THE ASTROPHYSICAL JOURNAL, 447:656–679, 1995 July 10

A MODEL FOR THE GALACTIC POPULATION OF SYMBIOTIC STARS WITH WHITE DWARF ACCRETORS

L. YUNGELSON,[1,2] M. LIVIO,[1] A. TUTUKOV,[2] AND S. J. KENYON[3]
Received 1994 October 28; accepted 1995 January 20

ABSTRACT

By means of a population synthesis code, we investigate the formation of symbiotic systems in which the hot components are assumed to be white dwarfs which are either burning hydrogen steadily or are in a post-nova "plateau" phase, in the evolution of exploding white dwarfs. Our estimate for the total number of symbiotic systems in the Galaxy, ~3000–30,000 (depending on different model assumptions), is compatible with observational estimates. The crucial parameter for the determination of the birthrate and number of symbiotic stars is the mass of the hydrogen layer which the white dwarf can accumulate prior to hydrogen ignition. We model the distributions of symbiotic stars over orbital periods, masses of the components, mass-loss rates by the cool components, and brightness of components, and we obtain a reasonable agreement with observations. We show that in systems which are the most efficient in producing the symbiotic phenomenon, the accretors have to capture up to ~30% of the matter lost by the cool component via a stellar wind. If the fraction of captured matter is significantly lower, it becomes impossible to explain even the lowest observational estimates of the number of symbiotic stars.

The theoretical estimate of the average rate of symbiotic novae is $\sim 0.1\ \mathrm{yr}^{-1}$, compatible with the observed one. The apparent normal chemical composition of symbiotic novae can be explained if the white dwarfs in these systems, which have systematically lower masses than in cataclysmic binaries, manage to preserve "buffer" helium layers between their CO cores and the accreted hydrogen envelopes.

Mass exchange in symbiotic systems does not lead to SN Ia's via the accumulation of a Chandrasekhar mass. However, if sub–Chandrasekhar-mass, double-detonation models indeed produce SN Ia's, then symbiotic systems can be the progenitors of $\lesssim \frac{1}{3}$ of the events. According to the model, SN Ia's in symbiotic binaries belong to young and intermediate-age populations ($t \lesssim 6 \times 10^9$ yr).

Subject headings: binaries: symbiotic — Galaxy: stellar content — novae, cataclysmic variables — stars: statistics — white dwarfs

1. INTRODUCTION

Symbiotic stars (SSs) are a heterogeneous group of variable stars with composite spectra. They typically exhibit both red and blue continua, strong TiO absorption bands, and high-excitation emission lines. Boyarchuk (1970, 1984) and Kenyon (1986) have published extensive reviews of their properties.

The modern model of SSs envisions a three-component system consisting of a binary star with a hot and a cool component and an H II region (Boyarchuk 1970). The cool component is a red giant (RG) or an asymptotic giant branch (AGB) star. In the majority of SSs the hot component is, most probably, a white dwarf (WD) or subdwarf or an accreting low-mass main-sequence star (Kenyon & Webbink 1984; Mürset et al. 1991). However, other types of hot components, e.g., low-mass helium stars or accreting neutron stars, are possible. Estimated masses of the WD components are in the range 0.3–1.1 $M_\odot$, the masses of the cool components are 1–3 $M_\odot$, and for systems with determined orbital periods, these are between 200 and 16,000 days. The variability of SSs may be due to thermonuclear runaways on the surface of an accreting WD (Tutukov & Yungelson 1976) or to variations in the accretion rate onto the hot component; the latter may be caused by an accretion disk instability (e.g., Paczyński & Rudak 1980; Duschl 1986) and/or variations in the mass-loss rate by the cool component (e.g., Bath 1977). It is also possible that the variability of some systems is due to short timescale variations in the emission from the ionized nebula, caused by variations in the mass-loss rate from the giant component (Nussbaumer & Vogel 1987). For the most recent discussion of the variability of SSs see Mikolajewska & Kenyon (1992).

Evolutionary origins of SSs of different kinds were considered by Tutukov & Yungelson (1976), Kenyon (1986), and Webbink (1988). Since some of the evolutionary scenarios leading to the type Ia supernovae (SN Ia's) suggest accreting WDs as the site of the explosive event, SSs have received special consideration as possible SN Ia progenitors (Whelan & Iben 1973; Tutukov & Yungelson 1976; Iben & Tutukov 1984a; Munari & Renzini 1992; Kenyon et al. 1993a; Munari 1994).

Kenyon & Webbink (1984) and Mürset et al. (1991) found several SSs which contain hot compact sources with $\log T_e \approx 4.6$–5.1 and $L \sim 10^3$–$10^4\ L_\odot$. These sources may be identified with WDs burning hydrogen in their surface layers. The WD may be in a stage of steady burning if $\dot{M}$ is confined to $\sim 10^{-8}$–$10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$, depending somewhat on the WD mass (e.g., Paczyński & Żytkow 1978; Iben & Tutukov 1989; Nomoto 1982), or it may be in the "plateau" luminosity stage after experiencing a hydrogen flash if the accretion rate is lower (for more details on the behavior of accreting WDs, see, e.g., Iben 1982; Livio 1995). The minimum estimated luminosity of the

[1] Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218; mlivio@stsci.edu.

[2] Postal address: Institute of Astronomy, 48 Pyatnitskaya Street, 109017 Moscow, Russia; lry, atutukov@inasan.rssi.ra.

[3] Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138; kenyon@cfa.harvard.edu.

hot components of observed SSs is $\sim 10\ L_{\odot}$. Kenyon & Webbink (1984) have shown that if such a luminosity is due to an accretion disk around a WD, then it is possible to reproduce the typical features of the SS UV spectra. On the other hand, examples of SSs in which the UV continuum originates from an accretion disk surrounding a WD are not found. However, because of the smallness of the sample of investigated SSs, one cannot exclude the possibility that some SSs are systems of the latter kind.

The minimum rate of inflow of matter from the RG into the H II region for which the nebulae is detectable (if it is ionized by a hydrogen-burning WD) is $\dot{M} \approx 10^{-8}$–$10^{7}\ M_{\odot}\ \mathrm{yr}^{-1}$ (Tutukov & Yungelson 1982; Kenyon & Webbink 1984; Nussbaumer & Vogel 1987).

In the present paper, we model the subpopulation of SSs with WDs as the hot components, using a population synthesis code. We attempt to find the evolutionary scenarios leading to the formation of systems containing helium (He), carbon-oxygen (CO), or oxygen-neon (ONe) WDs and RGs or AGB stars with mass-loss/accretion rates that may give rise to the symbiotic phenomenon, and to estimate their birthrates and lifetimes. This data will allow us to evaluate the number of SSs in the Galaxy, and to obtain their distributions over such potentially "observable" parameters as the orbital periods, masses of the components, their luminosities, and mass-loss rates. The model will also provide us with the frequency of symbiotic novae and supernovae (for specific progenitor models) in symbiotic systems.

The present paper continues a series of papers aimed at modeling different constituents of the population of binary stars in the Galaxy, based on a unified scenario approach. The objects that have been considered include supernovae in binaries (Tutukov, Yungelson, & Iben 1992; Tutukov & Yungelson 1994), binary WDs (Tutukov & Yungelson 1992; Yungelson et al. 1994; Tutukov & Yungelson 1994), neutron stars formed in binaries (Tutukov & Yungelson 1993a, b, 1994), single and binary cores of planetary nebulae (Yungelson & Tutukov 1993; Yungelson, Tutukov, & Livio 1993), and X-ray binaries (Iben, Tutukov, & Yungelson 1995a, b).

In § 2 we present our assumptions and describe some details of the modeling algorithm. In § 3 we discuss the main results in relation to the SSs themselves and to SSs as possible progenitors of SN Ia's and conclusions follow.

2. THE MODEL

Our goal is to construct a model of the subpopulation of SSs that contain accreting WDs as the hot sources. Below we describe our algorithm and give some computational details that are important for the understanding of the essence of our modeling. Additional details may be found in Yungelson et al. (1993).

2.1. *Scenario Code*

For the modeling of the population of binary stars we use a population synthesis program, which combines the data on the birthrates of binaries depending on the masses of their components and their separations with an analytical description of the evolution of single and binary stars. The latter is based on results of full-scale evolutionary computations. Similar codes have been used by a number of authors (e.g., Rappaport, Di Stefano, & Smith 1994; de Kool 1990) for different populations. We explore the evolution of binaries that are born with initial masses of both components between 0.8 and 10 $M_{\odot}$ and initial separations from 10 to $10^5\ R_{\odot}$ (as we shall see later, in fact, only systems with initial separations smaller than $\sim 10^4\ R_{\odot}$ contribute to the population of SSs in our model). This phase space of initial parameters was divided into cells by a grid with $\Delta \log (M_1/M_{\odot}) = 0.0125$, $\Delta \log (A/R_{\odot}) = 0.05$, and $\Delta q = 0.05$ (where M_1 is the mass of the primary, A is the separation, and q is the mass ratio). For each "cell" with a given set of (M_1, A, q) we followed all the evolutionary transformations, including mass-loss and mass transfer events up to the formation of a WD plus main-sequence (MS) star pair. The evolution of each pair was then followed up to the formation of a RG or AGB star, which loses mass via a Reimers-type stellar wind. At this phase, the conditions necessary for the symbiotic phenomenon were checked. The symbiotic stage was assumed to terminate when the cool component overfills its Roche lobe or when all of its hydrogen envelope is lost via the stellar wind and it transforms into a nucleus of a planetary nebula.

The following subsections describe some of the input data and assumptions used for the modeling.

2.2. *Birthrate of Binaries*

Based on results of numerous statistical investigations of spectroscopic and visual binaries, the present rate of formation of binaries can be written, as first suggested by Iben & Tutukov (1984a), as

$$d^3\nu = 0.2 d\log (A/R_{\odot}) M_1^{-2.5} dM_1 f(q) dq\ \mathrm{yr}^{-1}\ , \qquad (1)$$

where M_1 is the mass of the primary in $M_{\odot}$, A is the semimajor axis of the orbit, $q = M_2/M_1$ is the mass ratio of the components, and $f(q) = Cq^{\alpha}$ is the initial distribution of binaries over q. The latter function satisfies

$$\int_0^1 f(q)\,dq = 1\ . \qquad (2)$$

Equation (1) assumes that the formation rate of stars with the initial mass of the primary exceeding 0.8 $M_{\odot}$ is $\sim 1\ \mathrm{yr}^{-1}$ (e.g., Phillips 1989) and that all stars are born in binaries with initial separations between 10 and $10^6\ R_{\odot}$. The assumption according to which all stars are born in binaries does not contradict the apparent rate of stellar duplicity because it can be shown (e.g., Tutukov & Yungelson 1992; Yungelson et al. 1993) that ~20%–30% of the initially close binaries end their lives as single objects due to mergers in different evolutionary stages (an initial fraction of binaries can of course be assumed, resulting in an appropriate scaling of our results). For the present study we assume that $\alpha = 0$ for all binaries (e.g., Popova, Tutukov, & Yungelson 1982; Mazeh et al. 1992; Goldberg & Mazeh 1994; Mazeh 1994). This assumption may overestimate the number of wide binaries with components of comparable mass (see, e.g., Vereshchagin et al. 1988). However, our numerical experiments show that their contribution to the population of SSs is not important (see Fig. 4 below). For a more thorough discussion of equation (1) and the relevant list of references see Yungelson et al. (1993).

2.3. *Evolutionary Lifetimes*

Stars with masses less than $\sim 2.5\ M_{\odot}$ develop degenerate helium cores after they leave the main sequence. The luminosities and radii of stars with highly condensed cores, as well as the rate of growth of the cores, are determined mainly by the masses of the cores M_{He} (Refsdal & Weigert 1970). For stars with a chemical composition close to solar, Iben & Tutukov

(1984a), using the results of evolutionary computations by Sweigart & Gross (1976) and Mengel et al. (1979), have derived the following approximations for the dependences of R, L, and $\dot{M}_{He}$ on M_{He} (all variables are expressed in solar units):

$$R \approx 10^{3.5} M_{He}^4 , \quad L \approx 10^{5.6} M_{He}^{6.5} , \quad \dot{M}_{He} \approx 10^{-5.36} M_{He}^{6.6} . \quad (3)$$

The mass of the helium degenerate core at the base of the RG branch M_{He0}, based on the same evolutionary computations of Sweigert & Gross (1976) and Mengel et al. (1979), can be approximated as

$$\log M_{He0} \approx -0.783 + 0.32 \log M + 0.112(\log M)^2 , \quad (4)$$

where M is the mass of the star in solar masses. A combination of equation (3) with equation (12) for the mass-loss rate (see § 2.4 below) provides the lifetimes of RG donors of SSs.

In the stage of core He burning, stars with masses below $\sim 2.5\ M_\odot$ have radii that are smaller than their maximum radii at the shell hydrogen burning stage (at the top of the RG branch), and this stage is unimportant for the stellar wind mass-loss history of the cool components of SSs. After the completion of core He burning, stars begin to evolve along the AGB. Stars with masses below $\sim 2.5\ M_\odot$ typically have $R \sim 30\ R_\odot$ when they enter the early AGB (E-AGB) stage, while more massive stars at this point have radii which are close to the radii they had at the end of the shell hydrogen stage (see, e.g., Lattanzio 1991). Using results of evolutionary computations by Lattanzio (1991), the E-AGB lifetime (in years) may be approximated as

$$\log \tau_{E\text{-}AGB} \approx 7.15 + 0.85(|\log M|) - 2.42(|\log M|)^2 , \quad (5)$$

where M is in solar masses. At the end of the E-AGB stage and the beginning of the thermally pulsing AGB (TP-AGB) stage, the masses of the CO cores of low- and moderate-mass stars ($M \lesssim 10\ M_\odot$) can be approximated as (M in solar masses)

$$\log M_{CO} = -0.27 + 0.36(\log M)^{2.5} \quad \text{for } M \geq 1\ M_\odot ,$$
$$\log M_{CO} = -0.27 - 0.36(|\log M|)^{2.5} \quad \text{for } M < 1\ M_\odot . \quad (6)$$

The luminosities and radii of stars that evolve along the AGB are related as (Iben & Tutukov 1984a)

$$R/R_\odot \approx 0.59(L/L_\odot)^{0.68} , \quad (7)$$

while for the dependence of the luminosity on the mass of the CO core in the TP-AGB stage one has the familiar Paczyński-Uus relation (Paczyński 1970; Uus 1970), which we adopt here with coefficients used previously by Iben & Tutukov (1984a):

$$L/L_\odot \approx 6 \times 10^4 (M_{CO}/M_\odot - 0.5) . \quad (8)$$

It must be noted that for the TP-AGB stage, equation (7) gives the *average* radius.

Thus, one may approximate the time dependence of the radii of E-AGB stars by an exponent, as suggested by Iben & Tutukov (1985; see also Iben & Renzini 1983),

$$R \approx R_0 e^{\beta t} , \quad (9)$$

with R and β determined from equations (5)–(8).

For stars in the TP-AGB stage that have an envelope hydrogen content of $X = 0.7$, equation (8) results in the following equation for the time dependence of the mass of the CO core:

$$M_{CO} \approx 0.5 + [(M_{CO})_0 - 0.5] \exp [t/(1.17 \times 10^6)] . \quad (10)$$

The lifetime in the TP-AGB stage may be obtained by integrating equation (10) with the initial mass of the core given by equation (6) and the final mass of the core given by the following approximation to the (graphic) Weidemann (1986) initial-final mass relation (M is in solar masses):

$$\log M_{CO} = -0.22 + 0.36(\log M)^{2.5} \quad \text{for } M \geq 1 ,$$
$$\log M_{CO} = -0.22 - 0.36(|\log M|)^{2.5} \quad \text{for } M < 1 . \quad (11)$$

Relation (11) fits well the interval between the minimum and maximum final masses allowed by the observations of WDs in the solar neighborhood (see Fig. 12 in Groenewegen & de Jong 1993). If the evolution in the AGB is aborted by a Roche lobe overflow (RLOF), the lifetime may be derived by using in the integration of equation (10) the core mass estimated from the radius of the star at RLOF by means of equations (7) and (8).

2.4. *Stellar Wind Mass Loss*

In the description of the stellar wind mass loss we followed the usual prescription of Reimers (1975) for the empirical mass-loss law:

$$\dot{M} = 4 \times 10^{-13} \eta \frac{RL}{M} M_\odot \text{ yr}^{-1} , \quad (12)$$

with R, L, and M being the stellar radius, luminosity, and mass, respectively (expressed in solar units). One has to realize that equation (12) is nothing more than a fit to the observed mass-loss rates in RGs, which was given its functional form to express the idea that a certain fraction of the energy generated inside the star is expended on lifting material from the stellar surface. We assumed that for stars which develop degenerate helium cores ($M \leq 2.5\ M_\odot$), equation (12) describes the mass-loss rate starting from the phase when the star begins to ascend the RG branch in the H-R diagram. For more massive stars, equation (12) was applied starting from the evolutionary phase after the exhaustion of helium in their cores, i.e., in the E-AGB and TP-AGB stages, because for them the RG stage is short and faint. The parameter η in equation (12) was adjusted to allow us to reproduce Weidemann's (1986) initial-final mass relation for single stars given by equation (11). The dependence of η on the initial stellar mass is shown in Figure 1. Stellar wind mass-loss rates corresponding to the values of η in Figure 1,

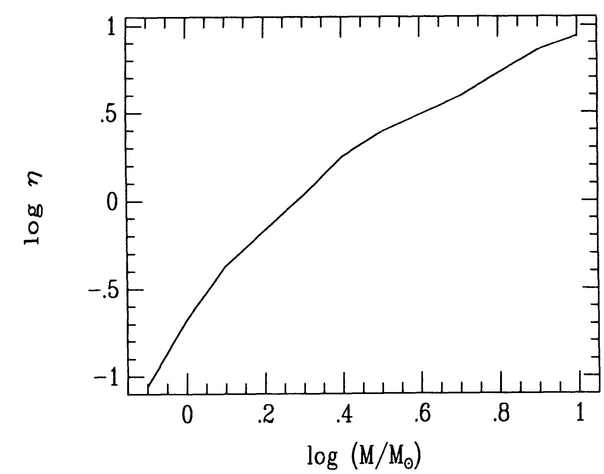

FIG. 1.—Dependence of the parameter of the Reimers mass-loss law (eq. [12]) on the mass of the star.

reproduce well both the rather low rates observed in RG stars and the very high loss rates in late AGB stars (see, e.g., estimates by Jura 1994, who gives $\dot{M} \sim 10^{-11}$–$10^{-10}\ M_\odot\ \mathrm{yr}^{-1}$ for RGs and $\dot{M} \sim 10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$ for very late post-Mira stars). In particular, with this η, a 1 $M_\odot$ star loses $\sim 0.15\ M_\odot$ in the shell hydrogen burning stage, in good agreement with estimates by Iben & Rood (1970) and Sweigart, Greggio, & Renzini (1990). For stars with masses below $\sim 3\ M_\odot$, which are the most efficient producers of SSs (see Fig. 4 below), the mass-loss rates derived by means of equations (3), (6)–(8), (11), and (12), and the data on η from Figure 1, are in reasonable agreement with the (rather scarce) reliable data on mass-loss rates from late-type stars as a function of luminosity and effective temperature (see, e.g., de Jager, Nienwenhuijzen, & van der Hucht 1988). For stars more massive than 3 $M_\odot$ (which are less important for the production of SSs), equation (12) somewhat overestimates $\dot{M}$. This reflects the fact that in deriving the value for η we averaged the mass-loss rate over the entire stellar lifetime, while for the most massive objects most of the mass is lost in the TP-AGB (and even there the mass-loss rate may be a steep function of the luminosity; e.g., Reid, Tinney, & Mould 1990; Frantsman 1986). However, as we shall show in § 3.2, for a quite wide range of mass-loss rates, the number of SSs is more sensitive to the total amount of mass lost by the donor than to the rate at which it is lost. The strong dependence of η on the stellar mass is a measure of the degree of inconsistency between the Reimers mass-loss law and the initial-final mass relation.

We do not consider in the present paper possible tidal enhancement of mass loss due to the presence of a close companion (Tout & Eggleton 1988), nor do we consider mass loss from tenous optically thin stellar atmospheres which overfill their Roche lobes (Plavec, Ulrich, & Polidan 1973).

2.5. *Accretion of Stellar Wind*

Three-dimensional gasdynamical calculations of the accretion from an inhomogeneous medium (e.g., Shima et al. 1985; Livio et al. 1986; see also discussions in Nussbaumer & Vogel 1987; Livio 1992; Bisikalo et al. 1994) have shown that for SSs the classical Bondi (1952) formula for accretion from the stellar wind is generally valid:

$$\dot{M} = 2\pi\zeta(GM)^2(V_{\rm rel}^2 + c_s^2)^{-3/2}\rho\ . \tag{13}$$

In equation (13) M is the mass of the accretor, ρ is the density of the stellar wind in the vicinity of the accretor, $V_{\rm rel}$ is the relative velocity between the hot star and the undisturbed part of the stellar wind, c_s is the sound speed, and $1 \lesssim \zeta \lesssim 2$ is a parameter. We have considered the case of supersonic accretion and set $\zeta = 2$. The mechanism that is responsible for the driving of winds from cool, late-type stars is still not fully understood. It may involve a two-step process, in which first matter is driven by pulsations, then cools down to form dust, and then the dust grains are accelerated by radiation pressure (Fadeev 1987). In any case, both empirical determinations of the wind velocity law for EG And (Vogel 1991) and numerical modeling of the mass-loss process (Sedlmayr, Dominik, & Gail 1988) show that the wind reaches a terminal velocity at a distance of ~ 10 stellar radii. In the calculation of $V_{\rm rel}$ we took into account the fact that the hot component may be located in the zone of stellar wind acceleration. To model the radial dependence of the velocity of the stellar wind, we assumed that

$$V_w = \alpha_w V_{\rm esc}\ , \tag{14}$$

where $V_{\rm esc}$ is the escape velocity at the surface of the donor, and we approximated α_w by

$$\alpha_w = \frac{0.04(r/R_d)^2}{1 + 0.04(r/R_d)^2}\ , \tag{15}$$

where r is the distance from the donor and R_d is the radius of the donor. This approximation gives a velocity law which is close to the familiar $V(r) = (1 - r/R_d)^\beta$, with $\beta = 3$, but it gives a finite velocity at $r/R_d = 1$. As the dependence of $\dot{M}$ on the parameter α_w may be rather strong (close to α_w^4), in § 3 we discuss the consequences of (1) assuming $\alpha_w = 1$ or (2) using the empirical approximation for $V(r)$ derived by Vogel (1991).

An additional enhancement of the accretion is possible if the stellar wind is emitted preferentially in the equatorial plane of the donor, where the accretor is located. We ignore this effect here. We also do not consider the effects of potential interactions between the winds of the two components and the influence of the ionizing radiation of the hot component on the wind acceleration and the accretion process.

2.6. *Efficiency of Hydrogen Accretion*

The traditional model of SSs (Tutukov & Yungelson 1976; Paczyński & Rudak 1980) envisions two types of behavior of WD accretors. In systems of the first type, the accretion rate is such that hydrogen burning at the surface of the WD is stable $[(dM/dt)_{\rm acc} \approx (0.5\text{–}2) \times 10^{-7}\ M_\odot\ \mathrm{yr}^{-1}$ for a 0.6 $M_\odot$ star]. In the systems of the second type, $(dM/dt)_{\rm acc}$ is below the rate necessary for steady burning, and these systems experience symbiotic nova eruptions. Variability with an amplitude which is smaller than in novae can be associated with variations in the accretion rate (see Mikolajewska & Kenyon 1992 for the most recent review). The traditional assumption is that steadily burning WDs consume the infalling hydrogen with 100% efficiency, while the erupting ones lose all their accreted envelopes and even a part of the underlying cores; i.e., "erosion" of WDs occurs.

However, there are circumstances in which this traditional picture has to be changed.

2.6.1. *Critical Ignition Mass*

Even if the accretion rate is below the steady burning range, a certain amount of matter has to be accumulated prior to the first explosion. This "critical mass" is a function of the temperature of the WD, its mass, and the accretion rate. Using the results of numerical modeling of nova eruptions for *relatively cold* ($T \sim 10^7$ K) WDs with masses between 0.75 and 1.2 $M_\odot$ (Schwartzman, Kovetz, & Prialnik 1994) we have derived the following average expression for $\Delta M_{\rm crit}$:

$$\frac{\Delta M_{\rm crit}}{M_\odot} \approx 2 \times 10^{-6}\left(\frac{M_{\rm WD}}{R_{\rm WD}^4}\right)^{-0.8}\ , \tag{16}$$

where $M_{\rm WD}$ is the mass of the WD (in grams) and $R_{\rm WD}$ is the radius of *zero-temperature* degenerate objects (Nauenberg 1972):

$$R_{\rm WD} = 1.12 \times 10^{-2}\ R_\odot[(M_{\rm WD}/M_{\rm Ch})^{-2/3} - (M_{\rm WD}/M_{\rm Ch})^{2/3}]^{1/2}\ , \tag{17}$$

with $M_{\rm Ch} = 1.433\ M_\odot$ and $R_\odot = 7 \times 10^{10}$ cm. Extrapolation of relation (16) to lower masses (0.4 and 0.6 $M_\odot$) agrees with results of numerical calculations (by the same code; Shara, Prialnik, & Kovetz 1993) to within a factor ~ 2.

A WD temperature of 10^7 K corresponds to an age of 10^9 yr (Iben & Tutukov 1984b). In fact, for most of the systems which become SSs, the time interval between the formation of the WD and the beginning of the symbiotic stage may be longer, up to 10^{10} yr (see Fig. 6b below). The temperature of a 10^{10} yr old WD is approximately 10^6 K (Iben & Tutukov 1984b). Qualitatively, the age dependence of $\Delta M_{\rm crit}$ may be described by three phases. In the first, the WD is still hot and it continues its contraction to the equilibrium radius given by equation (17). In this phase, we can guess that $\Delta M_{\rm crit}$ decreases, following more or less equation (16). In the second phase, the temperature continues to drop, the radius almost does not change, being close to the equilibrium value, and the critical mass depends mainly on the accretion rate (and the WD temperature). In the third phase, the heat conductivity of WD matter becomes so high that most of the gravitational energy released in the accretion process leaks inward. This may significantly increase $\Delta M_{\rm crit}$. The border between the first and second phases depends on the WD mass. For example, for a typical $M_{\rm WD} = 0.6\ M_\odot$, it is close to 3×10^7 yr (Iben & Tutukov 1984c, 1986). For a 0.3 $M_\odot$ WD, the equilibrium radius probably cannot be reached in Hubble time (Iben & Tutukov 1986). The border between the second and the third phases has not been studied systematically yet. Taking the inner temperature of a WD as an indicator of its age we can illustrate the behavior of $\Delta M_{\rm crit}$ in the third phase by the results obtained by Kovetz & Prialnik (1995): for a 0.75 $M_\odot$ WD accreting at 10^{-9} $M_\odot$ yr^{-1}, a variation in T from 64×10^6 to 5×10^6 K results in an increase in $\Delta M_{\rm crit}$ from 3×10^{-5} to 2.3×10^{-4} $M_\odot$. An example of the very significant increase in $\Delta M_{\rm crit}$ for the oldest WDs can be found in the calculations for a 0.6 $M_\odot$ WD with $T \approx 10^6$ K accreting at 10^{-10} and 10^{-11} $M_\odot$ yr^{-1} (Iben & Tutukov 1992), which show that the critical ignition mass for such very old and cold WDs may be more than an order of magnitude larger than that given by equation (16).

The dependence of $\Delta M_{\rm crit}$ for hydrogen ignition (in the first flash) on the temperature may strongly influence the birthrate of SSs because our crucial assumption is that stars which do not accumulate at least one $\Delta M_{\rm crit}$ do not become nuclear burning powered SSs.

If the first flash occurs, then in the course of subsequent flashes. $\Delta M_{\rm crit}$ may also vary because of changes in the temperature of the WD and the chemistry of its envelope (e.g., Shara et al. 1993). In the absence of self-consistent sets of computations covering a wide enough range of initial WD temperatures, we use for our estimates equation (16) and perform numerical experiments which show the effect of variations of $\Delta M_{\rm crit}$ on our results (see also § 3.4 and Table 3 for more details).

Equation (16) also neglects the dependence of $\Delta M_{\rm crit}$ on the accretion rate, which is not very strong. For example, Iben's (1982) model calculations for a $\sim 1\ M_\odot$ WD suggest a dependence of $\sim (dM/dt)^{-0.3}$. In fact, this dependence is not important for the estimate of the total number of SSs in the Galaxy and their overall parameters. Variations in the accretion rate may influence the observed variability of SSs.

2.6.2. *Accumulation Efficiency*

The efficiency of hydrogen consumption can never be 100%. If the star explodes, some mass is lost dynamically. After the explosion, the star spends a certain time in a "plateau" stage, where its luminosity is given by the following approximation for the core mass–luminosity relation for cold WDs accreting hydrogen (Iben & Tutukov 1989):

$$L/L_\odot \approx 4.6 \times 10^4 (M_{\rm core}/M_\odot - 0.26)\ . \qquad (18)$$

Stars that burn hydrogen stably are also in a plateau state. The lifetime in the plateau stage is determined by the amount of hydrogen available on the surface of the WD. It should be noted that relation (18) differs from the usual Paczyński-Uus relation (8), which was originally derived for AGB stars that have *both hydrogen- and helium-burning shells.*

Hot stars with radiative envelopes are supposed to have intense radiation-driven stellar winds. To first order, the rate of the mass loss in the wind may be estimated as

$$\dot{M} = \frac{L}{V_{\rm esc}\, c}\ , \qquad (19)$$

where L is the luminosity, given by equation (18), $V_{\rm esc}$ is the escape velocity, and c is the speed of light. If stars accrete at a rate which is higher than that for stable hydrogen burning, they expand and can reach RG dimensions. In cataclysmic variables, the existence of the Roche lobe sets limits to the expansion, a common envelope (CE) forms, and most of the envelope of the WD is ejected during the CE phase. In contrast, in SSs, with their large orbital separations, the WDs would expand to RG radii. However, for large radii, the radiatively driven stellar wind is effective (eq. [19]) and it restricts the possible expansion. Stellar wind mass loss limits the amount of nuclear fuel that is available for hydrogen shell burning and determines the lifetime in the plateau stage. At the same time, the presence or absence of a CE results in the difference in the light curves between classical and symbiotic novae.

The same type of reasoning about the efficiency of matter accumulation can be applied to helium-burning flashes. In accreting WDs, the helium shell which accumulates below the hydrogen shell is initially "inert." Helium does not burn until a critical mass is accumulated. This critical mass is $\sim$0.1–0.2 $M_\odot$ (for initially cold WDs and $\dot{M} \lesssim 3 \times 10^{-8}\ M_\odot$ yr^{-1}; Fujimoto & Taam 1982; Iben & Tutukov 1991; Woosley & Weaver 1994; Limongi & Tornambé 1991; Tutukov & Khokhlov 1992). After the critical mass of He is accumulated, the helium shell detonates, giving rise to an event that possibly may be identified with a SN Ia (Livne 1990; Tutukov & Khokhlov 1992; Woosley & Weaver 1994). We shall discuss SSs as potential progenitors of SN Ia's in § 3. (Another possibility for SN Ia's in SSs is offered in principle by the accumulation of mass over $M_{\rm Ch}$.) After the first shell explosion, the star (if it survives) becomes "hot," and subsequent He flashes require the accumulation of orders of magnitude lower critical mass. If as a result of the helium flashes the star experiences even a mild expansion, then the stellar wind again limits the amount of helium available for burning and, hence, the lifetime in the helium-burning stage. It is also important to note that the inert helium layer can serve as a "buffer" between the hydrogen envelope and the CO (or ONe) core, thus preventing (for at least a certain number of flashes) the enrichment of the envelope by heavy elements and thus affecting in this way the strength of the H flashes (i.e., novae).

In Figure 2, adapted from Iben & Tutukov (1985), we show for a 1 $M_\odot$ WD and different accretion rates, the ratio $\alpha_{\rm H}$ between the rate of accumulation of helium due to hydrogen burning (in the presence of a stellar wind) and the rate of accretion of matter by the WD $\dot{M}_{\rm acc}$. In the same figure, the

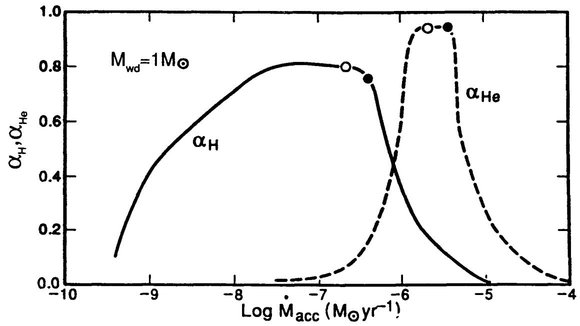

FIG. 2.—Efficiency of conversion of hydrogen into helium (α_H) and helium into carbon and oxygen (α_{He}) vs. the accretion rate, for a 1 $M_\odot$ WD (Iben & Tutukov 1994). Dots indicate the borders of the region of stable hydrogen burning.

ratio between the rates of accumulation and "consumption" of helium is also shown (the latter can be applied to the situation after the first He flash). Figure 2 is based on results of modeling of a 1 $M_\odot$ accreting WD in the quasi-static approximation (Iben 1982; Iben & Tutukov 1989). We mark on the curves in Figure 2 the region of stable burning. Even in this region, the efficiency of accumulation never exceeds 0.8, because of the stellar wind (eq. [19]). We should note that some deviations from the curves in Figure 2 are obtained when hydrodynamical calculations are performed (e.g., Prialnik & Livio 1995); however, these do not change any of our conclusions in a significant way. Nevertheless, because the efficiency of accretion is crucial for estimates of SN Ia rates, this problem will have to be addressed by future (possibly multidimensional) calculations.

In our calculations, we use the dependence of α_H on $\dot{M}_{acc}$ for the determination of the lifetimes of stars in the symbiotic stage. In the absence of calculations of α_H and α_{He} for stars with masses different from 1 $M_\odot$ (because of the lack of homogeneous sets of computations for different M_{WD} and $\dot{M}_{acc}$ which include all the above-mentioned effects), we simply scale the α_H and α_{He} curves with the maximum stable hydrogen burning rate given by equation (18). For accretion rates below $10^{-9.5}$ $M_\odot$ yr^{-1}, when strong nova flashes may be expected, we set, somewhat arbitrarily, the efficiency of accretion equal to 0.03 (this can in fact be regarded as an upper limit).

The most important feature exhibited by Figure 2 is the fact that if both hydrogen and helium burning occur, then even under the most favorable conditions, no more than $\sim$15% of the accreted mass can be converted into carbon and oxygen and be added to the core mass. This peculiarity severely restricts the possibility of producing supernovae by SSs, not only because the efficiency of conversion of H into C and O is low, but also because the efficiency of accretion of stellar wind in SSs is, as a rule, low itself (see Fig. 5 below). Of course, the estimate of the fraction of mass added to the underlying core is strongly model dependent (see, e.g., Kato, Saio, & Hachisu 1989 for an estimate of the rate of core mass growth in WDs accreting He).

Recently, Kato & Hachisu (1995) argued that the application of OPAL opacities for the calculation of the decay phase of novae results in optically thick winds that are about an order of magnitude more powerful than given by equation (19). While this could perhaps solve the problem of classical novae duration without involving CEs, the extremely short decay times obtained by Kato & Hachisu seem to contradict the observational data on symbiotic novae with relatively massive ($\sim$1 $M_\odot$) WDs, which exhibit outbursts lasting for decades (e.g., AG Peg; although for relatively short timescales the mass-loss rate can definitely be higher than that given by eq. [19]; Vogel & Nussbaumer 1994).

2.7. *Common Envelope Phases*

If at RLOF the star has a radiative envelope or its convective envelope is shallow and the mass ratio of the components q exceeds $\sim$0.8, mass is transferred to the companion conservatively; otherwise, mass exchange proceeds through a CE on a thermal ($0.5 \lesssim q \lesssim 0.8$) or dynamical timescale (Paczyński, Ziolkowski, & Żytkow 1969; Paczyński & Sienkiewicz 1972; Paczyński 1976; Tutukov, Fedorova, & Yungelson 1982; Hjellming & Webbink 1987; Hjellming 1989). Spiraling-in inside the CE results in a reduction in the separation of the components and at least a partial ejection of the CE. For the change in the separation of the components, we adopted the formalism suggested by Tutukov & Yungelson (1979), which assumes that the real spiraling-in begins when the CE extends to a radius approximately as large as the separation and that mass is ejected from the gravitational field of both components. In this case, in the limit of completely nonconservative evolution (Tutukov & Yungelson 1979),

$$\frac{(M_1 + M_2)(M_1 - M_{1R})}{A_0} = \alpha_{ce}(M_{1R} M_2)\left(\frac{1}{A_f} - \frac{1}{A_0}\right), \quad (20)$$

where M_{1R} is the final mass of the donor and A_0 and A_f are the initial and final semimajor axes of the orbit. The efficiency parameter, α_{ce}, is defined as the ratio of the binding energy of the ejected material and the difference in the orbital energy of the binary, between the beginning and the end of the spiraling-in process (see Iben & Livio 1993, for a detailed discussion of the physical processes affecting the value of α_{ce}). Its value is highly uncertain, but values in the range 0.1–1 seem to be consistent with the properties of a number of post-CE objects (see, e.g., Yungelson et al. 1993). In the extreme case of a small initial separation and a low mass ratio of the components, the CE stage may result in the coalescence of components. The masses of the WDs that arise after CE events are given by equations (3), (7), and (8), which relate the radii of stars at RLOF with the masses of their cores.

For a more thorough review of the CE problem see Iben & Livio (1993).

3. RESULTS AND DISCUSSION

3.1. *Evolutionary Scenarios for the Formation of Symbiotic Stars*

The evolution of binaries which results in the formation of SSs with wind-fed WD accretors was discussed, e.g., by Tutukov & Yungelson (1976), Kenyon (1986), and Webbink (1988). Here we shall restrict ourselves to the most important points.

The usual picture of the population of binary stars envisions the existence of "close" and "wide" binaries. Binaries in the first class have separations that allow RLOF by the initially more massive component of the system, while in the systems of the second class neither component ever suffers RLOF. The border between close and wide systems depends on the mass of the primary star and mass ratio of the components in any particular system because it is determined by the maximum radius that can be attained in the course of evolutionary expansion of a star of a given mass. If one takes into account stellar wind mass loss and a Jeans mode of loss of systemic angular momentum via the stellar wind, it appears that a certain fraction of the binaries that would have been close in the absence of stellar wind mass loss become wide. This may be especially important for the most massive close binaries in the sample under consideration, which may lose up to ~85%–90% of their initial mass.

If a system is wide (or became wide due to stellar wind mass loss), its evolution prior to the formation of a configuration which is potentially able to develop the symbiotic phenomenon follows the following sequence of evolutionary stages (see Fig. 3, scenario III): (1) MS + MS star, (2) RG + MS star, (3) core helium burning RG + MS star, (4) AGB star + MS star, (5) WD + MS star, and (6) WD + RG.

In a binary system, this sequence may be aborted in stages 1, 2, and 4. A partial exception are stars with initial masses $M \leq 2.5\ M_\odot$, which have at the onset of core He burning radii that are larger than at the end of E-AGB stage. Therefore, if these stars did not fill their Roche lobes in the shell hydrogen burning stage, they can overfill the Roche lobe only in a fairly advanced TP-AGB stage (see, e.g., Fig. 31 in Iben & Tutukov 1984a).

It follows from the discussion above that if one omits minor differences arising, e.g., from the evolutionary state of the secondary at the onset of RLOF, then all the progenitors of SSs proceed along one of three major "routes." These are shown in Figure 3, together with the birthrates of SSs that are formed in each scenario.

Scenarios I and II occur in systems that were initially close and remained close in spite of angular momentum losses via the stellar wind. In scenario I, the primary overflows its Roche lobe in the RG or AGB stage and transforms into a He, CO, or ONe WD.

In scenario II, the initial mass ratio of the components in the close system is above 0.8. This allows a stage of conservative mass exchange, which also results in the formation of a WD. The figure is slightly oversimplified for systems with initial masses of the components between ~2.5 and ~10 $M_\odot$, where the formation of a helium star is possible, as an intermediate stage between the RG and the WD. At this stage, an appearance of the symbiotic phenomenon can also be imagined if the helium star is hot enough and the matter inflow rate into the circumbinary medium is high enough. The rate of formation of helium stars with MS companions at separations which allow the MS star to evolve into a RG is ~0.004 yr^{-1}. Even if helium stars live longer than their giant companions, the number of such systems is more than 30 times smaller than the number of systems with WD accretors. The masses of the hot components in these systems and their stellar wind mass-loss rates are too low for the Wolf-Rayet star phenomenon. The observed counterparts of these systems are, most probably, binary hot subdwarfs (see, e.g., Tutukov & Yungelson 1987, 1990).

Scenario III occurs in systems that were initially wide or became wide because of a Jeans mode angular momentum loss. In these systems only CO accretors are formed.

In all the scenarios, the symbiotic phenomenon becomes apparent when the secondary component evolves into a RG or an AGB star.

The SS stage may terminate in a twofold way. In scenarios I and II, the cool component either overfills its Roche lobe or exhausts its envelope via a stellar wind. In all our SS systems that filled their Roche lobes at the termination of the SS stage, the mass ratio of the components, $M_{\rm cool}/M_{\rm hot}$, was higher than

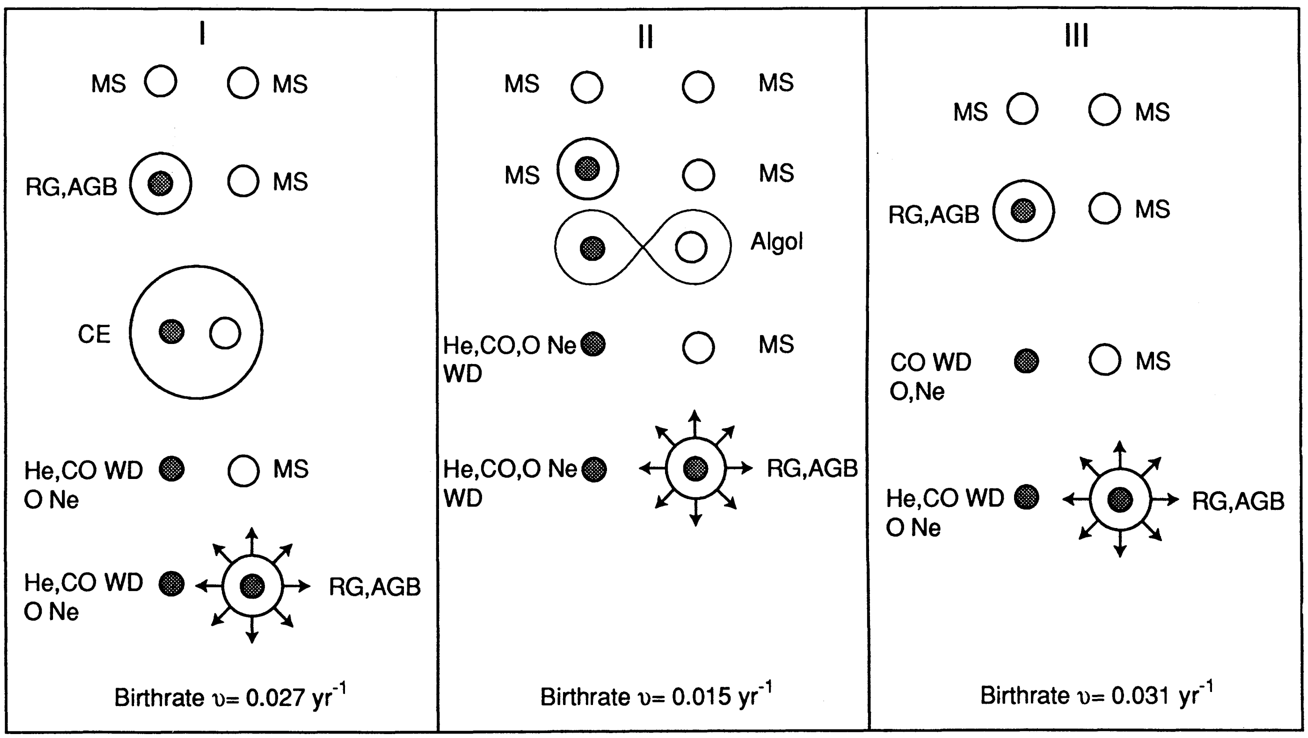

FIG. 3.—Evolutionary scenarios for the formation of SSs. *Open circles*, Unevolved MS stars. *Filled circles*, Helium or carbon-oxygen cores of evolved stars (RGs AGB stars) or WDs. *Large circles surrounding both components*, CEs. "*Roche lobes*," Semidetached (Algol) systems. Arrows indicate stages with strong stellar wind of one of the components, in which the symbiotic phenomenon is possible. For each scenario, the birthrate of SSs through it is indicated (for the standard model).

~0.9. Because the cool components have, as a rule, deep convective envelopes, CEs are expected to form. The separations of the components were large, and therefore, the formation of a double-degenerate system may be expected in most cases, and coalescence of the components is avoided. If the cool components lose their envelopes via a stellar wind, then the SSs transform into wide double-degenerate systems in all three scenarios.

We should mention that prior to the formation of a WD (present hot component), a stage of accretion onto a MS star is possible. Some of the SSs may be in this stage (see Kenyon & Webbink 1984; Mikolajewska & Kenyon 1992).

Figure 4*a* shows, for the "standard" assumptions listed in § 2, the dependence of the number density of stars which may exhibit the symbiotic phenomenon on the initial mass of the primary component and the separation $\partial^2 N/(\partial \log M_1 \partial \log A)$. Systems with $\log (M_1/M_\odot) \lesssim 0.5$ and $\log (A/R_\odot) \lesssim 1.5$ pass through the stage of low-mass semidetached Algols, which produce He WDs with masses up to $\sim 0.45\, M_\odot$ (scenario II in Fig. 3). Systems with $\log (M_1/M_\odot) \lesssim 0.7$ and $\log (A/R_\odot) \lesssim 2.3$ pass through a stage of a massive Algol, which produces helium stars (case B of mass exchange), which then evolve into low-mass CO dwarfs (also scenario II). Wider systems of all masses either experience RLOF by the primary (scenario I) or only a wind mass-loss stage (scenario III). It is evident that only a minor fraction of all the binary systems are able to produce a SS. Figure 4*a* also shows that most SSs descend from stars with initial primary masses not exceeding $\sim 3\, M_\odot$. The reasons for this are: (1) the initial mass function for primaries (eq. [1]), which effectively cuts off massive primaries, and (2) the increase in the separation of the components due to a Jeans mode angular momentum loss, which operates with an increasing efficiency, with an increasing relative mass loss (both in the presymbiotic and symbiotic stages).

In Figure 4*b* we show the dependence of the number density of SSs on the parameters of the progenitors, for systems in which it is possible to produce a luminosity of the hot source of $L \geq 10\, L_\odot$ by the release of gravitational energy (in the process of accretion onto a WD). While there are no examples of such purely "accretion-fed" systems containing WDs among the observed SSs, one cannot exclude their existence a priori. The most interesting feature exhibited by Figure 4*b* is the absence of post-Algol systems. Therefore, if accretion-fed SSs with WDs exist, they almost certainly do not contain helium WDs because of the low mass and low luminosity of the latter (for more discussion see § 3.4).

If one considers the distribution of pre-SSs over q, it appears that low-mass progenitors of SSs have q close to 0.8–1, but the range in q becomes wider (extends to $q \sim 0.5$ for $M \approx 3\, M_\odot$) with increasing M_1.

3.2. *Symbiotic Stage of a Binary System*

Following Tutukov & Yungelson (1976) and Paczyński & Rudak (1980), two classes of SSs are traditionally discussed:

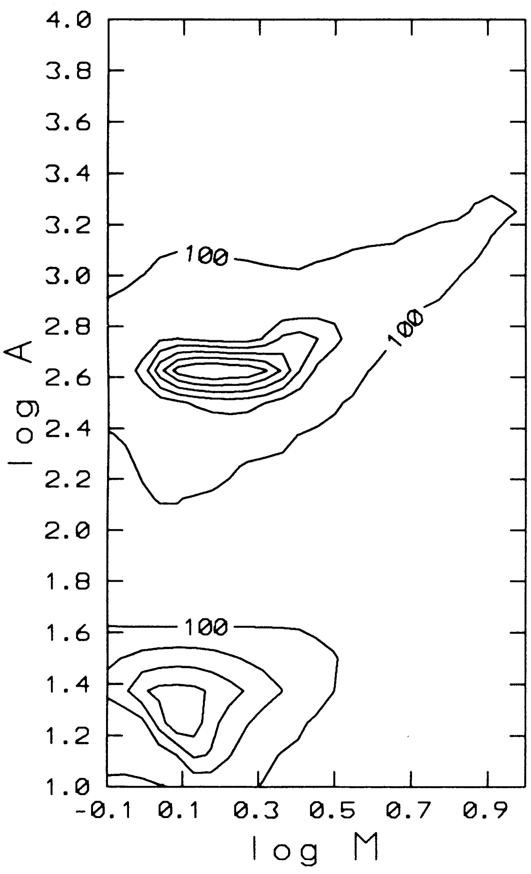

FIG. 4*a*

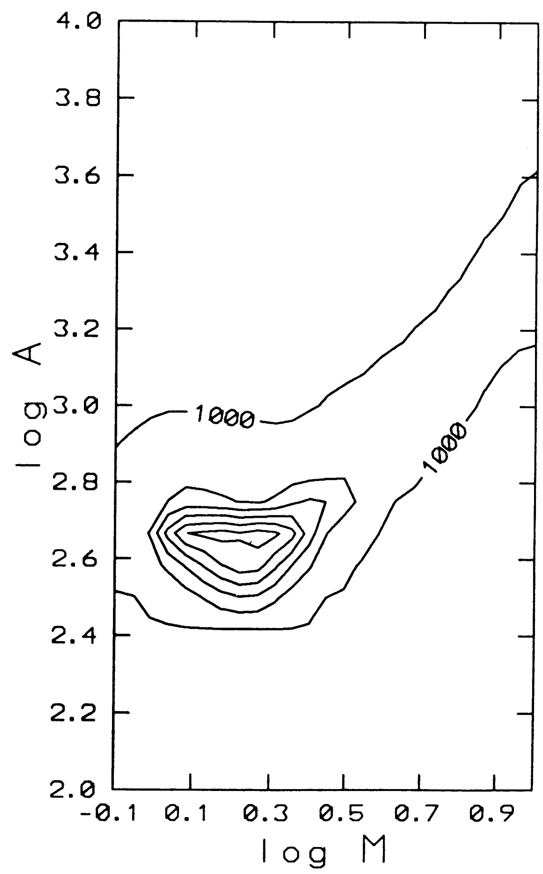

FIG. 4*b*

FIG. 4.—(*a*) Distribution of progenitors of SSs in the standard model in the (initial mass M, initial separation A)-plane. A and M are in solar units. Contour lines display the number density $\partial^2 N/(\partial \log M\, \partial \log A)$ with a step equal to 4000. (*b*) Same as in (*a*), for the accretion model; the step is equal to 30,000.

"ordinary" SSs, which are assumed to burn hydrogen steadily, and symbiotic novae, which experience thermonuclear runaways in their surface hydrogen layers, due to accretion at rates below $(dM/dt)_{st}$, given by the following approximation to the Iben & Tutukov (1989) results (M_{WD} is in solar masses):

$$\log \dot{M}^{st}_{crit} \approx -9.31 + 4.12M_{WD} - 1.42(M_{WD})^2 \ M_{\odot} \ yr^{-1} . \quad (21)$$

Let us examine, for example, the history of mass loss and accretion in a system consisting of a 0.65 $M_{\odot}$ WD and a 1 $M_{\odot}$ RG, at a separation of 400 $R_{\odot}$ (after the formation of the WD). Such a system is quite typical, it may be formed from a 2.5 $M_{\odot}$ + 1 $M_{\odot}$ system with an initial separation of $\sim$650 $R_{\odot}$ (orbital period $\sim$2.8 yr), in which the primary experienced RLOF in the AGB stage (see Figs. 4*a*, 7, and 8). As $t = 0$ we choose the instant when the star reaches the base of the RG branch in the H-R diagram. In the first 10^9 yr of the following evolution, the mass-loss rate by the donor increases from $\sim 10^{-12.2}$ to $\sim 10^{-8.4}$ $M_{\odot}$ yr^{-1}. However, because the system is rather wide and the ratio of accreted to lost matter is determined mainly by the factor A/R_d in equation (15), the accretion rate changes from $\sim 10^{-17.7}$ to $\sim 10^{-10}$ $M_{\odot}$ yr^{-1}. The history of the system is shown in Figure 5*a*, in which we show the donor mass-loss rate (*thick solid lines*), the accretion rate (*thin solid lines*), and the rate of conversion of hydrogen into helium (*thin dashed lines*). At $t = 1.00223 \times 10^9$ yr, when the WD has accumulated $\sim 0.21 \times 10^{-3}$ $M_{\odot}$ of matter, it experienced the first H flash. The accretion rate at this point was $\sim 10^{-10}$ $M_{\odot}$ yr^{-1}, the flash was strong, and the system, if observed would have been identified with a symbiotic nova. After the first and each subsequent nova explosion the WD spends a certain time in the plateau stage. This stage is important for our problem because during this time interval the WD is bright and it may be identified with the hot component of a symbiotic system. In the next 4.8×10^6 yr, the WD experienced a series of nova explosions of decreasing strength (because $\dot{M}_{acc}$ increased). Finally, at $t = 1.00799 \times 10^9$ it entered the stage of steady hydrogen burning with a luminosity corresponding to the plateau value given by equation (18). From this point on, the system remained in a state of "stationary" SS.

We suggest that stars in the "plateau" stage (both postnovae and steady burning ones) and stars in the post–hydrogen-exhaustion stage (as long as $L_{hot} \gtrsim 10\ L_{\odot}$) may be identified with the hot components of SSs.

The total time spent in the plateau stage is given by

$$t_{on} \approx 6.9 \times 10^{10} \int_0^T \frac{\alpha_H(\dot{M}_{acc}, M_{hot})\dot{M}_{acc}}{L(M_{hot})} dt \ yr , \quad (22)$$

where $\dot{M}_{acc}$ is the accretion rate in $M_{\odot}$ yr^{-1}, L is given by equation (18), and T is the lifetime from the base of RG branch to the end of the symbiotic stage. After each flash, the star remains in an "on" state until the amount of hydrogen in its envelope decreases below a certain minimum level and hydrogen burning ceases. However, the system remains observable as a SS until the WD cools to a temperature at which its luminosity becomes $L_{WD} \lesssim 10\ L_{\odot}$. The latter value of "threshold" luminosity for the symbiotic phenomenon is inferred by the minimum estimated values of L_{hot} in observed SSs (Mikolajewska & Kenyon 1992; Mürset et al. 1991). We should note that symbiotic systems which undergo steady hydrogen burning and some postnova systems may be identified as supersoft X-ray sources (e.g., Greiner, Hasinger, & Kahabka 1991; Schaeidt, Hasinger, & Trümper 1993; van den Heuvel et al. 1992). The "observable" cooling stage after the exhaustion of hydrogen is several (up to $\sim$10) times longer than the "on" stage, depending on the mass of the dwarf and accretion rate (see, e.g., Iben 1982). When the accretion rate becomes higher than $\sim 6.1 \times 10^{-9}$ $M_{\odot}$ yr^{-1}, the system under consideration remains "symbiotic" all the time between flashes because with this accretion rate a luminosity of 10 $L_{\odot}$ can be supported by accretion alone.

The value of t_{on} given by equation (11) is probably an upper limit for the duration of the "on" stage. One of the reasons may be that if the source is very hot, the interaction of its stellar wind with the stellar wind of the donor may prevent the accretion of the latter. The radiation pressure from the WD may act in the same direction. This important aspect of the SS

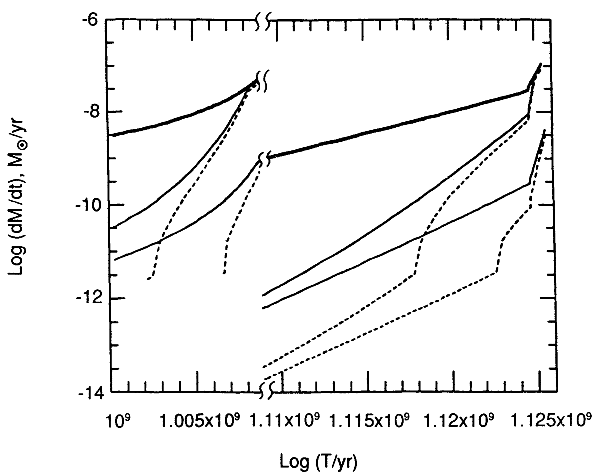

FIG. 5*a*

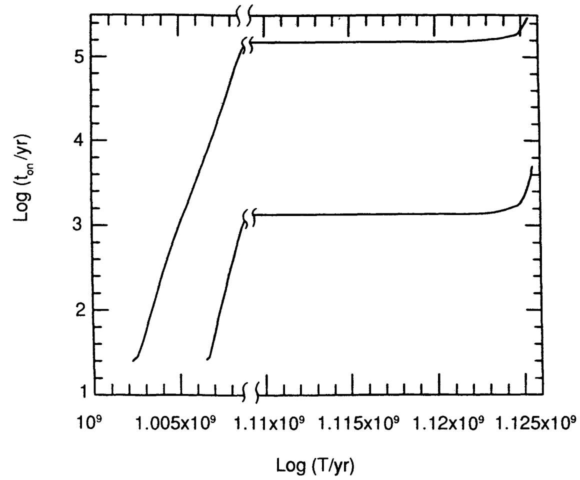

FIG. 5*b*

FIG. 5.—(*a*) Temporal variations of the mass-loss rate by the donor (*thick solid lines*), accretion rate by the WD (*thin solid lines*), and the rate of conversion of hydrogen into helium in a system which initially contained a 0.65 $M_{\odot}$ WD and a 1 $M_{\odot}$ RG donor. The initial separation was equal to 400$R_{\odot}$. The upper set of lines corresponds to a variable efficiency of accretion α_w as given by equation (14); the lower set of lines corresponds to $\alpha_w = 1$. (*b*) Time spent in the "on" state vs. total evolutionary lifetime. The discontinuity in the lifetime corresponds to the core helium burning stage, which almost does not contribute to the population of SSs.

problem has not been studied yet (see though Stevens, Blondin, & Pollock 1992; Dgani 1993; Dgani, Walder, & Nussbaumer 1993).

In Figure 5*b* we show how t_{on} accumulates during the evolution of the system under consideration. While the actual time spent in the symbiotic state may be several times longer than indicated by Figure 5*b*, stars with accretors in the plateau (or close to the latter phases) dominate the sample of observed SSs (due to selection effects; see § 3.4). A binary star with a RG donor remains in the symbiotic state until, at $t = 1.0089 \times 10^9$ yr, He burning commences in the donor's core. For the next $\sim 10^8$ yr, the star has a radius and luminosity that are lower than the ones it had at the final phase of hydrogen shell burning in the RG stage. The donor at this stage may be identified with a RG clump star. At this epoch, the accretor experiences infrequent, but violent (because of the low accretion rate) nova flashes. It almost does not contribute to the total population of SSs, and we therefore skip this stage in Figures 5*a* and 5*b*. The same applies to most of the E-AGB stage, when both the mass-loss rate and the accretion rate are low. The symbiotic phenomenon (as is seen from Fig. 5*b*) becomes prominent again for the last $\sim 5 \times 10^6$ yr of the donor's life, at the very end of the E-AGB stage and in the TP-AGB stage ($t > 1.0204 \times 10^9$ yr). Out of a total of 2.9×10^5 yr spent in the "on" stage, for 1.5×10^5 yr the donor is a RG and for 1.4×10^5 yr it is an AGB star. The evolution of the system terminates when the donor (with a mass now of 0.681 $M_\odot$) overflows its Roche lobe. It is worth mentioning that at the end of the RG and TP-AGB stages of the donor, when it is close to its tidal lobe, the accretion of the stellar wind matter becomes almost conservative, due to the low relative velocity of the stellar wind in the vicinity of the WD (see eq. [15]). The fact that a significant fraction of the SSs are in a stage of almost conservative mass exchange, with donors close to their tidal lobes, was expected by Webbink (1988) from a semiqualitative analysis of the origin of SSs.

Because the donor at the point of RLOF possesses a deep convective envelope, the RLOF has to result in the formation of a CE (the mass transfer is unstable). Ultimately, the system will become a double degenerate.

Also in Figure 5, we show the difference in the symbiotic properties of the same initial system if we assume that the velocity of the donor stellar wind in the vicinity of the accretor is equal to V_{esc}, i.e., if we relax the assumption about the existence of a zone of wind acceleration (see eqs. [14] and [15]). Clearly, the symbiotic phenomenon becomes much less prominent because the efficiency of accretion decreases by about an order of magnitude. As we shall show below, one cannot reduce the efficiency of accretion by too much because then it becomes impossible to explain the number of SSs in the Galaxy.

Table 1 shows the properties of a symbiotic system with typical masses of the donor (1 $M_\odot$) and WD accretor (0.65 $M_\odot$) (see Figs. 8 and 13), as a function of the separation of the components. In this Table, for each initial separation of the components, log A_0, we give the following data (all data is in solar units or years): the final separation of the components log A_f, i.e., the separation at which the donor overflows its Roche lobe or becomes a WD itself; M_{df} gives the mass of the donor at the instant of RLOF (or the mass of the WD produced by the donor); M_{af} is the final mass of the accretor; T, t_{on}, and t_b give, respectively, the total lifetime of the system starting from the RG branch base, the total time spent in the "on state, and the time during which the accretion luminosity of the WD exceeds 10 $L_\odot$; ΔM_d, ΔM_a, and ΔH are, respectively, the amounts of mass lost by the donor, accreted by the WD, and converted into helium; finally, N_{nova} gives the number of hydrogen-burning flashes experienced by the accretor in a given system. (The spacing in A_0 in Table 1 was chosen in order to show how N_{nova} varies with A_0.) We estimate the rate of symbiotic novae in the Galaxy in § 3.5 and show that a single system is *not able* to produce hundreds of novae in its lifetime. In systems with log $(A_0/R_\odot) \lesssim 2.55$ the cool components of SSs are shell hydrogen burning RG. In systems with $2.55 \lesssim \log (A_0/R_\odot) \lesssim 2.65$ the donors pass through shell hydrogen burning, core helium burning, and a part of the double-shell (AGB) stages. In systems with log $(A_0/R_0) \lesssim 2.65$ the donors pass through a complete evolutionary route up to the formation of a WD. In systems with log $(A_0/R_\odot) \gtrsim 3.35$ the accretors do not accumulate enough mass even for one

TABLE 1

DEPENDENCE OF PARAMETERS OF NUCLEAR-POWERED SSs ON SEPARATION OF THE COMPONENTS

log A_0	log A_f	M_{df}	M_{af}	T	t_{on}	t_b	ΔM_d	ΔM_a	ΔH	N_{nova}
1.000......	1.000	0.999	0.650	5.73(8)	4.91(1)	0.00(0)	−7.93(−4)	4.26(−4)	1.78(−4)	2.17(0)
1.250......	1.251	0.998	0.650	8.19(8)	1.02(2)	0.00(0)	−2.41(−3)	8.82(−4)	2.12(−4)	4.15(0)
1.600......	1.602	0.993	0.650	9.47(8)	2.62(2)	0.00(0)	−7.48(−3)	2.27(−3)	2.24(−4)	1.07(1)
1.850......	1.854	0.894	0.652	9.83(8)	5.38(3)	0.00(0)	−1.61(−2)	5.21(−3)	1.61(−3)	2.39(1)
2.100......	2.107	0.966	0.656	9.99(8)	2.17(4)	0.00(0)	−3.44(−2)	1.10(−2)	5.91(−3)	5.11(1)
2.350......	2.364	0.930	0.666	1.01(9)	5.88(4)	1.76(6)	−7.01(−2)	2.28(−2)	1.58(−2)	1.08(2)
2.550......	2.575	0.872	0.681	1.01(9)	1.13(5)	1.76(6)	−1.28(−1)	4.01(−2)	3.08(−2)	1.95(2)
2.600......	2.666	0.677	0.735	1.03(9)	2.95(5)	3.66(6)	−3.23(−1)	1.10(−1)	8.52(−2)	5.13(2)
2.650......	2.726	0.650	0.729	1.03(9)	2.76(5)	2.33(6)	−3.50(−1)	1.03(−1)	7.92(−2)	3.01(2)
2.700......	2.788	0.612	0.733	1.03(9)	2.89(5)	2.10(6)	−3.88(−1)	1.12(−1)	8.35(−2)	1.82(2)
2.750......	2.847	0.612	0.707	1.03(9)	2.03(5)	1.62(6)	−3.88(−1)	8.09(−2)	5.70(−2)	1.28(2)
2.800......	2.904	0.612	0.685	1.03(9)	1.29(5)	1.10(6)	−3.88(−1)	4.87(−2)	3.53(−2)	9.29(1)
2.850......	2.960	0.612	0.699	1.03(9)	6.95(4)	6.23(5)	−3.88(−1)	2.50(−2)	1.87(−2)	7.32(1)
2.950......	3.064	0.612	0.655	1.03(9)	1.91(4)	3.62(5)	−3.88(−1)	7.44(−3)	5.16(−3)	3.45(1)
3.000......	3.115	0.612	0.653	1.03(9)	1.03(4)	2.02(5)	−3.88(−1)	4.32(−3)	2.84(−3)	1.99(1)
3.100......	3.216	0.612	0.651	1.03(9)	3.21(3)	0.00(0)	−3.88(−1)	1.64(−3)	9.87(−4)	7.56(0)
3.200......	3.316	0.612	0.650	1.03(9)	9.89(2)	0.00(0)	−3.88(−1)	7.23(−4)	4.67(−4)	3.32(0)
3.300......	3.416	0.612	0.650	1.03(9)	5.84(2)	0.00(0)	−3.88(−1)	3.58(−4)	2.81(−4)	1.66(0)
3.350......	3.466	0.612	0.650	1.03(9)	4.37(2)	0.00(0)	−3.88(−1)	2.60(−4)	2.35(−4)	1.21(0)

NOTE.—Numbers in parentheses give the powers of the decimal exponent.

hydrogen-burning flash and never have an accretion luminosity which is high enough for the generation of the symbiotic phenomenon. In systems with log $(A_0/R_\odot) \gtrsim 2.6$ and log $(A_0/R_\odot) \gtrsim 2.9$ the rate of accretion by the WDs is below the lower limit for stationary hydrogen burning. In these systems, only symbiotic novae are expected to exist. In systems within the rather narrow range of initial separations of $2.6 \lesssim$ log $(A_\odot/R_\odot) \lesssim 2.9$ both symbiotic novae and steady hydrogen burners may exist. Several important conclusions may be drawn from Table 1:

1. The hot components never accrete more than $\sim 30\%$ of the matter lost by the donor during its lifetime. Accretion is most efficient in systems in which the cool components reach the top of the RG branch and the AGB.
2. The average efficiency of conversion of accreted hydrogen into helium may reach $\sim 80\%$. Again, it is achieved by systems that are wide enough to allow the donors to evolve to the top of the RG branch or the AGB. In the systems which are most efficient in producing SSs, most of the hydrogen is burned in the steady regime or in mild flashes, in which the accretors are relatively compact and do not have strong stellar winds.
3. Successful symbiotic systems may experience several hundreds of nova-scale flashes during their lifetimes. The issue of the evolution of the abundances in symbiotic nova ejecta is discussed briefly in § 3.5.
4. The release of gravitational energy during accretion can feed the symbiotic phenomenon in about an order of magnitude more systems than hydrogen burning.
5. The range of orbital separations (and hence, periods) of symbiotic novae may be rather narrow.

In § 2.5 we described our assumption on the radial dependence of the velocity of the cool component's stellar wind (eq. [14]). It is evident from Figure 5 and Table 1 that the accretion of wind material and the production of the symbiotic phenomenon is most effective in systems in which the donors have tenuous envelopes and are close to their Roche lobes. For these stars $A/R_d \sim 2.5$ and the wind velocity at the position of the accretor is only ~ 0.2 of its terminal velocity. Since Bondi-Hoyle accretion depends sensitively on the wind velocity, this becomes an important factor in determining the efficiency of accretion.

In the absence of a complete theory of stellar winds from giants, we have to treat the radial dependence of the wind velocity as one of the parameters of our problem. To this end, we performed computations for a 1 $M_\odot$ + 0.65 $M_\odot$ system, under the assumption that the wind velocity at entering the Bondi-Hoyle cylinder is always equal to its terminal velocity. This assumption provides us with a lower bound for the efficiency of accretion in our binary system. The results of these calculations are shown in Figure 5 (*lower set of curves*) and in Table 2. The efficiency of accretion is reduced to less than $\sim 1\%$. The lower accretion rate results in a less efficient conversion of hydrogen into helium (see Fig. 2). As a result, the number of SSs that can be produced (even with the most favorable separations of the components) is reduced by about a factor 30, as compared with the standard case (Table 1). This means that there may exist a certain threshold for the efficiency of accretion, below which it would be impossible to explain the number of SSs in the Galaxy. Steady-burning WDs were not encountered in this model.

As another extreme case, we performed computations with a wind velocity law like the one suggested by Vogel (1991) for EG And. This empirical law gives very low velocities in the vicinity of the donor, and thus, it results in more efficient accretion. Nevertheless, the birthrate of SSs with CO accretors changes very little (~ 0.045 yr^{-1}). This is a consequence of the fact that the donors in this case overfill their Roche lobes in an earlier evolutionary stage. On the other hand, the birthrate of systems with He accretors almost doubles in this case (as a consequence of the close separations in [progenitor] WD + MS star systems after the CE phase). Thus, an application of this velocity law to all the systems results in a poorer agreement between the model predictions and the observations. This exercise demonstrates once more, however, the importance of the wind velocity law for population synthesis models.

3.3. *Birthrate and Number of Symbiotic Stars*

Table 3 shows the birthrates and numbers of possible progenitors of SSs and of stars that may be identified with SSs, under different assumptions on the parameters that enter the model. First, we give the total birthrate and number of MS binary stars with both components more massive than 0.8 $M_\odot$ and of pairs composed of a WD accompanied by a MS star with $M_{MS} \geq 0.8$ $M_\odot$. Table 3 shows that (1) almost all MS + MS systems with secondaries satisfying $M \geq 0.8$ $M_\odot$ survive the first mass exchange or CE episode and (2) only $\sim 25\%$ of these systems may become SSs.

As a "standard" case we consider the case in which the efficiency of accretion is taken as shown in Figure 2 and take into consideration the wind acceleration zone; ΔM_{crit} for igni-

TABLE 2

DEPENDENCE OF PARAMETERS OF NUCLEAR-POWERED SSs ON SEPARATION OF THE COMPONENTS
(in the absence of a wind acceleration zone)

log A_0	log A_f	M_{df}	M_{af}	T	t_{on}	t_b	ΔM_d	ΔM_a	ΔH	N_{nova}
2.150......	2.160	0.960	0.650	1.00(9)	2.87(1)	0.00(0)	−4.01(−2)	2.49(−4)	1.96(−4)	1.25(0)
2.400......	2.422	0.914	0.650	1.01(9)	2.68(2)	0.00(0)	−8.58(−2)	5.99(−4)	2.63(−4)	2.83(0)
2.550......	2.586	0.865	0.651	1.01(9)	1.12(3)	0.00(0)	−1.35(−1)	1.01(−3)	5.02(−4)	4.65(0)
2.600......	2.699	0.656	0.652	1.03(9)	5.05(3)	0.00(0)	−3.44(−1)	3.21(−3)	1.52(−3)	1.47(1)
2.650......	2.760	0.621	0.652	1.03(9)	6.34(3)	4.00(4)	−3.79(−1)	3.53(−3)	1.87(−3)	1.62(1)
2.700......	2.816	0.612	0.652	1.03(9)	6.22(3)	4.18(4)	−3.88(−1)	3.32(−3)	1.81(−3)	1.53(1)
2.850......	2.966	0.612	0.651	1.03(9)	2.75(3)	0.00(0)	−3.88(−1)	1.71(−3)	8.81(−4)	7.92(0)
3.000......	3.116	0.612	0.650	1.03(9)	1.06(3)	0.00(0)	−3.88(−1)	8.74(−4)	4.87(−4)	4.01(0)
3.150......	3.266	0.612	0.650	1.03(9)	5.53(2)	0.00(0)	−3.88(−1)	4.45(−4)	3.10(−4)	2.06(0)
3.300......	3.416	0.612	0.650	1.03(9)	3.54(2)	0.00(0)	−3.88(−1)	2.25(−4)	2.14(−4)	1.08(0)

NOTE.—Numbers in parentheses give the powers of the decimal exponent.

TABLE 3

Birthrates and Numbers of SSs and Supernova Rates under Different Assumptions on SS Model

Model (1)	ν (yr^{-1}) (2)	N (3)	R_{Sup} (yr^{-1}) (4)
MS + MS ($M \geq 0.8\ M_\odot$)	0.28	0.55×10^9	3.0×10^{-3}
WD + MS ($M_{MS} \geq 0.8\ M_\odot$)	0.27	0.93×10^9	...
Nuclear SS ($\alpha_{ce} = 1$)	0.073	3370	1.1×10^{-3}
Nuclear SS($\alpha_{ce} = 1$, $V_c \leq 12.0$ mag)	...	37	...
Nuclear SS ($\alpha_{ce} = 0.5$)	0.064	2600	5.4×10^{-3}
Nuclear SS ($\alpha = -1$)	0.050	1880	4.8×10^{-4}
Accretion SS	0.019	13000	9.0×10^{-4}
Accretion SS ($V_c \leq 12$ mag)	...	190	...
Nuclear SS ($\alpha_H = 1$)	0.073	3600	9.7×10^{-4}
Nuclear ($V_c \leq 12.0$ mag)	...	40	...
Nuclear SS (reduced α_H)	0.073	1400	0.0
Nuclear SS (3 times increased ΔM_{crit})	0.042	2140	1.0×10^{-3}
Nuclear SS (10 times increased ΔM_{crit})	0.021	1720	1.0×10^{-3}
Nuclear SS (10 times reduced ΔM_{crit})	0.12	4200	8.7×10^{-4}

Note.—In the first row of the table we give the SN Ia rate due to merger of double degenerates, as estimated by the population synthesis code with the same assumptions as in the standard nuclear model (see Yungelson et al. 1994). The factor α is the power of the distribution function of mass ratios of (q) in eq. (2); α_{ce} is the CE energy conversion efficiency factor in eq. (20); α_H is the efficiency of conversion of accreted hydrogen into helium (Fig. 2); ΔM_{crit} is the critical mass for hydrogen ignition in eq. (16).

tion of hydrogen is given by equation (16). The CE parameter is taken to be $\alpha_{ce} = 1$. As the "number" of SSs in Table 3 we give the product of the birthrate of systems which experience at least one hydrogen flash and their "on" time (envelope hydrogen burning time; given by eq. [21]). Clearly, this number should be regarded as a lower limit to the actual number of SSs with bright hot WDs in the Galaxy, because after hydrogen is exhausted, the star spends a longer time (by up to a factor 10) with $L \gtrsim 10\ L_\odot$, and it may be hot enough to excite the nebulosity. Out of the 3300 SSs, approximately 50% have He accretors and the remaining ones have CO/ONe accretors. The birthrates of the two groups are $\nu_{He} \approx 0.024\ yr^{-1}$ and $\nu_{CO} \approx 0.049\ yr^{-1}$. However, helium WDs have longer lifetimes in the "on" state because of the larger critical masses which are required for flashes and their lower luminosities. Consequently, the number of systems of the two kinds of WDs are comparable: 1540 and 1760 (see also Fig. 7 below).

The fact that a considerable fraction of the symbiotic stars in the standard model contain He accretors should not be considered unexpected. The formation of such systems was shown to occur in the very first studies on the evolution of close binaries (Kippenhahn, Kohl, & Weigert 1967; see Iben & Tutukov 1986; Tutukov & Yungelson 1992 for the latest reviews). Their existence is clearly indicated by the presence of a low-mass component in the observed distribution of WDs over masses (Bergeron, Saffer, & Liebert 1992).

In Figure 6*a* we show the contribution of systems of different ages to the present annual birthrate of SSs under the assumption of a constant star formation rate (equal to the present Galactic rate $\dot{M}_G$). (This histogram may also be treated as showing the variation of the SS formation rate in a galaxy with an "instantaneous" star formation burst and a mass equal to the product of $\dot{M}_G$ and the Hubble time.) It is evident that most SSs belong to the young and intermediate-age population. Therefore, they can contribute significantly to the UV emission from early-type galaxies only in the case of a strongly enhanced star formation rate in the first $\sim 5 \times 10^{10}$ yr of their history. Figure 6*b* shows the contribution of systems with WDs of different chemical composition and age (in other words, with different time intervals between the formation of the WD and the first hydrogen flash) to the present rate of SS formation. A significant fraction of the WDs are rather old ($\Delta t \gtrsim 3 \times 10^9$ yr). This means that equation (16) may significantly underestimate the masses of the hydrogen layers which have to be accumulated prior to the first flash, as was already discussed in § 2.6. We discuss possible implications of this fact below. The double-peaked distribution of helium WDs over Δt reflects the existence of two channels of formation: through quasi-conservative mass exchange and through CEs.

Symbiotic stars are usually detected due to the presence of features typical for ionized nebulae in the spectra of giants. This allows us to assume that the simplest selection effect that governs the observed sample of SSs is the visual magnitude of the cool component V_c. Inspection of the catalog of SSs (Kenyon 1986) reveals that the number of objects increases up to $V_c \approx 12.0$ mag. Therefore, we consider 12.0 mag as the limiting stellar magnitude of the sample and compute the number of model objects with $V_c \leq 12.0$ mag. For an estimate of V_c we used the core mass–luminosity and core mass–radius relations given by equations (3), (7), and (8) and bolometric corrections for giants from Allen (1973). We derive a number of 37 objects with $V_c \leq 12.0$ mag in our standard model. This number agrees reasonably well with the observed number of ~ 50 SSs with $V \leq 12.0$ mag.

The total number of SSs in the Galaxy has to be (in our standard model) at least 3300 and may be as high as $\sim 30{,}000$ if we take into account the cooling stage of hydrogen-burning WDs and the possibility of powering SSs by the release of gravitational energy due to accretion. At least ~ 40 of them have to be observed with $V \lesssim 12.0$ mag. There are about 200 observed SSs with $V \lesssim 15.0$ mag. Estimates of the total number of SSs in the Galaxy range from several thousands

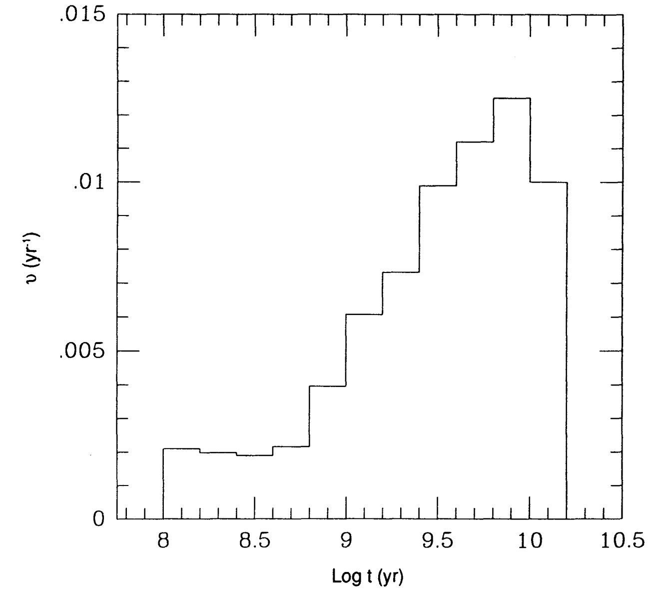

FIG. 6a

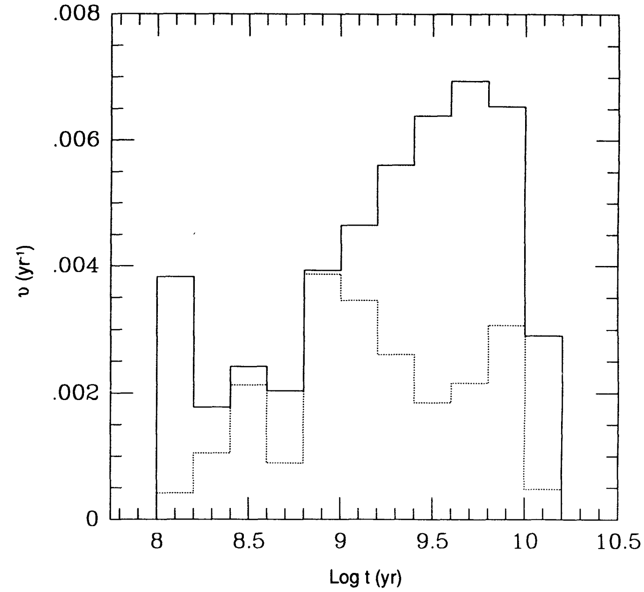

FIG. 6b

FIG. 6.—(*a*) Contribution of stars of different age to the present birthrate of SSs. The age of a symbiotic system is the sum of the lifetime of the present accretor (prior to becoming a WD) and the time necessary for the accumulation of the first critical ignition layer (or of the age of the WD at first flash). (*b*) Age of carbon-oxygen (*solid line*) and helium (*dotted line*) WDs at the instant of first flash.

(Boyarchuk 1970) to ~30,000 (Kenyon 1994) or up to ~300,000 (Munari & Renzini 1992), depending on the assumptions on the distance to typical SSs and on observational selection. The latter number is based on an ad hoc estimate of a 90% incompleteness factor of the sample of already detected SSs within 1 kpc of the Sun and, in our opinion, has to be treated with caution. Thus, our standard model provides a reasonable estimate of the number of SSs in the Galaxy. Of course, one has to keep in mind the fact that the criteria for classifying a star as a symbiotic system are rather uncertain, and the selection effects for them may be numerous. The numbers cited above include, apart from SSs with WD hot sources, also SSs with MS accretors and possibly other kinds of systems.

The total number of SSs in the standard model depends only weakly on the CE parameter α_{ce}. A reduction of α_{ce} to 0.5 reduces the birthrate and number of SSs by ~20% (Table 3).

Another parameter entering the model, which comes from statistical studies of binaries, is their distribution over the mass ratio of the components (eq. [1]). In order to test the sensitivity of the results to variations of the parameter of this distribution α, we performed a computation with $\alpha = -1$ for $0.1 \leq q \leq 1$. As the entries in Table 3 show, such a variation in the initial mass-ratio distribution does not cause any dramatic changes in the overall birthrates and number of SSs, which remain compatible with observational estimates. The decrease in ν and N as compared to the $\alpha = 0$ case are caused by the relative decrease in the number of stars with initial masses of the secondaries above $0.8\ M_\odot$.

As we have already mentioned, the lower limit for the luminosity of the hot components of SSs may be as low as $10\ L_\odot$. In Table 3, we give the birthrate and number of stars in which the luminosity due to accretion onto WDs is $\geq 10\ L_\odot$ (under "Accretion SS"; these systems correspond to entries in column t_b in Tables 1 and 2). The birthrate of SSs in the "accretion" model is lower than in the standard model, because, as Figure 5 and Table 1 show, the outbursts can occur at much lower accretion rates than the rate of $\sim 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$ which corresponds to $L \sim 10\ L_\odot$ for a typical $M_{\mathrm{WD}} \approx 0.65\ M_\odot$. Thus, only ~25% of the SSs can exhibit the symbiotic phenomenon continuously, oscillating between an "on" state due to hydrogen burning and a state of an accretion-powered source. In fact, the number of accretion SSs may be considered as a lower limit to the number of SSs in the Galaxy. However, the luminosities of the WDs in the accretion model are peaked around $10\ L_\odot$ (see Fig. 12), while the observed sample of SSs contains predominantly SSs of "nuclear" origin. The model "observed" sample of accretion SSs with $V_c \leq 12.0$ mag contains 190 stars. Again, we cannot consider this number as contradicting observations because most of these systems have relatively low luminosity hot components and will not be detected as SSs. Moreover, the selection by the stellar magnitude of the cool components is certainly not the only selection factor.

In Table 4, we show the distribution of SSs in the standard model over the accretion rate. The numbers in columns (2)–(4) indicate the numbers of stars in 0.5 dex–wide bins centered on the $\dot{M}$-values indicated in the first column. N_{tot} is the total number of systems containing RGs and WDs, in which the latter are accreting at the given rate. N_{ss} is the number of systems in which the WD experienced at least one hydrogen flash; i.e., it gives the total number of SSs that are in "on" and "off" states. N_{on} is the number of stars that accrete at the given rate and are in an "on" state. Table 4 clearly shows that almost all binary systems in which the WDs are able to accrete from the stellar wind of the donor with a rate $\dot{M} \gtrsim 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ are SSs in "on" or "off" states. Under the most favorable accretion rates, $t_{\mathrm{on}} \approx t_{\mathrm{off}}$ (cf. cols. [3] and [4]).

One of the important parameters entering the model is the efficiency of conversion of hydrogen into helium, α_{H}, which is based on a rather crude extrapolation. We performed a

TABLE 4

DISTRIBUTION OF RG + WD SYSTEMS OVER ACCRETION RATE

$\log \dot{M}$ ($M_\odot$ yr^{-1}) (1)	N_{tot} (2)	N_{ss} (3)	N_{on} (4)
−14.25......	0.224(7)	0.341(2)	9.954(−8)
−13.75......	0.180(7)	0.122(3)	0.214(−6)
−13.25......	0.148(7)	0.189(4)	0.174(−4)
−12.75......	0.126(7)	0.197(5)	0.122(−2)
−12.25......	0.110(7)	0.366(5)	0.339(−2)
−11.75......	0.990(6)	0.103(6)	0.423(−1)
−11.25......	0.853(6)	0.234(6)	0.284(0)
−10.75......	0.628(6)	0.239(6)	0.249(1)
−10.25......	0.377(6)	0.199(6)	0.276(2)
−9.75......	0.190(6)	0.109(6)	0.344(3)
−9.25......	0.881(5)	0.724(5)	0.770(3)
−8.75......	0.349(5)	0.310(5)	0.435(3)
−8.25......	0.150(5)	0.140(5)	0.334(3)
−7.75......	0.612(4)	0.587(4)	0.340(3)
−7.25......	0.241(4)	0.227(4)	0.377(3)
−6.75......	0.754(3)	0.749(3)	0.315(3)
−6.25......	0.477(3)	0.453(3)	0.261(3)
−5.75......	0.899(2)	0.846(2)	0.460(2)

numerical experiment in which we have set α_H = constant = 1 for all the stars which accrete at a rate below the maximum accretion rate for steady burning $\dot{M}_{st}^{max}$. This exercise can give us an upper limit to the number of SSs in the "on" state, because we neglect mass loss due to violent flashes, while accretion rates at which α_H is reduced strongly below 1 due to a stellar wind from extended envelopes, $\dot{M} \gtrsim \dot{M}_{st}^{max}$, are not achieved. As the results given in Table 3 show, this increase in the efficiency of hydrogen consumption influences the results only by ∼25%. This is due to the fact that accretion becomes more efficient as the star approaches its Roche lobe; i.e., most SSs are produced by systems with cold components close to the top of the RG branch or the AGB. At this evolutionary stage, $\dot{M} \gtrsim 10^{-7}$ $M_\odot$ yr^{-1}, and the accretion efficiency is higher than 10% (Fig. 5*a*). This results in a typical accretion rate $\gtrsim 10^{-8}$ $M_\odot$ yr^{-1} for which α_H is the highest (Fig. 2). This also means that in a significant fraction of the observed SSs, hydrogen burns stably or they experience rather mild flashes.

In our standard model, in the computation of α_H we assumed that the WD mass-loss rate is given by equation (19). According to this equation, mass loss is most efficient when the WD has its most extended envelope. However, the stellar wind may still be strong when the WD is close to the turnoff of its evolutionary track, at $T_{eff} \sim 10^5$ K. To model the possible effects of an enhanced stellar wind, we computed a model in which we assumed that for the same amount of accreted mass, the stellar wind removes 10 times more matter than in the standard case. This assumption corresponds to an efficiency of hydrogen consumption of $\alpha'_H = \alpha_H/(10 - 9\alpha_H)$, where the unprimed value is shown in Figure 2. In this model, the number of SSs is reduced approximately by a factor 2.5 compared to the standard model because of the shortening of the "on" stage, but it still does not contradict the observations [see the item "Nuclear SS (reduced α_H)" in Table 3].

An additional uncertainty in our model stems from the uncertainty in ΔM_{crit} (eqs. [16] and [17]) as discussed in § 2.6. The mass-radius relation (17) gives radii of zero-temperature degenerate configurations, while at the age of $\sim 10^8$–10^9 yr when the WDs begin to accrete in protosymbiotic systems, their radii may still be somewhat larger. Higher radii of the WDs may result in higher ΔM_{crit} as compared with equation (16). Regretably, existing data on the temperature dependence of ΔM_{crit} does not cover a sufficiently wide range for obtaining a quantitative relation. Therefore, we performed numerical experiments in which we assumed that ΔM_{crit} is 3 and 10 times higher than given by equation (16). An increase in ΔM_{crit} results in a drop in the birthrate and number of SSs, which is much more pronounced for systems with He accretors: $\nu_{He} = 0.008$ yr^{-1}, $\nu_{CO} = 0.034$ yr^{-1}, $N_{He} = 485$, $N_{CO} = 1655$ and $\nu_{He} = 0.001$ yr^{-1}, $\nu_{CO} = 0.020$ yr^{-1}, $N_{He} = 33$, $N_{CO} = 1687$ in the two cases, respectively (Table 3). The disappearance of most systems with He dwarfs is due to the fact that after the formation of the WDs, these systems have relatively moderate separations and the donors in them overfill their Roche lobes before they attain high mass-loss rates. Thus, the low-mass accretors in these systems do not have enough time to accumulate hydrogen even for a single flash. Wider systems with CO accretors are less influenced, and because the total amount of hydrogen available for burning does not change, the total number of SSs in experiments with an increased ΔM_{crit} is not in conflict with observations.

On the other hand, ΔM_{crit} may be also less than given by equation (16), e.g., because of a high $\dot{M}$. A dependence on $\dot{M}$, although not very strong, is known to exist (e.g., Iben 1982; Schwartzman et al. 1994; Kovetz & Prialnik 1994). To test the possible influence of this effect, we computed a model in which we assumed that the actual ΔM_{crit} is 10 times lower than given by equation (16). The results for this model are also listed in Table 3. As one could expect, the birthrate of SSs in the nuclear model is increased by ∼60%, while the number of SSs is increased only by ∼30%. The reasons for this are clear: we permit explosions at lower amounts of accumulated matter, while the total amount of available matter gain does not change. The rate of novae in this model is obviously increased (by a factor of 10 as compared to the standard model). However, the number of actual SSs in this model again is not in conflict with observational estimates.

3.4. *Distributions of Symbiotic Stars over Different Parameters*

Orbital periods and masses of the hot components.—In Figure 7*a* we show the distribution of nuclear SSs in the standard model over their orbital periods, and in Figure 8*a* the distribution of their hot components' masses. These figures refer to the total sample of Galactic SSs. In Figures 7*b* and 8*b* we show the distributions over P and M_{hot} for the model "observed" samples (with $V_c \leq 12.0$ mag). The two maxima in both distributions are due to systems with He accretors ($M \leq 0.5$ $M_\odot$) and CO accretors ($M \geq 0.5$ $M_\odot$), respectively, as can be seen in Figure 9, in which we display the "correlation" between the periods and accretor masses in model SSs. Systems with He WDs belong to the short-period group ($P \leq 250$ days). As Figures 7 and 9 show, our model allows for the existence of numerous SSs with rather short (∼60–100 days) orbital periods. Among the observed SSs, the shortest known period is ∼200 days (TX CVn) and the maximum of the distribution is close to 1000 days. A question that needs to be asked is whether the model short-period systems can indeed be identified with SSs. Possible uncertainties in the radii given by equation (3) are not sufficient to shift the maximum of the distribution for He dwarf systems to ∼1000 days. A possible selection effect acting against the discovery of short-period systems is that the RGs may be hard to find. For example, in T

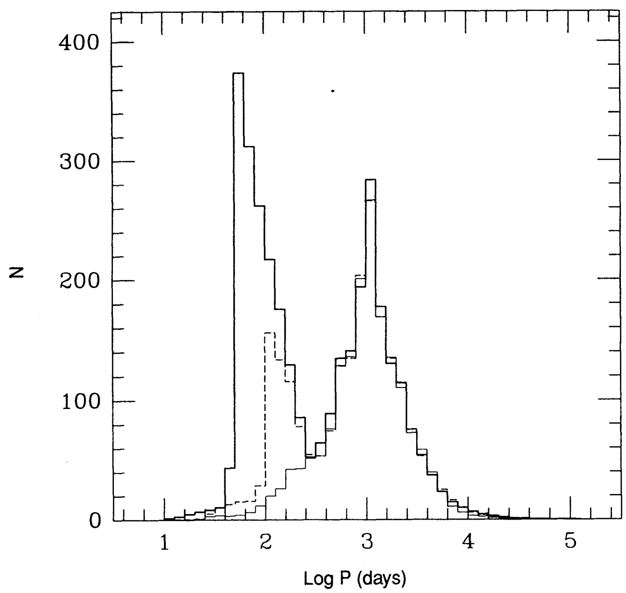

FIG. 7a

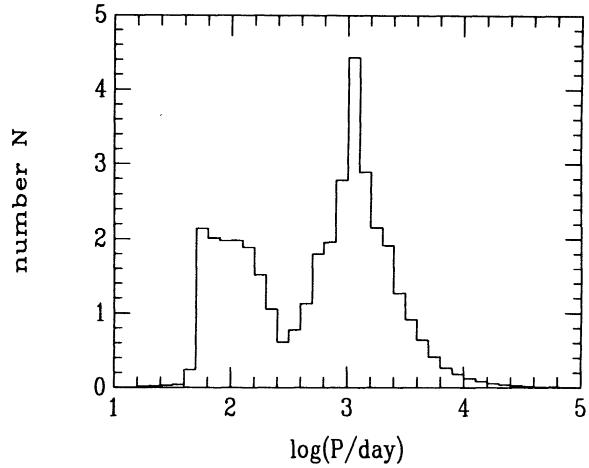

FIG. 7b

FIG. 7.—Number distribution of SSs as a function of the orbital periods for the standard (nuclear) model. (*a*) Total sample: *thick solid line*, standard model; *dotted line and thin solid line*, samples with 3 and 10 times increased ΔM_{crit}, respectively. (*b*) Subsample of standard model with donors brighter than $V_c = 12.0$ mag.

CrB (with an orbital period of 230 days), the RG which fills its Roche lobe is an M3 giant. For periods of $\sim$100 days, however, the RGs would fill their Roche lobes at a much earlier spectral type. If the spectral type is earlier than K4, no TiO absorption would be visible and it would be hard to classify the system observationally as a SS. Some of these systems may look like BQ[ ] stars or something different altogether.

On the other hand, we have to remember that in defining a system as "symbiotic" we rely essentially on whether the accreted mass is equal to ΔM_{crit} (eq. [16]). If we reject, for example, systems with helium accretors, then the remaining ones, with CO accretors, have orbital periods just in the observed range, between $\sim$200 (TX CVn) and $\sim$16,000 days (R Aqr), with a peak close to 1000 days. As we mentioned in § 2.6, it is quite possible that ΔM_{crit} actually may be higher than given by equation (16). In § 3.3 (see also Table 3) we showed that increasing ΔM_{crit} results in the disappearance of systems with He WDs from model samples. In Figure 7*a* we show the distributions over orbital periods for samples generated under the assumption that ΔM_{crit} is larger by factors of 3 and 10. As we can see, this ad hoc assumption solves the problem of short-period SSs. However, since such an assumption still needs to be justified by model calculations of accreting WDs; in the rest of the paper we do not exclude systems with He WDs from the discussion.

In Figures 10 and 11 we show the distributions over *P* and M_{hot} for the SSs in which the luminosity of the hot components (at the level $L \geq 10\ L_\odot$) is supported entirely by accretion (the accretion model). This sample does not contain short-period systems with helium accretors. This can be understood as a consequence of several factors: the lower efficiency of generation of luminosity by accretion, due to the lower masses and higher radii of He WDs as compared to the CO ones; lower rates of mass loss by the cool components, which have to have

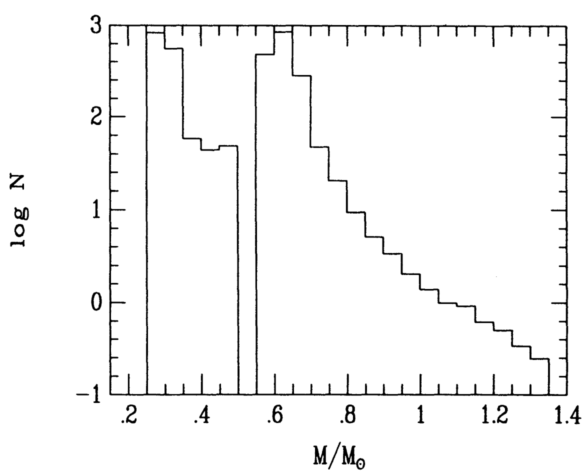

FIG. 8a

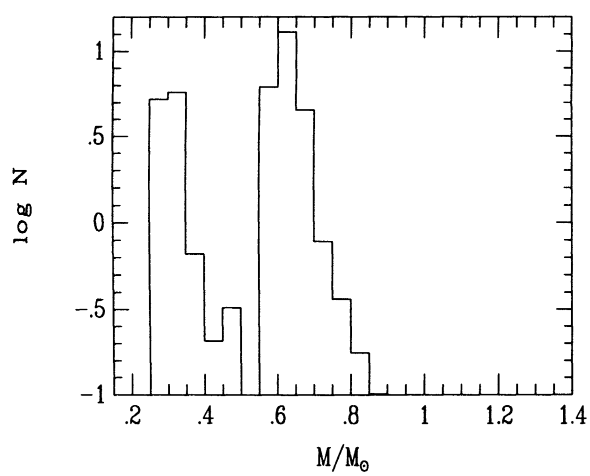

FIG. 8b

FIG. 8.—Number distribution of SSs as a function of the WD mass for the standard (nuclear) model. (*a*) Total sample. (*b*) Subsample with donors brighter than $V_c = 12.0$ mag.

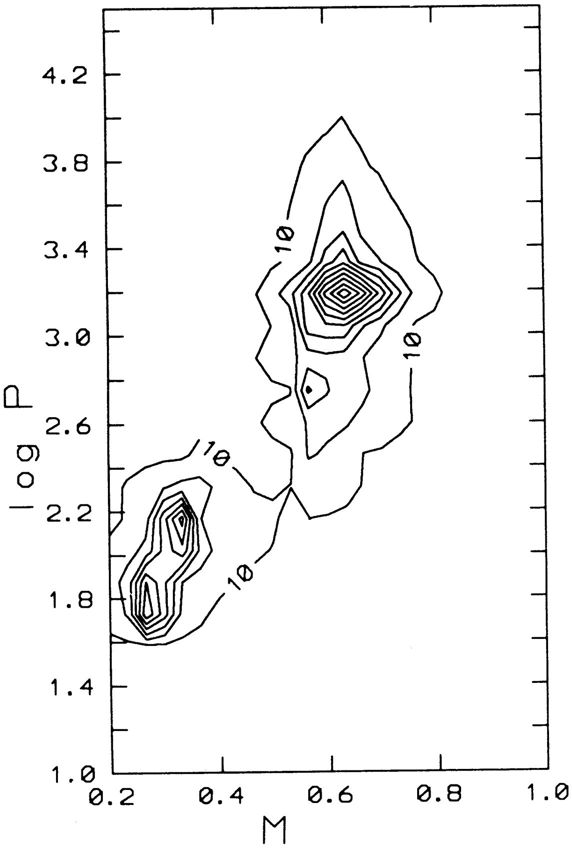

FIG. 9.—Distribution of SSs in the standard model in the (WD mass M, orbital period P)-plane. P is in days, M in $M_{\odot}$. Contour lines display the formation rate $\partial^2 \nu/(\partial \log M \, \partial \log P)$ with a step equal to 40.

a mass lower than 2.5 $M_{\odot}$. Also, because of the short orbital periods obtained after the CE phase, donors in systems with He accretors overfill their Roche lobes and terminate the SS stage before attaining a high $\dot{M}$. As a result, if there exist SSs with accretion-fed WDs, all of them have to contain CO or ONe WDs.

Unfortunately, the masses of the components in symbiotic systems are very poorly known. Both the spectroscopic orbits and the radial velocity data for SSs are not very reliable (see, e.g., discussions in Boyarchuk 1970, 1984; Kenyon 1986; Mürset & Nussbaumer 1994). A direct determination exists for the eclipsing binary EG And (Vogel, Nussbaumer, & Monier 1992). In this system, $M_h = 0.4 \pm 0.1$ $M_{\odot}$. This value of M_h suggests the presence of a helium WD in the "cooling" stage in this system.

Luminosities of the hot components.—Figure 12 shows the distribution of the model SSs over the luminosities of the hot components. Both "nuclear-fed" and "accretion-fed" samples are shown. This figure indicates that one can expect the existence of three distinct groups of SSs: nuclear-fed, which have high and intermediate luminosities (with CO and He WDs, respectively), and low-luminosity SSs (with CO dwarfs fed by accretion). The relative frequency of the groups, as indicated in Figure 12, may be misleading because, as we have already mentioned, the burning stage in the nuclear-fed systems is followed by a longer cooling stage (of course, still-unknown selection effects may also act).

The masses of the hot components may be estimated from their luminosities under the assumption that they are in the plateau stage and equation (18) is valid. This assumption is probably true for the WDs in symbiotic novae, and it gives a lower limit for the mass of the WDs in nuclear SSs that were not observed as novae (if they are in the cooling stage). For the accretion-fed SSs one may infer the accretion rate from the luminosity. One has to note that there exists a certain discrepancy in the data on the luminosities of the WDs (compare, e.g., the data of Mürset et al. 1991; Mürset & Nussbaumer 1994; Mikolajewska & Kenyon 1992). This discrepancy may be attributed to different assumptions used for the derivation of effective temperatures and bolometric corrections (Kenyon et al. 1993b). However, the observational data show hints of the existence of three groups of hot components: with $L_h \gtrsim 10{,}000$ $L_{\odot}$, $L_h \lesssim 2000$ $L_{\odot}$, and $L_h \sim L_{\odot}$. For nuclear SSs this may be translated into two groups in WD mass, namely, high-mass CO WDs and low-mass He ones (see Figs. 8 and 12), and a group of cooling objects. For example, in the symbiotic nova AG Peg, a determination of the mass from its peak luminosity (~ 3400 $L_{\odot}$; Mürset et al. 1991) by means of equation (18) gives $M_h \approx 0.33$ $M_{\odot}$, which is typical for He WDs. For V1016 Cyg, with $L \approx 20{,}000$ $L_{\odot}$, equation (18) gives $M_h \approx 1.1$ $M_{\odot}$.

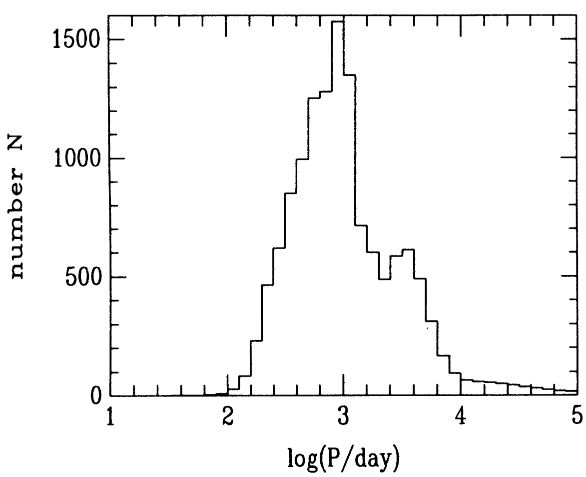

FIG. 10*a*

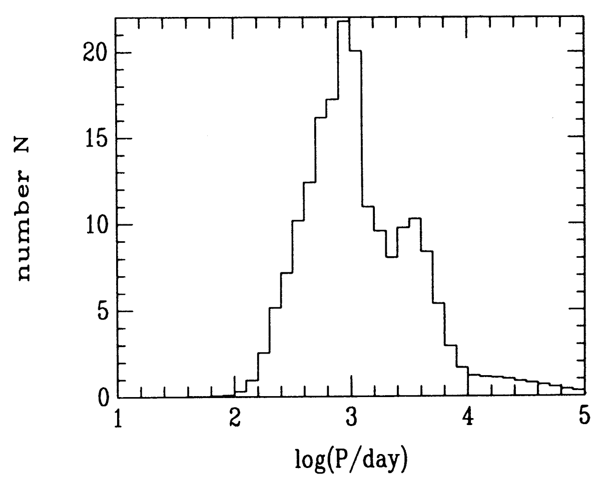

FIG. 10*b*

FIG. 10.—Number distribution of SSs as a function of the orbital period for the accretion model. (*a*) Total sample. (*b*) Subsample with donors brighter than $V_c = 12.0$ mag.

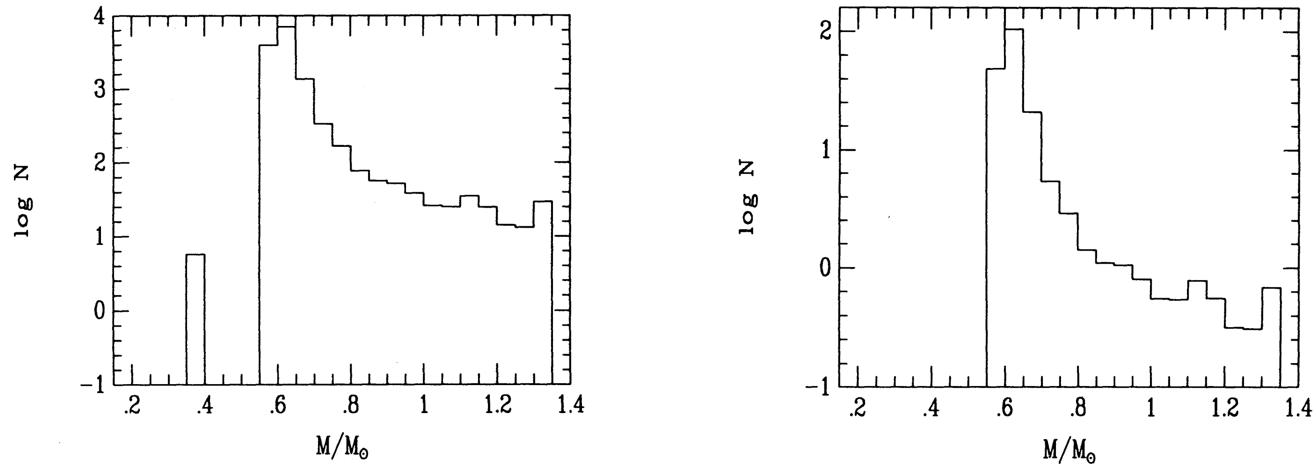

FIG. 11a

FIG. 11b

FIG. 11.—Number distribution of SSs as a function of the WD mass for the accretion model. (*a*) Total sample. (*b*) Subsample with donors brighter than $V_c = 12.0$ mag.

Properties of the donors.—In Figure 13 we show the distributions of the SSs over the masses of the cool components for the standard model and for the accretion model (for the total sample and for the sample limited by selection [$V_c \leq 12.0$ mag]). In the standard model, the group of donors which is peaked at log $(M_c/M_\odot) \approx -0.15$ has CO companions, while the more massive donors [the group peaked at log $(M_c/M_\odot) \approx 0.25$] have He companions. This difference is due to several circumstances. A considerable portion of the SSs with He donors evolve through the quasi-conservative "Algol" channel (scenario II in Fig. 3), which increases the mass of the possible donor and the separation of the components, thus allowing for a longer SS stage with a higher $\dot{M}$. In the systems with CO WDs, which form through CEs or are quasi-wide (scenarios I and III in Fig. 3), the dominance of low-mass donors is a reflection of the power-law initial mass function of the primaries (eq. [1]). The distribution of SSs over $q = M_c/M_h$ for the standard model and the accretion model is shown in Figure 14. Again, the double-peaked distribution in the standard model reflects the existence of systems with CO accretors ($q \sim 1$–2) and He accretors ($q \gtrsim 4$).

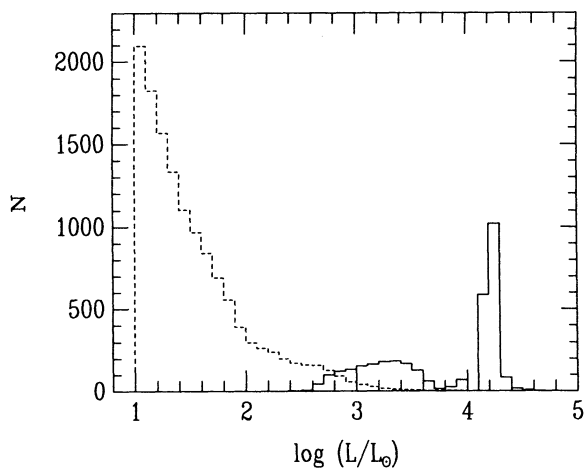

FIG. 12.—Number distribution of SSs as a function of the luminosity of the WD for the standard (*solid line*) and accretion (*dashed line*) models.

As for the hot components, data on the masses of the cool stars are scarce. For EG And Vogel et al. (1992) suggest $M_c \approx$ 1–2 $M_\odot$ (for $M_h = 0.4 \pm 0.1$ $M_\odot$). EG And is the SS with the lowest known luminosity ($L \sim 10$ $L_\odot$); thus, it is more probably a nuclear SS in the cooling interflash state than an accretion-fed one. However, our model for both types of systems allows for its rare combination of parameters. For a few other systems, an estimate for M_c may be inferred from the mass ratio of the components, typically $M_c/M_h \sim 3$. This implies that M_c must be between 1 and 2 $M_\odot$. If the cool components are normal late M giants, their masses have to be close to or slightly above 1 $M_\odot$. This data also agrees with our model.

In Figure 15 we show the distribution of SSs over the mass-loss rates from the cool components, along with the distribution over $\dot{M}_c$ for the observed SSs (Seaquist, Krogulec, & Taylor 1993). In nuclear SSs, the range in $\dot{M}_c$ is wider than in accretion SSs (as was demonstrated in Tables 1 and 2) because the WDs are able to accumulate the critical mass for explosion earlier than the accretion luminosity attains the value of 10 $L_\odot$. From the high mass-loss rate side, the limits are similar because they are simply the highest mass-loss rates accessible by supergiants. The low- and high-$\dot{M}$ groups are due to RGs and AGB stars. The former are mainly components of systems with He accretors. If systems with He accretors are excluded from the sample (e.g., due to a higher $\Delta M_{\rm crit}$), the peak at $\dot{M} \geq 10^{-8}$ $M_\odot$ yr^{-1} disappears, while the number of systems with $10^{-7.5} \leq \dot{M} \leq 10^{-8}$ $M_\odot$ yr^{-1} is reduced by $\sim$50%. In the magnitude-limited samples for both the nuclear and accretion models, the low-$\dot{M}$ group is reduced relatively to the other because of the lower luminosities of RGs. The model predictions are in very good agreement with the observed $\dot{M}_c$.

Figure 16 shows the distribution of the donors over the ratio of their radii to the radii of their respective Roche lobes $R/R_{\rm cr}$, for the magnitude-limited samples (they do not differ qualitatively from the distributions for the whole sample). As one could expect (it was already indicated by Fig. 5), the most efficient producers of the symbiotic phenomenon have donors close to their tidal lobes, while a small fraction of the stars that are very far from their tidal lobes are also able to produce SSs. It is difficult to compare Figure 16 with the observational data directly because the notion of a stellar radius is uncertain for

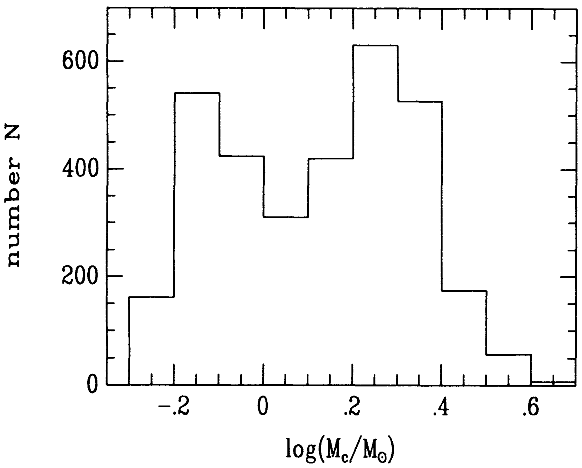

FIG. 13*a*

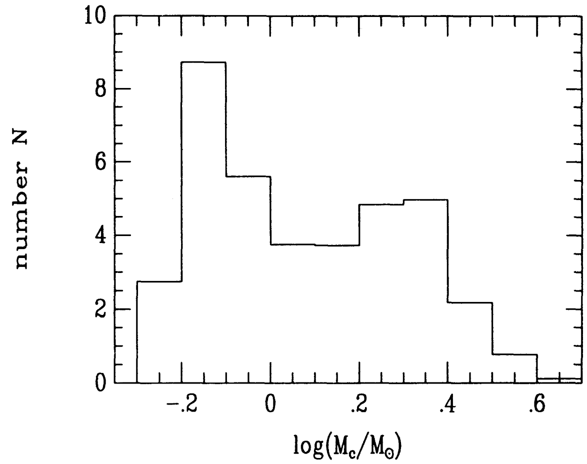

FIG. 13*b*

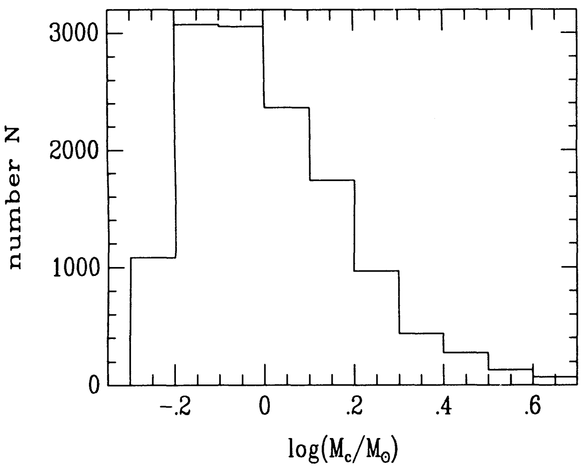

FIG. 13*c*

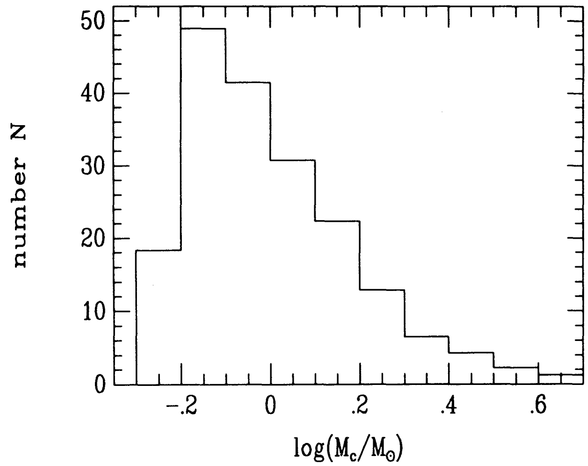

FIG. 13*d*

FIG. 13.—Number distribution of SSs as a function of the donor mass for (*a*) the total sample in the standard (nuclear) model, (*b*) the subsample of the standard model limited to $V_c \leq 12.0$ mag, (*c*) the total sample in the accretion model, and (*d*) the subsample of the accretion model limited to $V_c \leq 12.0$ mag.

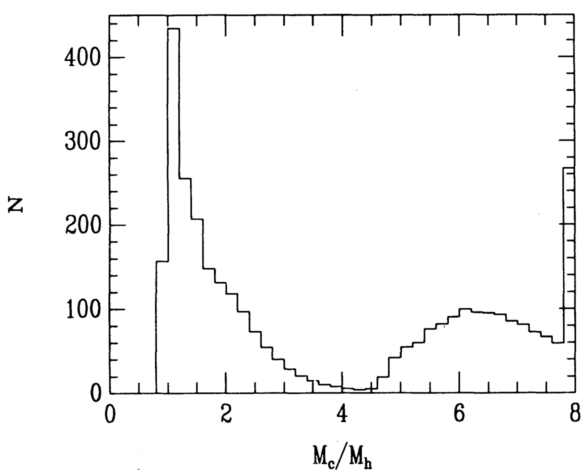

FIG. 14*a*

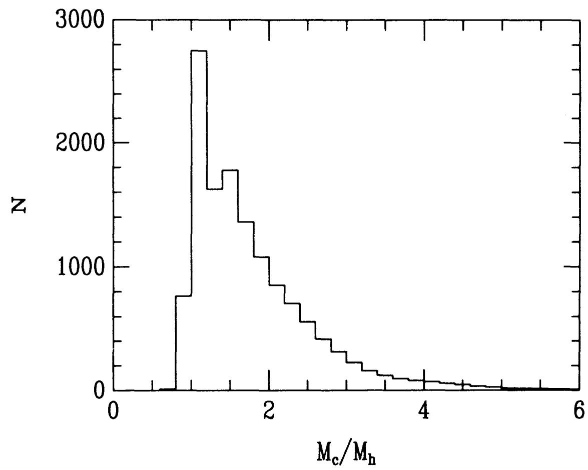

FIG. 14*b*

FIG. 14.—Number distribution of SSs as a function of the mass ratio of the cool and hot components for the total samples in (*a*) the standard (nuclear) model and (*b*) the accretion model.

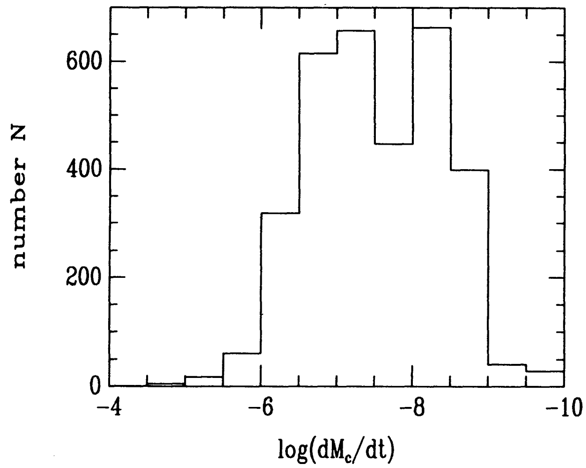

FIG. 15*a*

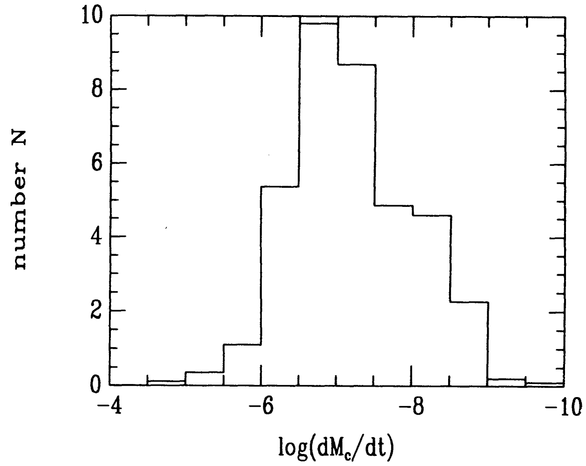

FIG. 15*b*

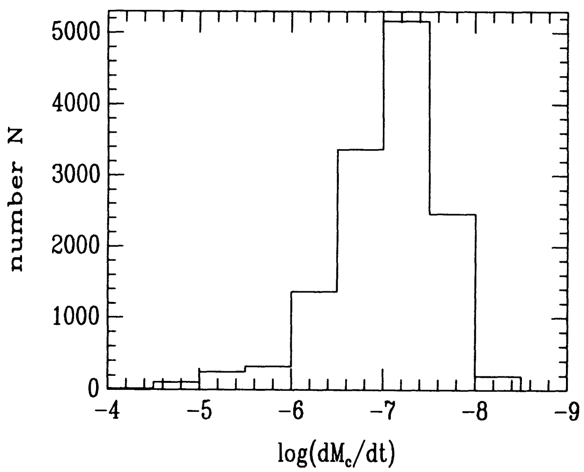

FIG. 15*c*

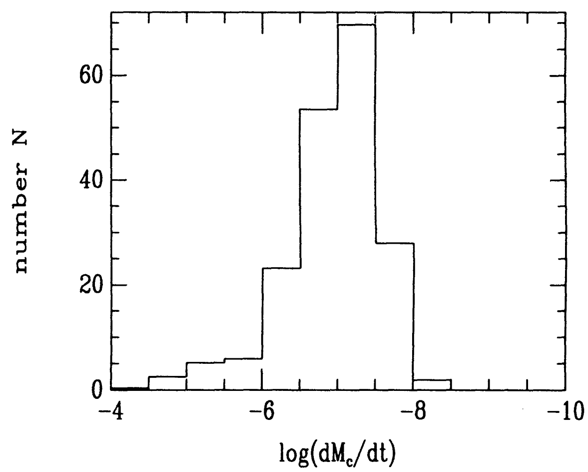

FIG. 15*d*

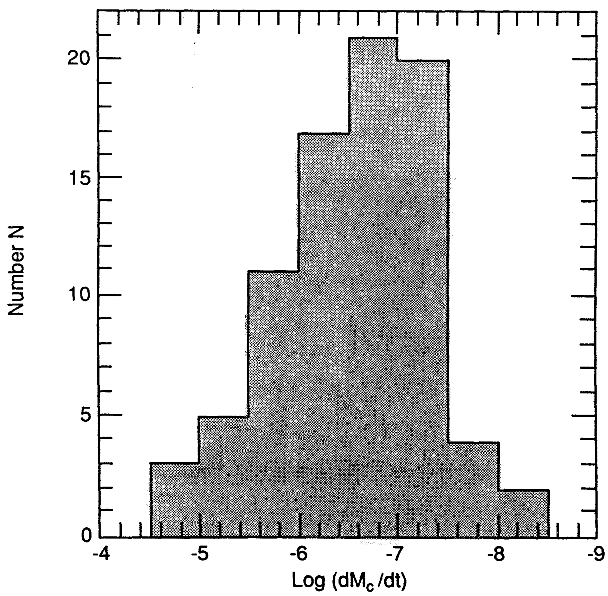

FIG. 15*e*

FIG. 15.—Number distribution of SSs as a function of the donor mass-loss rate for (*a*) the total sample in the standard (nuclear) model, (*b*) the subsample of standard model limited to $V_c \leq 12.0$ mag, (*c*) the total sample in the accretion model, and (*d*) the subsample of the accretion model limited to $V_c \leq 12.0$ mag. (*e*) Observed distribution from Seaquist et al. (1993).

stars in the latest stages of their evolution: these stars have nonstationary tenuous "atmospheres" above their formally (at optical depth $\tau = \frac{2}{3}$) defined photospheric radii. Stars in the TP-AGB stage may quasi-periodically approach the Roche lobe during thermal pulses, when their radii may be significantly larger than those given by equations (7) and (8). In any case, the closer the star is to RLOF, the higher is its mass-loss rate, and thus the more likely it is to produce a SS.

If most of the SSs really have donors that are close to their Roche lobes, then one can use the known orbital periods for estimates of stellar radii, since for Roche lobe filling stars $R \propto P^{2/3}M^{1/3}$, while M may be inferred from the spectral type of the donor.

Figure 17 shows the distribution over the absolute visual stellar magnitude of the donors (for the magnitude-limited samples) for the nuclear and accretion models. These magnitudes are higher than the $M_v \approx 0.0$ to -0.5 mag that could be assigned to the cool components on the basis of their optical spectral types (being usually early M III; e.g., Boyarchuk 1984). However, Kenyon & Gallagher (1983) showed that in some SSs the spectra of the donors have much stronger CO bands than would have been expected on the basis of their spectral types. The strength of the CO band increases with luminosity, and thus, the donors of SSs may be brighter than common field giants.

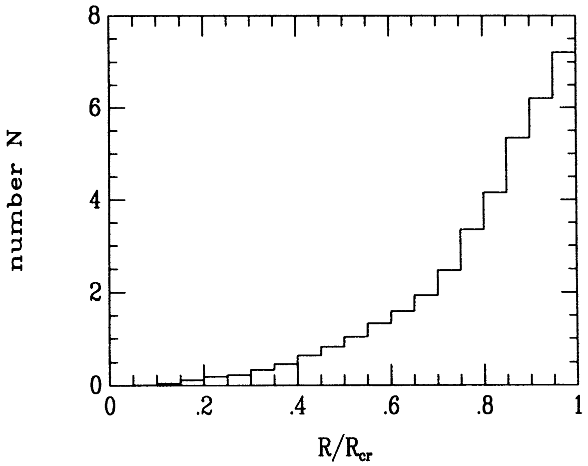

FIG. 16a

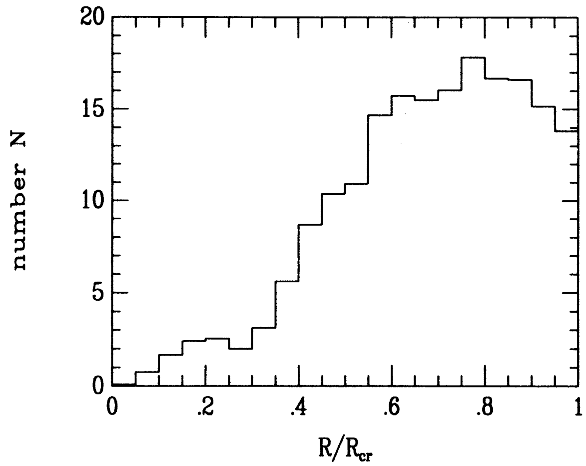

FIG. 16b

FIG. 16.—Number distribution of SSs as a function of the ratio of the donor radius and Roche lobe radius for the subsamples of (*a*) the standard and (*b*) the accretion models limited to $V_c \leq 12.0$ mag. The distributions for the subsamples do not differ qualitatively from the distributions for total samples.

3.5. "*Stationary*" *Symbiotic Stars and Symbiotic Novae*

As we have mentioned in § 3.3, our model allows us to estimate the relation between steady hydrogen burning SSs and symbiotic novae. The total rate of symbiotic novae in the Galaxy in our standard model is $R_{\rm Nova} \approx 1.6 \text{ yr}^{-1}$. A comparison with the total birthrate of SSs gives an average number of nova outbursts experienced by a SS to be of order 20. However, if we address the question of which stars explode as novae in symbiotic systems, by means of selecting systems with WDs accreting below the "steady-burning" limit given by equation (21), we discover the following. Helium accretors experience during their lifetime an average of about four flashes. The first occurs when H burning commences after the first $\Delta M_{\rm crit}$ is accumulated. The "on" time is long in this case. After the first flash, He WDs accrete steadily or very slowly accumulate the next critical H layer and form a population of low-luminosity SSs. Out of the total rate of ~ 1.6 novae yr^{-1}, 1.5 are due to CO(ONe) WDs, while only 0.1 are due to the helium accretors. AG Peg may be a rare example of the latter.

Accreting CO(ONe) WDs experience on the average ~ 30 nova-scale flashes during their lifetimes as SSs. Among the CO(ONe) WDs, 60% are in a postnova state and 40% are burning H steadily. Figure 18 shows the distributions over the length of the "on" stages after particular flashes, $\tau_{\rm on} \approx 6.9 \times 10^{10} \alpha_{\rm H} \Delta M_{\rm crit}/L$ yr, for the postnovae and all SSs (for steady-burners we consider their entire lifetime as a single flash). The two distinct groups are again due to CO and He accretors.

The outbursts last longer than 30 yr. In comparing our "on" times with observations, one has to bear in mind the fact that observed flashes of symbiotic novae have two phases: a relatively short peak in which the radius of the flaring star reaches $\sim 100\ R_\odot$ and a long decline phase of still-uncertain (because in no symbiotic novae did the hot component return to the initial WD size) duration in which the radius of the star is several tens of $R_\odot$ (see Mürset et al. 1991; and Mürset & Nussbaumer 1994). Therefore, the disagreement with the so-called durations of symbiotic novae is possibly only apparent. Moreover, long-lasting outbursts may have a "fine structure"

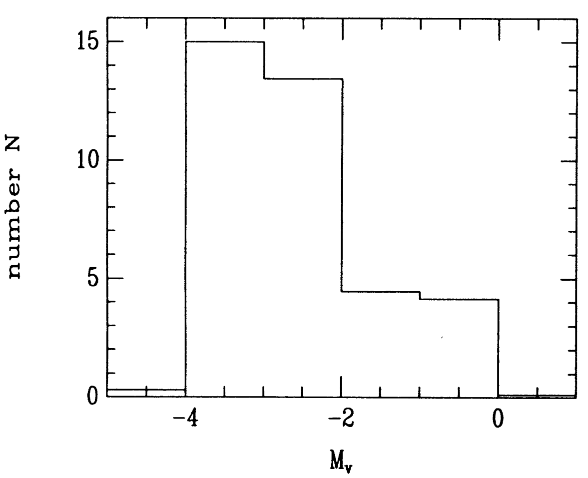

FIG. 17a

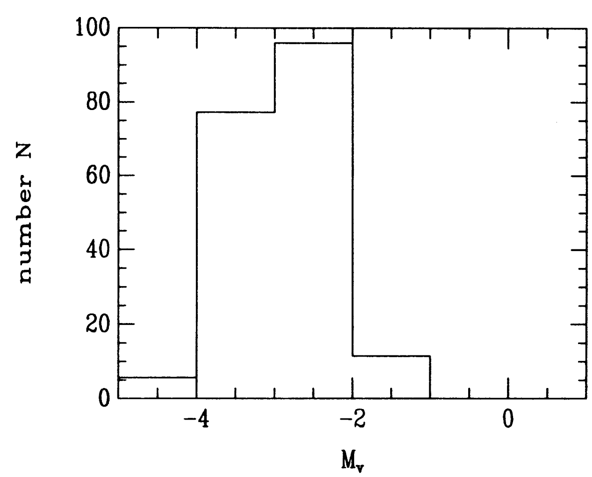

FIG. 17b

FIG. 17.—Number distribution of SSs as a function of the absolute visual magnitude of the donor for (*a*) the subsample of the standard model limited to $V_c \leq 12.0$ mag and (*b*) the subsample of the accretion model limited to $V_c \leq 12.0$ mag.

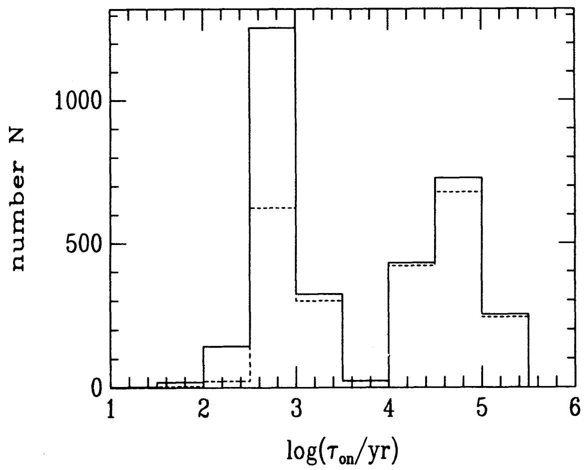

FIG. 18.—Number distribution of SSs in the standard model as a function of the " on " times: *solid line*, all SSs; *dashed line*, post–symbiotic novae.

on a timescale of years or tens of years (e.g., Shara et al. 1993; Sion & Ready 1992). During these "suboutbursts" the luminosity of the star may vary by an order of magnitude; thus, it may mimic the symbiotic behavior. The average total length of the outbursts is ~ 1000 yr in the first group and $-50{,}000$ yr in the second; therefore, observationally, postoutburst stars will be indistinguishable from stationary burners.

In the estimates of the " on " times we depend on equation (16) for ΔM_{crit}, which may not be very accurate. The inaccuracy in ΔM_{crit} may change the estimates of the total number of nova outbursts and of τ_{on}, but it does not seriously affect the total number of SSs, which in our model is a function of the accreted mass and the accretion rate (see § 3.3 and Table 3).

Among the observed SSs, symbiotic novae (with the exception of AG Peg and possibly PU Vul) have typically high ($\gtrsim 10{,}000\ L_\odot$) luminosities, while ordinary SSs are much dimmer ($\lesssim 1000\ L_\odot$). This does not contradict our finding that 40% of the CO WDs are burning H stably. Some of the steady-burning WDs may mimic novae due to variations in the accretion rate (e.g., BF Cyg). It is also possible that if a steadily burning WD has a strong stellar wind, the latter may temporarily prevent accretion. This may result in quasi-periodic variations in the WD's luminosity and radius, which again mimic nova-type variability. According to Paczyński & Rudak (1980), WDs which accrete with a rate which is variable, but still within the limits of $\dot{M}$ for stationary H burning, will experience large variations in their radii, at an almost constant bolometric luminosity. This also may result in the large variability of SSs.

Our model predicts the possibility of symbiotic novae in short-period systems. This is not observed, or these systems are not identified as SSs for reasons which are not fully understood. The solution may be in the exclusion of the He accretors from the sample. The reason for exclusion may be the strong dependence of ΔM_{crit} on the WD mass and age, as discussed in §§ 3.3 and 3.4. This point may be clarified by computations of models of accreting WDs of a very low mass (0.25–0.35 $M_\odot$).

An important peculiarity of symbiotic novae is their chemical composition. To the best of our knowledge, all seven known SSs apparently have solar abundances (although nitrogen enrichments may have been seen in PU Vul; Vogel & Nussbaumer 1992), while in classical novae all cases with more or less reliable abundance determinations show enrichments in heavy elements, helium, or both (Livio & Truran 1994). The clues to this difference may be in the sequence of explosive events (Iben & Tutukov 1994; Iben, Fujimoto, & MacDonald 1992) and in the properties of symbiotic systems. A " newborn " WD in a future SS retains at its surface a thin shell of helium ($\sim 0.001\ M_\odot$). This layer prevents initially the interaction of the exploding hydrogen shell with the CO (or ONe) core; therefore, at least initially, ejecta will have nearly solar abundances. One may image several possibilities to preserve this layer and to avoid enrichments in heavy elements.

1. As detailed calculations show, for a relatively massive WD, despite gradual erosion of the layer by consequent explosions, solar abundances are retained for at least ~ 100 outbursts (Prialnik & Livio 1995). However, the most efficient producers of symbiotic novae, according to our model, experience more explosions, up to ~ 500 (Table 1). Thus, high metallicity in most ejecta could be expected. It is possible, that the calculations overestimate the rate of erosion. If the WDs in symbiotic binaries are typically of lower masses than those in classical novae (e.g., Ritter et al. 1991), then detailed calculations which include diffusion-induced convection tend to produce lower enrichments (e.g., Schwartzman et al. 1994), and, in general, the erosion of the helium layer occurs more slowly (Prialnik & Livio 1994).

2. If the helium layer is not eroded and its accumulation is possible, then He ignites when at least 0.1–0.2 $M_\odot$ of it is accumulated (Tutukov & Khokhlov 1992; Weaver & Woosley 1994). Helium burning, if it occurs at all and if it does not disrupt the WD, results in mixing of the surface layers and the possible loss of the whole He barrier. Therefore post–He-flash novae may have ejecta enriched in heavy elements. The ratio of metal-rich to normal novae is then equal to the ratio of novae occurring after He flashes to the number of explosions that occur prior to them. In our standard model, which assumes that all the CO WDs which are more massive than 0.6 $M_\odot$ experience helium flashes after the accretion of 0.1 $M_\odot$ of helium, this ratio is ~ 700. If we assume (after Tutukov & Khokhlov 1992) that for 0.6–0.8 $M_\odot$ mild He burning is possible, this ratio will be much lower. If helium ignition results in a double detonation with a disruption of the WD for all the WDs more massive than 0.6 $M_\odot$ (Weaver & Woosley 1994), then metal-rich symbiotic novae are excluded completely.

3. Shear mixing may play a role in generating heavy-element enrichments (e.g., MacDonald 1983; Sparks & Kutter 1987; Livio & Truran 1987; Fujimoto 1988). Then the absence of accretion disks in SSs may result in no enrichments.

This issue will have to be further clarified by accurate abundance determinations in symbiotic novae.

The rate of symbiotic novae of ~ 1.6 yr^{-1} can be compared with the total rate of classical novae in the Galaxy of $R_{CN} \sim 20$ yr^{-1} (Della Valle & Livio 1994). An inspection of the Reference Catalogue of Novae (Duerbeck 1987) shows that the number of discovered classical novae begins to decline rapidly from 8.0 mag onward. If we consider 8.0 mag as the limiting stellar magnitude of the sample, we arrive at a rate of 0.09 yr^{-1} symbiotic novae within this magnitude range. The latter number is in agreement with the fact that seven symbiotic novae have been discovered during the last 100 yr. We have to note that (1) our number may be an underestimate and (2) that certainly, many symbiotic novae were missed because, in fact, a systematic study of these objects started no more than 50 years ago. We can also obtain a rate of $\sim 0.1\ \mathrm{yr}^{-1}$ if we take 4 kpc as a limiting distance for the detected Galactic symbiotic novae (see Table 4 in Mürset & Nussbaumer 1994) (for a Galactic radius 15 kpc and a thickness of the Galactic disk of 400 pc). The model predicts that currently we have to observe in the Galaxy 14 symbiotic novae in the postflash state with $V \lesssim 8.0$ mag. In fact, there are six observed symbiotic novae with such magnitudes (see Fig. 3 in Mürset & Nussbaumer 1994). We consider this agreement as almost surprisingly good.

3.6. *Symbiotic Stars and Supernovae Ia*

All the models for SN's Ia involve accreting WDs. Within this general framework, there exist models in which the WD is driven to the Chandrasekhar limit and models which involve sub-Chandrasekhar-mass WDs. The Chandrasekhar-mass models can be further subidivided into single-degenerate models (Whelan & Iben 1973), in which the WD accretes mass from a nondegenerate companion, and double-degenerate models (Iben & Tutukov 1984a; Webbink 1984), in which two WDs (with a total mass exceeding the critical mass) merge. Because of the fact that searches for double-degenerate systems have yielded rather negative results (see, however, discussion of this issue by Yungelson et al. 1994), the interest in single-degenerate scenarios has increased considerably. In this context, Munari & Renzini (1992) have revived the interest in symbiotic systems as potential progenitors. Kenyon et al. (1993a) have shown that while it is unlikely that SSs can produce SN Ia's via the Chandrasekhar-mass models, SSs can be considered as promising candidates for sub-Chandrasekhar-mass models, *if these models indeed produce SN Ia's.*

Recently, some calculations suggested that the off-center detonation of helium in WDs in the mass range 0.6–0.9 $M_\odot$, after accreting 0.15–0.2 $M_\odot$, can give rise to explosions that resemble SN Ia's in many respects (e.g., Livne 1990; Limongi & Tornambé 1991; Woosley & Weaver 1994; Pinto 1994). For the purposes of the present study, we shall ignore for the moment the question of whether such sub-Chandrasekhar-mass WDs indeed produce SN Ia's and we shall consider only the question of the statistics.

In the standard model, the frequency of events in which the WD reached the Chandrasekhar limit was about $1 \times 10^{-6}\ \mathrm{yr}^{-1}$. The main reason for this low rate was the deficiency in SSs with massive WDs (see Figs. 4 and 8). This deficiency in itself is a consequence of the fact that in massive systems the separation between the components increases due to mass loss.

Next, we calculated the rate of events in which 0.1 $M_\odot$ of helium accumulated on WDs with masses not lower than 0.6 $M_\odot$, in the entire population. These rates are given in column (4) of Table 3, for different model assumptions (line 1 gives the rate of WD mergers with total mass exceeding M_{Ch} from the calculations of Yungelson et al. 1994). As we can see, the rate does not exceed $\sim 0.001\ \mathrm{yr}^{-1}$, which is about one-third of the estimated SN Ia rate (e.g., Wheeler & Harkness 1990; van den Bergh & Tammann 1991). The rates that we obtained should definitely be considered as upper limits since calculations suggest that, in fact, the accumulation of $\sim$0.15–0.20 $M_\odot$ of helium is required for an explosion (e.g., Iben & Tutukov 1991; Woosley & Weaver 1994) and, furthermore, the calculations seem to require $M_{\mathrm{WD}} \gtrsim 0.7\ M_\odot$ (Woosley & Weaver 1994). Thus, SSs appear not to constitute the main class of progenitors for SN Ia's, but their contribution could be significant

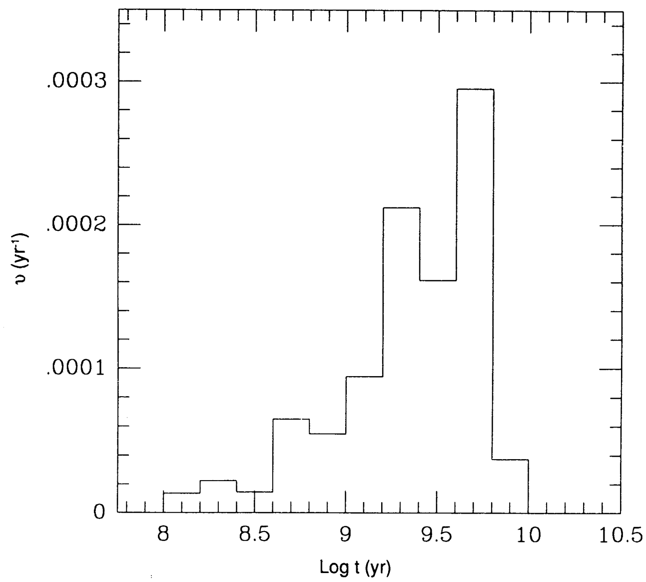

FIG. 19.—Contribution of stars of different age to the present rate of sub-Chandrasekhar supernovae in symbiotic binaries.

(from a statistical point of view), should sub-Chandrasekhar models prove to be viable candidates for SN Ia explosions.

Figure 19 shows the contribution of stars of different ages to the present rate of occurrence of sub-Chandrasekhar SNs in SSs, under the assumption of a constant star formation rate (or the history of these SNs for an instantaneous star formation) for the standard model. It is immediately evident that sub-Chandrasekhar SNs in SSs can occur only in young and moderate-age populations. The contribution of very old stars cannot be very significant, even if the star formation rate was enhanced in the first 5×10^9 yr of the galactic history (Gallagher, Hunter, & Tutukov 1984). Thus, they have to be almost absent in elliptical galaxies. The shape of the histogram reflects the lifetimes of the most efficient donors of SSs, which have initial masses higher than 1 $M_\odot$.

4. CONCLUSIONS

1. We performed a detailed study of the formation of symbiotic systems with WDs as the hot component, based on the population synthesis approach to the evolution of binaries. In this study we assumed that the hot components of SSs are WDs which are either steady hydrogen burning objects or objects in a postnova "plateau" phase in the evolution of exploding WDs. This approach allowed us to determine which evolutionary routes result in the formation of SSs and to estimate their birthrates and number in the Galaxy. The total number of SSs predicted by the model, $\sim$3000–30,000 (depending on different model assumptions), is compatible with observational estimates by Boyarchuk (1984) and Kenyon (1986, 1994), while it is an order of magnitude lower than the estimate of Munari & Renzini (1992).

2. We show that among the WD components of SSs, a significant fraction may be low-mass helium WDs, if ΔM_{crit} for the first hydrogen shell flash is given by equation (16). Systems with He WDs must have systematically lower periods than the systems with CO or ONe WDs. If this group of SSs really exists, then there may be a correlation between the masses of the WDs and the orbital periods of SSs. However, If ΔM_{crit} for the first flash is higher than given by equation (16) (e.g., even by

merely a factor of 3), the number of SSs with He WDs can be significantly reduced, in agreement with current observations. ΔM_{crit} for the first flash may be higher than the "standard" value due to very low temperatures of the WDs or high radii of the WDs which are still contracting to their final radius (eq. [17]).

3. There has to exist a certain threshold efficiency for accretion in binary systems containing WD accretors and RG (or supergiant) donors. With accretion rates below this threshold, it becomes impossible to explain even the lowest observational estimates of the number of SSs. For the production of SSs, it is important for the accretor to be located in the zone of acceleration of the stellar wind of the donor. This hypothesis must be further verified by gasdynamical calculations.

4. The average rate of symbiotic novae in the solar vicinity obtained in the model is compatible with the observed one ($\sim$0.1 yr^{-1}). Again, this rate is directly dependent on ΔM_{crit} for hydrogen flashes given by equation (16).

5. The apparent normal chemical composition of the symbiotic novae may be a consequence of the existence of a helium layer at the surface of accretors in the overwhelming majority of SSs. This layer can prevent the enrichment of the hydrogen layer by CNO-group elements from the core until it is eroded. The erosion process may be slower in symbiotic novae than in classical novae, due to lower WD masses. In addition, calculations show that diffusion-induced convection produces lower levels of enrichments on relatively low-mass WDs.

6. Symbiotic systems *do not* lead to SN's Ia via Chandrasekhar-mass models. *If* sub–Chandrasekhar-mass double-detonation models indeed produce SN Ia's, then (even under the most generous assumptions) symbiotic systems can be the progenitors of not more than about one-third of the events. The supernovae resulting from SSs in this way have to be relatively rare in elliptical galaxies because their progenitors belong to an intermediate-age population.

7. Some symbiotic systems may be identified as supersoft X-ray sources.

8. The two main areas of uncertainty in our models are in the process of accretion from a stellar wind and in the evolution of low-mass accreting WDs. Future theoretical work on these two subjects will be able to reduce the uncertainties involved.

The authors acknowledge fruitful discussions with A. A. Boyarchuk, D. V. Bisikalo, I. Iben, Jr., and Yu. L. Frantsman. We are grateful to S. V. Sushko for her help in the preparation of the manuscript. We thank the referee, M. Vogel, for his detailed report, which helped to improve the manuscript. This study was supported by Russian Foundation for Fundamental Research grant 93-02-2893, International Science Foundation grant MPT000, and ESO C&EE Programme grant A-01-019, and by NASA grants NAGW-2678 and GO 5255. L. Y. acknowledges the support and hospitality of the Space Telescope Science Institute, where part of this study was performed.

REFERENCES

Allen, C. W. 1973, Astrophysical Quantities (London: Athlone)
Bath, G. T. 1977, MNRAS, 178, 203
Bergeron, P., Saffer, R. A., & Liebert, J. 1992, ApJ, 394, 228
Bisikalo, D. V., Boyarchuk, A. A., Kuznetsov, O. A., Popov, Yu. P., & Chechetkin, V. M. 1994, AZh, 71, 1
Bondi, H. 1952, MNRAS, 112, 195
Boyarchuk, A. A. 1970, in Eruptivnyje Zvezdy, ed. A. A. Boyarchuk & R. E. Gershberg (Moscow: Nauka), 113
———. 1984, Astrophys. Space Sci. Rev., 3, 123
de Jager, C., Nienwenhuijzen, H., & van der Hucht, K. A. 1988, A&AS, 72, 259
de Kool, M. 1990, ApJ, 358, 189
Della Valle, M., & Livio, M. 1994, A&A, 286, 786
Dgani, R. 1993, A&A, 271, 527
Dgani, R., Walder, R., & Nussbaumer, H. 1993, A&A, 267, 155
Duerbeck, H. W. 1987, Space Sci. Rev., 45, 1
Duschl, W. J. 1986, A&A, 163, 61
Fadeev, Yu. A. 1987, in Circumstellar Matter, ed. I. Appenzeller & C. Jordan (Dordrecht: Reidel), 515
Frantsman, Yu. L. 1986, Astrofizika, 24, 131
Fujimoto, M. Y. 1988, A&A, 198, 163
Fujimoto, M. Y., & Taam, R. E. 1982, ApJ, 260, 249
Gallagher, J. S., Hunter, D., & Tutukov, A. V. 1984, ApJ, 284, 544
Goldberg, D., & Mazeh, T. 1994, ApJ, 394, 592
Greiner, J., Hasinger, G., & Kahabka, P. 1991, A&A, 246, L17
Groenewegen, M. A. T., & de Jong, T. 1993, A&A, 267, 410
Hjelming, M. 1989, Space Sci. Rev., 50, 155
Hjellming, M., & Webbink, R. 1987, ApJ, 318, 794
Iben, I., Jr. 1982, ApJ, 259, 244
Iben, I., Jr., Fujimoto, M. Y., & MacDonald, J. 1992, ApJ, 388, 521
Iben, I., Jr., & Livio, M. 1993, PASP, 105, 1373
Iben, I., Jr., & Renzini, A. 1983, ARA&A, 21, 271
Iben, I., Jr., & Rood, R. T. 1970, ApJ, 161, 587
Iben, I., Jr., & Tutukov, A. V. 1984a, ApJS, 54, 335
———. 1984b, ApJ, 282, 615
———. 1984c, ApJ, 311, 742
———. 1985, ApJS, 58, 661
———. 1986, ApJ, 311, 742
———. 1989, ApJ, 342, 430
———. 1991, ApJ, 370, 615
———. 1992, ApJ, 389, 369
———. 1993, ApJ, 418, 343
———. 1995, ApJ, submitted
Iben, I. Jr., Tutukov, A. V., & Yungelson, L. R. 1995a, ApJS, in press
———. 1985b, ApJS, in press
Jura, M. 1994, ApJ, 422, 102
Kato, M., & Hachisu, I. 1995, ApJ, submitted
Kato, M., Saio, H., & Hachisu, I. 1989, ApJ, 340, 509
Kenyon, S. J. 1986, The Symbiotic Stars (Cambridge: Cambridge Univ. Press)
———. 1994, Mem. Soc. Astron. Italiana, 65, 135
Kenyon, S. J., & Gallagher, J. S. 1983, AJ, 88, 666
Kenyon, S. J., Livio, M., Mikolajewska, J., & Tout, C. A. 1993a, ApJ, 408, L81
Kenyon, S. J., Mikolojewska, J., Mikolajewski, M., Polidan, R. S., & Slovak, M. H. 1993b, AJ, 106, 1573
Kenyon, S. J., & Webbink, R. F. 1984, ApJ, 279, 252
Kippenhah, R., Kohl, K., & Weigert, A. 1967, Z. Astrophys., 66, 58
Kovetz, A., & Prialnik, D. 1995, ApJ, in press
Lattanzio, J. 1991, ApJS, 76, 215
Limongi, M., & Tornambé, A. 1991, ApJ, 371, 317
Livio, M. 1992, in IAU Symp. 151, Evolutionary Process in Interacting Binary Stars, ed. Y. Kondo et al. (Dordrecht: Kluwer), 185
———. 1995, in Millisecond Pulsars: A Decade of Surprise, ed. A. Fruchter, M. Tavani, & D. Backer (San Francisco: ASP), 105
Livio, M., Soker, N., de Kool, M., & Savonije, G. M. 1986, MNRAS, 222, 235
Livio, M., & Truran, J. W. 1987, ApJ, 318, 316
———. 1994, ApJ, 425, 797
Livne, E. 1990, ApJ, 354, L53
MacDonald, J. 1983, ApJ, 273, 289
Mazeh, T. 1994, A&A, 282, 801
Mazeh, T., Goldberg, D., Duquennoy, A., & Mayor, M. 1992, ApJ, 401, 265
Mengel, J. G., Sweigart, A. V., Demarque, P., & Gross, P. G. 1979, ApJS, 40, 733
Mikolajewska, J., & Kenyon, S. J. 1992, MNRAS, 256, 177
Munari, U. 1994, Mem. Soc. Astron. Italiana, 65, 157
Munari, U., & Renzini, A. 1992, ApJ, 397, L87
Mürset, U., & Nussbaumer, H. 1994, A&A, 282, 586
Mürset, U., Nussbaumer, H., Schmidt, H. M., & Vogel, M. 1991, A&A, 248, 458
Nauenberg, M. 1972, ApJ, 175, 417
Nomoto, K. 1982, ApJ, 253, 798
Nussbaumer, H., & Vogel, M. 1987, A&A, 182, 51
Paczyński, B. 1970, Acta Astron., 20, 47
———. 1976, in Structure and Evolution of Close Binary Stars, ed. P. Eggleton et al. (Dordrecht: Reidel), 75
Paczyński, B., & Rudak, B. 1980, A&A, 82, 349
Paczyński, B., & Sienkiewicz, R. 1972, Acta Astron., 16, 231
Paczyński, B., Ziolkowski, J., & Żytkow, A. N. 1969, in Mass Loss from Stars, ed. M. Hack (Dordrecht: Reidel), 237
Paczyński, B., & Żytkov, A. N. 1978, ApJ, 222, 604

Phillips, J. P. 1989, in IAU Symp. 131, Planetary Nebulae, ed. S. Torres-Peimbert (Dordrecht: Kluwer), 425
Pinto, P. 1994, private communication
Plavec, M., Ulrich, R. K., & Polidan, S. 1973, PASP, 85, 769
Popova, E. I., Tutukov, A. V., & Yungelson, L. R. 1982, Ap&SS, 88, 55
Prialnik, D., & Livio, M. 1995, ApJ, submitted
Rappaport, S., Di Stefano, R., & Smith, J. D. 1994, ApJ, 426, 692
Refsdal, S., & Weigert, A. 1970, A&A, 6, 426
Reid, N., Tinney, C., & Mould, J. 1990, ApJ, 348, 98
Reimers, D. 1975, Mem. Soc. R. Sci. Liege, 8, 369
Ritter, H., Politano, M. J., Livio, M., & Webbink, R. F. 1991, ApJ, 376, 177
Schaeidt, S., Hasinger, G., & Trümper, J. 1993, A&A, 270, L9
Schwartzman, E., Kovetz, A., & Prialnik, D. 1994, MNRAS, 269, 323
Seaquist, E. R., Krogulec, M., & Taylor, A. R. 1993, ApJ, 410, 260
Sedlmayr, E., Dominik, C., & Gail, H.-P. 1988, in Atmospheric Diagnostics of Stellar Evolution: Chemical Peculiarity, Mass Loss and Explosion, ed. K. Nomoto (Berlin: Springer), 167
Shara, M., Prilanik, D., & Kovetz, A. 1993, ApJ, 406, 220
Shima, E., Matsuda, T., Takeda, H., & Sawada, K. 1985, MNRAS, 217, 367
Sion, E. M., & Ready, C. J. 1992, PASP, 104, 87
Sparks, W. M., & Kutter, G. S. 1987, ApJ, 321, 394
Stevens, I. R., Blondin, J. M., & Pollock, A. M. T. 1992, ApJ, 386, 265
Sweigart, A. V., Greggio, L., & Renzini, A. 1990, ApJ, 364, 527
Sweigart, A. V., & Gross, P. G. 1976, ApJS, 36, 405
Tout, C. A., & Eggleton, P. P. 1988, ApJ, 386, 197
Tutukov, A. V., Fedorova, A. V., & Yungelson, L. R. 1982, Soviet Astron. Lett., 8, 198
Tutukov, A. V., & Khokhlov, A. M. 1992, Soviet Astron., 36, 401
Tutukov, A. V., & Yungelson, L. R. 1976, Astrophysics, 12, 321
———. 1979, in Mass Loss and Evolution of O-Type Stars, ed. C. de Loore & P. S. Conti (Dordrecht: Reidel), 401
———. 1982, in The Nature of Symbiotic Stars, ed. M. Friedjung & R. Viotti (Dordrecht: Reidel), 283
Tutukov, A. V., & Yungelson, L. R. 1990, AZh, 67, 109
———. 1987, in 2d Conf. Faint Blue Stars, ed. A. G. D. Philips et al. (Schenectady: Davis), 435
———. 1992, Soviet Astron., 36, 266
———. 1993a, AZh, 70, 812
———. 1993b, MNRAS, 260, 675
———. 1994, MNRAS, 268, 871
Tutukov, A. V., Yungelson, L. R., & Iben, I., Jr. 1992, ApJ, 386, 197
Uus, U. 1970, Nauch. Inf., 17, 32
van den Bergh, S., & Tammann, G. 1991, ARA&A, 29, 363
van den Heuvel, E. P. J., Bhattacharya, D., Nomoto, K., & Rappaport, S. A. 1992, A&A, 262, 97
Vereschchagin, S. V., Kraicheva, Z. T., Popova, E. I., Tutukov, A. V., & Yungelson, L. R. 1988, Ap&SS, 142, 245
Vogel, M. 1991, A&A, 249, 173
Vogel, M., & Nussbaumer, H. 1992, A&A, 259, 525
———. 1994, A&A, 284, 145
Vogel, M., Nussbaumer, H., & Monier, R. 1992, A&A, 260, 156
Webbink, R. F. 1984, ApJ, 277, 355
———. 1988, in The Symbiotic Phenomenon, ed. J. Mikolajewska et al. (Dordrecht: Kluwer), 311
Weidemann, V. 1986, in Late Stages of Stellar Evolution, ed. S. Kwok & S. P. Pottash (Dordrecht: Reidel), 347
Wheeler, C. J., & Harkness, R. P. 1990, Rep. Progr. Phys., 53, 1467
Whelan, J., & Iben, I., Jr. 1973, ApJ, 186, 1007
Woosley, S. E., & Weaver, T. A. 1994, ApJ, 423, 371
Yungelson, L. R., Livio, M., Tutukov, A. V., & Saffer, R. A. 1994, ApJ, 420, 336
Yungelson, L. R., & Tutukov, A. V. 1993, in Planetary Nebulae, ed. R. Weinberger & A. Acker (Dordrecht: Kluwer), 389
Yungelson, L. R., Tutukov, A. V., & Livio, M. 1993, ApJ, 418, 794