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ON POSSIBLE IMPLICATIONS OF ORBITAL PARALLAXES OF WIDE ORBIT BINARY PULSARS AND THEIR MEASURABILITY

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ABSTRACT

A complete theoretical description of the parallactic terms in the timing formula for a binary pulsar is presented. It is shown that the terms depend not only on the annual motion of Earth but also on the orbital motion of the binary pulsar as well. It is proved that measurements of the effect of orbital parallax can serve a useful purpose, both in determination of pulsar distances, and also in the measurement of companion masses. Estimates of expected magnitudes of the corresponding timing signals for the most appropriate binary pulsar candidates are tabulated, and prospects for the measurement of orbital parallaxes are discussed.

Subject headings: binaries: general — pulsars: individual (PSR B1259−63, PSR J1713+0747, PSR J2019+2425) — stars: fundamental parameters

1. INTRODUCTION

It is well known that measurements of annual trigonometric parallax caused by Earth's orbital motion give model independent distances to stellar objects. In the case of pulsars, parallax measurements allow us to estimate the average electron density along the line of sight. With this information, one can calibrate the distance scale based on pulse dispersion. Previous measurements of the annular pulsar parallaxes have been reported by Salter, Lyne, & Anderson (1979), Backer & Sramek (1982), Gwinn et al. (1986), and Bailes et al. (1990). These measurements were based on the VLBI technique and are applicable for pulsars having both second and millisecond periods. The independent method of using pulsar timing data to measure the annual pulsar parallaxes has been discussed by Blandford, Narayan, & Romani (1984), and a similar idea was proposed independently by Kuzmin & Kuzmin (1988). Rawley, Taylor, & Davis (1988) tried to detect the timing parallax of PSR B1937+21, but their null result served only to set an upper limit. The first successful measurement of the annual parallax of PSR B1855+09 using timing data was recently reported by Ryba & Taylor (1991). Further development of this method, in application to PSR B1855+09 and PSR B1937+21, is described in Kaspi, Taylor, & Ryba (1994). These investigations show obviously that parallax determinations by the pulsar timing technique are very difficult and can be relied on only for those pulsars which meet stringent criteria for timing accuracy. Nevertheless, the rapid growth of the population of millisecond pulsars reveals new candidates such as J2019+24, J1713+07, and J0437−47 which display unprecedented accuracy in their pulse arrival times. It seems not unreasonable to hope that the number of pulsar parallax measurements will increase rapidly in the not so far distant future, with a corresponding improvement of calibration for the distance scale in the Galaxy.

It is worthwhile to note that in the case of a binary pulsar, the parallactic shift of the pulsar in the sky is caused not only by the annual motion of Earth about the Sun, but also by the orbital motion of the pulsar itself. Measurements of the orbital timing parallaxes in binary pulsars open an additional possibility for the determination of distances to the pulsars and masses of the pulsar companions (Kopeikin 1992). An accurate measurement of the masses of binary pulsars along with their distance and kinematics offers the best quantitative checks of the formation scenarios of millisecond binary pulsars.

2. MEASURABILITY AND APPLICATIONS OF PULSAR ORBITAL PARALLAXES

The modern method of the precise determination of neutron star masses is currently based upon the measurement of any two relativistic effects in the orbital motion of binary pulsars (Taylor 1992). Unfortunately the method can be applied only for binary pulsars having "relativistic" orbits and/or an angle of orbital inclination i sufficiently close to 90°. For binary pulsars having large orbital periods, the application of the relativistic effects to the mass determination is questionable due to their smallness.

However, in the event that a binary pulsar has a wide orbit and is not too far from Earth, its orbital parallax could have a measurable value. In that situation, one might hope to determine the apparent orbit of the binary pulsar in the same fashion as is sometimes possible with visual double stars (Couteau 1981). Such a determination would impose an independent numerical restriction on the angle of inclination of the pulsar's orbit to the plane of the sky and therefore restrict possible values of masses of the pulsar and its companion. If the pulsar's companion was also visible as a pulsar, or if a relativistic effect was measurable, then a separate determination of mass of each star would be possible. On the other hand, for systems in which the pulsar's companion can be identified in the visible wavelength range, the parameter of the variable Doppler velocity of the companion might be obtained from spectral studies. In combination with the parameters obtained from the timing observations, this would provide an alternate route to mass determinations.

According to Ryba & Taylor (1991), the magnitude of annual trigonometric parallax is $\Delta_{\pi\odot} \sim (1 \text{ a.u.})^2/(2cd)$. For a pulsar in the ecliptic at a distance $d = 1$ kpc, the value of $\Delta_{\pi\odot}$ is approximately 1.2 μs. Thus, the pulsar is accessible for measurement if the timing accuracy ϵ is equal to or better than 1 μs. It therefore follows that the determination of pulsar parallaxes can be obtained only for millisecond pulsars. In the case of binary pulsars, the additional orbital parallax is split in two terms: (1) the pure orbital parallax $\Delta_{\pi P} \sim a_p^2/(2cd) = \pi_P/\sin^2 i$ and (2) the mixed annual-orbital parallax $\Delta_{pM} \sim (1\ \text{a.u.})a_p/(cd) = \pi_M/\sin i$, where a_p is the semimajor axis of the pulsar's orbit with respect to the barycenter of the binary system, $\pi_P = cx^2/(2d)$, $\pi_M = (1\ \text{a.u.})x/d$, and $x = a_p \sin i/c$ is the projected semimajor axis of the binary system's orbit. Note that Δ_{pM} is larger than $\Delta_{\pi P}$ for those pulsars whose semimajor axes a_p exceed 1 a.u.

At present, several binary pulsars are known which have semimajor axes large enough for possible determination of orbital timing parallaxes. We have selected the most appropriate candidates from the list of millisecond pulsar binaries and have tabulated the lower limits for the expected magnitude of the corresponding timing signals. We have also estimated the upper limits for annual parallaxes. The results are presented in Table 1.

The table shows that the magnitude of orbital parallaxes may be marginally accessible to measurement in PSR B1259−63, PSR J2019+2425, and PSR J1713+0747. In particular, it should be noted that we have used the minimal values of distances for PSR J1713+0747 and PSR B1855+09 in our calculations in accordance with the low limits on annual parallaxes given by Camilo, Nice, & Taylor (1993) and Kaspi et al. (1994), respectively. Our estimates show that it makes sense to develop an exact analytical model for measurement of orbital pulsar parallaxes and include it in existing timing software packages. Moreover, it is not difficult to see that the orbital motion of PSR B1259−63 is directly accessible for observations by the VLBI technique that is equivalent to a measurement of the timing parallax of the system. The maximal magnitude of the orbital parallactic shift in the sky for this pulsar reaches 0.003/sin i arcsec.

3. TIMING FORMULA

The timing formula for the wide orbit millisecond binary pulsars can be summarized in the equation

$$T = \tau - \tau_0 + \Delta_C - D/f^2 + \Delta_{R\odot} + \Delta_{\pi\odot} + \Delta_{E\odot} + \Delta_{S\odot} - \Delta_R - \Delta_{\pi P} - \Delta_{\pi M}\,, \quad (1)$$

where we consider all nonseparable parameters as being absorbed in the redefinitions of separately measurable ones (Damour & Taylor 1992; Kopeikin 1994). Here T is the pulsar proper time; τ is the proper time of observer; τ_0 is an initial epoch of observations; Δ_C represents time offsets between the local observatory clock and international atomic time (TAI); D/f^2 is the dispersive delay; $\Delta_{R\odot}$, $\Delta_{\pi\odot}$, $\Delta_{E\odot}$, and $\Delta_{S\odot}$ are "Roemer," "Parallax," "Einstein," and "Shapiro" propagation delays within the solar system; and Δ_R, $\Delta_{\pi P}$, $\Delta_{\pi M}$ are the analogous terms for the binary pulsar's orbit. Expressions for $\Delta_{R\odot}$, Δ_R, $\Delta_{E\odot}$, and $\Delta_{S\odot}$ are well known (Damour & Taylor 1992). We are especially interested in the parallactic terms $\Delta_{\pi\odot}$, and $\Delta_{\pi P}$, $\Delta_{\pi M}$ which take the form (Kopeikin 1992)

$$\Delta_{\pi\odot} = -\frac{1}{2cd}(\boldsymbol{K}_0 \times \boldsymbol{r})^2\,, \quad (2)$$

$$\Delta_{\pi P} = \frac{1}{2cd}(\boldsymbol{K}_0 \times \boldsymbol{r}_p)^2\,, \quad (3)$$

$$\Delta_{\pi M} = -\frac{1}{cd}(\boldsymbol{K}_0 \times \boldsymbol{r}_p)(\boldsymbol{K}_0 \times \boldsymbol{r})\,. \quad (4)$$

In these terms, c is the speed of light, d is the radial distance between the binary and solar systems, $\boldsymbol{r} = (X, Y, Z)$ is the solar system barycentric coordinates of the geocenter (Standish 1982), $\boldsymbol{r}_p$ is the radius vector of the pulsar with respect to the binary system's barycenter, $\boldsymbol{K}_0$ is the unit vector pointing from the solar system's barycenter toward that of the binary system's, and the multiplication cross denotes an ordinary vector crossproduct. The radius-vector $\boldsymbol{r}_p$ is given in Brumberg (1991):

$$\boldsymbol{r}_p = a_p[\boldsymbol{i}Q(u) + \boldsymbol{j}R(u)]\,, \quad (5)$$

where

$$Q(u) = \cos\omega(\cos u - e) - (1 - e^2)^{1/2}\sin\omega\sin u\,, \quad (6)$$

$$R(u) = \sin\omega(\cos u - e) + (1 - e^2)^{1/2}\cos\omega\sin u\,, \quad (7)$$

$$\boldsymbol{i} = \cos\Omega\boldsymbol{I}_0 + \sin\Omega\boldsymbol{J}_0\,, \quad (8)$$

$$\boldsymbol{j} = \cos i(-\sin\Omega\boldsymbol{I}_0 + \cos\Omega\boldsymbol{J}_0) + \sin i\boldsymbol{K}_0\,. \quad (9)$$

In the above expressions, u is the eccentric anomaly, the angles ω and Ω are the longitudes of periastron and ascending node respectively, $(\boldsymbol{I}_0, \boldsymbol{J}_0, \boldsymbol{K}_0)$ is the triad of unit vectors attached to the barycenter of the binary system and related to the commonly adopted equatorial coordinates (α, δ) by

$$\boldsymbol{I}_0 = (-\sin\alpha, \cos\alpha, 0)\,, \quad (10)$$

$$\boldsymbol{J}_0 = (-\cos\alpha\sin\delta, -\sin\alpha\sin\delta, \cos\delta)\,, \quad (11)$$

$$\boldsymbol{K}_0 = (\cos\alpha\cos\delta, \sin\alpha\cos\delta, \sin\delta)\,. \quad (12)$$

4. ANNUAL PARALLAX

The annual parallax term $\Delta_{\pi\odot}$ has been well known for a long time (Kuzmin & Kuzmin 1988; Ryba & Taylor 1991). It is

TABLE 1

MAGNITUDE OF ORBITAL AND ANNUAL-ORBITAL PARALLAXES

PSR J	PSR B	P (s)	d (kpc)	x (s)	$\Delta_{\pi\odot}$ (μs)	π_P (μs)	π_M (μs)	sin i	References
1302−6350......	1259−63	0.047	2.3	3450	0.53	24.41	7.08	>0.9	1
1312+18	1310+18	0.033	18.5	84.17	0.07	0.00	0.02	...	2
1623−2631......	1620−26	0.011	2.2	64	0.55	0.01	0.16	...	3
2019+2425......	...	0.004	0.9	38.77	1.35	0.01	0.20	...	5
1955+2908......	1953+29	0.006	3.5	33	0.35	0.00	0.04	...	4
1713+0747......	...	0.005	0.8	32.34	1.52	0.01	0.20	<0.96	6
1857+0943......	1855+09	0.005	0.7	9.23	1.73	0.00	0.06	0.9992	7
0437−4715......	...	0.006	0.15	3.37	8.09	0.00	0.10	...	8
2317+1439......	...	0.003	1.46	2.3	0.83	0.00	0.02	...	9

REFERENCES.—(1) Kochanek 1993; (2) Kulkarni et al. 1991; (3) Lyne et al. 1988; (4) Boriakoff et al. 1983; (5) Nice et al. 1993; (6) Camilo et al. 1994; (7) Kaspi et al. 1994; (8) Johnston et al. 1993; (9) Camilo et al. 1993.

parameterized as follows:

$$\Delta_{\pi\odot} = -\frac{1}{2cd}\left[(X \cos\delta \cos\alpha + Y \cos\delta \sin\alpha + Z \sin\delta)^2 - r^2\right]. \quad (13)$$

The annual parallaxes are accessible for measurements in PSR J1713+07 (Camilo et al. 1993) and PSR B1855+09 (Kaspi et al. 1994) and could in principle be measurable in PSR B1937+21 (Kaspi et al. 1994), PSR J2019+24, and PSR J0437−47 (Johnston et al. 1993).

5. ORBITAL PARALLAX

After a straightforward calculation, the pure pulsar orbital parallax term $\Delta_{\pi P}$ can be expressed as

$$\Delta_{\pi P} = \frac{cx^2}{2d}\left\{\csc^2 i - \frac{1}{2} + \frac{1}{2}e^2(1 + \sin^2\omega - 3\csc^2 i) - 2e(\csc^2 i - \sin^2\omega)(\cos u - e) + (1 - e^2)^{1/2}\sin 2\omega\left(e \sin u - \frac{1}{2}\sin 2u\right) + \frac{1}{2}\left[\cos 2\omega + e^2(\csc^2 i + \cos^2\omega)\right]\cos 2u\right\}, \quad (14)$$

where a_p and e are the semimajor axis and eccentricity of the binary pulsar orbit, $x = a_p \sin i/c$, and i and ω are the orbital inclination and longitude of periastron, respectively. One concludes immediately from equation (14) that the constant and periodic terms depending exceptionally on $\cos u$ and $\sin u$ are absorbed in the astrometric and Keplerian parameters of the binary system. Thus, the coefficients of the functions in equation (14) which depend periodically on time as $\sin 2u$ and $\cos 2u$ are the only separately measurable parameters. Their measurement will deliver new physical information regarding the numerical values of distance to the pulsar and the sine of orbital inclination. Therefore, measurement of the pure orbital parallax $\Delta_{\pi P}$ will admit a separate determination of the distance d to the pulsar as well as the angle of the orbital inclination i, but only in the case when $e \neq 0$. Such a situation is realized in the case of PSR B1259−63 (see Table 1) and can be used to extend the calibration of the distance scale in the Galaxy to larger space intervals.

Additional information may be extracted from the independent measurement of the annual-orbital parallax $\Delta_{\pi M}$.

6. ANNUAL-ORBITAL PARALLAX

Let us introduce the following notations:

$$\Delta_{I_0} = (\boldsymbol{r} \cdot \boldsymbol{I}_0) = -X \sin\alpha + Y \cos\alpha, \quad (15)$$

$$\Delta_{J_0} = (\boldsymbol{r} \cdot \boldsymbol{J}_0) = -X \sin\delta \cos\alpha - Y \sin\delta \sin\alpha + Z \cos\delta. \quad (16)$$

The expressions on the right-hand sides of equations (15) and (16) are known functions of time and can be precisely computed using the modern ephemerides (Standish 1982). The mixed annual-orbital parallax term $\Delta_{\pi M}$ can then be presented in the form

$$\Delta_{\pi M} = \frac{x}{d}\left[(\Delta_{I_0}\sin\Omega - \Delta_{J_0}\cos\Omega)R(u)\cot i - (\Delta_{I_0}\cos\Omega + \Delta_{J_0}\sin\Omega)Q(u)\csc i\right]. \quad (17)$$

It is not difficult to see that the structure of the annual-orbital parallax term is such that if $\Delta_{\pi M}$ is omitted from the timing formula, we would measure

$$x^{\rm obs} = x^{\rm intrinsic}\left[1 + \frac{\cot i}{d}(\Delta_{I_0}\sin\Omega - \Delta_{J_0}\cos\Omega)\right], \quad (18)$$

and

$$\omega^{\rm obs} = \omega^{\rm intrinsic} - \frac{\csc i}{d}(\Delta_{I_0}\cos\Omega + \Delta_{J_0}\sin\Omega). \quad (19)$$

This will induce the systematic periodic variations in the parameters $x^{\rm obs}$ and $\omega^{\rm obs}$ with annual period. Probably, oscillations in these parameters could be responsible for the nonrandom behavior in the residual phases of PSR J2019+24 (Nice, Taylor, & Fruchter 1993) if $\sin i$ were equal to or less than 0.1.

Again one can see from equations (14) and (17) that the observations of $\Delta_{\pi P}$ and/or $\Delta_{\pi M}$ admit a separation of the orbital inclination i and the semimajor axis a_p under condition that the distance d is already known. It is possible to measure the longitude of the ascending node Ω as well. Taking into account the numerical values of the inclination angle i, as well as the mass function $f(m_p, m_c)$, one will be able to obtain more reliable estimate of the ratio $K = f(m_p, m_c)/\sin^3 i = m_c^3(m_c + m_p)^{-2}$. The mass of the pulsar's companion can then be evaluated, assuming for example, that the pulsar's mass is equal to 1.4 $M_\odot$.

Measurements of the pulsar orbital and annual-orbital parallaxes are equivalent to the determination of the apparent orbit of the binary pulsar as in the case of visual double stars (Couteau 1981). Such observations would improve the mass estimates of stars in binary pulsars even in the event that the orbit is close to face-on. In such situation, the classical method of the stellar mass determination does not work so well, due to the large uncertainty in the numerical value of the mass function.

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