

STEPS TOWARD THE HUBBLE CONSTANT. X. THE DISTANCE OF THE VIRGO CLUSTER CORE USING GLOBULAR CLUSTERS

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ABSTRACT

New data for Secker's (1992) unbiased sample of 100 globular clusters in the Milky Way exhibit a nearly Gaussian luminosity function with peak (turnover) luminosities of $\langle M_B \rangle_0 = -6.90 \pm 0.11$, $\sigma(M) = 1.07$ mag, and $\langle M_V \rangle_0 = -7.60 \pm 0.11$, $\sigma(M) = 1.07$. These are derived from revised Galactic globular cluster distances determined from an RR Lyrae calibration of $M_V(\text{RR}) = 0.30([\text{Fe}/\text{H}]) + 0.94$ based on the Oosterhoff-Arp-Preston period-metallicity effect. The new calibration averages 0.25 mag brighter than is generally in current use.

Halo clusters in M31 give $\langle M_B \rangle_0 = -7.01 \pm 0.20$, $\sigma(M) = 0.89$, and $\langle M_V \rangle_0 = -7.70 \pm 0.20$, $\sigma(M) = 0.89$ for the globular cluster luminosity function (GCLF) using a Cepheid modulus for M31 of $(m-M)^\circ = 24.44$ (Madore & Freedman 1991) with a foreground reddening of $E(B-V) = 0.08$. Use of the brighter RR Lyrae magnitudes for the MW calibrators nearly eliminates the marginal 0.2–0.3 mag difference between the MW and the M31 GCLFs obtained earlier by Racine & Harris (1992) and Secker (1992).

The combined globular cluster luminosity function for the MW plus M31 gives $\langle M_B \rangle_0(\text{turnover}) = -6.93 \pm 0.08$ and $\langle M_V \rangle_0(\text{turnover}) = -7.62 \pm 0.08$, with a realistic external error of ~ 0.2 mag.

Applying this local calibration to the E galaxy Virgo cluster data gives the modulus of the Virgo cluster core as $(m-M)^\circ = 31.64 \pm 0.25$ ($D = 21.3 \pm 2.7$ Mpc; external error). This value is based on the observed mean GCLF turnover luminosity of $\langle B \rangle_0 = 24.64 \pm 0.07$ for three galaxies as determined by Harris et al. (1991). The modulus is only marginally increased if NGC 4365 is accepted as a likely Virgo cluster member and if new data for NGC 4636 are included. We have used the precept that the mean absolute turnover luminosity of the GCLF is universal, that is, does not depend on a second parameter. The suggestion (Secker & Harris 1993) that the turnover luminosity varies with the dispersion of the GCLF, based on their a priori adoption of a short-distance-scale modulus of the Virgo cluster as $(m-M)^\circ = 30.89$, is discussed. Contrary, independent evidence that the modulus of the Virgo cluster core is $(m-M)^\circ = 31.7$ is reviewed.

The Hubble constant, based on (1) the distance to the Virgo cluster core determined from the calibration of the globular cluster method given here, and (2) the redshift of the Virgo core of $v(\text{cosmic}) = 1179 \pm 17$ km s⁻¹ relative to the Machian frame of the cosmic microwave background, freed from all local streaming motions (Jerjen & Tammann 1993), is $H_0(\text{global}) = 55 \pm 7$ km s⁻¹ Mpc⁻¹ (external error).

Subject headings: distance scale — galaxies: individual (Virgo) — galaxies: distances and redshifts — galaxies: star clusters

1. INTRODUCTION

1.1. *An Originally Perceived Difference between Globular Clusters in the Milky Way and in M31*

At the time of Hubble's (1932) identification of globular clusters in M31 even the initial data provided a clue to the error in Hubble's (1929) distance to that galaxy. Hubble had adopted photometric data for the Milky Way (MW) globular clusters based on Shapley's (1930) cluster distances. He then compared the resulting MW globular cluster luminosity function (GCLF) with the LF for the objects in M31 using his own photometry and his 1929 Cepheid distance to M31 as $m-M = 22.2$. (He in fact used $m-M = 22.0$ for M31 in his 1932 analysis.)

Hubble found a difference, writing in his abstract "the objects in M31 are fainter than the galactic globular clusters by an amount varying from about 0.75 to 1.95 magnitude according to the interpretation of the data."

Shapley's distances to the Galactic globular clusters were based on RR Lyrae absolute magnitudes of about $M_V = 0$,

whereas Hubble's M31 distance was based on Shapley's calibration of the classical Cepheid P - L relation which Baade (1952) eventually showed was too faint by about 1.5 mag.

Baade's discovery of the error began with his failure to resolve RR Lyrae variables in the disk of M31 with the Palomar 200 inch reflector. Either the RR Lyrae variables have absolute magnitudes closer to $M_V = +1.5$ than to $M_V = 0$, or the distance to M31 was about 1.5 mag farther than Hubble had determined.

Baade (1944) had previously revised Hubble's M31 modulus upward to $m-M = 22.4$ by correcting the Mount Wilson Catalog apparent magnitude scale in Selected Area 68 (Seares, Kapteyn, & van Rhijn 1930) by 0.45 mag together with an adjustment by -0.23 mag to the Cepheid P - L relation zero point.

However, his larger and more fundamental correction of an additional 1.8 mag came from the bold conjecture that the zero point of the Cepheid P - L relation was grossly in error. His evidence was that the calibration of the RR Lyraes should

remain at about $M_V = 0$, based on the main-sequence-fitted distance to M3 (Sandage 1953), and therefore that the error had to reside with the Cepheids. An important historical review of these and other associated results was written by Baade (1956) as his acceptance lecture for the Catherine Wolf Bruce Gold Medal of the ASP.

With his revision upward of the M31 distance, Baade (1952) could state to the Rome IAU meeting, as recorded by Fred Hoyle, secretary of Commission 28, "many notable implications followed immediately from the corrected distances: the globular clusters in M31 and our own Galaxy now come out to have similar luminosities; our Galaxy may now come out to be somewhat smaller than M31".¹

It is easy with hindsight to suggest that if the 1932 globular cluster discrepancy had been heeded, and if the "principle of the uniformity of nature," so often emphasized by Hubble in applying Cepheids to the Local Group galaxies, had been believed, then the early M31 Cepheid distance of $m - M = 22$ could have been questioned. To be sure, a value judgement was required in 1932 as to whether to question the Cepheid $P-L$ relation or the universality of the GCLF. That solution had to await progress in an understanding of the Cepheid calibration (Baade 1956).

We are faced with circumstances in the present literature that are also subject to different interpretations. (1) A similar but considerably smaller discrepancy exists in comparisons of galaxy distances from Cepheids and from RR Lyrae stars. Although these differences are within $2\sigma(M)$ statistics in each case, they have been emphasized by their respective authors, leading to apparent (supposed) differences in the absolute magnitudes of globular clusters in the Galaxy, in M31, and in the Virgo cluster E galaxies, even using the second-parameter calibration of the globular cluster turnover luminosity (Secker & Harris 1993). (2) The justification of the second-parameter calibration (Secker 1992; Secker & Harris 1993) rests on their assumption of the Virgo Cluster modulus of $(m - M)^\circ = 30.89$ as suggested by Jacoby et al. (1992).

Because we question this modulus on the basis of data not considered by Secker & Harris, we also question the evidence for a correlation of $M(\text{turnover})$ of the GCLF on the dispersion of the luminosity function. We set out the case that the evidence is not strong enough to compromise the globular cluster method in such a fundamental way. By adopting the universality of the GCLF, we recover from the current globular cluster data a Virgo cluster distance that is consistent with other evidence for the long distance scale.

The arrangement of this paper is to (1) discuss again the calibration of the turnover luminosity of the GCLF using the new calibration of the RR Lyrae variables as a function of $[\text{Fe}/\text{H}]$ for the Galactic clusters, (2) combine the resulting new Milky Way calibration with data for Secker's (1992) list of halo clusters in M31, calibrated using the Cepheid distance to M31 of $(m - M)_{AV} = 24.68$ and $(m - M)_{AB} = 24.76$, (3) apply this combined calibration, with no dependence on dispersion, to the extant data on globular clusters in five Virgo cluster E and

S0 galaxies to derive a Virgo cluster core modulus of $m - M = 31.64 \pm 0.25$ (external), and (4) show that the resulting modulus is consistent with a large body of independent evidence that favors the long scale.

1.2. The Present Literature

Partial entrance to the extensive current literature on the globular cluster LFs in the Galaxy and in M31 can be found in papers by Racine & Shara (1979), Hanes (1980), Battistini et al. (1987), Elson & Walterbos (1988), Fusi Pecci (1988), Harris (1988, 1991), Harris et al. (1991, hereafter HAPV), Racine & Harris (1992), and Secker (1992). These contain references to previous reviews, searches, catalogs, and summaries.

In an analysis of unbiased local samples of clusters, Racine & Harris (1992) conclude that "the GCLF [in M31] is similar in shape to the Milky Way cluster distribution, though it has a narrower dispersion and its turnover (peak frequency luminosity) is almost 0.3 mag brighter." Racine & Harris based their distance scale for MW clusters on (1) an RR Lyrae absolute magnitude of $M_V(\text{RR}) = 0.6$ mag with no dependence on $[\text{Fe}/\text{H}]$, and (2) an apparent V modulus for M31 of $(m - M)_{AV} = 24.57$.

In a parallel study, Secker (1992) obtained $\langle M_V \rangle_0 = -7.29 \pm 0.13$ for the peak luminosity² of the Galaxy cluster LF compared with $\langle M_V \rangle_0 = -7.51 \pm 0.15$ for the M31 turnover luminosity of the M31 globular clusters using $(m - M)_{AV} = 24.51$ for the apparent V M31 modulus. The difference is 0.22 ± 0.20 mag, in the sense that the M31 clusters are brighter. Secker's precept of the Galactic globular cluster distance scale was based on his adoption of the RR Lyrae star absolute magnitude calibration used by Harris et al. (1991) of $M_V(\text{RR}) = 0.20([\text{Fe}/\text{H}]) + 1.00$. This ranges between 0.2 and 0.3 mag fainter over the relevant metallicity range than magnitudes from equation (1) given later (§ 2).

In the same way as Racine & Harris and as Secker (1992), Secker & Harris (1993) conclude that the M31 clusters are 0.26 ± 0.20 brighter than those in the Galaxy using a Galaxy distance scale based on the same HAPV scale as above.

The difference of ~ 0.3 mag in the turnover luminosity of the GCLF in the MW and in M31, even if barely significant, would, of course, be eliminated if the $M_V(\text{RR})$ calibration were made brighter by this amount, provided that the modulus of M31 is kept at the classical Cepheid value. The latter provision is reasonable because of the excellent agreement between the Cepheid calibrations of Sandage & Tammann (1969), Feast & Walker (1987), and Madore & Freedman (1991). Only the calibration of Gieren, Barnes, & Moffett (1993) is marginally brighter by ~ 0.2 mag.

One of the purposes of this paper is to set out a calibration of the MW globular cluster LF using the brighter calibration of the RR Lyrae luminosities based on the Oosterhoff-Arp-Preston effect, which we adopt in § 2. Our second purpose is to discuss the precepts of Secker (1992) and Secker & Harris (1993) in their use of the globular cluster method to calibrate the GCLF, based on their a priori assumption that the core of the Virgo cluster is at $(m - M)^\circ = 30.89$. The calibration, so made, cannot then be used to determine the distance to Virgo cluster galaxies (Kissler et al. 1994, Ajhar, Blakeslee, & Tonry 1994) because the argument would be circular.

¹ Shapley, who was in the audience, requested more information from Baade on his reasoning. Upon returning from the IAU Shapley wrote a remarkable summarizing report emphasizing the globular cluster discrepancy, how he had puzzled over it some years before, and how he had taken measures to check the photometry for the LMC data (Shapley 1953). Shapley's review is instructive as an historical document in illuminating the prevalent precepts before Baade's bold revision, keeping in mind that Shapley's paper was written after he knew of Baade's result.

² The subscript zero, following the notation of Harris (1988), stands for turnover magnitudes of the GCLF, rather than absorption-free magnitudes, which are denoted by superscript.

In an early review Harris (1988) outlines the standard operating precepts needed for the simplest application of the method of photometric parallaxes. Concerning the globular clusters he writes, "The validity of the GCLF method rests on the operating assumption that the globular clusters have similar luminosities everywhere, i.e. that they have a universal LF." After discussing data that justify this view he continues, "*the intrinsic scatter in M_B^o [the turn over luminosity] among large elliptical galaxies is at the level of ~ 0.2 magnitudes, and the LF dispersion is not sensibly different from one galaxy to another*" (his italics). He then writes, "We would like to ask whether the new Virgo data can be successfully matched to the calibrating Local Group GCLF. Since the method must assume that M_B^o and σ are intrinsically similar in both systems, we have now (thankfully!) lost virtually all freedom to adjust the matchup in arbitrary ways. By hypothesis, we are required to take our calibrating function $n(M_B)$, move it faintward by an amount $(\langle B \rangle_o - \langle M_B \rangle_o)$ (having now directly observed $\langle B \rangle_o$), and then scale it to normalize the total populations in the two samples."

By using this rigid approach, Harris (1988) obtained the distance modulus of the Virgo cluster core as $(m - M)^o = 31.7$. When combined with the tie-in to the global (Machian frame) redshift of the Virgo cluster core of $v(\text{cosmic}) = 1179 \pm 17 \text{ km s}^{-1}$, freed from all streaming motions, a Hubble constant of $H_0 = 54 \pm 3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (internal error) follows. This $v(\text{Virgo})_{\text{cosmic}}$ has been measured (Jerjen & Tammann 1993) using a method that has considerably higher weight than the earlier estimates of the Virgo "infall" (e.g., Tammann & Sandage 1985 for a review) used by Harris (1988) and by HAPV.

A change in the precepts of the method and of the absolute magnitude calibration of the turnover was made by Secker (1992) and by Secker & Harris (1993). They forced a Virgo distance of $(m - M)^o = 30.89$ on the globular cluster data and assumed, in addition, the Virgo galaxy NGC 4365 to lie in the background. We take up the latter proposition again in § 4.2.

They placed major reliance on the planetary nebula and the surface brightness fluctuation methods as distance indicators to the Virgo cluster. Their assumption neglected the contrary evidence, both from the Tully-Fisher method corrected for observational selection bias and the analysis of seven other methods that require the long distance scale (Kraan-Korteweg, Cameron, & Tammann 1988; Sandage 1988; Fouqué et al. 1990; Sandage & Tammann 1990; Tammann 1988, 1992, 1993; Sandage 1993c).

This led Secker & Harris to introduce a second-parameter calibration of $M(\text{turnover})$ depending on the dispersion, $\sigma(M)$, of the GCLF (their Fig. 10). However, their derived correlation of $M(\text{turnover})$ with $\sigma(M)$ depends on their a priori adopted small Virgo distance. In addition, their resulting calibration could not then be used to determine the Virgo cluster distance from the globular cluster method because, as said, such a determination would be circular (Kissler et al. 1994; Ajhar et al. 1994).

In what follows we apply the GCLF method in the pure form pioneered by Harris (1988). The calibration rests on the cluster data for M31 and the Milky Way alone. Relying in first approximation on the uniformity of nature, it assumes a fixed $M(\text{turnover})$ independent of the dispersion. This, with the brighter calibration of $M_V(\text{RR})$ needed to explain the Oosterhoff-Arp-Preston period-metallicity correlation, gives a distance to the Virgo cluster core based on a combination of

Population I (Cepheids in M31) and Population II (RR Lyrae variables). The value of the Hubble constant follows by dividing the redshift of the Virgo cluster core freed from all local velocity anomalies (Sandage & Tammann 1990; Jerjen & Tammann 1993) by the distance to the cluster core.

Two side benefits, however insignificant, are the following: (1) the supposed difference of $0.26 \pm 0.20 \text{ mag}$ between the M31 and the MW calibration suggested by Secker (1992) and again by Secker & Harris (1993) is reduced to 0.11 ± 0.15 , due mostly to the use of the new RR Lyrae absolute magnitudes, and (2) the difference of $0.31 \pm 0.33 \text{ mag}$ between the giant ellipticals and the M31 and MW calibrators, suggested by Secker & Harris, is also eliminated.

2. ADOPTED ABSOLUTE MAGNITUDE CALIBRATION OF RR LYRAE STARS

We have calculated revised integrated absolute magnitudes of the Milky Way globular clusters determined from new distances to individual clusters based on (1) the observed level of their horizontal branches (HB) in the V photometric band (the HB is horizontal in V), and (2) an adopted absolute magnitude for the RR Lyrae instability strip on the HB. The new determination of $M_V(\text{RR})$ was made as follows.

The calibration problem started with an attempt to understand the Oosterhoff (1939, 1944) period dichotomy for cluster RR Lyrae stars, later shown to be correlated with metallicity (Arp 1955; Preston 1959), and spread into a near continuum (Sandage 1982) when account is taken of the nonmonotonic HB morphology as $[\text{Fe}/\text{H}]$ is varied (Renzini 1983; Castellani 1983; Sandage 1993a, b, hereafter S93a, b). An explanation of the period ratios of the Oosterhoff groups in terms of a luminosity difference as a function of metallicity began in the late 1950s using a simple model, made first at constant temperature (Sandage 1958; Sandage, Katem, & Sandage 1981, Fig. 13).

The explanation continued to be viable when even the first HB models showed such a luminosity difference (Iben & Rood 1970; Rood 1973; Sweigart & Gross 1976), but the details eluded a precise agreement between observations and theory when the theoretical models were in fact analyzed at constant temperature (Caputo 1988; Catelan 1992). A large literature developed on the apparent differences between the predictions of the models and the observations (e.g., Sweigart, Renzini, & Tornambé 1987). It is now known that the difference is not real but is due to reading the models at a constant temperature rather than along a line parallel to the temperature-luminosity correlation of the edge of the instability strip in the HR diagram (S93b). The requirement to do this is as follows.

Suppose that a luminosity difference in fact exists between the two Oosterhoff groups (or its generalization as a continuum; S93a, Figs. 1, 10, and 11). If so, and because the instability strip in the HR diagram slopes toward lower temperatures at higher luminosities at the rate of about $d \log T_e / d \log L = -0.06 \pm 0.02$ (e.g., Iben & Huchra 1971; Iben 1971; Stellingwerf 1975; Caputo & Castellani 1975; Fernie 1990; Morgan 1992), this temperature difference with luminosity must be taken into account in comparing the models with the *observed* Oosterhoff-Arp-Preston period-metallicity correlation.

Factoring this change of $\langle T_e \rangle$ (Simon & Clement 1993; S93b) with luminosity into the pulsation equation, and noting that the period-metallicity correlation at the fundamental blue edge of the instability strip is $d \log P / d ([\text{Fe}/\text{H}]) =$

-0.12 ± 0.02 , gives the RR absolute magnitude calibration that is self-consistent with all the data for the generalized Oosterhoff problem as

$$M_V(\text{RR}) = 0.30([\text{Fe}/\text{H}]) + 0.94, \quad (1)$$

which we adopt. The details are slightly more complicated because we need also the $T_e = f([\text{Fe}/\text{H}])$ and the mass $= g([\text{Fe}/\text{H}])$ relations at the fundamental blue edge of the strip, but these are found directly from the observations (details in S93b).

The calibration of equation (1) is between 0.2 and 0.3 mag brighter than most $M_V(\text{RR})$ assumptions in the current literature such as the Baade-Weisselink data summarized from many working groups. The B/W data give approximately $M_V(\text{RR}) = 0.25([\text{Fe}/\text{H}]) + 1.09$ (e.g., Sandage & Cacciari 1990; S93b; Carney, Storm, & Jones 1992).

Independent evidence exists that the bright revision contained in equation (1) is necessary:

1. Walker (1992) shows that the absolute magnitude of the RR Lyrae stars in the disk of LMC must be $\langle M_V \rangle = 0.44$ at $[\text{Fe}/\text{H}] = -1.9$ which is "some 0.3 mag brighter than suggested by statistical parallax analysis of Galactic field RR Lyraes and from parallaxes of subdwarfs" (his abstract).

2. Saha et al. (1992), in comparing the modulus for IC 1613 from Cepheids and from RR Lyrae stars state that "the distance modulus from [RR Lyrae stars] is smaller than that derived from Cepheids by an amount which is dependent upon the RR Lyrae zero-point adopted, and may be as large as 0.3 mag." The RR Lyrae calibration they had used was $M_V(\text{RR}) = 0.77$ independent of $[\text{Fe}/\text{H}]$.

3. Lee, Freedman, & Madore (1993) reach the same conclusion in a review of all the same type of data from many new programs such as those by Saha & Hoessel (1990) for NGC 185, Saha, Hoessel, & Mossman (1990) for NGC 147, Pritchett & van den Bergh (1987) for M31, etc. They conclude that the Carney et al. (1992) calibration of $M_V(\text{RR}) = 0.15([\text{Fe}/\text{H}]) + 1.01$ is too faint by 0.21 mag.

4. The zero point of theoretical models of the HB (Sweigart & Gross 1976; Sweigart 1987; Sweigart et al. 1987; Dorman 1992, 1993 as examples of a more extensive literature) all require the brighter luminosity for the zero age HB for $[\text{Fe}/\text{H}] > -1.2$, and with evolution from the ZAHB (e.g., Lee 1990) for $[\text{Fe}/\text{H}] < -1.3$ (see Figs. 10 and 11 of S93b).

We adopt the new $M_V(\text{RR})$ calibration of equation (1) because (1) it provides a consistent understanding of the Oosterhoff period problem for RR Lyrae stars, based as it is on the theoretical pulsation equation (no appeal made to theoretical HB models), (2) it does now agree with such HB models when they are read at *different* temperatures along the slope of the instability strip's correlation of L with T_e , and (3) its increased luminosity by between 0.2 and 0.3 mag agrees with the four independent points above.

3. LUMINOSITY FUNCTIONS FOR SECKER'S BIAS-FREE GLOBULAR CLUSTER SAMPLES IN THE MW AND M31

We have adopted Secker's (1992) choice (his Table 1) of MW globular clusters that constitutes an unbiased list "treating the Galaxy as if it were being viewed from the outside." The 100 clusters in Secker's list are set out in Table 1. Other data, which will differ in detail from Secker's because of the different sources, are as follows. Column (2) shows the observed mean V

magnitude of the HB listed by Peterson (1993) who used many references, the most complete still being the catalog of Webbink (1985). Column (3) shows the metallicity adopted by Djorgovski (1993), taken mostly from Zinn & West (1984) but supplemented by the measurements from Armandroff & Zinn (1988) and by the many others listed by Djorgovski for the few additional clusters. Column (4) gives our absolute magnitude of the HB based on equation (1) for the RR Lyrae stars. Columns (5) to (7) give the integrated apparent V magnitude of the cluster, the measured (or sometimes inferred) $B - V$ color, and the adopted $E(B - V)$ reddening, all taken from Peterson's (1993) table.

Column (8) is our derived absorption-free integrated V absolute magnitude found from $M_V = V(\text{GC}) - V(\text{HB}) + M_V(\text{RR})$, that is, by subtracting column (2) from the sum of columns (4) and (5), the absorption canceling out. The corresponding absorption-free integrated M_B absolute magnitude is found from column (8) by applying the reddening-free $(B - V)^\circ$ color found by subtracting column (7) from column (6).

The observed V and $B - V$ data for the adopted sample of 82 M31 halo clusters were taken from Secker's (1992) Table 2. They were changed to absolute magnitudes by adopting the absorption-free M31 modulus as $(m - M)^\circ = 24.44$ (Madore & Freedman 1991, their Table 1) and a foreground reddening of $E(B - V) = 0.08$. We justify this reddening value from the several sources as (1) $E(B - V) = 0.06$ to 0.08 from van den Bergh (1964, 1969), (2) $E(B - V) = 0.11 \pm 0.02$ from McClure & Racine (1969), (3) 0.08 from Burstein & Heiles (1984), (4) 0.06 from $A_V = 0.18$ by Freedman & Madore (1990) in field IV of Baade & Swope (1963), and (5) 0.05 from the difference between the reddening-free mean $\langle (B - V) \rangle = 0.70$ color for the Table 1 globulars in the MW and the observed $\langle B - V \rangle = 0.77$ mag color of the M31 halo clusters from Table 2 of Secker, the difference corrected for the difference between the mean metallicity of the MW and the M31 clusters ($\langle [\text{Fe}/\text{H}] \rangle = -1.35$ for the MW clusters and $\langle [\text{Fe}/\text{H}] \rangle = -1.22$ for the M31 clusters based on Harris [1991, his Table 2]). These are all smaller than the original value of $E(B - V) = 0.16$ from field IV of Baade & Swope (1963), which is almost certainly too high.

Hence, the adopted apparent modulus of the M31 halo aggregate is $(m - M)_{AV} = 24.68$ for V and $(m - M)_{AB} = 24.76$ for B magnitudes.

The mean apparent magnitude of the M31 sample of 82 clusters by Secker (1992) is $\langle V \rangle = 16.98 \pm 0.11$ with $\sigma(V) = 0.89 \pm 0.11$. Applying the individual observed $B - V$ colors to the listed V magnitudes gives individual B magnitudes for those clusters with observed colors. For the 18 clusters with no observed colors we adopt the mean color from the other data of $\langle (B - V) \rangle = 0.77$. The average apparent B magnitude of the sample of 82 M31 halo clusters is $\langle B \rangle_0 = 17.75 \pm 0.11$ with $\sigma(M) = 0.89$ again.

Applying the adopted M31 apparent distance moduli to these values gives the absorption-free turnover luminosity of the M31 GCLF as

$$\langle M_V \rangle_{0 \text{ M31}} = -7.70 \pm 0.20, \quad (2)$$

$$\sigma(M) = 0.89 \text{ mag}$$

in V , and

$$\langle M_B \rangle_{0 \text{ M31}} = -7.01 \pm 0.20, \quad (3)$$

with the same dispersion in B , where the error is an estimated external value.

TABLE 1
STECKER'S SAMPLE OF 100 OUTLYING CLUSTERS IN THE MILKY WAY

Name (1)	$V(\text{HB})$ (2)	$[\text{Fe}/\text{H}]$ (3)	$M_V^{\circ}(\text{RR})$ (4)	$V(\text{GC})$ (5)	$(B-V)$ (6)	E_{B-V} (7)	M_V° (8)	M_B° (9)
AM 4	18.20	(-1.05)	0.63	15.9	(0.75)	0.06	-1.67	-0.98
Pal 13	17.70	-1.79	0.40	13.8:	0.73	0.05	-3.50	-2.82
E3	15.8:	(-1.05)	0.63	11.35	(0.97)	0.28	-3.82	-3.13
E452SC	16.66	(-1.05)	0.63	12.0	(1.00)	0.31	-4.03	-3.34
Pal 12	17.09	-1.14	0.60	11.71	1.05	0.02	-4.78	-3.75
N6496	14.90	-0.48	0.80	8.60	0.98	0.16	-5.50	-4.68
Pal 11	17.30	-0.7	0.73	9.80:	1.27:	0.34	-6.77	-6.08
Pal 5	17.50	-1.47	0.50	11.75	(0.72)	0.03	-5.25	-4.56
N6838	14.44	-0.58	0.77	8.39	1.09	0.28	-5.28	-4.47
Pal 8	18.90	-0.48	0.80	10.89	1.22	0.32	-7.21	-6.31
Arp 2	18.21	(-1.05)	0.63	12.3	(0.80)	0.11	-5.28	-4.59
N7492	17.63	-1.82	0.39	11.15:	0.42	0.01	-6.09	-5.68
Pal 15	19.95	(-1.05)	0.63	14.2	(1.09)	0.40	-5.12	-4.43
N4147	17.03	-1.80	0.40	10.35	0.59	0.02	-6.28	-5.71
N6642	16.30	-1.29	0.55	8.89	1.09	0.37	-6.86	-6.21
N6535	15.85	-1.75	0.42	9.27	0.94	0.33	-6.16	-5.55
N6366	15.70	-0.99	0.64	9.53:	1.46	0.65	-5.47	-4.66
N6256	18.20	(-1.05)	0.63	11.29:	1.68	0.80	-6.28	-5.40
N2298	16.11	-1.85	0.33	9.39	0.78	0.15	-6.39	-5.76
N6544	15.00	-1.56	0.47	7.48:	1.44	0.74	-7.05	-6.35
N6352	15.15	-0.51	0.79	7.79	1.08	0.19	-6.57	-5.68
N6528	16.67	+0.12	0.90	9.58	1.52	0.62	-6.19	-5.29
N6426	18.00	-2.20	0.28	10.90	1.04	0.37	-6.82	-6.15
Rp 106	17.85	(-1.05)	0.63	10.9	(0.93)	0.24	-6.32	-5.63
N6325	17.30	-1.44	0.51	10.15	1.67	0.88	-6.64	-5.85
N4833	15.45	-1.86	0.38	8.35	0.96	0.32	-6.72	-6.08
N288	15.31	-1.40	0.52	8.08:	0.66	0.03	-6.71	-6.08
N6362	15.34	-1.08	0.62	8.05	0.85	0.10	-6.67	-5.92
N6342	17.40	-0.62	0.75	9.52	1.28	0.44	-7.13	-6.29
N6717	15.73	-1.32	0.54	8.35	1.00	0.25	-6.84	-6.09
N5927	16.60	-0.30	0.85	8.00	1.30	0.46	-7.75	-6.91
N6171	15.70	-0.99	0.64	7.76	1.10	0.33	-7.30	-6.53
N6453	17.70	-1.53	0.48	10.22	1.33	0.65	-7.00	-6.32
N5466	16.60	-2.22	0.27	9.16	0.67	0.00	-7.17	-6.50
N6101	16.60	-1.81	0.40	9.17	0.69	0.04	-7.03	-6.38
N6144	16.6:	-1.75	0.42	9.00	0.95	0.32	-7.18	-6.55
N6981	16.99	-1.54	0.48	9.22	0.74	0.03	-7.29	-6.58
N5897	16.30	-1.68	0.44	8.42	0.74	0.06	-7.44	-6.76
N6304	16.15	-0.59	0.76	8.27	1.33	0.52	-7.12	-6.31
N6235	16.66	-1.40	0.52	8.93	1.05	0.36	-7.21	-6.52
N6397	12.90	-1.91	0.37	5.25	0.73	0.18	-7.28	-6.73
N5053	16.67	-2.3	0.25	9.00	0.63	0.01	-7.42	-6.80
N3201	14.75	-1.61	0.46	6.91	0.97	0.21	-7.38	-6.62
N6517	18.0:	-1.34	0.54	10.10	1.76	1.08	-7.36	-6.68
N6779	16.20	-1.94	0.36	8.37	0.86	0.20	-7.47	-6.81
N6934	16.82	-1.54	0.48	8.86	0.75	0.11	-7.48	-6.84
N6121	13.35	-1.33	0.54	5.35	1.03	0.40	-7.46	-6.83
N6584	15.90	-1.54	0.48	7.91	0.76	0.12	-7.51	-6.87
N6254	14.65	-1.60	0.46	6.64	0.89	0.28	-7.55	-6.94
N6681	15.60	-1.51	0.49	7.79	0.72	0.06	-7.32	-6.66
N6712	16.25	-1.01	0.64	8.10	1.15	0.46	-7.51	-6.82
N6652	16.70	-0.89	0.67	8.54	0.94	0.10	-7.49	-6.65
N6809	14.35	-1.82	0.39	6.28	0.72	0.11	-7.68	-7.07
N6553	16.5:	-0.29	0.85	8.26	1.73	1.00:	-7.39	-6.66
N7006	18.72	-1.59	0.46	10.60	0.75	0.05	-7.66	-6.96
N5694	18.50	-1.92	0.36	10.21	0.70	0.10	-7.93	-7.33
N6752	13.85	-1.54	0.48	5.32	0.67	0.04	-8.05	-7.42
IC 4499	17.80	-1.50	0.49	10.11	0.91	0.25	-7.20	-6.54
N1261	16.75	-1.31	0.55	8.34	0.70	0.04	-7.86	-7.20
N4372	15.60	-2.08	0.32	7.20	1.10	0.45	-8.08	-7.43
N7099	15.10	-2.13	0.30	6.89	0.60	0.05	-7.91	-7.36
N6760	16.50	-0.52	0.78	8.98	1.68	0.62	-6.74	-5.68
N5634	17.75	-1.82	0.39	9.48	0.68	0.05	-7.88	-7.25
N6284	16.85	-1.40	0.52	8.87	1.01	0.28	-7.46	-6.73
N6293	16.40	-1.92	0.36	8.32	0.96	0.36	-7.72	-7.12

TABLE 1—*Continued*

Name (1)	$V(\text{HB})$ (2)	$[\text{Fe}/\text{H}]$ (3)	$M_V^o(\text{RR})$ (4)	$V(\text{GC})$ (5)	$(B-V)$ (6)	E_{B-V} (7)	M_V^o (8)	M_B^o (9)
N4590	15.60	-2.09	0.31	7.27	0.62	0.05	-8.02	-7.45
N6139	17.80	-1.65	0.45	9.05	1.40	0.75	-8.30	-7.65
N6093	15.86	-1.68	0.44	7.31	0.84	0.20	-8.11	-7.47
N1904	16.20	-1.69	0.43	7.70	0.64	0.01	-8.07	-7.44
N6273	16.95	-1.68	0.44	6.81	1.01	0.37	-9.70	-9.06
N362	15.40	-1.27	0.56	6.84	0.78	0.04	-8.00	-7.26
N6723	15.40	-1.09	0.61	6.84	0.77	0.03	-7.95	-7.21
N6229	18.10	-1.54	0.48	9.36	0.72	0.00	-8.26	-7.54
N6333	16.10	-1.78	0.41	7.79	0.97	0.35	-7.90	-7.28
N5946	17.50	-1.37	0.53	8.44	1.29	0.56	-8.53	-7.80
N6626	15.68	-1.44	0.51	6.90	1.08	0.38	-8.27	-7.57
N6341	15.05	-2.24	0.27	6.52	0.62	0.02	-8.26	-7.66
N6864	17.45	-1.32	0.54	8.59	0.87	0.16	-8.32	-7.61
N6218	14.90	-1.61	0.46	6.05	0.83	0.18	-8.39	-7.74
N5286	16.20	-1.79	0.40	7.37	0.89	0.24	-8.43	-7.78
N1851	16.10	-1.36	0.53	7.10	0.76	0.02	-8.47	-7.73
N5986	1650	-1.67	0.44	7.61	0.89	0.26	-8.45	-7.82
N6541	15.10	-1.83	0.39	6.31	0.75	0.13	-8.40	-7.78
N5824	18.60	-1.87	0.38	9.09	0.75	0.14	-9.13	-8.52
N6656	14.15	-1.75	0.42	5.16	0.99	0.36	-8.57	-7.94
N6205	14.95	-1.65	0.45	5.84	0.67	0.02	-8.66	-8.01
N6356	17.67	-0.62	0.75	8.24	1.13	0.30	-8.68	-7.85
N6539	16.60	-0.66	0.74	8.89	1.83	0.94	-6.97	-6.08
N5272	15.65	-1.66	0.44	6.30	0.69	0.01	-8.91	-8.23
N5024	16.94	-2.04	0.33	7.66	0.65	0.00	-8.95	-8.30
N5904	15.11	-1.40	0.52	5.68	0.71	0.03	-8.91	-8.23
N6316	17.80	-0.47	0.80	8.11	1.40	0.52	-8.89	-8.01
N7089	16.05	-1.62	0.45	6.56	0.67	0.02	-9.04	-8.39
N7078	15.86	-2.15	0.30	6.26	0.68	0.05	-9.30	-8.67
N6402	17.50	-1.39	0.52	7.61	1.27	0.59	-9.37	-8.69
N104	14.06	-0.71	0.73	4.02	0.88	0.04	-9.31	-8.47
N2808	16.19	-1.37	0.53	6.24	0.90	0.24	-9.42	-8.76
N6715	17.71	-1.42	0.51	7.68	0.85	0.15	-9.52	-8.82
N6388	16.90	-0.74	0.72	6.80	1.18	0.34	-9.38	-8.54
N5139	14.52	-1.59	0.46	3.85	0.8	0.15	-10.21	-9.56

The corresponding value for the MW clusters obtained by averaging columns (8) and (9) of Table 1 are

$$\langle M_V \rangle_{0\text{MW}} = -7.60 \pm 0.11, \quad (4)$$

$$\sigma(M) = 1.07 \pm 0.11$$

in V , and

$$\langle M_B \rangle_{0\text{MW}} = -6.90 \pm 0.11 \quad (5)$$

with the same dispersion in B , both values neglecting the five faintest clusters. This is justified by inspection of the distribution of the data in Table 1. It is clear that the faint wing is non-Gaussian.

The middle panel of Figure 1 shows the distribution of the B -band M31 cluster data. Because the top and middle panels are so similar, they are combined in the bottom panel which we adopt to be the calibration of the local GCLF. Treating the combined data with proper weights as a single sample gives the peak absolute magnitudes in both V and B as

$$\langle M_V \rangle_{0\text{M31+MW}} = -7.62 \pm 0.08 \quad (6)$$

with

$$\sigma(M) = 1.02 \pm 0.08$$

and

$$\langle M_B \rangle_{0\text{M31+MW}} = -6.93 \pm 0.08 \quad (7)$$

with the same dispersion in B .

The average of the $\langle M_V \rangle_0$ and $\langle M_B \rangle_0$ turnover magnitudes obtained by Secker & Harris (1993), giving the M31 and MW data equal weight, is $\langle M_V \rangle_0 = -7.42$ and $\langle M_B \rangle_0 = -6.73$. The mean difference between our calibration here (eqs. [6] and [7]) and that of Secker & Harris (eqs. [8] and [9]) is 0.20 mag, our calibration being brighter. This is primarily a consequence of our brighter adopted absolute magnitude for the RR Lyrae stars and for our different precept of the $(m-M)_{\text{AV}}$ modulus for M31.

4. THE GLOBULAR CLUSTER DISTANCE TO THE VIRGO CORE

4.1. The Virgo Cluster Turnover Luminosity

Three sets of data exist for the GCLF in E galaxies in the direction of the Virgo cluster.

1. Photometry in the Gunn g band by Cohen (1988) gave cluster LFs for NGC 4406, NGC 4472, and NGC 4486.

2. Photometry in the B band was obtained for NGC 4486 by van den Bergh, Pritchett, & Grillmair (1985).

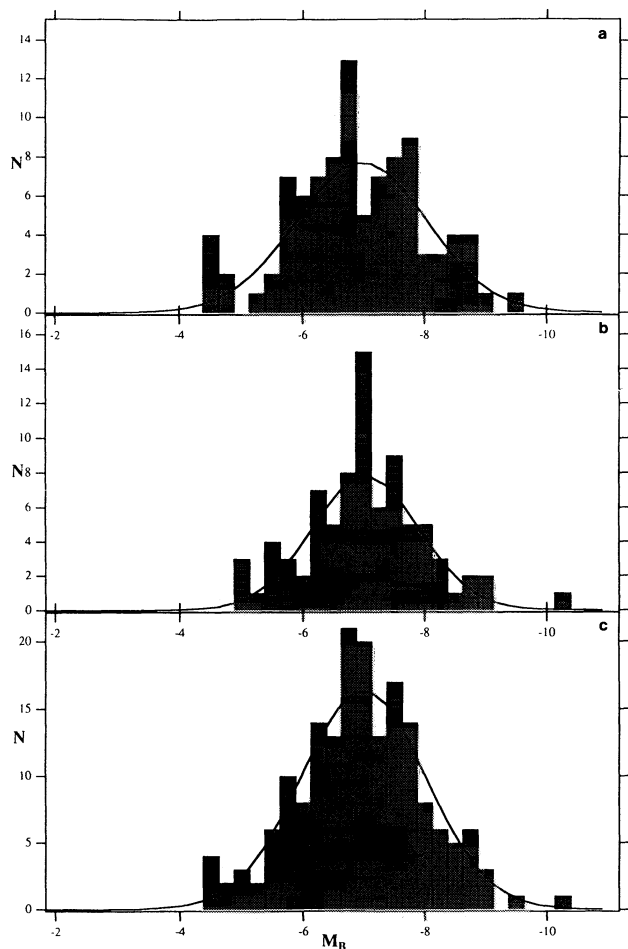


FIG. 1.—(a) The luminosity function in M_B for the Milky Way sample of 100 clusters in Secker's (1992) list selected as those clusters "that an outside observer would most easily detect." The cluster distance scale is based on the apparent magnitude of the horizontal branch and on the RR Lyrae calibration of equation (1), based on the Oosterhoff period relation via the pulsation equation. (b) The luminosity function of a representative sample of M31 halo clusters also from Secker, based on an apparent V modulus of $(m - M)_{AV} = 24.68$. (c) The combined luminosity function of panels (a) and (b). The turnover absolute magnitude is $\langle M_B \rangle_0 = -6.93 \pm 0.08$ as eq. (7) of the text. The V -band absolute magnitude of the turnover is $\langle M_V \rangle_0 = -7.62 \pm 0.08$ from eq. (6), based on the individual $(B - V)_0$ colors in Table 1.

3. Photometry also in the B band for NGC 4365, NGC 4472, and NGC 4649 was obtained by Harris et al. (1991). Harris (1988), reviewing the data by HAPV (that had not been published at the time) adopted $\langle B \rangle_0 = 24.7 \pm 0.25$ for the average turnover luminosity for the four galaxies NGC 4365, 4472, 4486, and 4649. The data for each galaxy were displayed in his Figure 5.

HAPV, reviewing the same data but using different statistical precepts, concluded that $\langle B \rangle_0 = 24.77 \pm 0.07$, $\sigma(M) = 1.46 \pm 0.07$. This is a mean of three ways to analyze the data (their Table 5). The first (their case a) is by leaving $\sigma(M)$ free between the data sets for the individual galaxies, making a Gaussian fit for each data set and averaging to give $\langle B \rangle_0 = 24.71 \pm 0.12$, with $\langle \sigma(M) \rangle = 1.46$. The second uses a single Gaussian fit keeping $\sigma(M)$ constant at 1.46 mag for the total sample. The result is $\langle B \rangle_0 = 24.77$. The third is a maximum-

likelihood solution again using $\sigma(M) = 1.46$. This gives $\langle B \rangle_0 = 24.78 \pm 0.06$.

The same data, but without NGC 4486, were reanalyzed by Secker & Harris (1993), again using a maximum-likelihood calculation but now (1) correcting in a more detailed way by simulations for background contamination, and (2) using bias corrections (a second-order effect, corrected to first order by HAPV) near the sample limit. Besides excluding NGC 4486, Secker & Harris also exclude NGC 4365 on the supposition that it is in the background, associated with the W cloud (Tonry, Ajhar, & Luppino 1990). Their two-galaxy solution gave $\langle B \rangle_0 = 24.45 \pm 0.10$.

Accepting for the moment their premise that NGC 4365 may be in the background, and therefore excluding it, but including the data for NGC 4486 used by HAPV of $\langle B \rangle_0(\text{N4486}) = 24.78 \pm 0.13$, gives the weighted mean from the three galaxies of Table 5 of HAPV (cf. also Table 2 here) as

$$\langle B \rangle_0 = 24.71 \pm 0.07, \quad (8)$$

which we adopt. We adopt a reasonable external error of ± 0.20 mag in the calculations that result in equation (9) in the next section. Also in the next section we reanalyze the data on the basis that NGC 4365 is in fact not in the background but is in the "core" of subcluster B.

4.2. The Distance Modulus to the Virgo Cluster

First, excluding NGC 4365, the three galaxies remaining in the analysis (NGC 4472, NGC 4486, and NGC 4649) are from different parts of the Virgo complex as defined by the adopted "cluster members" in the Virgo cluster catalog (Binggeli, Sandage, & Tammann 1985, hereafter BST). NGC 4472 is the central E galaxy in the Virgo subcluster B (Binggeli, Tammann, & Sandage 1987, hereafter BTS) and is the brightest E galaxy in the Virgo complex. NGC 4486 is the brightest E galaxy in Virgo subcluster A. NGC 4649 is an outlier of the subcluster A.

The question whether cluster A and B are at the same distance has been discussed from the kinematic point of view by BTS. From five relative distance indicators the modulus difference is at a level of only 0.1 mag (Tammann 1988). We therefore use the combined data of the three mentioned galaxies as defining the average distance of the E galaxy Virgo "core." We assume that the analysis of the cosmic (undisturbed) redshift of this complex, freed from all streaming motions as derived by the method of Jerjen & Tammann (1993), refers to this mean distance.

Applying the mean absolute magnitude of the turnover luminosity of $\langle M_B \rangle_0 = -6.93 \pm 0.15$ (estimated external error) from equation (7) to the adopted Virgo turnover magnitude in equation (8) with its adopted external error of 0.2 mag gives a Virgo cluster modulus (neglecting any possible, small amount of Galactic absorption) as

$$(m - M)^0 = 31.64 \pm 0.25 \text{ (external)}, \quad (9)$$

or a distance of 21.3 ± 2.7 Mpc.

An alternative interpretation is to assume NGC 4365 to be a member of the Virgo cluster. It lies near the core of the Virgo subcluster B (Binggeli, Popescu, & Tammann 1993). Its seemingly large distance according to Tonry et al. (1990) is probably incorrect due to its exceptionally high metallicity, indicated by its large Mg_2 index (Tammann 1992). The method, as initially applied, is sensitive to metallicity. Moreover, considering a self-consistent Virgocentric infall model (Kraan-Korteweg 1986) one expects the galaxy, with a heliocentric velocity of

TABLE 2
THE TURNOVER LUMINOSITIES OF THE GLOBULAR CLUSTER LUMINOSITY FUNCTIONS
AND DERIVED VIRGO CLUSTER MODULI

GALAXY	GAUSS FIT ($\sigma = 1.46$)			t_5 FIT		
	m_B^o	m_V^o	$(m-M)_{\text{Virgo}}$	m_B^o	m_V^o	$(m-M)_{\text{Virgo}}$
Galaxy.....	-6.90 ^a	-7.60 ^a	...	same	same	...
	± 0.11	± 0.11	...	same	same	...
M31	-7.01 ^a	-7.70 ^a	...	same	same	...
	± 0.20	± 0.20	...	same	same	...
MW + M31.....	-6.93 ^a	-7.62 ^a	...	same	same	...
	± 0.08	± 0.08	...	same	same	...
NGC 4365	25.15	...	32.08	25.31	...	32.24
	± 0.18	...	± 0.20	± 0.35	...	± 0.36
NGC 4472	24.70	...	31.63	24.61	...	31.54
	± 0.11	...	± 0.14	± 0.15	...	± 0.17
NGC 4486	24.78	...	31.71
	± 0.13	...	± 0.15
NGC 4636	24.22	31.84	...	24.11	31.73
	...	± 0.1	± 0.13	...	± 0.1	± 0.16
NGC 4649	24.65	...	31.58	24.33	...	31.26
	± 0.14	...	± 0.16	± 0.13	...	± 0.15
Weighted mean $(m-M)_{\text{Virgo}}$	31.75 \pm 0.07			31.55 \pm 0.09		

^a The absolute turnover luminosity $\langle M \rangle_0$ from eqs. [2]–[7] is given. $\langle M \rangle_0$ is determined by treating σ as a free parameter.

$1240 \pm 12 \text{ km s}^{-1}$, either to lie in the Virgo cluster (100%) or at a distance of 78% in front or 139% in the back of the cluster. From the D_n - σ distance (Faber et al. 1989) by far the most probable case is that it belongs to the cluster. Its Hubble type (E) gives further support to assume that it lies in a high-density region.

We want further to include NGC 4536 (S0) for which Kissler et al. (1994) have determined a visual turnover magnitude for its globular cluster system. Forcing a Gaussian with the generally adopted dispersion of $\sigma = 1.46 \text{ mag}$ to their data, we obtain $\langle V \rangle_0 = 24.22 \pm 0.1$. (Their value is $\langle V \rangle_0 = 24.1 \pm 0.1$ for the turnover magnitude.) The distance modulus for NGC 4636 then becomes $(m-M)^o = 31.84 \pm 0.13$ using the calibration of equation (6). This is in agreement³ with the result of equation (9).

The data for all five galaxies are set out in Table 2. They give a mean-weighted Virgo modulus of

$$(m-M)^o = 31.75 \pm 0.07.$$

Secker (1992) and Secker & Harris (1993) have proposed to use a t_5 fit instead of a Gaussian for the determination of the turnover magnitude. Using the $\langle B \rangle_{t(5)}$, respectively $\langle V \rangle_{t(5)}$ values from the original authors, we find (cf. Table 2) a weighted Virgo modulus of $(m-M)^o = 31.55 \pm 0.09$, which we take to be the same as equation (9) to within statistics.

As a compromise, we adopt the Virgo modulus given in equation (9). This is similar to that derived by Harris (1988) whose value was $(m-M) = 31.7$, and to that derived by HAPV who derived $(m-M) = 31.6$, but adopted 31.5. Hence the long

distance scale is derived by all parties using the simple version of the globular cluster method.

The result of this paper is not that we have derived a larger distance to the Virgo core than had been derived by Harris and his collaborators in their first discussions, but rather that we have believed this distance, whereas Secker & Harris did not. Our fundamentally different precept is that equation (7) applies to the Virgo globulars as well as to the local calibrators, and that adopting the contrary assumption of Secker & Harris opens the method of distances via globular clusters to a circularity.

5. DISCUSSION

5.1. Is the Turnover Luminosity Universal?

The adopted Virgo modulus of $(m-M)^o = 31.64 \pm 0.25$ is, as shown in the previous section, stable against different fitting procedures and the inclusion of different galaxies. Of course, this method stands or falls with the assumption that the turnover luminosity of the luminosity function of the outer GCs is the same for the calibrating spirals (Milky Way, M31) as for the five E/S0 target galaxies in Virgo.

Several doubts have been raised against the basic assumption. The most direct is the fact that the bell-shaped GCLFs in the Milky Way and M31 are significantly narrower than those in the Virgo ellipticals. But of course, that alone does not allow any conclusions on the mean luminosities. The agreement of the turnover luminosity of two spirals (MW, M31) is impressive, as is the agreement within the errors between the five Virgo ellipticals for which $\langle B \rangle_0$ values are available.

A caveat against the universality of the turnover luminosity was suggested by Wagner, Richtler, & Hopp (1991) who found from B, V photometry that the total GC system of NGC 1399 in the Fornax cluster has a very red mean color of $(B-V)^o = 0.87 \pm 0.04$. This is redder than $(B-V)^o = 0.81$ for the GCs of NGC 4486 from Cohen (1988), even neglecting a possible correction of any small amount of Galactic reddening. It is also significantly redder than the mean color of $(B-V)^o = 0.70 \pm 0.01$ of the 100 outer Galactic globulars in

³ Kissler et al. (1994) derive a distance modulus of $(m-M) = 31.2$ for NGC 4636 by adopting $\langle V \rangle_0(\text{turnover}) = 24.1$ and a calibration of $\langle M_V \rangle_0 = -7.1 \pm 0.3$. This turnover absolute magnitude is 0.52 mag fainter than our calibration in equation (6). Kissler et al. justify their use of such a faint calibration of the V -band turnover absolute magnitude by adopting the assumption of Secker & Harris that the Virgo cluster has a modulus of 30.89. But, of course, the distance modulus of Virgo cannot then be obtained in reverse from the globular cluster data because their calibration of $\langle M_V \rangle_0$ is itself based on the assumed cluster distance. The Kissler et al. argument is circular.

Table 1, and than the mean color of $(B-V)^0 = 0.69 \pm 0.01$ for Secker's sample of outer GCs in M31. However, the globulars of NGC 1399 become bluer with increasing distance from the galaxy center (Wagner et al. 1991, Fig. 8); the 150 outer GCs have a mean color of $(B-V)^0 = 0.67 \pm 0.02$ (no Galactic reddening assumed). The color gradient and the relative blueness of the outer GCs in NGC 1399 is also confirmed by Bridges, Hanes, & Harris (1991). As for the *outer* globular clusters there is, therefore, no evidence that their mean colors vary from galaxy to galaxy. But even if they do, the conclusion that the GCLF is universal, despite differences in the color distribution from galaxy to galaxy has been presented by Ajhar, Blakeslee, & Tonry (1994).

A direct test whether the turnover luminosity depends on the Hubble type is provided by nearby dwarf ellipticals (dEs). The five GCs in the Fornax dwarf system with known B , V photometry (Harris & Racine 1979) have mean absolute magnitudes of $\langle M_B \rangle_0 = -7.01 \pm 0.41$ and $\langle M_V \rangle_0 = -7.61 \pm 0.43$ in fortuitous agreement with the calibration in equations (6) and (7) from the Galaxy and M31. Here, the Fornax distance is assumed to be $(m-M)_{AV} = 21.25$ and with $E(B-V) = 0.03$, $(m-M)_{AB} = 21.28$ based on a mean horizontal branch (RR Lyrae) magnitude of $\langle B \rangle = 21.60 \pm 0.04$ for the five clusters and a mean metallicity of $[Fe/H] = -1.96$ (Buonanno et al. 1985), as well as on the calibration in equation (1).

The sample can be increased by 12 GCs with known B and V in NGC 147, NGC 185 (Harris & Racine 1979), and NGC 205 (Harris & Racine 1979; Battistini et al. 1987). Only the peculiar cluster, Hubble V in NGC 205, is excluded (Da Costa & Mould 1988). The mean apparent distance moduli are assumed to be the same as for M31 (cf. § 3). One then obtains $\langle M_B \rangle_0 = -6.69 \pm 0.24$ and $\langle M_V \rangle_0 = -7.31 \pm 0.23$. The differences to the adopted calibration of equations (6) and (7) amount to $\Delta \langle M_B \rangle_0 = 0.24 \pm 0.25$, and $\Delta \langle M_V \rangle_0 = 0.31 \pm 0.24$, which are not significant.

The problem of the universality of $\langle M \rangle_0$ can also be approached from the other direction by noting that the assumption of universality leads to a Virgo cluster distance that is in close agreement with the independent methods set out in § 6.2, and discussed elsewhere (Tammann 1992; Sandage 1993c). A discrepancy between the turnover luminosity in spirals and ellipticals by more than ~ 0.2 mag would destroy this agreement.

5.2. Is There a Dependence of the Turnover Luminosity on the Dispersion of the GCLF?

Brief reference was made in §§ 1.1 and 1.2 to the suggestion of Secker & Harris (1993) that the turnover luminosity, $\langle M \rangle_0$, depends on the dispersion of the GCLF. These authors concluded that $\langle M \rangle_0$ is not universal but that it becomes fainter as the dispersion increases (their Fig. 10).

To carry forward an investigation of this precept one must of course give up the assumption of a constant dispersion for all Virgo E's and then make a free fit to the data, determining the turnover apparent magnitude, $\langle m \rangle_0$, and σ_B *simultaneously* for each galaxy. In what follows we use a Gauss function to fit the observations instead of the t_5 function of Secker & Harris (1994). This convention does not affect the conclusion.

We adopt the *free-fit* values for $\langle m \rangle_0$ and σ_B from § 3 for the MW and M31, from HAPV for the four Virgo ellipticals, and from Kissler et al. (1994) for NGC 4636. The latter galaxy was observed in V but the parameters were converted to the B system by adopting $\langle m_B \rangle_0 = \langle m_V \rangle_0 + 0.69$, and $\sigma_B = \sigma_V$.

We now argue that the dependence of $\langle M_B \rangle_0$ on σ_B advocated by Secker & Harris (1993) rests on three very subtle assumptions.

1. They conclude that the variation in σ_B in the Virgo galaxies is in fact significant. However, the variation of the σ_B values from 1.46 is only marginal, considering the individual errors. Indeed, Harris (1988) concluded "the LF dispersion is not sensibly different from one galaxy to another." Note that Ajhar et al. (1994) also used a constant value of σ_R to fit their data for Virgo cluster E's, although we disagree⁴ with their adopted calibration of $\langle M_R \rangle_0 = -7.5$.

2. As discussed before, Secker & Harris adopt a priori the small Virgo distance, and further assume NGC 4365 to be in the background. If, instead, we adopt the revised parameters from the sources cited above (§ 3, HAPV, and Kissler et al.), any remaining correlation between $\langle M_B \rangle_0$ and σ_B hinges entirely on NGC 4486 and NGC 4365, both of which have large errors in their data. Having (marginally) larger σ_B , their $\langle M_B \rangle_0$ values are fainter by 0.62 ± 0.48 mag and 0.70 ± 0.46 mag than the remaining five galaxies in Table 2. This is a weak signal, insufficient in our opinion to abandon universality.

3. Any search for a dependence of $\langle M_B \rangle_0$ on σ_B rests on the assumption that the fitting provides *uncorrelated* values of these parameters. For a complete GC population, that is, where the bright and faint wings of the distribution are observed, that would be the case. However, for the GCs in Virgo, only the bright wing of the LF is determined (Fig. 14 of Ajhar et al. 1994), leaving an appreciable margin as to the turnover luminosity. This automatically has the result that if one chooses increasing values of the dispersion, then the turnover luminosity is pushed to fainter magnitudes by fitting only the bright wing.

The actual procedure is to find $\langle m \rangle_0$ for different *preselected* σ -values and to then perform the χ^2 test. One adopts the $(\sigma, \langle m \rangle_0)$ pair which lies at the valley of the χ^2 variation. A typical illustration of the situation is given by McLaughlin, Harris, & Hanes (1994, their Table 10). When they vary the σ_V of the GCLF from 1.50 to 2.10 mag, the turnover luminosity varies between $\langle m_V \rangle_0 = 23.65$ and 25.25 mag, becoming systematically fainter for larger σ -values.

Hence, σ , $\langle m \rangle_0$, and the error in $\langle m \rangle_0$ are always correlated unless the turnover point is exquisitely fixed by the observations. We believe that this effect is the main reason why NGC 4486 as well as NGC 4365 have seemingly unusual GCLFs.

However, this discussion on the possible peculiarity of NGC 4486 and NGC 4363 is only of academic interest here because

⁴ Our value for the R -band turnover absolute magnitude, based on the precepts of this paper, is $\langle M_R \rangle_0 = -8.12 \pm 0.13$ using $\langle V-R \rangle = 0.5 \pm 0.1$ (Ajhar et al. 1994, Fig. 9) and our adopted calibration of $\langle M_V \rangle_0(\text{turnover}) = -7.62 \pm 0.08$ from eq. (6). The difference of 0.62 mag between the Ajhar et al. zero point and the value we adopt in eq. (6) is essentially the difference between the long and short distance scales.

It is to be noted that the Ajhar et al. value of $\langle M_R \rangle_0(\text{turnover}) = -7.5$ contradicts the calibration using the clusters in M31 and the MW. This is the consequence of the short distance scale adopted by Tonry. The fundamental problem with Tonry's scale is seen directly from the discussion by Ajhar et al. (1994) in which, like Secker & Harris, they force the short distance scale of $(m-M)_{\text{Virgo}} \sim 30.9$ on the data to obtain their adopted $\langle M \rangle_0$ value. They must then admit that the calibration through M31 and the MW does not apply to the Virgo cluster ellipticals because the two zero points differ by 0.62 mag. Hence, the argument by Ajhar in fact requires that the GCLF cannot be universal, yet they argue to the contrary. We agree with their conclusion of universality but disagree with their calibration of $\langle M_R \rangle_0(\text{turnover}) = -7.5$. Either the GCLF is universal or it is not. If it is, then $H_0 \sim 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

the adopted Virgo distance from equation (9) depends only weakly on NGC 4486 due to the large error in $\langle m \rangle_0$, while NGC 4365 is not used at all.

6. THE VALUE OF THE HUBBLE CONSTANT

6.1. H_0 From the Present Discussion

The cosmic redshift of the Virgo core has been determined, freed from all velocity anomalies, by tying the Virgo cluster to distant groups and clusters that have themselves been corrected to the velocity frame of the cosmic microwave background (Tammann & Sandage 1985; Sandage & Tammann 1990; Jerjen & Tammann 1993). The latest solution by Jerjen & Tammann gives the cosmic redshift of Virgo relative to the CMB as

$$v_{(\text{cosmic})} = 1179 \pm 17 \text{ km s}^{-1}. \quad (10)$$

Dividing equation (10) by the distance from equation (9) as $D = 21.3 \pm 2.7$ gives

$$H_0 = 55 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (11)$$

where the error is external, based on the quoted error in equations (9) and (10).

6.2. Other Methods Favoring the Long Distance Scale

The distance scale derived here is based in principle only on Population II objects, that is, RR Lyrae stars and globular clusters. We have used for the local calibration of the globular cluster luminosities also the Population I Cepheid distance of M31, but our Virgo distance modulus would be changed by only 0.03 mag if we had based the local calibration solely on the Galactic globular clusters.

We would be remiss not to point out the agreement here with the distance scales derived from eight other methods. These are (1) angular sizes of galaxies in local distance-limited samples if the diameters of M31 and M101 are average (van der Kruit 1986; Tammann 1992; Sandage 1993c,d), (2) the Tully-Fisher method applied to the unbiased distance-limited 500 km s⁻¹ galaxy sample (Kraan-Korteweg & Tammann 1979), as calibrated by local group galaxies that themselves are calibrated by Cepheids (Sandage 1988, 1994), (3) the distance to the Virgo cluster derived from the Tully-Fisher method when corrected for observational selection bias (Kraan-Korteweg et al. 1988; Fouqué et al. 1990; Sandage, Tammann, & Federspiel

1995), (4) supernovae of Type Ia as calibrated directly from Cepheid distances (Sandage et al. 1992, 1994; Saha et al. 1994, 1995), (5) model calculations of three supernovae of Type Ia in the Virgo cluster (Müller & Höflich 1992; Branch & Khokhlov 1994) and of the late phases of two such Virgo objects (Ruiz-Lapuente et al. 1994), (6) supernovae of Type II as analyzed by Branch (1988), Branch et al. (1981), and Schmidt, Kirshner, & Eastman (1993) in their first analysis,⁵ (7) the $D_n - \sigma$ method using the bulges of the Galaxy, M31, and M81 as calibrators (Dressler 1987) as rediscussed by Tammann (1988) and Sandage & Tammann (1990), and (8) the two new physical methods concerned with gravitational lenses (e.g., Dahle, Maddox, & Lilje 1994), and the distortion of the microwave background seen through the centers of X-ray clusters (Birkinshaw 1994; Jones 1994).

It is also known why the Tully-Fisher method, when applied to incomplete, flux-limited samples in groups and clusters (Pierce & Tully 1992) that have been calibrated with the distance-limited samples of the local calibrators, give the incorrect short scale (Kraan-Korteweg et al. 1988; Federspiel, Sandage, & Tammann 1994; Sandage et al. 1995).

Of all the methods to the distance scale, the only two that currently favor the short distance scale are the planetary nebulae luminosity function and the surface brightness fluctuation method. There are reasons to suspect the planetary nebulae method because it uses the *maximum* of the PN luminosity function that is not sufficiently sharp at the bright end, giving rise to the sample-population-normalization problem (Bottinelli et al. 1991; Tammann 1993). Hence, it only remains how to adjudicate the single method of surface brightness fluctuations, not sufficiently understood to date, that favors the short distance scale.

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⁵ (However, the latter group changed their distances in a second analysis [Schmidt et al. 1994a, b] by using a different value of the dilution factor by which the emergent flux differs from a blackbody at the same temperature; the distances based on their dilution factor are thought to be too small by a factor of ~ 1.6 , according to calculations by others [Branch 1994] and similar difficulties with the expanding atmosphere models in interpreting the observational data are also discussed by Best & Wehrse 1994 and by Hafner, Baschek, & Wehrse 1994).

REFERENCES

- Ajhar, E. A., Blakeslee, J. P., & Tonry, J. L. 1994, *AJ*, 108, 2087
 Armandroff, T., & Zinn, R. 1988, *AJ*, 96, 92
 Arp, H. C. 1955, *AJ*, 60, 317
 Baade, W. 1944, *ApJ*, 100, 137
 ———. 1952, in *Trans. IAU VIII (Rome 1952 meeting)* (Cambridge: Cambridge Univ. Press), 397, 398
 ———. 1956, *PASP* 68, 5 (Bruce Medal Lecture)
 Baade, W., & Swope, H. H. 1963, *AJ*, 68, 435
 Battistini, P., Bonoli, F., Bracciosi, A., Federici, L., Fusi Pecci, F., Marano, B., & Borngen, F. 1987, *A&AS*, 67, 447
 Best, M., & Wehrse, R. 1994, *A&A*, 284, 507
 Binggeli, B., Popescu, C. C., & Tammann, G. A. 1993, *A&AS*, 98, 275
 Binggeli, B., Sandage, A., & Tammann, G. A. 1985, *AJ*, 90, 1681 (BST)
 Binggeli, B., Tammann, G. A., & Sandage, A. 1987, *AJ*, 94, 251 (BTS)
 Birkinshaw, M. 1994, in *Present and Future of the Cosmic Microwave Background*, ed. J. L. Sanz et al., in press
 Bottinelli, L., Gouguenheim, L., Paturel, G., & Terrikorpi, P. 1991, *A&A*, 252, 550
 Branch, D. 1988, in *Extragalactic Distance Scale*, ed. S. van den Bergh & C. J. Pritchet (ASP Conf. Series 4), 146
 ———. 1994, Lecture at the Seventh Marcel Grossmann Meeting (Stanford July 1994)
 Branch, D., Falk, S. W., McCall, M., Rybski, P., Umoto, A. K., & Wills, B. J. 1981, *ApJ*, 244, 780
 Branch, D., & Khokhlov, A. M. 1994, preprint; Symposium in honor of Sterling Colgate (Dordrecht: Elsevier Science)
 Bridges, T. Y., Hanes, D. A., & Harris, W. E. 1991, *AJ*, 101, 469
 Buonanno, R., Corsi, C. E., Fusi Pecci, F., Hardy, E., & Zinn, R. 1985, *A&A*, 152, 65
 Burstein, D., & Heiles, C. 1984, *ApJS*, 54, 33
 Caputo, F. 1988, *A&A*, 189, 70
 Caputo, F., & Castellani, V. 1975, *Mem. Soc. Astron. Ital.*, 46, 303
 Carney, B. W., Storm, J., & Jones, R. V. 1992, *ApJ*, 386, 663
 Castellani, V. 1983, *Mem. Soc. Astron. Ital.*, 54, 141
 Catelan, M. 1992, *A&A*, 261, 457
 Cohen, J. G. 1988, *AJ*, 95, 682
 Da Costa, G. S., & Mould, J. R. 1988, *ApJ*, 334, 159
 Dahle, H., Maddox, S. J., & Lilje, P. B. 1994, *Univ. Oslo*, preprint 19
 Djorgovski, S. G. 1993, in *Structure and Dynamics of Globular Clusters*, ed. S. G. Djorgovski & G. Mevian, ESO preprint 932, 39
 Dorman, B. 1992, *ApJS*, 81, 221
 ———. 1993, in *The Globular Cluster-Galaxy Connection*, ed. G. H. Smith & J. P. Brodie (ASP Conf. Ser. 48), 198
 Dressler, A. 1987, *ApJ*, 317, 1

- Elson, R. A., & Walterbos, R. A. M. 1988, *ApJ*, 333, 594
- Faber, S. M., Wegner, G., Burstein, D., Davies, R. L., Dressler, A., Lynden-Bell, D., & Terlevich, R. J. 1989, *ApJS*, 69, 763
- Feast, M. W., & Walker, A. R. 1987, *ARA&A*, 25, 345
- Federspiel, M., Sandage, A., & Tammann, G. A. 1994, *ApJ*, 430, 29
- Fernie, J. D. 1990, *ApJ*, 354, 295
- Fouqué, P., Bottinelli, L., Gouguenheim, L., & Paturel, G. 1990, *ApJ*, 349, 1
- Freedman, W. L., & Madore, B. F. 1990, *ApJ*, 365, 186
- Fusi Pecci, F. 1988, in *IAU Symp. 126, Globular Cluster Systems in Galaxies*, ed. J. Grindlay & A. G. Davis Philip (Dordrecht: Reidel), 543
- Gieren, W. P., Barnes, T. G., & Moffett, T. J. 1993, *ApJ*, 418, 135
- Hafner, M., Baschek, B., & Wehrse, R. 1994, *A&A*, in press
- Hanes, D. A. 1980, in *Globular Clusters*, NATO Adv. Study Inst., ed. D. Hanes & B. Madore (Cambridge: Cambridge Univ. Press), 213
- Harris, W. E. 1988, in *The Extra Galactic Distance Scale*, ed. S. van den Bergh & C. J. Pritchet (ASP Conf. Series 4), 231
- . 1991, *ARA&A*, 29, 543
- Harris, W. E., Allwright, J. W. B., Pritchet, C. J., & van den Bergh, S. 1991, *ApJS*, 76, 115 (HAPV)
- Harris, W. E., & Racine, R. 1979, *ARA&A*, 17, 241
- Hubble, E. 1929, *ApJ*, 69, 103
- . 1932, *ApJ*, 76, 44
- Iben, I. 1971, *PASP*, 83, 697
- Iben, I., & Huchra, J. 1971, *A&A*, 14, 293
- Iben, I., & Rood, R. T. 1970, *ApJ*, 161, 587
- Jacoby, G. H., et al. 1992, *PASP*, 104, 599
- Jerjen, H., & Tammann, G. A. 1993, *A&A*, 273, 354
- Jones, M. 1994, *Ap. Lett. Comm.*, in press
- Kissler, M., Richtler, T., Held, E. V., Wagner, S. J., & Capaccioli, M. 1994, *A&A*, 287, 463
- Kraan-Korteweg, R. C. 1986, *A&AS*, 66, 255
- Kraan-Korteweg, R. C., Cameron, L. M., & Tammann, G. A. 1988, *ApJ*, 331, 620
- Kraan-Korteweg, R. C., & Tammann, G. A. 1979, *Astron. Nach.*, 300, 181
- Lee, M. G., Freedman, W. L., & Madore, B. F. 1993, *ApJ*, 417, 553
- Lee, Y.-W. 1990, *ApJ*, 363, 159
- Madore, B. F., & Freedman, W. L. 1991, *PASP*, 103, 933
- McClure, R. D., & Racine, R. 1969, *AJ*, 74, 1000
- McLaughlin, D. E., Harris, W. E., & Hanes, D. A. 1994, *ApJ*, 422, 486
- Morgan, S. M. 1992, in *IAU Colloq. 139, New Perspectives on Stellar Pulsation and Pulsating Variable Stars*, ed. J. M. Nemec & J. M. Matthews (Cambridge: Cambridge Univ. Press), 307
- Müller, E., & Höflich, P. 1992, *MPA preprint*, 709
- Oosterhoff, Th. 1939, *Observatory*, 62, 104
- . 1944, *Bull. Astr. Inst. Netherlands*, 10, 55
- Peterson, C. J. 1993, in *Structure & Dynamics of Globular Clusters*, ed. S. G. Djorgovski & G. Meylan, ESO preprint no. 932, 13
- Pierce, M., & Tully, R. B. 1992, *ApJ*, 387, 47
- Preston, G. 1959, *ApJ*, 130, 507
- Pritchet, C. J., & van den Bergh, S. 1987, *ApJ*, 346, 517
- Racine, R., & Harris, W. E. 1992, *AJ*, 104, 1068
- Racine, R., & Shara, M. 1979, *AJ*, 84, 1694
- Renzini, A. 1983, *Mem. Soc. Astron. Ital.*, 54, 335
- Rood, R. T. 1973, *ApJ*, 184, 815
- Ruiz-Lapuente, P., Kirshner, R. P., Phillips, M. M., Challis, P. M., Schmidt, B. P., Filippenko, A. V., & Wheeler, J. C. 1994, *Harvard preprint* 3855
- Saha, A., Freedman, W. L., Hoessel, J. G., & Mossman, A. E. 1992, *AJ*, 104, 1072
- Saha, A., & Hoessel, J. 1990, *AJ*, 99, 97
- Saha, A., Hoessel, J. G., & Mossman, A. E. 1990, *AJ*, 100, 108
- Saha, A., Labhardt, L., Schwengeler, H., Macchetto, D., Panagia, N., Sandage, A., & Tammann, G. A. 1994, *ApJ*, 425, 14
- Saha, A., Sandage, A., Labhardt, L., Schwengeler, H., Tammann, G. A., Panagia, N., & Macchetto, D. 1995, *ApJ*, 438, 8
- Sandage, A. 1953, *AJ*, 58, 61
- . 1958, in *Stellar Populations*, ed. D. O'Connell (Specola Vaticana), *Ri. Astr.*, 5, 41
- . 1982, *ApJ*, 252, 553
- . 1988, *ApJ*, 331, 605
- . 1993a, *AJ*, 106, 687 (S93a)
- . 1993b, *AJ*, 106, 703 (S93b)
- . 1993c, *ApJ*, 402, 3
- . 1993d, *ApJ*, 404, 419
- . 1994, *ApJ*, 430, 13
- Sandage, A., & Cacciari, C. 1990, *ApJ*, 350, 645
- Sandage, A., Katem, B. N., & Sandage, M. 1981, *ApJS*, 46, 41
- Sandage, A., Saha, A., Labhardt, L., Schwengeler, H., Tammann, G. A., Panagia, N., & Macchetto, D. 1994, *ApJ*, 424, L11
- Sandage, A., Saha, A., Tammann, G. A., Panagia, N., & Macchetto, D. 1992, *ApJ*, 401, L7
- Sandage, A., & Tammann, G. A. 1969, *ApJ*, 157, 683
- . 1990, *ApJ*, 365, 1
- Sandage, A., Tammann, G. A., & Federspiel, M. 1995, *ApJ*, in press
- Schmidt, B. P., Kirshner, R. P., & Eastman, R. G. 1993, in *Observational Cosmology*, ed. G. Chincarini, A. Iovino, T. Maccacaro, & D. Maccagni (ASP Conf. Series 51), 30
- Schmidt, B. P., et al. 1994a, *AJ*, 107, 1444
- . 1994b, *ApJ*, 432, 42
- Seares, F. H., Kapteyn, J. C., & van Rhijn, P. J. 1930, *Mount Wilson Catalog of Photographic Magnitudes in Selected Areas 1-139* (Carnegie Institution Pub. 402, Carnegie Institution of Washington)
- Secker, J. 1992, *AJ*, 104, 1472
- Secker, J., & Harris, W. E. 1993, *AJ*, 105, 1358
- Shapley, H. 1930, *Star Clusters*, Harvard Monograph (Cambridge: Harvard Univ. Press)
- . 1953, *Proc. Natl. Acad. Sci.*, 39, 349
- Simon, N. R., & Clement, C. M. 1993, *ApJ*, 410, 526
- Stellingwerf, R. F. 1975, *ApJ*, 195, 441
- Sweigart, A. V. 1987, *ApJS*, 65, 95
- Sweigart, A. V., & Gross, P. G. 1976, *ApJS*, 32, 367
- Sweigart, A. V., Renzini, A., & Tornambè, A. 1987, *ApJ*, 312, 762
- Tammann, G. A. 1988, in *The Extragalactic Distance Scale*, ed. S. van den Bergh & C. J. Pritchet (ASP Conf. Series 4), 282
- . 1992, *Phys. Scripta*, T43, 31
- . 1993, in *IAU Symp. 155, Planetary Nebulae*, ed. R. Weinberger & A. Acker (Dordrecht: Kluwer), 515
- Tammann, G. A., & Sandage, A. 1985, *ApJ*, 294, 81
- Tonry, J. L., Ajhar, E. A., & Luppino, G. A. 1990, *AJ*, 100, 1416
- van den Bergh, S. 1964, *AJ*, 69, 610
- . 1969, *ApJS*, 19, 145
- van den Bergh, S., Pritchet, C., & Grillmair, C. 1985, *AJ*, 90, 595
- van der Kruit, P. 1986, *A&A*, 157, 230
- Wagner, W., Richtler, T., & Hoop, U. 1991, *A&A*, 241, 399
- Walker, A. R. 1992, *ApJ*, 390, L81
- Webbink, R. 1985, in *IAU Symp. 113, Dynamics of Star Clusters*, ed. J. Goodman & P. Hut (Dordrecht: Reidel), 541
- Zinn, R., & West, M. 1984, *ApJS*, 55, 45