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UPPER LIMIT OF THE ANGULAR VELOCITY OF NEUTRON STARS

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ABSTRACT

The maximum angular velocity of uniformly rotating neutron stars is investigated. If we consider a hot neutron star stage after its birth from supernova explosions, an upper limit of the angular velocity for uniformly rotating neutron stars becomes a little smaller than that obtained by the equation of state for cold neutron matter. Since the equatorial radius of hot neutron stars becomes large, the angular velocity for such configurations cannot become so large. Therefore the hot neutron star stage sets the limit to the angular velocity of uniformly rotating cold neutron stars if we consider an evolution without losing the angular momentum during the cooling stage. We have studied cold and hot neutron stars whose temperature ranges from 0.75 MeV to 9 MeV. We assume hot neutron stars to be isothermal and in hydrostatic equilibrium. Using some equations of state of the hot neutron gas and the two-dimensional numerical code to get equilibrium structures of rapidly rotating relativistic stars, we have solved sequences of rotating neutron stars up to the mass-shedding state. For hot neutron stars of temperature of 9 MeV with a very soft equation of state, we get a stable model in a critical state with the angular velocity of 8.0×10^3 s^{-1}, which corresponds to a neutron star with a rotational period of 0.8 ms and a gravitational mass of 1.6 $M_\odot$. If this neutron star cools down to a cold neutron star of 0.75 MeV without losing its angular momentum, the angular velocity becomes 7.0×10^3 s^{-1}. It corresponds to the rotational period of 0.9 ms. We conclude that 1 ms is a rough lower limit of the rotational period of neutron stars born from the explosion of massive stars ($M > 10$ $M_\odot$).

Subject headings: dense matter — equation of state — stars: interiors — stars: neutron — stars: rotation

1. INTRODUCTION

Rapidly rotating neutron stars can be used to determine equations of state (EOSs) for high-density matter (see, e.g., Friedman, Ipser, & Parker 1984, 1986, 1989; Eriguchi, Hachisu, & Nomoto 1994). This can be done because the maximum angular velocity of rotating neutron stars depends crucially on the EOS if the angular velocity becomes very large. The angular velocity can be larger for the softer EOS because the softer EOS gives models with a smaller equatorial radius. In this respect, if we could find very short period pulsars, we would be able to exclude stiffer EOSs by which we could not construct rotating models with the observed period.

At the present time, however, even the fastest pulsar, PSR 1937+21, rotates rather slowly compared with the fastest models which can be constructed numerically by using various EOSs (see, e.g., Friedman et al. 1984, 1986, 1989; Eriguchi et al. 1994). This may be because we have not found faster pulsars yet, though they exist, or there may be other physical reasons why there are no pulsars which rotate faster than PSR 1937+21. In this paper we will study the second possibility from the neutron star formation scenario.

Neutron stars are widely considered to be born through supernova explosions or accretion-induced collapses of white dwarfs. Concerning the supernova explosion scenario, the exact relations between supernova explosions and neutron stars have not been fully explored yet because numerical computations have not given birth to neutron stars from explosions of massive stars (e.g., Bethe 1993). Thus the relation between supernova explosions and resulting neutron stars remains one of the major problems of the final stage of stellar evolution toward supernova explosions (e.g., Bethe 1990; Nomoto & Hashimoto 1988; Woosley & Weaver 1988). In fact, we cannot tell definitely whether SN 1987A and/or SN 1993J have produced neutron stars or black holes either from observations or from theories at present.

Although the situation concerning the origin of neutron stars is as mentioned above, we may expect many neutron stars are born through supernova explosions. During formation stages, the structure of neutron stars will change in a very small timescale of order of 1 s. If we assume neutron stars to be spherical, proto–neutron stars will contract rapidly and cool down to hot neutron stars in 0.1 s to 1 s after the bounce. Although the cooling mechanism of proto–neutron stars is very complicated due to the neutrino diffusion, the temperature of proto–neutron stars ranges from 50 to 100 MeV, and the radius is about 100 km. Hot neutron stars ($T \sim 10$ MeV) will cool down to cold neutron stars ($T \sim 0$ MeV) within a timescale of seconds to a day.

Therefore we may consider that hot neutron star stages will last rather long compared to rotational periods or dynamical timescales, which are of order of milliseconds, so that we will assume hot neutron stars are in equilibrium states. In this case

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the EOS of finite temperatures is extremely important to investigate the structure of hot neutron stars (e.g., Kunihiro et al. 1993). The thermal effect on the EOS of neutron star matter becomes important when the temperature of the star exceeds some MeV ($\sim 10^{10}$ K). The equation of state of high temperatures is different from that of cold states. In general, high temperatures will make the radius of the hot neutron stars larger. Neutron stars with a larger equatorial radius cannot rotate rapidly. This possibility will set a limit to rotation rates of neutron stars.

In the present paper, we will study an upper limit of the angular velocity of uniformly rotating hot neutron stars by computing their critical configurations due to mass shedding. In particular, we present results for a very soft EOS because the softest one gives an upper limit of the angular velocity (see, e.g., Friedman et al. 1984, 1986, 1989; Eriguchi et al. 1994). We found that 0.9 ms is the minimum rotational period of neutron stars which evolve from hot neutron stars of temperature of ~ 10 MeV.

2. ASSUMPTIONS AND RESULTS

2.1. *Assumptions*

Since thermal structures of hot neutron stars have not been fully established due to the uncertainty of the EOS, we will assume that the temperature distribution in the most part of neutron stars is isothermal. However, temperature of the crust of the hot neutron star must be much cooler than the inner hotter region. Thus the temperature of regions where the density is below a certain value is assumed to be zero. Here, for the low-density region ($\rho < 10^{10}$ g cm^{-3}), we adopt the EOS of Baym, Pethick, & Sutherland (1971, hereafter BPS). Although this assumption is very crude, it will be accurate enough to find the upper limit of the angular velocity. This is because the transition region is very thin and will not affect the global structures of stars such as the mass and the radius.

As for the rotation of neutron stars, it is expected that the rotation will become uniform before the star cools down to $T \sim 0$ K. Thus we assume that the temperature at which uniform rotation is established is $T \sim 10$ MeV. Therefore, we treat uniformly rotating isothermal hot neutron stars with a temperature of ≤ 10 MeV.

2.2. *Spherical Hot Neutron Stars*

Before we discuss the effect of rotation on the structures of hot neutron stars, we will summarize the structure of spherical neutron stars with high temperatures. For the equation of state at high temperatures, we will adopt three different EOSs: (1) the EOS of Friedman & Pandharipande (1981, hereafter FP), (2) Model I EOS of Oyamatsu (1993) modified by taking into account the finite temperatures, and (3) the stiff EOS for finite temperatures of Sumiyoshi & Toki (1994, hereafter RMF). Model I and RMF EOSs are phenominological ones which fit masses, radii of nuclei, and saturation property of nuclear matter. These two EOSs are used to take into account the present uncertainties about the equation of state of the neutron star matter. For Model I EOS, the effect of finite temperatures is included in the kinetic energy by assuming the Fermi-Dirac distribution for that of a single particle. We have, however, neglected the excitation in the bulk matter energy because below the temperature of 50 MeV, the potential energy of the neutron gas is dominated by the bare interaction of the nuclear force. It should be noted that Model I and FP differ only for very high density regions. The slope of FP is steeper than that of Model I. In the RMF EOS, a nonlinear σ potential is included to reduce incompressibility at nuclear density and to reproduce nuclear property well (Sumiyoshi & Toki 1994).

We will show three EOSs in Figure 1, EOSs of FP and Model I, with $T = 0$, 3, 6, 10, and 20 MeV are plotted by solid lines and dot-dashed lines, respectively. Those for RMF are shown by the dashed lines. The degree of softness (or stiffness) of the EOS can be seen from Figure 1 which is the pressure versus the density for several constant temperatures. Beyond $\sim 5 \times 10^{14}$ g cm^{-3}, thermal effect on the pressure can be neglected due to strong degeneracy of neutrons so that lines converge to a single line irrespective of the temperature.

In Figure 2, gravitational masses of neutron stars are shown against the central density. The maximum masses for each EOS are 1.55 (Model I), 1.95 (FP), and 2.9 (RMF) $M_{\odot}$, respectively. The masses of neutron stars are governed mainly by the EOS for the regions with $\rho > 5 \times 10^{14}$ g cm^{-3}. As long as the temperature remains below 10 MeV, masses of neutron stars do not depend on temperatures so significantly due to the degeneracy of the neutron gas.

In Figure 3, gravitational masses of neutron stars are plotted against the radius. This figure shows that the radius depends on the temperatures contrary to the mass. As seen from these figures, Model I (Oyamatsu 1993) is the softest, FP is intermediate, and RMF is the stiffest EOS.

2.3. *Rapidly Rotating Hot Neutron Stars*

We solve structures of rapidly rotating neutron stars with the Model I EOS by Oyamatsu (1993) which is very soft as shown in the previous section. As discussed in the Introduction, the softer EOS is crucial to determine the upper limit of the angular velocity of rotating neutron stars. Thus we will be able to set limit to the angular velocity by using the Model I EOS by Oyamatsu (1993) because it is the softest. In order to

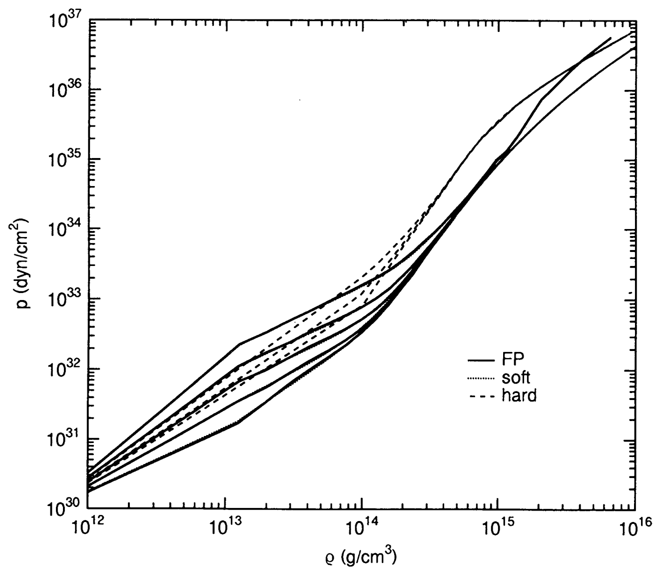

FIG. 1.—The energy density and the pressure relation, i.e., the equations of state by different authors. FP denotes the equation of state by Friedman & Pandharipande (1981), Model I by Oyamatsu (1993) which is the softest, and RMF by Sumiyoshi & Toki (1994) which is the stiffest. Curves correspond to different temperatures: from upper to lower curves, 0, 3, 6, 10, 13, 16, and 20 MeV both for Model I and for FP. For RMF, the curves correspond to 3.1, 6.1, 9.7, 13.7, 15.3, and 19.3 MeV.

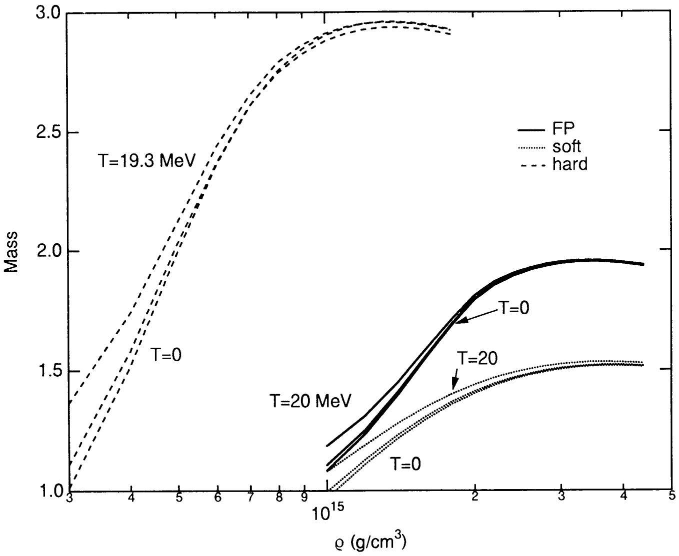

FIG. 2.—Same as in Fig. 1 except for the gravitational mass–central energy density relation.

solve rapidly rotating and general relativistic structures, we have used the code developed by Komatsu, Eriguchi, & Hachisu (1989a, b) and extended to neutron stars (Eriguchi et al. 1994). One equilibrium model is constructed by computing equilibrium structure for a fixed central energy density and a given ratio r_p/r_e of the polar radius (r_p) to the equatorial radius (r_e). By changing the ratio r_p/r_e we have a sequence of equilibrium models. After obtaining one sequence, we solve other sequences by changing the central energy density.

The realistic evolution of rotating neutron stars can be approximated as follows. During the evolution we assume that the total angular momentum is conserved and that no baryon mass is lost from the neutron star. If we find two models with the same total angular momentum and the same baryon mass, one for the high temperature and the other for the low tem-

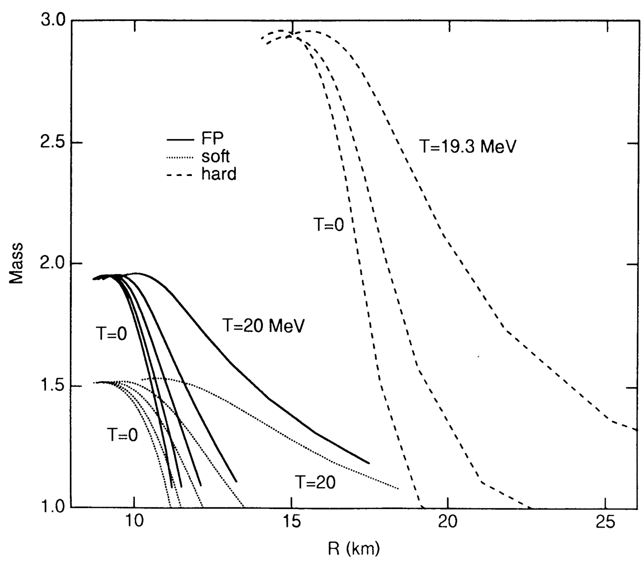

FIG. 3.—Same as in Fig. 1 except for the gravitational mass–radius relation.

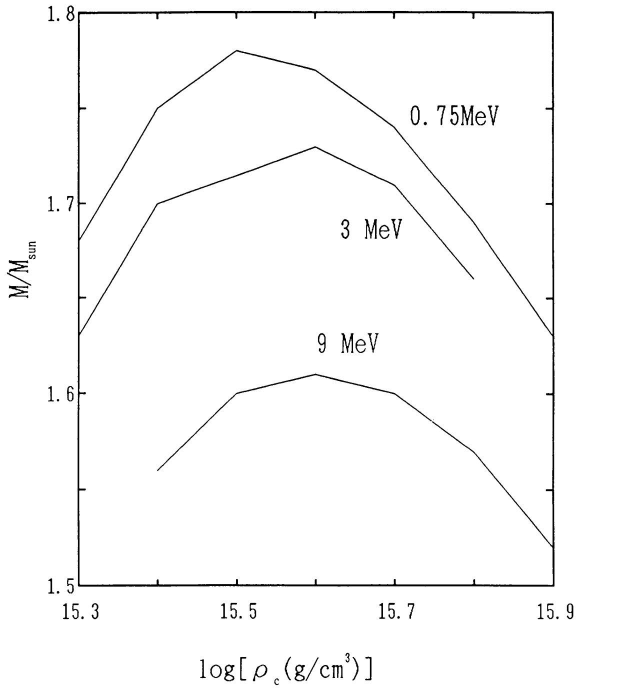

FIG. 4.—The sequences of the gravitational mass of the neutron star against the central energy density for the mass-shedding state are shown for $T = 0.75$, 3.0, and 9.0 MeV.

perature, we can consider the high-temperature neutron star will evolve to the low-temperature neutron star. As for the initial state of the evolution, we select the most rapidly rotating model from stable configurations, because the angular momentum for that model is the largest. The maximum angular velocity for stable configurations is determined by the following conditions: (1) the gravitational mass of the model is extreme against the central energy density on the equilibrium sequence with the constant total angular momentum, i.e.,

$$\partial M/\partial \rho_c \Big|_{J=\mathrm{const}} = 0 \,, \tag{1}$$

where M, ρ_c, and J are the gravitational mass, the central energy density, and the total angular momentum, respectively; and (2) the model is in a critical state, i.e., at the mass-shedding state, beyond which no equilibrium states are allowed due to the centrifugal force. In Figure 4 the sequence of the gravitational mass of the neutron star against the central energy density for the mass-shedding state are shown.

In Table 1 are shown the results for two models of temperatures of 0.75 MeV and 9 MeV, which correspond to the cold and hot neutron stars, respectively. The angular momentum is 8.9×10^{48} g cm^2 s^{-1}, and the rest mass is 1.77 $M_{\odot}$. The hot model is in a critical state where the mass begins to shed

TABLE 1

PHYSICAL QUANTITIES FOR SEVERAL MODELS

Model	T (MeV)	ρ_c (g cm^{-3})	$M/M_{\odot}$	Ω (s^{-1})	P (ms)
Model I	9.0	4.0e15	1.61	8.1e3	0.8
	0.75	2.5e15	1.59	7.0e3	0.9
Critical Model	0.75	2.5e15	1.75	1.0e4	0.6

from the equator, i.e., in a mass-shedding state. It should be noted that this angular velocity is relatively small owing to the soft equation of state. This comes from the fact that the radius is large due to the high temperatures. Thus the angular momentum of the hot neutron stars is small.

As seen in Table 1, if the neutron stars are settled down to a uniform rotation with a temperature of 10 MeV, it will evolve to a rotating neutron star with a rotational period of 0.9 ms. In Table 1 the critical state for a temperature of 0.75 MeV and a central energy density $\rho = 2.5 \times 10^{15}$ g cm^{-3} is also tabulated. Thus the neutron stars which experience a very hot stage will rotate with the angular velocity of $\frac{2}{3}$ of the critical one or the Kepler angular velocity.

3. DISCUSSION AND CONCLUSION

3.1. *Upper Limit of the Angular Velocity Due to Secular Instabilities or Other Reasons*

Rapidly rotating stars are considered to suffer from the secular instability due to the gravitational radiation (see, e.g., Chandrasekhar 1970). However, this instability has been shown to be stabilized by the existence of the viscosity for stars at homogeneous density (see, e.g., Detweiler & Lindblom 1977; Lindblom & Detweiler 1977). For compressible models, using Newtonian gravity, some authors (Ipser & Lindblom 1991; Yoshida & Eriguchi 1994) have shown that the viscosity will stabilize the gravitational radiation-induced instability for most temperatures. However, for some range of temperatures, models remain unstable even if the viscosity is employed.

However, the models which will suffer from the instability due to the gravitational radiation must rotate with the angular velocity larger than 95% of the angular velocity of the critical state. It implies that this stability criterion is not severe, compared to the criterion obtained in this paper.

Neutron stars with large angular velocity may exist in binary systems. In a binary system, slowly rotating neutron stars can be accelerated by accretion from secondary stars. In this circumstance it is natural to consider that neutron stars rotate with a critical angular velocity or a Kepler angular velocity. Thus for neutron stars in binary systems, there can be pulsars with a rotational period shorter than 1 ms.

3.2. *Isothermality*

Clearly the isothermal assumption of the star breaks down at the crust where the radiative diffusion dominates toward the surface. Then we should take into account the detailed physics determining the structure of the crust (e.g., Lattimer & Swesty 1992). Even for the inner part of a neutron star ($\rho > 5 \times 10^{14}$ g cm^{-3}), our assumption will break down if the thermal conduction is not fast enough to establish an isothermal configuration. As far as the whole structure of a neutron star, such as mass and angular momentum, is concerned, the structure is mainly determined from the states at the central region with $\rho > 5 \times 10^{14}$ g cm^{-3} so that the EOS at the crust affects the whole structures very little. Then our treatment will be justified if the thermal conduction is fast enough.

3.3. *Softness of the Equation of State*

The EOS we have used is very soft. There are several arguments that the EOS may be very soft. Shigeyama, Nomoto, & Hashimoto (1988) have constructed a model of the explosion of SN 1978A on the basis of light curve modeling. The resulting explosive nucleosynthesis of their analysis is consistent with the observation (Hashimoto, Nomoto, & Shigeyama 1989). In their model, the inner core of 20 $M_\odot$ star is chosen to be 6 $M_\odot$. The explosion is assumed to leave the baryon mass of 1.6 $M_\odot$ core which consists of an iron core of 1.4 $M_\odot$. The rest, 0.2 $M_\odot$, is assumed to be composed of silicon and sulfur which accrete to the iron core. The 1.6 $M_\odot$ of the baryon mass corresponds to ~ 1.4 $M_\odot$ gravitational mass.

Some authors (e.g., Brown, Bruenn, & Wheeler 1992) suggested that a black hole of 1.6 $M_\odot$ will be formed from the explosion of SN 1987A. This possibility can be marginally consistent with the EOS of Model I with a finite temperature. It is because the EOS cannot allow a mass larger than 1.5 $M_\odot$ for neutron stars. On the other hand, since the stiff EOS such as that of RMF can sustain ~ 3.0 $M_\odot$ as neutron stars even if the star does not rotate (see Fig. 2), black holes with mass less than 1.6 $M_\odot$ or so will not be formed. Thus very stiff EOSs must be excluded. Recent study of the presupernova evolution suggests that the iron core of the massive star of 10–70 $M_\odot$ ranges from 1.1 to 1.8 $M_\odot$ (Hashimoto & Nomoto 1994). Furthermore, Tsujimoto et al. (1994) have derived the upper mass of ~ 50 $M_\odot$ for the massive stars that result in neutron stars. Actually, for $M > 40$ $M_\odot$, a huge mass loss reduces even the core mass of the massive star (Weaver & Woosley 1993). Therefore, a very soft EOS is compatible with the present study of the stellar evolution scenario.

If the softening due to the pion or kaon condensation occurs during cooling stages of hot neutron stars, the EOS would become more soft than that of Model I. It should be noted that EOSs which are too soft are excluded because they will not sustain spherical neutron stars with a mass of 1.44 $M_\odot$ of the binary pulsar PSR 1913+16 (Taylor & Weisberg 1989). Then slightly stiffer EOSs should be required. A detailed study of the relation between the maximum angular velocity and the degree of the stiffness of EOS will be the subject of the next paper in this series (Hashimoto, Oyamatsu & Eriguchi 1994).

3.4. *Conclusions*

We conclude that the maximum angular velocity cannot exceed about 7.0×10^3 s^{-1}. In other words, the period of a pulsar cannot be shorter than about 0.9 ms. Therefore, most single pulsars rotate with periods of larger than roughly 1 ms.

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