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THE FUNDAMENTAL PLANE CORRELATIONS FOR GLOBULAR CLUSTERS

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ABSTRACT

In the parameter space whose axes include a radius (core, or half-light), a surface brightness (central, or average within the half-light radius), and the central projected velocity dispersion, globular clusters lie on a two-dimensional surface (a plane, if the logarithmic quantities are used). This is analogous to the "fundamental plane" of elliptical galaxies. The implied bivariate correlations are the best now known for globular clusters. The derived scaling laws for the core properties imply that cluster cores are fully virialized, homologous systems, with a constant (M/L) ratio. The corresponding scaling laws on the half-light scale are different, but are nearly identical to those derived from the "fundamental plane" of ellipticals. This may be due to the range of cluster concentrations, which are correlated with other parameters. A similar explanation for elliptical galaxies may be viable. These correlations provide new empirical constraints for models of globular cluster formation and evolution, and may also be usable as rough distance-indicator relations for globular clusters.

Subject headings: galaxies: star clusters — galaxy: globular clusters: general — methods: statistical

1. INTRODUCTION

Globular clusters (GCs) have a unique place among the families of stellar systems. They are relatively nearby and are resolved, and we can study them in some detail. Their physical properties, including the characteristic timescales, span a vast dynamical range, thus practically guaranteeing that a large spectrum of dynamical evolution stages will be present. From GCs, we may learn about the old stellar populations and gravitational physics of other, more distant or more complex stellar systems, such as galactic nuclei. Analysis of correlations among their properties can yield fundamental clues for our understanding of their formation and dynamical evolution. This, in turn, may lead to new interpretations of correlations of galaxian properties and their implications for galaxy formation and evolution.

Similar approaches for elliptical galaxies have been very fruitful in the past, including the discovery of a so-called "fundamental plane" (FP) of elliptical galaxy properties (cf. Djorgovski 1992a, b, 1994, for reviews and references). The FP represents a set of bivariate correlations which provide hints for, and constraints on the models of formation of elliptical galaxies. These correlations also represent possible distance indicator relations for ellipticals. In its most popular form, the FP is given as a bivariate correlation between the radius, surface brightness, and velocity dispersion.

In this *Letter* we present evidence for an analogous family of bivariate correlations for GCs, both for their core and half-light properties. This FP of GC properties represents an interesting constraint on the physics of globular clusters and provides new hints about the nature of the FP of elliptical galaxies.

2. THE DATA AND ANALYSIS

The data used here are from the compilations by Peterson (1993), Pryor & Meylan (1993), Trager, Djorgovski, & King (1993, 1995), and Djorgovski (1993b). A more complete analysis is presented by Djorgovski & Meylan (1994). Here we concentrate on a subset of GC parameters, including: log of the core radius in parsecs, r_c; log of the half-light radius in parsecs, r_h; the central surface brightness in the V band, $\mu_V(0)$; the average surface brightness in the V band within the r_h, $\langle\mu_V\rangle_h$; and log of the central velocity dispersion in kilometers per second, σ. These are the equivalents of the variables used in the studies of the FP of elliptical galaxies. We have such data for 56 clusters, σ being the most restricting quantity (note that we do not have available velocity dispersions averaged out to r_h, but only the central values). There is no reason to believe that there are selection effects present, which would bias the derived correlations in any significant way. The data are better for the core parameters than for the half-light parameters. We note that the clusters with a post–core-collapse morphology (Djorgovski & King 1984, 1986) are treated here just as if they are a high-concentration end of the King (1966) model sequence, which turns out to be an excellent approximation.

Since GCs are subject to a number of evolutionary processes and factors, both internal (e.g., the core collapse) and external (e.g., tidal shocks), their properties may be correlated in a complex manner. This calls for a multivariate statistical analysis (MVA). A pioneering study of GC properties using MVA was done by Brosche & Lentes (1984). Subsequent work includes the papers by Djorgovski (1991), and Djorgovski & Meylan (1994). Other studies used MVA effectively to address problems in the GC research (Djorgovski & Santiago 1992; Fusi Pecci et al. 1993; Djorgovski, Piotto, & Capaccioli 1993; Santiago & Djorgovski 1993).

Consider a distribution of data points in an N-dimensional parameter space of N input observables. If there are any correlations present in the data, then data points will not fill the entire N-dimensional volume, but rather a volume of a lower dimensionality, D. The statistical dimensionality $D \leq N$ of the data manifold gives the true number of independent factors which determine the properties of objects, regardless of how many independent measured quantities are there. Once this is established, one can look for the correlations between observables using various optimization techniques. As a rule, the

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correlations will be D-variate; i.e., any of the input quantities could be correlated to within the measurement errors with some combination of D others. The monograph by Murtagh & Heck (1987) is very useful in describing the techniques used here.

We first apply the principal component analysis (PCA) in order to find the statistical dimensionality of the data set, i.e., the number of independent factors contributing to the total sample variance in all measured quantities. Djorgovski & Meylan (1994) did this for a full data set of a dozen observed GC parameters, and obtained a statistical dimensionality of $D > 4$ or 5. However, if only the photometric, structural, and dynamical parameters of clusters are used, the statistical dimensionality is $D = 3$, even though there are five or six independently measured input quantities (Djorgovski & Meylan 1994). This is exactly as expected from a family of objects described by King (1966) models.

However, if we consider data subsets consisting of the central velocity dispersion, radius, and surface brightness, their statistical dimensionality is $D = 2$. The first two eigenvectors (principal axes) of the data ellipsoid account for 98.4% or 97.1% of the total sample variance, for the core and half-light parameters respectively; the remainder is accountable by the measurement errors. The first eigenvector alone accounts for some $\frac{3}{4}$ of the total variance, but the second one is still highly significant, indicating that two dimensions are necessary. In other words, data sit on a tilted plane in the three-dimensional space of these observables (luminosity can also be added or substituted for the surface brightness or radius). This is equivalent to the situation for elliptical galaxies, and the result implies directly that there is indeed a FP of GC properties. Note that mixing the core and the half-light parameters or adding the concentration index increases the dimensionality back to $D = 3$.

In order to find the specific expressions for the implied bivariate correlations, we first search for an optimal mix of any two variables [e.g., σ and $\mu_V(0)$] which maximizes their correlation coefficients with the third one (e.g., r_c). We then perform a least-squares fit which accounts for the errors in both coordinates. These fits are discussed below.

3. A FUNDAMENTAL PLANE OF GC PROPERTIES

Figure 1 shows the simple pairwise (monovariate) correlations between the core and the half-light properties, and three examples of the newly derived bivariate correlations.

The correlation between the core radius, r_c, and the central surface brightness $\mu_V(0)$, has been noted before by Kormendy (1985) and by Djorgovski & Meylan (1994). Also, r_h and $\langle\mu_V\rangle_h$ are by necessity correlated, since $\langle\mu_V\rangle_h$ is derived by dividing half a cluster's luminosity by πr_h^2 (see also van den Bergh, Morbey, & Pazder 1991). The correlations of cluster velocity dispersions, σ, with surface brightness [either $\mu_V(0)$ or $\langle\mu_V\rangle_h$] have been discussed by Djorgovski (1993a) and Djorgovski & Meylan (1994). As already found by Djorgovski (1991), core radii and concentrations play the roles of a "second parameter" in these velocity dispersion correlations, in the sense that clusters with smaller cores and/or higher concentrations have higher central velocity dispersions at a given luminosity or surface brightness.

The new bivariate correlations represent the views of the FP of GCs edge-on. The improvement in bivariate correlations over the best monovariate ones is relatively modest, although it is clearly present. The improvement over the other monovariate correlations is rather striking. These are the best nontrivial correlations for GCs now known.

For the core parameters, we obtain a bivariate least squares solution in the form:

$$\log r_c = (1.8 \pm 0.15)[\log \sigma + 0.24\mu_V(0)] - (8.6 \pm 0.6) , \quad (1)$$

which corresponds to a scaling law:

$$r_c \sim \sigma^{1.8\pm0.15} I_0^{-1.1\pm0.1} . \quad (2)$$

Fitting through surface brightness (a better and more stable fit), we get

$$\mu_V(0) = (-4.9 \pm 0.2)(\log \sigma - 0.45 \log r_c) + (20.45 \pm 0.2) , \quad (3)$$

which can be solved through radius to a scaling law:

$$r_c \sim \sigma^{2.2\pm0.15} I_0^{-1.1\pm0.1} . \quad (4)$$

The average of these two solutions is remarkably close to the scaling law expected from the Virial Theorem:

$$r_c \sim \sigma^2 I_0^{-1} (M/L)^{-1} . \quad (5)$$

Thus, equation (4) is consistent with globular cluster cores being virialized systems with a universal and constant (M/L) ratio to within the measurement errors, neither of which comes as a surprise. Nevertheless, this is the first such demonstration for globular clusters. One could, in principle, concoct models in which σ, $\mu_V(0)$, and a variable (M/L) are coupled in such a way that produces the observed equation (4), but that seems artificial and would require some fine tuning. Assuming $(M/L) =$ const. seems more natural. The comparison of equations (4) and (5) actually provides a more stringent constraint on $(M/L) =$ const. than the direct computations of the (M/L) suggest (cf. Pryor & Meylan 1993).

This is somewhat surprising, since a small deviation from $(M/L) =$ const. may be expected due to the mass segregation effects, as the heavier, dark stellar remnants would tend to sink toward the cluster center. While observations of surface brightness profiles and velocity dispersions are largely dominated by the red giants and subgiants, both the velocity dispersions and radial density distributions of stars dominating the light are affected by the presence of other stellar mass species in a nontrivial manner. Furthermore, we know that some differences in stellar mass function slopes do exist among the globular clusters, at last in their less crowded regions where such measurements are performed (McClure et al. 1986; Djorgovski et al. 1993). However, these differences may be mostly tidally induced, and not very strong in the cores. Careful dynamical modeling may be needed in order to estimate the importance of these effects, and it is possible that equation (4) may provide nontrivial constraints on the degree of mass segregation and stellar mass function differences in GC cores.

For the half-light parameters, we do not get an improved bivariate fit through radius, due to the strong coupling of measurement errors in r_h and $\langle\mu_V\rangle_h$, but only through the surface brightness, viz.,

$$\langle\mu_V\rangle_h = (-4.1 \pm 0.2)(\log \sigma - 0.7 \log r_h) + (19.8 \pm 0.1) , \quad (6)$$

which can be solved through radius to a scaling law:

$$r_h \sim \sigma^{1.45\pm0.2} I_h^{0.85\pm0.1} . \quad (7)$$

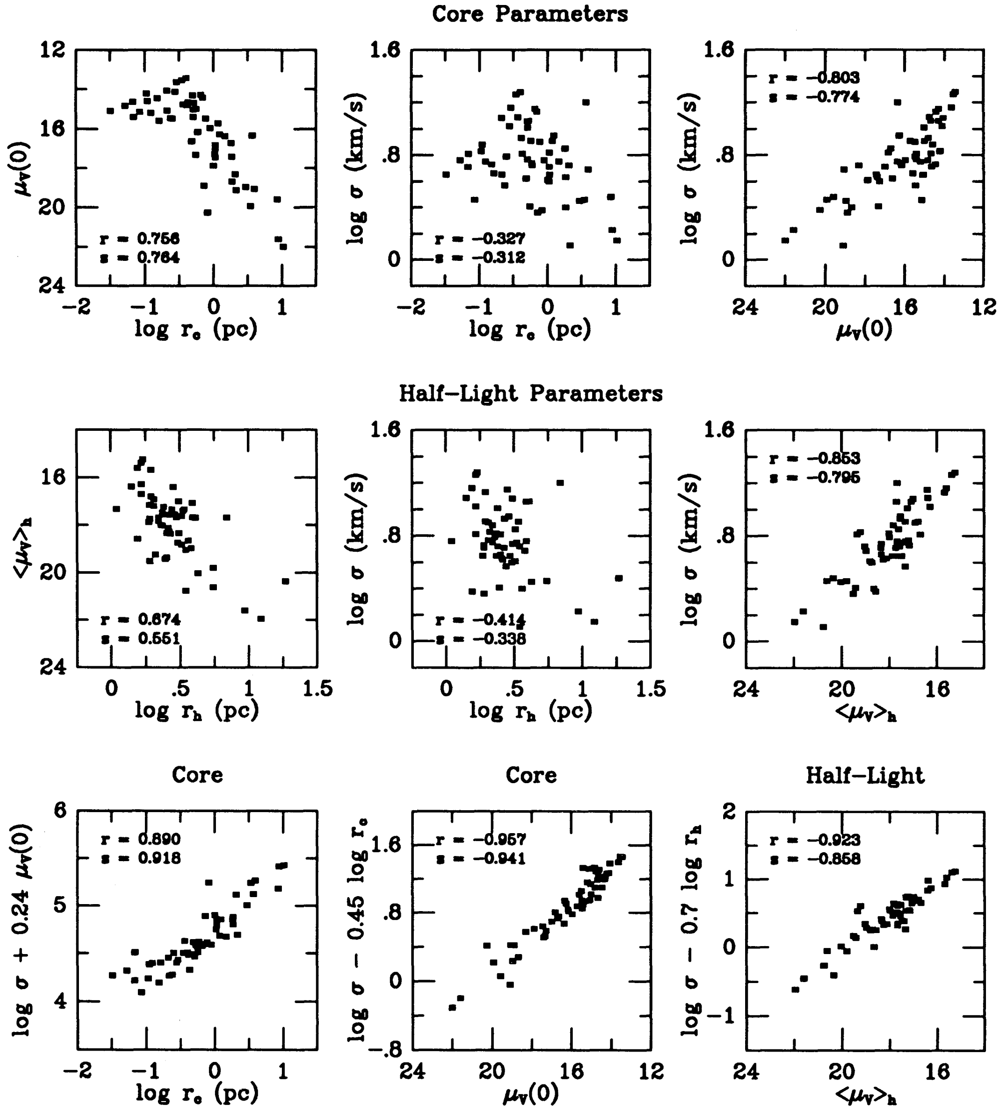

FIG. 1.—The pairwise correlations between the input observables, radii, surface brightness, and central velocity dispersions, for the core parameters (*top row*), and the half-light parameters (*middle row*). The bottom row shows two bivariate correlations for the core data, and one for the half-light data set. These represent the views of the FP of globulars edge-on. Pearson linear regression (r) and Spearman rank (s) correlation coefficients are shown in each panel.

This is different from the core parameters (but perhaps only on a marginally significant level). However, it is nearly identical to the equivalent FP solutions for elliptical galaxies, which is typically found out to be

$$R \sim \sigma^{1.4 \pm 0.2} I^{-0.8 \pm 0.1} . \quad (8)$$

This is a very intriguing result. Interpreted in terms of the change in the (M/L) ratios, as it is commonly done for the FP of ellipticals (cf. Faber et al. 1987; Djorgovski 1988; 1992a, b), it would imply a scaling law $(M/L) \sim M^{0.16 \pm 0.05}$. This does not seem to be physically reasonable for GCs, and it is also in a direct contradiction to our result on the FP of the core properties.

An alternative explanation, which was also proposed for the FP of elliptical galaxies (Djorgovski 1992a, b; Djorgovski & Santiago 1993), is simply that GCs are not a homologous family of objects, and that the mapping of the true median radii, surface densities, and mean velocity dispersions into the observed quantities r_h, $\langle \mu_V \rangle_h$, and σ varies along the GC sequence. We know this to be true, since there is indeed a shape parameter in the King models sequence. For example, we do not have the velocity dispersions averaged within r_h, but use here the central values, and the relation between the two would be a function of the cluster concentration (as well as other, ill-constrained factors, such as the velocity anisotropy). Cores, on the other hand, are structurally very similar. The deviation of the scaling law in equation (7) from that in equation (5), which is derived from the pure Virial Theorem and the assumption of homology, is then almost certainly due to a systematic change in cluster concentrations, which is correlated with their other properties (Djorgovski & Meylan 1994). This suggests that a similar explanation may be at work for the elliptical galaxies.

The new FP correlations may be used as rough distance-indicator relations for GCs, since only the radii are distance dependent. The implied accuracy in distances from the present data is only $\sim$70%–80%, but that may be improvable. While this is not competitive with the traditional methods which employ color-magnitude diagrams, it might provide a useful check in some situations, e.g., for the heavily obscured clusters, etc.

4. CONCLUDING REMARKS

We have demonstrated that the characteristic radii, surface brightness, and central velocity dispersions for GCs form statistically two-dimensional manifolds, both on the core and half-light scales. This "fundamental plane" of GC properties produces the best correlations now known for GCs. For the cores, the derived scaling laws imply that these are fully virialized systems, with uniform, constant (M/L) ratios. On the half-light scale, the derived scaling laws are different, and nearly identical to those derived for the elliptical galaxies. At least in the case of GCs, this may be explained by the change in cluster concentrations along the King model sequence. A more detailed modeling of this effect is clearly desirable.

While the FP of GC properties on the half-light scale is tantalizingly similar to that of the elliptical galaxies, it would probably be a mistake to assume that they reflect entirely the same physics. For example, the metallicities and colors of ellipticals participate in their FP (de Carvalho & Djorgovski 1989), whereas metallicities of GCs do not correlate with any other property (Djorgovski 1993a; Djorgovski & Meylan 1994).

It is also not obvious that these correlations should have the shape they do after some 10^{10} yr or more of dynamical evolution in the tidal field of our Galaxy (cf. Chernoff & Weinberg 1990, and references therein), or how they may constrain the initial conditions of GC formation (cf., e.g., Fall & Rees 1985; Larson 1988; or Murray & Lin 1992). Indeed, Brosche & Lentes (1984) have already seen a relation between the cluster tidal radii and distances to the Galactic center. The new scaling laws derived here may provide strong observational constraints for models of GC formation and evolution.

Finally, it would be of a great interest to explore if such correlations apply for GC systems in other galaxies, e.g., the Magellanic Clouds, or M31, both of which are within the reach of the available observing technology.

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