which depends on the extinction law $A_y(d)$ adopted for each field of the survey. For the densities calculated with the $1/V'_m$ summing method, the density-weighted volume accessible to each object in the survey must be calculated, and this quantity also depends on the $A_y(d)$ law for each survey field. For this task, we have thus used the extensive compilation of Neckel & Klare (1980), which appears of superior quality to the earlier compilations. Nevertheless, the impact of these diagrams is smaller here than in Downes's work, since (i) the absolute magnitudes used in the density calculations are previously, and independently, constrained, and (ii), in the case of the $1/V'_m$ summing of § 4.3, the diagrams are used in a regime where the distances considered are large enough for the extinction to be properly described by a constant value. Indeed, our calculations of D_0 reveal only a modest sensitivity, *once M_y values have been assigned*, to the particular source of extinction data.

As mentioned earlier, Saffer & Liebert (1995) have essentially completed their spectroscopic reobservations of the complete sample of hydrogen-rich subdwarfs in the KPD survey. This is a significant development, as our work here has assumed all along that the original sample isolated by Downes (1986)—of which we consider only those members brighter than $V \sim 15.0$—is (i) as complete, at faint magnitudes as stated, and (ii) contains only hydrogen-rich subdwarfs. The work of Saffer & Liebert (1995) now permits a quantitative evaluation of point (ii). The preliminary results of their classification work show that KPD 0311+4801, KPD 0721−0026, and KPD 2022+2033 should be removed from the bright sample considered here, while KPD 2024+5303 has not yet been reobserved.[4] The impact of these latest results for our final values of D_0 can be most simply evaluated with the $1/V'_m$ summing technique used in § 4.3, leaving out the three reclassified stars. With the Schulz (1978) photometric calibration, the value of D_0, averaged over the four sequences and the two choices of z_e of Table 4, goes down from 4.1×10^{-7} pc^{-3} to 3.4×10^{-7} pc^{-3}; with the Olson (1974) calibration (Table 5), the average drops from 4.4×10^{-7} pc^{-3} to 3.7×10^{-7} pc^{-3}. In both cases, thus, the reclassification work of Saffer & Liebert (1995) leads to a slight lowering, of the order of $\sim 15\%$, of the space density estimated above on the basis of Downes's original sample.

[4] Note that under these circumstances, the parameters derived in Table 2 for the three misclassified objects may be especially inaccurate.

We are grateful to the Kitt Peak National Observatory for its long-term support of the observational program which served as the basis for this investigation, and to F. Allard, P. Bergeron, R. F. Green, and R. Lamontagne for their contribution in securing the observations. We are also indebted to R. A. Downes for supplying the essential finding charts for the then-unpublished KPD catalog, and to R.A. Saffer for graciously sharing with us the preliminary results of the Saffer & Liebert spectroscopic reinvestigation of the KPD sample. We are also grateful to both for reading and commenting on an earlier version of this manuscript. This work was supported in part by the NSERC Canada and by the Fund FCAR (Québec). One of us (B. V.) benefitted from the status of Chercheur collégial, granted by the Fund FCAR, while this work was being carried out.

REFERENCES

Allard, F., Wesemael, F., Fontaine, G., Bergeron, P., & Lamontagne, R. 1994, AJ, 107, 1565
Bergeron, P., Fontaine, G., Lacombe, P., Wesemael, F., Crawford, D. L., & Jakobsen, A.-M. 1984, AJ, 89, 374
Bixler, J. V., Bowyer, S., & Laget, M. 1991, A&A, 250, 370
Downes, R. A. 1986, ApJS, 61, 569
FitzGerald, M. P. 1968, AJ, 73, 983
Gilmore, G., & Reid, N. 1983, MNRAS, 202, 1025
Green, R. F. 1980, ApJ, 238, 685
Green, R. F., & Liebert, J. 1987, in IAU Colloq. 95, The Second Conference on Faint Blue Stars, ed. A. G. D. Philip, J. Liebert, & D. S. Hayes (Schenectady: Davis), 261
Green, R. F., Schmidt, M., & Liebert, J. 1986, ApJS, 61, 305
Greenstein, J. L., & Sargent, A. I. 1974, ApJS, 28, 157
Heber, U. 1986, A&A, 155, 33
Heber, U., Hunger, K., Jonas, G., & Kudritzki, R. P. 1984, A&A, 130, 119
Liebert, J., Saffer, R. A., & Green, E. M. 1994, AJ, 107, 1408
Lucke, P. B. 1978, A&A, 64, 367
Moehler, S., Richtler, T., de Boer, K. S., Dettmar, R. J., & Heber, U. 1990, A&AS, 86, 53
Nandy, K., Thompson, G. I., Carnochan, D. J., & Wilson, R. 1978, MNRAS, 184, 733
Neckel, Th., & Klare, G. 1980, A&AS, 42, 251
Olson, E. C. 1974, PASP, 86, 80
Reid, N., Wegner, G., Wickramasinghe, D. T., & Bessell, M. S. 1988, AJ, 96, 275
Saffer, R. A. 1991, Ph.D. thesis, Univ. of Arizona
Saffer, R. A., Bergeron, P., Koester, D., & Liebert, J. 1994, ApJ, 432, 351
Saffer, R. A., & Liebert, J. 1995, in Proc. Ninth European Workshop on White Dwarf Stars, ed. D. Koester & K. Werner (Berlin: Springer), in press
Schmidt, M. 1975, ApJ, 202, 22
Schulz, H. 1978, A&A, 68, 75
Seaton, M. J. 1979, MNRAS, 187, 73p
Slettebak, A., & Brundage, R. K. 1971, AJ, 76, 338
Theissen, A., Moehler, S., Heber, U., & de Boer, K. S. 1993, A&A, 273, 524
Villeneuve, B., Wesemael, F., Fontaine, G., Carignan, C., & Green, R. F. 1995, ApJ, 446, 646 (Paper I)
Wesemael, F., Auer, L. H., Van Horn, H. M., & Savedoff, M. P. 1980, ApJS, 43, 159
Wesemael, F., Fontaine, G., Bergeron, P., Lamontagne, R., & Green, R. F. 1992, AJ, 104, 203