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ON THE LUMINOSITY FUNCTION, LIFETIMES, AND ORIGIN OF BLUE STRAGGLERS IN GLOBULAR CLUSTERS

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ABSTRACT

We compute theoretical evolutionary tracks of blue stragglers created by mergers. Two formation scenarios are considered: mergers of primordial binaries, and stellar collisions. These two scenarios predict strikingly different luminosity functions, which are potentially distinguishable observationally. Tabulated theoretical luminosity functions and lifetimes are presented for blue stragglers formed under a variety of input conditions. We compare our results with observations of the blue straggler sequences in 47 Tucanae and M3. In the case of 47 Tuc, the luminosity function and the formation rate are compatible with the hypothesis that the blue stragglers formed through the collision of single stars. Mergers of primordial binaries are only marginally consistent with the data, and a significant enhancement of the collision cross section by binary–single-star encounters appears to be ruled out. In the case of M3, we find that the innermost blue stragglers have a luminosity function significantly different from that of the outer stragglers, thus confirming earlier suggestions that there are two distinct populations of blue stragglers in this cluster. The inner stragglers are preferentially brighter and bluer, as would be expected if they were made by collisions, but there are so many of them that the collision rate would need to be enhanced by interactions involving wide binaries. The luminosity function of the outer stragglers is almost identical to the predictions of mergers from primordial binaries and is inconsistent with the collision hypothesis.

Subject headings: celestial mechanics, stellar dynamics — globular clusters: individual (M3, 47 Tucanae) — stars: blue stragglers — stars: evolution — stars: luminosity function, mass function

1. INTRODUCTION

Blue stragglers have been a thorn in the side of students of stellar structure and evolution since they were first identified by Sandage (1953) four decades ago. Their position in the color-magnitude diagram suggests that they are main-sequence stars with masses exceeding the current turnoff mass. The observed concentration of blue stragglers toward the centers of clusters (Nemec & Harris 1987; Sarajedini 1992, 1993; Fusi Pecci, Ferraro, & Cacciari 1993b and references therein) is generally consistent with such high masses. The challenge blue stragglers present is to explain why they have not evolved to become giants as all stars of their mass in globular clusters should long since have done.

The mechanisms which have been invoked for keeping blue stragglers on the main-sequence have recently been reviewed by Stryker (1993). One widely discussed mechanism is mixing, in which hydrogen from the surface layers is mixed into the stellar core, prolonging the main-sequence lifetime of the straggler. However, no plausible mixing mechanism has yet been proposed. Furthermore, recent spectroscopic studies of blue stragglers in open clusters have shown that the surface gravities are inconsistent with the predictions of mixing models (Schönberner & Napiwotzki 1994; but see also Pritchet & Glaspey 1991). In globular clusters, even complete mixing would not provide sufficient fuel to power the brightest blue stragglers at their current luminosities over the more than 10 Gyr lifetime of the clusters. Instead, most recent workers now favor the merger hypothesis, in which two low-mass stars have recently merged to produce a relatively unevolved massive object. Two kinds of mergers have been discussed: the merger of binary systems through contact or mass transfer, and direct collisions between stars (e.g., Bailyn 1992; Fusi Pecci et al. 1993a). We will refer to the former as "binary mergers" and the latter as "collisional mergers."

Recent theoretical and observational developments have led to a resurgence of interest in the blue straggler problem. Dynamical models have shown that binary stars and stellar interactions are crucial to the dynamical evolution of globular clusters (Hut et al. 1992). Since binary evolution and stellar collisions have emerged as the most promising candidates for producing blue stragglers, blue stragglers may provide a critical link between problems of stellar structure and evolution and problems in stellar dynamics. Observationally, the advent of CCDs, coupled with improved seeing from ground-based observatories and the launch of the *Hubble Space Telescope* (*HST*), have led over the past few years to an order-of-magnitude increase in the number of known blue stragglers. This observational work has culminated recently in observed luminosity functions of blue stragglers which contain hundreds of stars (Sarajedini 1992, 1993; Fusi Pecci et al. 1993b). The general shape of these luminosity functions strongly support

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the merger hypothesis, in that the number of stragglers falls off abruptly at a luminosity equal to that of main-sequence stars of twice the turnoff mass. Given these new data, the time would appear ripe to compute detailed evolutionary tracks for merger remnants, from which theoretical luminosity functions and lifetimes could be determined.

The general problem of constructing theoretical luminosity functions for blue stragglers created by mergers can be divided into three distinct steps. First, the nature of the merger product must be determined for each possible combination of input stars. Second, one must determine the rate of mergers for each pair of input stars considered. Finally, one must calculate the trajectory and timescale of the merged star's evolution through the blue straggler region of the color-magnitude diagram (CMD). With this information in hand, one can determine the evolution of blue stragglers and hence predict their observed luminosity function. From the lifetimes of the merger products, the birthrate required to produce a given number of observed blue stragglers can also be inferred.

In this paper we present a first attempt to calculate theoretical blue straggler luminosity functions and lifetimes according to the above scheme. We consider both binary and collisional mergers. All three of the steps listed above involve complex physical processes which are not fully understood, and much more work needs to be done before definitive formation rates and luminosity functions can be calculated. Nevertheless, our calculations provide a basis for understanding the nature of the problem, and a comparison with observations yields intriguing results.

In § 2 we discuss our assumptions regarding the outcomes of binary and collisional mergers, and describe a parameterization of the relative merger rates as a function of mass in terms of an "effective mass function." Section 3 describes detailed evolutionary models of merger products, computed with the Yale stellar evolution code. Section 4 presents the resulting luminosity functions for both binary and collisional mergers, and discusses the physical basis of the differences between them. Section 5 shows how our results can be used to infer birthrates of blue stragglers. In § 6 we compare our theoretical luminosity functions to the recently discovered blue straggler sequence in 47 Tuc. Our results are consistent with a collisional origin for these blue stragglers but are mildly inconsistent with a primordial binary origin at a 90% confidence level. The required formation rate of blue stragglers is compatible with collisions between single stars; a significant enhancement of the cross section from single-star–binary encounters seems to be ruled out in this case. In § 7 we examine the blue straggler sequence in M3 and find significant differences between the luminosity functions of the stragglers in the inner and outer regions of the cluster. The inner population is consistent with a collisional origin, although in this case an enhanced collision rate is required. By contrast, the outer cluster population has proportionally fewer bright stragglers, which are likely to be made from primordial binaries. Finally, in § 8 we critically examine the current state of the theory and of the observational data, and suggest areas where improvements are needed.

2. MERGER PRODUCTS AND RELATIVE MERGER RATES

2.1. *Results of Collisional Mergers*

A number of authors have performed smooth particle hydrodynamics models of direct collisions between stars (e.g., Rasio & Shapiro 1991; Davies, Benz, & Hills 1991). In particular, Benz & Hills (1987) have modeled collisions between main-sequence stars in globular clusters. They find that the result of such a collision is a merged star with negligible mass loss and complete mixing. If one or both of the colliding stars are main-sequence stars near the turnoff, considerable helium will have been produced in the core. After the merger, this helium will be mixed throughout the merger remnant (Benz & Hills 1987), resulting in stars with anomalously high helium abundance. (While a more realistic treatment might result in less mixing, due to atomic weight and density gradients ignored in Benz & Hills's polytropic models, complete mixing represents at the least an interesting limiting case, so we will assume this result for the purposes of this paper.) In some cases, particularly for the most massive merger products, the enhanced He abundance will have a significant impact on the star's position in the CMD (Larson 1965; Bailyn 1992). The need to follow the evolution of stars with such unusual chemistry is what has led us to calculate the new evolutionary tracks described in § 3.

2.2. *Results of Mergers of Primordial Binaries*

The evolution of contact binary systems is not well understood, but it seems most likely that the less massive of the two components will gradually lose material to the more massive, leading to a more extreme mass ratio and eventual coalescence (Webbink 1979). The chemistry of blue stragglers formed in this way will also be different from that of a typical cluster star. We assume, as Webbink (1979) suggests, that the originally less massive star (the secondary) is decanted onto the primary. This process is similar to what is expected for a semidetached system. In both cases, the nuclear-burning products of the secondary are mixed throughout the envelope of the merger product, enhancing its helium abundance. Since only the less massive star contributes to this effect, binaries with nearly equal mass will have more helium enhancement than those with more extreme mass ratios, and the enhancement will be considerably less than that of collisional mergers. The nuclear-burning products of the primary will presumably remain in the core of the merger product. Thus a primordial binary merger will start its life as a more massive chemically inhomogeneous star, similar to a star which has already evolved partway through its main-sequence lifetime.

An intermediate case between collisional and binary mergers may result from near collisions. Main-sequence stars which approach each other closer than $3R_*$ are expected to become bound due to tidal capture (Fabian, Pringle, & Rees 1975; Press & Teukolsky 1977). However, such a bound system will not persist as a binary, since the energy generated by tidal friction will cause the component stars to expand considerably (Ray, Antia, & Kembhavi 1987; Kochanek 1992). Whether the chemistry of the resulting merger product will resemble that of a binary merger or a direct collision is currently unclear.

Another situation which might result in stars in the blue straggler region is collisions between giants and main-sequence stars. Such collisions might result in the stripping of the giant's envelope, leaving behind a small degenerate helium core with a small residual envelope. Such remnants may appear on a faint blue extension of the horizontal branch (Bailyn et al. 1992 and references therein). Ordinarily such stars are bluer than the blue stragglers, but in some cases they may be confused. For the purposes of this paper, however, we will assume that the observed blue straggler sequences are uncomtaminated by the horizontal branch and its extension.

2.3. *Relative Merger Rates*

To generate a luminosity function for blue stragglers, it is essential to know the relative rates at which stars of different masses merge. In this paper we will parameterize the complex effects involved by using an "effective mass function" described by a power-law index x. We will assume that the two input stars are drawn at random from a power-law distribution of masses such that

$$\frac{dN(m)}{dm} = m^{-(1+x)} , \tag{1}$$

with m extending from 0.1 $M_\odot$ up to the current main-sequence turnoff.

It is important to realize that the effective mass function for mergers will not be equal to the observed mass function of a given cluster. In the case of collisional mergers, the relevant mass function will be that of the dense central regions of the cluster, where collisions are most likely to occur. Both multimass King models (Pryor, Smith, & McClure 1986) and Fokker-Planck models of cluster evolution (e.g., Chernoff & Weinberg 1990) suggest that the power-law index of stars in the region where collisions are frequent may be as low as $x \lesssim -2$. Thus the population from which the input stars for collisions are drawn is biased toward massive stars.

In contrast, the stars which form primordial binaries are drawn from the initial mass function (IMF) of the cluster. Stellar IMFs have power-law indices which favor low-mass stars (i.e., positive values of x as defined by eq. [1]). In the case of globular clusters, the IMF is expected to be steeper than the currently observed global mass functions, owing to the evaporation of low-mass stars from the cluster. Thus the currently observed cluster global mass function indices of $0 \le x \le 2$ (Pryor et al. 1986; McClure et al. 1986) serve as a lower bound on the power-law index of the IMF. The processes which form binaries may bias the mass function of the primordial binaries relative to the IMF in an unknown way, and binary evolution may further bias the mass function of stars which enter into contact. Since the magnitude and sign of these effects are unknown, we will consider a variety of effective mass functions for binary mergers.

3. EVOLUTIONARY TRACKS

We have used the Yale stellar evolution code (Guenther et al. 1992) to construct evolutionary sequences of stars with compositions appropriate for 47 Tuc and M3. The nuclear reaction rates used were those of the best standard model of Bahcall & Pinsonneault (1992). We used Kurucz (1991) opacities for temperatures below 10^4 K; above 10^4 K, we used OPAL opacities of Iglesias & Rogers (1991) for the Anders & Grevesse (1989) mixture of heavy elements with the meteoritic iron abundance. For metal-poor stars, the α-capture elements are known to be enhanced with respect to the Sun (Barbuy 1992); here we have assumed that $[\alpha/\mathrm{Fe}] = +0.2$ for 47 Tuc and $+0.4$ for M3. Combined with the assumed [Fe/H] of -0.7 and -1.7, this yields total metal abundances of 0.005 and 0.000625, respectively. The difference in the mixture of heavy elements was neglected in the opacities but was included in the nuclear reaction rates and mean molecular weight.

The solar calibration gives a mixing length of 1.7; because we are primarily interested in the luminosities of blue stragglers relative to the turnoff point, our model predictions are not sensitive to the outer-layer physics or convection theory. We assumed initial He abundances of 0.235 for M3 and 0.245 for 47 Tuc, which assumes $\Delta Y/\Delta Z \approx 2$. We also computed models with enhanced initial helium of 0.305 and 0.315, respectively, for the M3 and 47 Tuc metallicity. Models with masses between 0.4 and 1.8 $M_\odot$ were computed at intervals of 0.1 $M_\odot$. Models with other masses and helium abundances could be determined by interpolation. The turnoff mass was taken to be the mass at which hydrogen was exhausted in the core at an age of 14 Gyr.

In the case of collisional mergers, the He abundance Y is determined by adding the initial helium to that produced over the lifetime of the two components. This new value of Y can be written in terms of the luminosities due to hydrogen burning of the input stars averaged over their lifetimes prior to the merger. Writing these time-averaged luminosities as $\langle L_1 \rangle$ and $\langle L_2 \rangle$, we have

$$Y = Y_0 + \tau_{10} \frac{0.10\, M_\odot}{M_1 + M_2} \frac{\langle L_1 \rangle + \langle L_2 \rangle}{L_\odot} (1 - Y_0) , \tag{2}$$

where Y_0 is the initial helium abundance, τ_{10} is the age of the cluster in units of 10^{10} yr, and M_1 and M_2 are the masses of the input components. For the binary mergers, equation (2) is modified to include only the mean luminosity of the less massive component, since only the nuclear-burning products of the secondary are mixed throughout the envelope in this case. Also, the track is started at the point where the evolution from the zero-age main-sequence (ZAMS) would have produced a total integrated luminosity equal to that of the more massive component over its entire lifetime. Thus the merger remnant starts as a partially evolved star with a core He content equal to that of the primary at the time of merger, and an envelope He content slightly enhanced by the nuclear-burning products of the secondary. Figure 1 shows several of our tracks for the case of $Z = 0.000624$, appropriate for a rela-

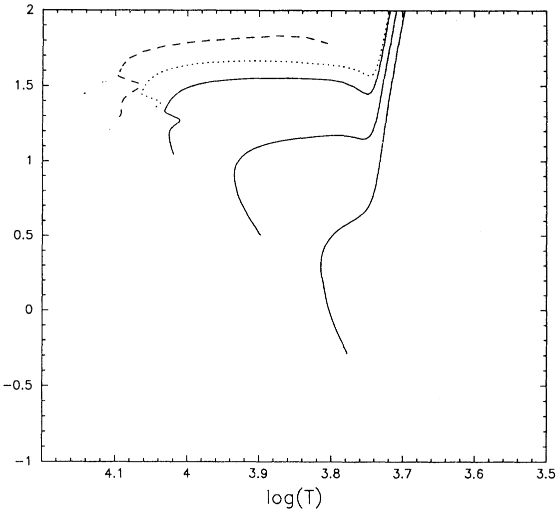

FIG. 1.—Evolutionary tracks for stars with $Z = 0.000624$. Solid lines show tracks for stars with masses 0.8 (at the main-sequence turnoff for an age of 14 Gyr) 1.2, and 1.6 $M_\odot$. The dashed line shows the track for collisional merger of two 0.8 $M_\odot$ stars, while the dotted line shows the track for primordial binary merger of two 0.8 $M_\odot$ stars. The offset between these two merger tracks and the 1.6 $M_\odot$ single-star track demonstrates the importance of the changes in chemistry due to the merger process for high-mass blue stragglers.

tively metal-poor globular cluster, together with tracks for binary and collisional mergers between two turnoff-mass stars.

4. LUMINOSITY FUNCTIONS

With the evolutionary tracks in hand, we compute luminosity functions for blue stragglers in the following way. We consider each combination of two masses between 0.1 $M_\odot$ and the main-sequence turnoff for which the total mass is greater than the main-sequence turnoff, varying the component masses by 0.01 $M_\odot$. The composition of the merger product is determined using the prescription given above, and its evolution is determined by iterating between the tracks described in § 3. For each such track we calculate the total time the model spends in each 0.1 interval in log $(L/L_{\rm to})$ (where $L_{\rm to}$ is the luminosity of the bluest point on the main-sequence turnoff) while log $T_{\rm eff} > 3.8$. Changes in the observed magnitude of blue stragglers as a function of cluster distance and age will scale to first order with the turnoff magnitude, so the use of $L_{\rm to}$ as a reference luminosity will allow us to ignore small errors in these parameters. The temperature limit forces the luminosity function to be computed from stars in the blue straggler region; almost all of the luminosity function is contributed by core H-burning stars, with a negligible contribution from massive stars crossing the Hertzsprung gap. The contribution of each merger is weighted by a factor proportional to the product of the effective mass function at each of the input masses, that is, by the relative probability that this particular combination of masses would be chosen out of that mass function. The overall blue straggler luminosity function is then computed from the weighted sums of all the tracks in each luminosity bin.

In Figure 2 we present theoretical blue straggler luminosity functions computed in this way. Results for both the binary and the collisional chemistry for $x = 1.35$ (Salpeter mass function) and $x = -2$ and for metal-poor and moderate metallicity are shown. Clearly, there are significant differences between the resulting luminosity functions. Decreasing negative values of x, i.e., mass functions which favor massive input stars, result in proportionally more massive, and hence brighter, remnants. For $x > 0$, the combination of low-mass input stars with the relatively short lifetimes of the more massive merger products means that the fraction of *observed* stragglers with $M \approx 2M_{\rm turnoff}$ is small. The changes in the luminosity function between the two different metallicities are due to the variation in the slope of the main-sequence as a function of metallicity. Table 1 gives luminosity functions for a variety of values of x in numerical form, for metallicities corresponding to 47 Tuc and M3.

The differences between binary and collisional chemistry are important only for relatively massive input stars, and thus become more obvious for negative values of x. In the case of $x = -2$, there is a readily apparent difference between the two kinds of mergers in the sense that the luminosity function for collisions is biased toward more luminous objects. This difference can be understood as the combination of two effects. First, the greater envelope He abundance of collisional mergers produces stragglers which are generally brighter and bluer (see Fig. 1). Second, the evolved cores of the binary mergers sharply decrease the lifetimes of merger products in which the more massive input star is near the main-sequence turnoff. This depresses the bright end of the luminosity function, where all binary mergers will have a relatively evolved core.

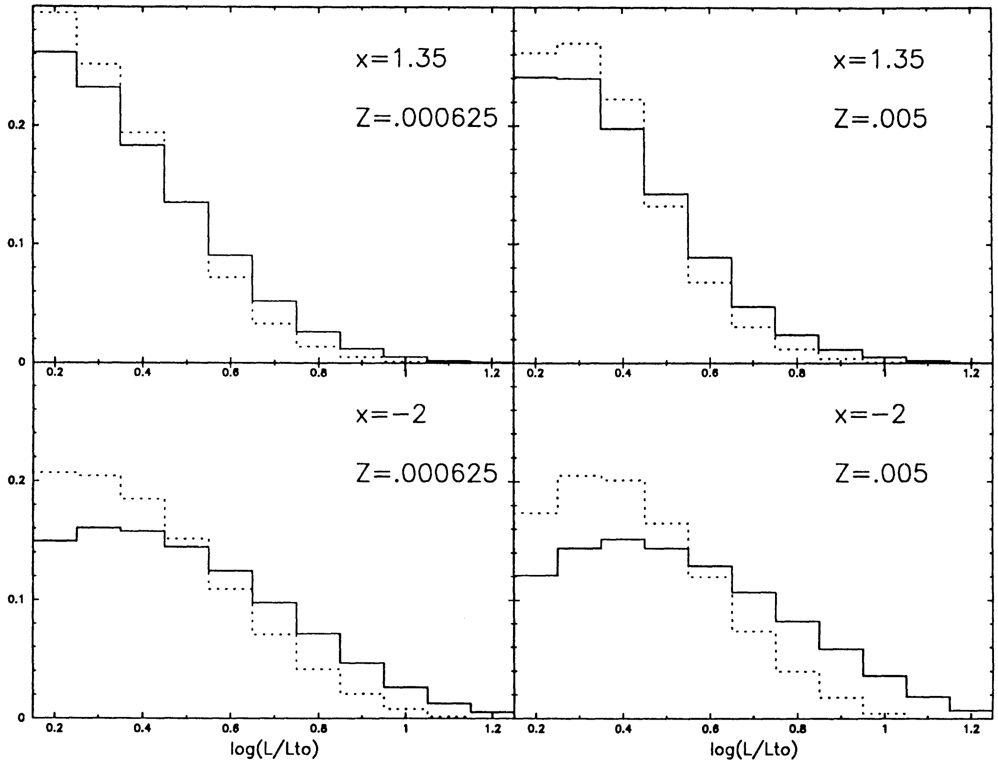

FIG. 2.—Theoretical luminosity functions of blue stragglers for various values of the effective mass function x and the metallicity Z. In all cases the solid line is for the collisional chemistry, while the dotted line is for primordial binary mergers.

TABLE 1

	$x = -2$		$x = 0$		$x = 2$	
log (L/L_{to})	Collisional	Primordial	Collisional	Primordial	Collisional	Primordial
			$Z = 0.005$			
0.2	0.191	0.174	0.195	0.227	0.260	0.277
0.3	0.123	0.205	0.207	0.246	0.252	0.280
0.4	0.131	0.201	0.187	0.217	0.200	0.225
0.5	0.130	0.164	0.149	0.147	0.138	0.124
0.6	0.123	0.120	0.107	0.088	0.081	0.059
0.7	0.106	0.073	0.069	0.045	0.039	0.024
0.8	0.084	0.040	0.042	0.020	0.018	0.009
0.9	0.059	0.018	0.024	0.007	0.008	0.003
1.0	0.035	0.004	0.012	0.002	0.003	0.000
1.1	0.018	0.000	0.005	0.000	0.001	0.000
			$Z = 0.000624$			
0.2	0.149	0.207	0.221	0.261	0.277	0.309
0.3	0.161	0.204	0.210	0.235	0.240	0.258
0.4	0.158	0.185	0.178	0.192	0.184	0.194
0.5	0.145	0.152	0.141	0.142	0.131	0.131
0.6	0.125	0.109	0.103	0.087	0.085	0.065
0.7	0.098	0.071	0.067	0.047	0.046	0.027
0.8	0.071	0.042	0.040	0.023	0.021	0.010
0.9	0.047	0.021	0.022	0.009	0.009	0.004
1.0	0.026	0.008	0.011	0.003	0.004	0.001
1.1	0.013	0.002	0.005	0.001	0.001	0.000

NOTE.—Bins are 0.1 wide in log (L/L_{to}), centered on the given value.

The difference between the blue straggler luminosity functions of collisional and primordial binary mergers is likely to be even more severe than is suggested by Figure 2, since the effective mass function for primordial binary mergers is likely to be significantly greater than that of collisional mergers. Both the lower input masses and the effects of the different chemistry discussed above contribute to a fainter luminosity function for binary blue stragglers as compared to collisional blue stragglers. As can be seen from our results, both effects are important. In §§ 6 and 7 we compare our predictions with data for two cases and find that the differences can be significant enough to constrain the origin of the observed blue straggler sequences.

5. BIRTHRATES OF BLUE STRAGGLERS

The evolutionary tracks we have computed tell us the time a given merger product spends in the blue straggler region. If we consider the merger of two stars with masses m_i and m_j, and write the time spent by the merger product as a blue straggler as T_{ij}, then

$$N_{\rm BS} = \left(\frac{\sum T_{ij} W_{ij}}{\sum W_{ij}}\right) R_{\rm merge} = \langle T \rangle R_{\rm merge} \,, \qquad (3)$$

where $N_{\rm BS}$ is the number of blue stragglers observed at any one time, W_{ij} is the relative frequency of mergers of stars with masses m_i and m_j (computed in this case from the effective mass function), and $R_{\rm merge}$ is the rate at which mergers occur. Note that $R_{\rm merge}$ in general includes mergers between stars which have sufficiently low mass that they never appear as blue stragglers; for these mergers $T_{ij} = 0$. We can use the above equation and our evolutionary tracks to estimate the merger rate from the observed number of blue stragglers (or vice versa). The quantity $\langle T \rangle$ can be computed for any particular value of x for either of the two formation scenarios considered here, and is plotted in Figure 3. From this plot, the merger rate between main-sequence stars required to produce a given observed number of blue stragglers can be determined for a given value of x. The sharp decrease in $\langle T \rangle$ for $x > 0$ is due to the increasing fraction of mergers which have too little mass to ever appear as blue stragglers.

The timescale for collisions, τ_c, for a given star in a cluster can be computed using equation (8) from Hills & Day (1976), given the mass, radius, velocity dispersion, and number density of the stars involved. This equation applies strictly to a population of identical stars; however, the modification required to apply it to a distribution of different stars is straightforward. In

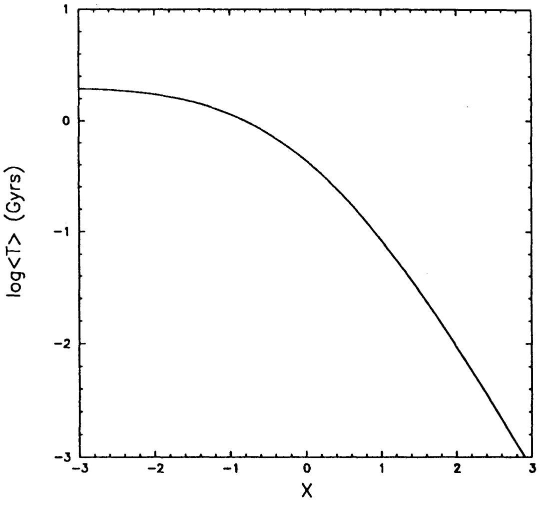

FIG. 3.—Low-metallicity collisional case of log $\langle T \rangle$ as a function of x. Other cases differ by less than 0.1 in the log from the curve shown.

the case of collisions between main-sequence stars, use of "typical" values for these parameters results in only small errors. Thus a rate of collisions between single stars can be estimated from quantities which can in principle be determined observationally, and compared to the rate required to support a given blue straggler population. The calculated rate is in fact a lower limit; Leonard (1989) has pointed out that encounters with binary stars will enhance the collision cross section if a significant number of wide binary stars exists.

6. COMPARISON WITH 47 Tuc

Despite the plethora of recently discovered blue straggler sequences (Sarajedini 1993; Fusi Pecci et al. 1993b), there are only a few data sets appropriate to compare with our theoretical luminosity functions. This is because crowding makes the completeness of most observed sequences difficult to determine. Since the incompleteness will vary with magnitude, sequences for which the completeness is not determined do not represent an accurate luminosity function.

One sample of blue stragglers in a globular cluster which is likely to be complete is the sequence in the center of 47 Tuc which has recently been discovered with *HST* (Paresce et al. 1991; Guhathakurta et al. 1992). The high spatial resolution of the cores of the stellar images allows *HST* to distinguish much fainter stars in the centers of globular clusters than can be done from the ground. Even more important from the point of view of blue stragglers is the possibility of imaging clusters in the UV. At 2200 Å the blue stragglers are the brightest stars in the cluster and stand out strongly against the background of red giants. Judging from the images presented by Paresce et al. (1991) and the UV CMDs presented by de Marchi et al. (1993), it seems unlikely that any blue stragglers have been missed. We therefore adopt the 19 blue stragglers brighter than $m_{342} = 17.3$ (de Marchi, Paresce, & Ferraro 1993) in the center of 47 Tuc as a complete sample of blue stragglers. The magnitude cutoff was chosen at a point above which confusion with photometric errors of main-sequence stars is unlikely. As discussed below, this magnitude cutoff corresponds to $\log (L/L_{\rm to}) = 0.25$, so we will truncate our theoretical luminosity functions at that point.

Since the output from the evolutionary tracks is in bolometric luminosity, a bolometric correction must be applied to the data before they are compared with the models. For standard photometric filters, the bolometric correction as a function of color is tabulated. However, the Faint Object Camera results are for UV filters for which bolometric corrections are not well known. Fortunately, spectral types between A0 and F5 have a bolometric correction for U magnitudes BC_U (defined as $\mathrm{BC}_U = M_{\rm bol} - M_U$) which is constant to within ± 0.1 mag. The constancy of BC_U in this spectral range is due to the competing effects of increasing temperature and increasing Balmer decrement as one goes to earlier spectral types. Since the blue stragglers in 47 Tuc are all in this spectral range (Paresce et al. 1991), differences in m_{342} are approximately equal to differences in bolometric magnitude. The main-sequence turnoff is somewhat redder, and BC_U for the turnoff will be about 0.1 more negative than for the stragglers. Since the main-sequence turnoff is at $m_{342} \approx 18.0$, we have

$$\log (L/L_{\rm to}) = 0.4(17.9 - m_{342}) \tag{4}$$

for a given blue straggler. This is obviously somewhat crude, but given that the results will be sorted into bins 0.1 wide in $\log (L/L_{\rm to})$ or 0.25 wide in magnitude, the inaccuracies will not greatly impact our results. We note that photometric errors are also at the ± 0.1 mag level, and that changes in the strength of the Balmer decrement due to the altered helium abundance may influence the bolometric correction.

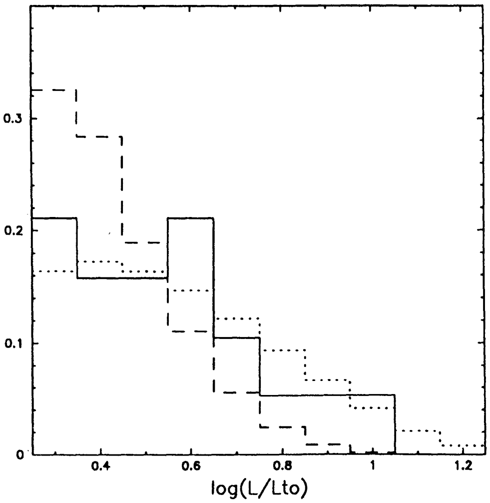

FIG. 4.—Luminosity functions for 47 Tuc. The solid line indicates the 19 observed blue stragglers, the dashed line indicates the prediction of the primordial binary merger scenario for $x = 0.2$, and the dotted line indicates the prediction of the collisional scenario for $x = -2$.

Figure 4 displays the resulting bolometric luminosity function of the observed stars. Also displayed are the theoretical luminosity functions computed from our simplified assumptions for primordial binary mergers with $x = 0.2$ (the observed mass function; Hesser et al. 1987) and for collisional mergers with $x = -2$. A Kolmogorov-Smirnov (K-S) test comparing our theoretical primordial binary luminosity function with the observations show that the two distributions are only marginally consistent (the probability of the K-S statistic of 0.28 occurring by chance is 10%). In contrast, the collisional merger hypothesis is clearly consistent with the data.

Because of the uncertainties in the merger process for primordial binaries, it seems prudent to explore other effective mass functions. Figure 5 plots the K-S statistic between the observed and calculated luminosity functions against the assumed power-law index for the effective mass function. Results for both collisional and binary mergers are displayed. For primordial binary mergers, negative values for x are favored. The relatively low number of observed stars means that x cannot be pinned down precisely. For collisional mergers, the value of $x = -2$, which is consistent with results obtained from fitting multimass King models to the kinematic and photometric data for this cluster (Meylan 1989), is consistent with the data.

Using $0 \geq x \geq -2$, and the collisional chemistry, we find $R_{\rm merge} = [(2\text{–}7) \times 10^7 \text{ yr}]^{-1}$, for $N_{\rm BS} = 19$ as observed in 47 Tuc. Note that we are including only stars with $\log (L/L_{\rm to}) > 0.25$ as blue stragglers in this case. This birthrate can be compared with the expected rate of single-star collisions in the observed region of 47 Tuc. Table 3 of Meylan (1989) gives values for the central mass density, fraction of compact objects, and radial velocity dispersion of a range of multimass King models which are compatible with the existing photometric and velocity data for 47 Tuc. For each of these models, one can then determine the rate of collisions between main-sequence stars.

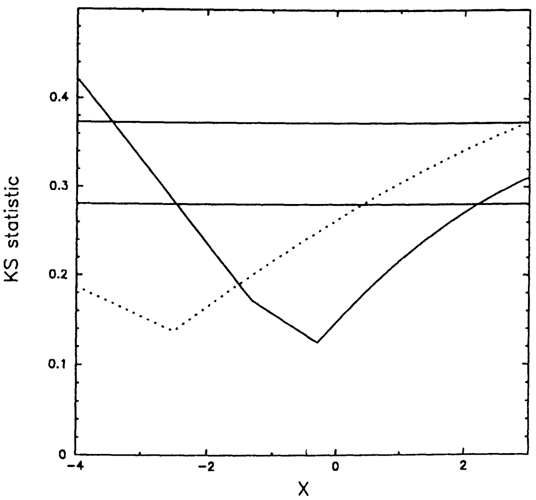

FIG. 5.—K-S statistic between the observed and predicted blue straggler luminosity functions of 47 Tuc, as a function of the effective mass function index *x*. The solid line represents the collisional scenario; the dotted line, the primordial binary merger scenario. The horizontal line at 0.280 shows the 90% confidence limit, and that at 0.374 shows the 99% confidence limit.

The total number of main-sequence stars in the portion of the cluster observed in the *HST* data is $N_{\rm ms} = n_{\rm ms}\,(\theta D)^2\, 2r_c$, where $n_{\rm ms}$ is the number density of main-sequence stars, θ is the angular size of the field of view ($=22''$), D is the distance of the cluster, and r_c is the core radius. Here we have assumed that the density of stars is constant inside 1 core radius, and drops to zero outside the core (the large range of acceptable parameters of Meylan's models suggests that the errors in this assumption are not critical). The rate of collisions R_c is then $R_c = N_{\rm ms}/\tau_c = [(1.5\text{–}7.0) \times 10^7\ {\rm yr}]^{-1}$. This value is consistent with the merger rate required to produce the observed blue stragglers quoted above. Thus direct collisions between main-sequence stars can provide the mergers necessary to produce the observed blue straggler sequence in 47 Tuc.

Leonard (1989) has suggested that the single-star collision cross section may be considerably enhanced by binary–single-star interactions. Since $R_c \approx R_{\rm merge}$, such an enhancement in the collision cross section is *not* required to explain the blue stragglers in the core of 47 Tuc. Indeed, an enhancement of more than a small factor would appear to produce many *more* blue stragglers than are observed. This result suggests that any wide binaries in the core of this dense cluster have either been disrupted or merged before the currently observed blue stragglers were formed. We thus concur with Ferraro, Fusi Pecci, & Bellazzini (1994), who suggest that 47 Tuc is unusual in that it has *fewer* blue stragglers than might be expected. This paucity of blue stragglers in the core of 47 Tuc would seem to indicate that the binary population has been severely depleted by stellar encounters. We note that while wide binaries have been observed in sparser clusters, there is some evidence for depletion in denser clusters (Pryor, Latham, & Hazen 1988; Pryor, Schommer, & Olszewski 1991).

7. M3

The original blue straggler sequence discovered by Sandage (1953) was in the globular cluster M3. This sequence has been the subject of two recent detailed ground-based studies (Ferraro et al. 1993, hereafter F93; Bolte, Hesser, & Stetson 1993) and an *HST* study covering the central regions (Guhathakurta et al. 1994). Both F93 and Bolte et al. (1993) observed large regions of the cluster under excellent conditions, and find that the large blue straggler population is unusual in that it is not centrally concentrated with respect to the other stellar sequences. Thus in this case mass segregation does not appear to be the dominant force in determining the global radial distribution of the stragglers, although it may affect the distribution of some portion of the population. There seems to be a bimodal radial distribution of stragglers, with a high frequency in the inner and outer regions. Bolte et al. (1993) and F93 therefore suggest that these two populations of stragglers may be associated with different formation mechanisms. The high quality of the available data, which suggests that the M3 straggler sequence may be complete, together with the large numbers of stragglers observed and their peculiar radial distribution, impel us to make a detailed comparison of these data with our models.

F93 note that the specific frequency of the blue stragglers drops in the range $100'' \lesssim r \lesssim 200''$. We will therefore use the photometry listed in F93 to examine two populations of blue stragglers: those lying between 20″ and 100″ from the cluster center, and those beyond 225″ from the cluster center. F93 claim to be complete for $V \leq 18.6$ in these regions. The main-sequence turnoff appears to be at $V \approx 19.2$ (F93; Buonanno et al. 1986), and the change in the bolometric correction for V magnitudes between the blue stragglers and the turnoff is 0.1 ± 0.05 (Lang 1991). Thus we will adopt $\log\,(L/L_{\rm to}) = 0.4(19.1 - V)$ for the blue stragglers in M3. Once again we note that our luminosity bins are 0.25 mag wide, so small errors are unimportant. The two resulting blue straggler luminosity distributions are plotted in Figure 6. A K-S test shows that these distributions differ at the 99% confidence level; this provides some corroborating evidence for the presence of two different populations of stragglers in this cluster.

In fact, this result is less robust than it at first appears. An examination of the luminosity function of the inner blue straggler sequence shows a sharp drop in the number of stragglers with $V \gtrsim 18$. As can be seen in Table 1, this lack of faint stragglers is difficult to reproduce in the context of merger

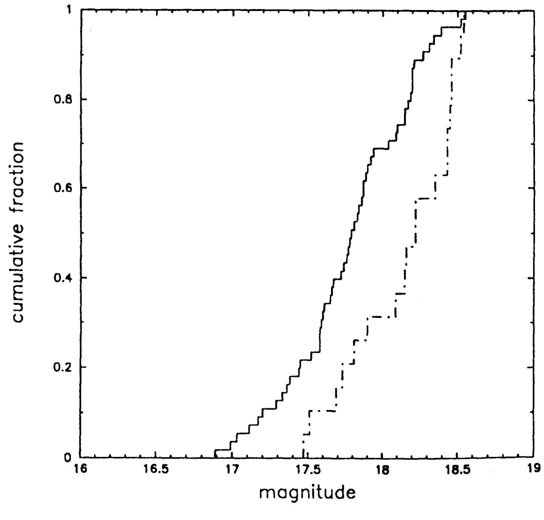

FIG. 6.—Cumulative luminosity functions for the observed inner and outer blue straggler sequences of M3. Inner stragglers are indicated by the solid line, while outer stragglers are indicated by the dot-dash line.

models even with large negative values of x, since the long lifetimes of the fainter mergers would require that orders of magnitude fewer of them be produced than the brighter stragglers. However an examination of Figure 3 of F93 shows that at the faint end of their magnitude limit, only the bluest stars are being counted as stragglers. Their Figure 5 shows that in the outer areas of the clusters, where the principal sequences are much better defined, many of the faint stragglers are relatively red, as would be expected if they are analogs of massive main-sequence stars. Thus, while Ferraro et al. may have complete photometry for $V \leq 18.6$, their list of blue stragglers may not be complete, since some of the faint, relatively red stragglers may be confused with ordinary main-sequence turnoff stars. This problem points out the dangers in the usual regions of the color-magnitude diagram used to define blue stragglers (e.g., Guhathakurta et al. 1994; Bolte et al. 1993; Sarajedini & Da Costa 1991): fainter stars are generally required to be *bluer* to qualify as blue stragglers, whereas theoretical expectations would suggest that fainter blue stragglers should be *redder* than their brighter counterparts. For the purpose of comparison with our theoretical results, we suggest that blue straggler sequences be considered complete *only* to luminosities where the blue straggler sequence is clearly separated from the subgiant branch. For this reason we will adopt a more stringent luminosity limit of log $(L/L_{\rm to}) \geq 0.45$ (i.e., $V < 18.0$) for completeness in the inner region. This new limit creates difficulties for the comparison between the straggler luminosity functions of the inner and outer region, since there are so few bright blue stragglers in the outer regions. However, we note that if the inner stragglers followed the luminosity function of the outer stragglers, there would have to be $\gtrsim 70$ blue stragglers in the magnitude range $18.6 \leq V \leq 18.0$, which seems unlikely.

In Figure 7 we compare the inner stragglers with log $(L/L_{\rm to}) \geq 0.45$ to our computed luminosity functions for low-metallicity stragglers. We find that the luminosity function is intermediate between the collisional and the primordial binary case. Since we are missing both the innermost blue stragglers, which may be relatively bright due to mass segregation, and the fainter ones, which provide the greatest leverage in distinguishing between the formation mechanisms, we are unable to come to any firm conclusions regarding the origin of the inner stragglers from their luminosity function. However, it is worth noting that there are well over 100 blue stragglers in this central region, in a cluster which is less dense and less massive than 47 Tuc. It is therefore unlikely that collisions between single stars alone can account for this large number. We therefore suggest that the core of M3 still contains a significant population of wide primordial binaries, which create an enhanced collision rate (Leonard 1989; Leonard & Fahlman 1991).

The blue stragglers found by Guhathakurta et al. (1994) are well separated from the subgiants for $V \leq 18.0$ all the way into the middle of the cluster. However, the small spatial coverage of the WF/PC I instrument means that there are only 15 blue stragglers reported in this magnitude range, so a detailed comparison with the theoretical luminosity function once again appears unwarranted. However, we note that nine of these stragglers lie in the luminosity range $0.45 \leq \log (L/L_{\rm to}) \leq 0.65$, while six have log $(L/L_{\rm to}) > 0.65$. This ratio of 1.5 is clearly compatible with the ratio of 1.0 predicted by the collisional hypothesis with $x = -2$, given the small number of observed stragglers involved, but is much more difficult to reconcile with the value of 3.4 predicted by the primordial hypothesis. Thus the innermost stragglers in M3 appear to have a luminosity function comparable to that of the stragglers in the core of 47 Tuc.

In contrast, the blue straggler luminosity function from the outer region of M3 is significantly biased toward the faint end compared to the inner region, as can be seen from Figures 6–8. This effect is in the right direction if the outer stragglers were formed from mergers of primordial binaries. As can be seen in Figure 8, the outer luminosity function is very similar to the predictions of the primordial binary scenario with $x = 0.8$, which is the global mass function of M3 (Pryor et al. 1986). The collisional hypothesis is ruled out at the 99% level for $x \leq -1.1$, and at the 90% level for $x \leq 1.2$. Only one of the stragglers has log $(L/L_{\rm to}) > 0.65$, while seven are in the range

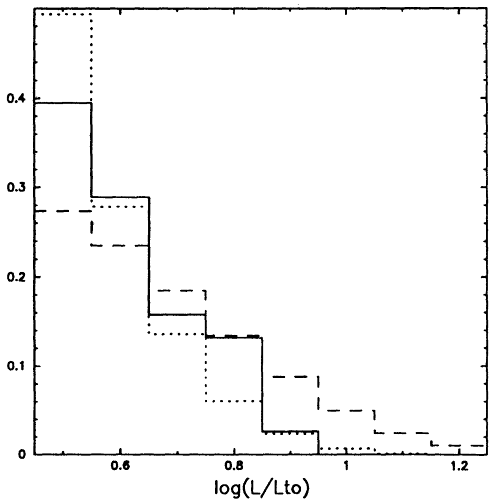

FIG. 7.—Luminosity functions for the inner blue stragglers in M3. The solid line represents the 38 observed stragglers, the dotted line indicates the prediction of the primordial binary merger scenario for $x = 0.8$, and the dashed line indicates the prediction of the collisional scenario for $x = -2$. The limitations of the observed data are discussed in the text.

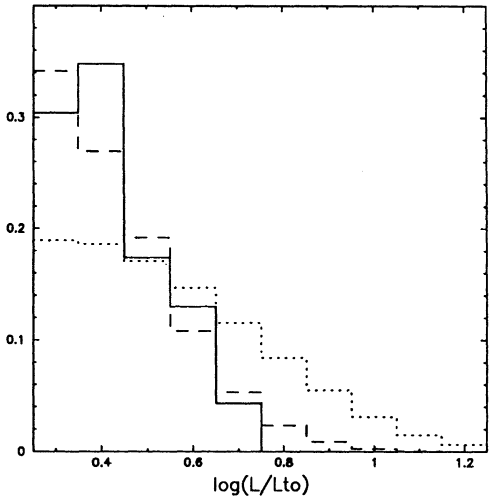

FIG. 8.—Luminosity function for the outer blue stragglers in M3. The solid line represents the 23 observed stragglers, the dashed line indicates the prediction of the primordial binary merger scenario, and the dotted line indicates the prediction of the collisional scenario.

$0.45 \leq \log (L/L_{\mathrm{to}}) \leq 0.65$. Thus this sequence of stragglers is clearly different from that in the core of M3 or 47 Tuc.

An overall explanation of the peculiar distribution of blue stragglers in M3 might then be as follows. The inner regions of the cluster have recently attained some critical density at which the primordial binary population is rapidly destroyed by interactions leading to collisions. In these regions the collision timescale is shorter than the binary evolution timescale, so collisions dominate the production of blue stragglers. In the outer parts of the cluster, where mass segregation has not progressed to the point where most of the primordial binaries have fallen to the center, the evolution of the primordial binaries creates a different kind of straggler. If the outlines of this scenario are correct, then M3 is in an unusual and possibly relatively short-lived phase of its dynamical evolution, during which the cluster's primordial "fuel" of binaries is being rapidly consumed. Sigurdsson, Davies, & Bolte (1994) have suggested that the stragglers in the outer regions of M3 may have been formed in the core and expelled in binary–single-star interactions. While this hypothesis can explain the bimodal radial distribution of the stragglers, it does not provide any obvious explanation for the differences in luminosity function, which would seem to indicate a profound difference in origin between the two sets of stragglers.

8. DISCUSSION

In this paper we have attempted to provide detailed theoretical models of merger products which can be compared with observed blue straggler sequences. Such a comparison can in principle lead to a quantitative test of the merger hypothesis and also to information about the dynamical state and binary populations of the host clusters. Clearly the conclusions which we draw here are suggestive rather than compelling. However, modest advances in the quality of the available data and in our understanding of several theoretical issues might well lead to much stronger results.

The major observational problem is simple to state: we require *complete* sequences of large numbers of blue stragglers to compare with the models. In practice, such data sets are not easy to assemble. Even with recent advances in detectors and resolution from ground-based telescopes, the portion of the color-magnitude diagram near the main-sequence turnoff often remains a muddle. To get a statistically useful sample of stragglers, it will be very helpful to use the fainter stragglers which overlap this region. We therefore strongly encourage observers to perform artificial star experiments to quantify the relevant selection and completeness effects. This is a lengthy and painful task, but without it any comparison with theoretical results will be suspect.

Judicious use of the *HST*, especially in UV filters, will be extremely powerful in some situations, as was the case in 47 Tuc. However, there are also some drawbacks to *HST*. First, the bolometric corrections and principal sequences are not yet well defined for the UV filters. More important, the field of view is small, so it will be difficult to map out large regions of clusters and to acquire large numbers of stragglers. Thus ground-based data taken in excellent seeing, coupled with artificial star experiments, will likely remain an important tool for the study of blue stragglers in globular clusters for some time to come.

From a theoretical standpoint, our parameterization of the mass spectrum of merger products by a power-law effective mass function is highly simplified. For the collisional case, more physically realistic input masses might be obtained from Fokker-Planck simulations of clusters. Coupled with *n*-body and smooth particle hydrodynamical simulations of the colliding stars, more realistic starting masses and chemical compositions could be obtained. The effects of including a population of binary stars, which might alter the collisional cross sections significantly (Leonard 1989; Leonard & Fahlman 1991), should also be taken into account. The situation with mergers of primordial binaries is even more complex. Not only is the initial binary distribution completely unknown, but a variety of effects such as the dependence of magnetic braking on the mass of the binary components would need to be taken into account in determining the true mass spectrum of binary mergers. The chemistry of binary mergers should also be modeled in more detail.

Some extensions of our method may yield further insight into the nature of blue stragglers and the dynamics of the clusters they inhabit. A detailed look at the surface rotation and chemical abundances as merger products evolve might be compared with observations. The distribution of the stragglers in color as well as in luminosity might be compared with our theoretical tracks. The contribution of blue horizontal-branch stars to the blue straggler region must be considered in more detail. Finally, as Fusi Pecci and others have pointed out (Fusi Pecci et al. 1993a; Bailyn 1994), it is clearly of interest to follow the evolution of blue stragglers beyond the main sequence, and to compare the results with the zoo of unusual evolved stars which inhabit globular clusters.

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