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THE LEAST-ACTION METHOD, COLD DARK MATTER, AND Ω

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ABSTRACT

Peebles has suggested an interesting technique, called the least-action method, to trace positions of galaxies back in time. This method applied on the Local Group galaxies seems to indicate that we live in an $\Omega \approx 0.1$ universe. We have studied a CDM N-body simulation with $\Omega = 0.2$ and $H = 50$ km s^{-1} Mpc^{-1} and compared trajectories traced back by the least-action method with the ones given by the center of mass of the CDM halos. We show that the agreement between these sets of trajectories is at best qualitative. We also show that the line-of-sight peculiar velocities of halos are underestimated. This discrepancy is due to orphans, i.e., CDM particles which do not end up in halos. We vary the value of Ω in the least-action method until the line-of-sight velocities agree with the CDM ones. The best value for this Ω underestimates one of the CDM simulations by a factor of 4–5.

Subject headings: dark matter — galaxies: clusters: general — galaxies: formation — Local Group

1. INTRODUCTION

The density parameter Ω is one of the most important parameters characterizing our universe. There are many methods of measuring Ω, but a factor of 10 uncertainty in its value remains. Recently Peebles suggested that it may be possible to estimate its value in the Local Neighborhood, by tracing Local Group galaxies back in time.

Peebles (1989, 1990, 1994) used the principle of least action to find complete trajectories for Local Group galaxies. The idea is to assume that galaxies growing out of small density perturbations in the early universe have negligible peculiar velocities with respect to the Hubble flow. This is a reasonable assumption, since we know that the microwave background has very small anisotropies. Using zero initial peculiar velocities as one boundary condition and the present positions of the galaxies as the other, trial orbits are iteratively varied so as to minimize the action. The method has been criticized, since the galaxies are treated as point particles throughout their history, even though the size of the galaxies must be comparable to their separation at early times. However, the least-action principle leaves the final velocities of the galaxies unconstrained, and its ability to reproduce the observed radial velocities remains a powerful test of the validity of the trajectories. For the Local Group galaxies, Peebles has obtained remarkable agreement between the observed radial velocities and those calculated from the least-action principle. Obtaining reliable trajectories for nearby galaxies might shed light into the origin and their angular momentum (Dunn & Laflamme 1993).

Although the least-action method (LAM) provides a powerful tool for investigating galaxy orbits, its predictions are only as good as the assumptions upon which they stand. These are (1) that galaxies initially have negligible peculiar velocities with respect to the Hubble flow, (2) galaxies can be represented as point particles throughout their history, (3) mergers have little effect on a galaxy's motion, and (4) light traces mass. One way to test the validity of some of these assumptions is to apply the LAM to a numerical simulation of the universe. In the simulation we have complete information about the particle trajectories, which we compare with the predictions made by the LAM. In this *Letter* we use a cold dark matter (CDM) simulation. Although CDM may not be able to reproduce all the observable features of our universe, it does at least represent a *possible* universe in which all the particles are governed by Hamiltonian dynamics.

In the first section we give details of the simulation we have used and comment on the groups that we have studied. Second, we compare the trajectories obtained from the CDM simulation and the LAM and compare their "line-of-sight" velocities. Finally we comment on the origin of the discrepancy.

2. THE LEAST-ACTION METHOD

Peebles's use of the LAM sèlects a set of classical trajectories for a group of galaxies (point masses) which are interacting through gravity, within the background of an expanding universe model. This method differs from the usual application of the least-action principle in that boundary conditions are applied to the beginning and end of each trajectory. The trajectories are constrained in such a way that

$$\delta \boldsymbol{x}_i = 0 \quad \text{at} \quad t = t_0 \,, \quad a^2\, d\boldsymbol{x}_i/dt \to 0 \quad \text{at} \quad a \to 0 \,, \qquad (2.1)$$

where a is the scale factor of the universe and $\boldsymbol{x}_i(a)$ is the trajectory of the ith galaxy in comoving coordinates. That is, the positions of the galaxies are fixed at the present epoch, and their peculiar velocities vanish as we approach the big bang. Trial trajectories for the galaxies are adjusted in order to find a stationary point in the action.

In this *Letter* a matter-dominated universe with no cosmological constant is assumed; thus

$$H\, dt = \frac{a^{1/2}\, da}{F^{1/2}} \,, \qquad (2.2)$$

where $F = \Omega + (1 - \Omega)a$, H is the present Hubble constant, and Ω is the density parameter. Following Peebles (1990), the action for particles moving in such a universe is

$$S = \int_0^{t_0} dt \left[\sum \frac{m_i a^2}{2} \left(\frac{dx_i}{dt} \right)^2 + \frac{G}{a} \sum_{i \neq j} \frac{m_i m_j}{|x_i - x_j|} + \frac{2}{3} \pi G \rho_b a^2 \sum m_i x_i^2 \right], \quad (2.3)$$

from which we can deduce the equation of motion,

$$a^{1/2} \frac{d}{da} a^{3/2} \frac{dx_i}{da} + \frac{(1 - \Omega)a^2}{2F} \frac{dx_i}{da} = \frac{\Omega}{2F} \left[x_i + \frac{R_0^3}{M_T} \sum_j \frac{m_j(x_j - x_i)}{|x_j - x_i|^3} \right]. \quad (2.4)$$

Here R_0 is the radius of a sphere which would enclose a homogeneous distribution of the total mass M_T of the group of galaxies considered, $R_0^3 = M_T[(4/3)\pi\rho_b^0]^{-1}$. Note that this equation is slightly different from the one used by Peebles, since we do not assume a flat ($k = 0$) universe.

It is very hard to have exact analytic solutions for the coupled system of equations (2.4). However, Peebles succeeded in obtaining approximate solutions using trial functions of the form

$$x_i(a) = x_i^0 + \sum_n C_i^n f_n(a) , \quad (2.5)$$

where x_i^0 are the present positions of the galaxies and the f_n are linearly independent functions chosen to satisfy the boundary conditions (2.1). In this *Letter* we take $f_n = a^n(1 - a)$ for $n = 0, \ldots, 4$. The classical solutions are obtained by introducing $x_i(a)$ in the action and iteratively modifying the coefficient C_i^n to obtain a stationary action. As Peebles did, we verify that the LAM solutions are good approximations to real solutions by evolving the classical equations of motion starting with the initial positions and velocities derived from the least-action solutions at $z = 60$.

3. CDM SIMULATION

In order to understand the limits of the LAM, it is important to compare it with some other method. We used a CDM, N-body simulation described by Kauffmann & White (1992). It is a particle-particle-particle mesh (P^{3}M) simulation with 262,144 particles, representing an $\Omega = 0.2$ universe. Scaled to $H = 50$ km s^{-1} Mpc^{-1}, it encompasses a size of 100 Mpc with particles of mass 5.2×10^{10} $M_\odot$.

We have studied a few groups containing 10 or so galaxy halos. The halos are determined by a friend-of-friend algorithm, applied to the final frame of the CDM model. They were chosen in order to match the conditions in the Local Group: two dominant galaxies with peculiar velocities toward each other, a mass ratio of roughly 4:3, somewhat isolated from high-density mass concentrations corresponding to rich clusters. Because of the limitations of dynamic range in the simulation, we could not find any such halos with a separation of 0.7 Mpc but had to go to approximately 2 Mpc. These halos also had masses approximately 5–10 times greater than M31 and the Milky Way. In addition to the two central galaxies, the galaxies around them up to a distance of 20 Mpc were selected to form a group. We have studied the effect of the spatial distribution of the halos, the influence of the nearby galaxies on their dynamics, and also the effect of particles not linked to any halo (orphans).

We have selected nine groups which had two halos of the order of 200 particles within 2 Mpc. Within each group, we analyzed the motions of the 10 nearest halos surrounding the central pair. All the groups had similar behavior; therefore, for brevity, we present the results of only one of them here.

4. COMPARISON

Once we have identified the galactic halos, we can use the LAM described in § 2 to trace them back in time. We can also trace back the particles making the halos in the final step of the CDM simulation. Figure 1 shows one projection of the halo trajectories. We can see that there is only a very rough agreement between the LAM trajectories and the CDM ones. It could be thought that the CDM trajectories are not the least-action ones but only trajectories which make the action stationary. We have verified that this is not the case.

Assuming that the CDM and LAM trajectories are in reasonable agreement, we can compare the line-of-sight velocities of these halos. It is seen from Figure 2 that with the same parameters Ω and H, the LAM overestimates the velocities. However, dividing Ω in the LAM by a factor of 4–5 (compare with the CDM simulation value), we obtain agreement between these velocities. This implies that the LAM could systematically underestimate Ω in a CDM universe. Why should this be the case?

First we consider the effect on the line-of-sight velocities of modifying Ω in the LAM. There are two factors to take into account to understand the consequences of this modification. The first factor is the time elapsed since the big bang. Increasing Ω decreases the elapsed time and thus increases the velocities. The second factor is the radius R_0 of equation (2.4), the radius of a sphere which would enclose a homogeneous distribution of the total mass. A larger Ω decreases R_0. Thus we do not have to go so far to gather the mass to make the halos, thus decreasing the velocities. These two factors conspire

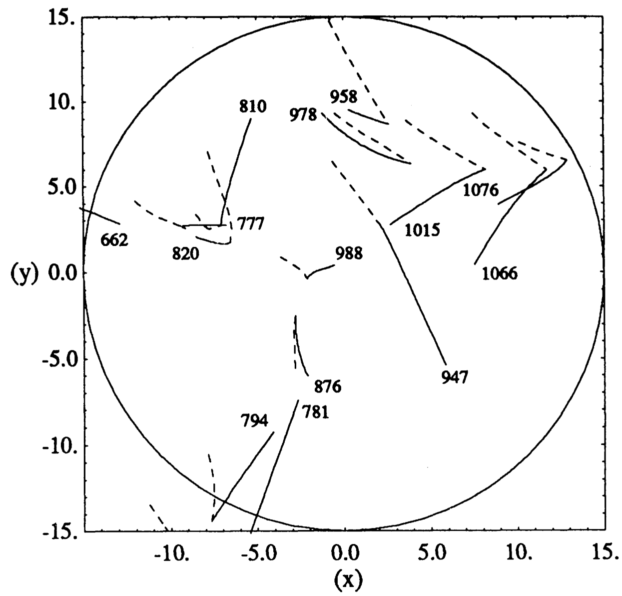

FIG. 1.—Projection of the CDM (*broken line*) and LAM (*solid line*) trajectories for galaxies in the chosen group. We can see that the agreement is at best qualitative. Notice that the CDM trajectories are typically shorter than the LAM ones. (Units are megaparsecs.)

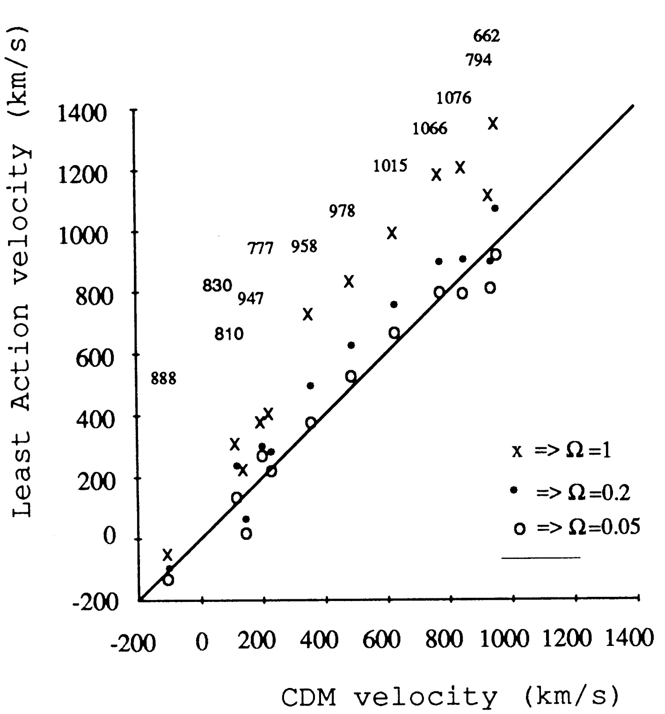

FIG. 2.—LAM line-of-sight velocities (with respect to one target galaxy of the chosen group) as a function of the CDM line-of-sight velocities. The best fit corresponds to an adjusted density parameter $\Omega \approx 0.05$ in the LAM, a factor of 4–5 lower than the CDM simulation parameter.

against each other, but a simple calculation for a 2-body system shows that the first one wins. Thus, as shown in Figure 2, when Ω is decreased in the LAM the velocities decrease.

We must now answer the question of why, for the same Ω and H, we have the LAM line-of-sight velocities being larger than the CDM one. The answer lies in the fact that in the CDM model there are orphans, i.e., CDM particles which are not linked to halos for a given choice of link scale. They have their most drastic effect in the early stage of the universe when the matter is pretty much homogeneous. If we do not include orphans, the force between the centers of mass of halos will be larger than if we had included them. A larger force implies that the halos would have to start at a larger distance from one another in order to end up in their fixed final positions (this can be seen in Fig. 1). Therefore, if we neglect orphans, as in the LAM, the line-of-sight velocities would typically be increased. In practice this is a transgression of our assumption 4, that light traces mass.

As mentioned earlier, we can also investigate the effect of the spatial distribution of the halos. Consider equation (2.4); all the terms are linear in (x_i) except the last one on the right. We have compared the contribution of this term, which we all, by abuse of language, the inhomogeneous component of the "force," when we take the sum over particles in different ways. First, in the LAM it is assumed that the halos interact as point sources, that is, the important part of the force only acts between the centers of mass of the halos, thus:

$$F_a^1 = \sum_b \frac{x_b - x_a}{|x_b - x_a|^3}, \qquad (4.1)$$

where a is the target galactic halo and the sum over b is over the center of mass of the nearby halos.

The second approach is obtained by summing over all the particles in each of the halos rather than just their center of mass. We have also divided by the number of particles of the target halo (N_a) to obtain the force on its center of mass. This will give an estimate of the effect of the higher multiple moment of the halos.

$$F_a^2 = \frac{1}{N_a} \sum_{a_i} \sum_{b_j} \frac{x_{b_j} - x_{a_i}}{|x_{b_j} - x_{a_i}|^3}, \qquad (4.2)$$

where the sum over a_i is over all particles of the targer halo and the one over b_j is over all particles of the halo b and then over all halos in our sample. This will essentially sum over everything except the orphans.

The third quantity is

$$F_a^3 = \frac{1}{N_a} \sum_{a_i} \sum_j \frac{x_j - x_{a_i}}{|x_j - x_{a_i}|^3}, \qquad (4.3)$$

where the sum a_i is over all particles of the target halo and the sum j is over all particles within 20 Mpc of the center of mass of the target group. This corresponds to the true force on the halo. To verify that the force had converged, we modified the distance of 20 Mpc to a shorter distance without significant change in the results (for the force on the last frame).

We have plotted the result in Figure 3, where the magnitude of the different "forces" is shown. From this figure we can see that at early times the force F_a^1 is overestimated by a factor of ∼2, which is not unexpected, since galaxies make poor approximations to point particles at early times. An investigation of the direction of the forces shows that the inclusion of higher multipoles of halos is still not a very good approximation. There are discrepancies and scatter between the direction of the vector F_a^2 and the true force F_a^3. We must therefore reject the suggestion of Branchini & Carlberg (1994) that the discrepancy between the CDM and LAM line-of-sight velocities might be due to neglecting the shape of the CDM halos.

We should also point out that another possible problem for the LAM is the existence of mergers. In one of our groups there was a significant merger, and for this halo the force was not very well represented by the one at its center of mass. However, for a merger to have an important effect, it must dissipate a significant fraction of the total kinetic energy of the system, as in the case of roughly equally massive halos which merge from

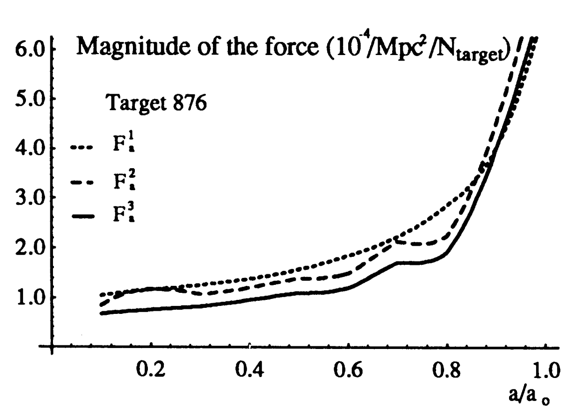

FIG. 3.—Plot of the inhomogeneous part of the "force" (eqs. [4.1]–[4.3]) F_a^1 and F_a^2 with respect to the true force F_a^3 on a typical halo. We see that the force on a halo is not well approximated by the force due to other halos; orphans (CDM particles not bound to halos) have an important contribution.

rather different directions. A detailed study of mergers in the CDM model is needed to know in a quantitative sense whether this is a potentially serious problem for the LAM.

5. CONCLUSION

In this *Letter* we have shown that Peebles's least-action method underestimates the value of Ω for a CDM universe by a factor of 4–5. The main discrepancy is due to neglecting the effect of orphans, CDM particles which are not members of any halos. They are scattered uniformly in the early stage of the universe and populate the voids between halos at the end of the simulation (this is not precise, since the distinction between halo particles and orphans depends on the link scale used to identify halos, and the orphan particles will gradually merge with the halos if the simulation is evolved further). The effect of the orphans is to reduce the force on the particles which will eventually form halos. Thus, in the presence of orphans, the protohalos must start at a shorter distance than that expected in Peebles's original suggestion. This is equivalent to failure of one of the key assumptions of the least-action method, that is, that light traces mass (at least at kiloparsec to megaparsec scales). We conclude that the analysis of the dynamics of the Local Group using the least-action method and a careful examination of its line-of-sight velocities does not exclude a closed universe.

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