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SODIUM IN WEAK G-BAND GIANTS

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ABSTRACT

Sodium abundances have been determined for eight weak G-band giants whose atmospheres are greatly enriched with products of the CN-cycling H-burning reactions. Systematic errors are minimized by comparing the weak G-band giants to a sample of similar but normal giants. If, further, Ca is selected as a reference element, model atmosphere–related errors should largely be removed. For the weak-G-band stars [Na/Ca] = 0.16 ± 0.01, which is just possibly greater than the result [Na/Ca] = 0.10 ± 0.03 from the normal giants. This result demonstrates that the atmospheres of the weak G-band giants are not seriously contaminated with products of ON cycling.

Subject headings: stars: abundances — stars: evolution — stars: giants

1. INTRODUCTION

Astrophysial interest in the element Na has been strongly piqued in recent years by several studies that have pointed to the large Na excesses (up to factors of 5 relative to Fe) revealed in some abundance analyses of F–K supergiants (Luck 1977; Boyarchuk & Lyubimkov 1983; Boyarchuk et al. 1988a, b). Boyarchuk & Lyubimkov (1983) speculated that the Na excess might be the result of the transformation of ^{22}Ne to ^{23}Na by the reaction ^{22}Ne(p, γ)^{23}Na, first suggested by Marion & Fowler (1957), operating during main-sequence core hydrogen burning and the subsequent post–main-sequence mixing of core and envelope, known as the first dredge-up. Predictions of the Na enrichment of F–K supergiants following the first dredge-up were made by Denisenkov (1989) and Dearborn (1992)—see the review by Lambert (1992). The observed enrichments considerably exceed the predictions. To our knowledge, three solutions to this puzzle have been suggested.

A factor controlling the predicted enrichment is the initial ^{22}Ne abundance. The solar system isotopic ratio of ^{20}Ne/^{22}Ne ≈ 14, combined with the relative Ne and Na abundances, implies a ratio of ^{22}Ne/^{23}Na ≈ 4 (Anders & Grevesse 1989); converting all the ^{22}Ne to ^{23}Na in the stellar core and diluting some fraction of this, depending on the depth of core penetration of the dredge-up, throughout the envelope would yield only a small Na excess at the stellar surface. Denisenkov (1988, 1989) had to invoke a ^{22}Ne abundance higher by a factor of 3–4 than the solar system value in order to reproduce the observed Na abundances. Since studies of Galactic cosmic-ray particles have found the isotope ratio ^{22}Ne/^{20}Ne to be 3 times above solar (Mewaldt et al. 1980), Denisenkov argued that this suggestion, although arbitrary, is not entirely unreasonable. Lambert (1992), however, noted that the excess ^{22}Ne in the cosmic rays probably reflects a contribution to the cosmic-ray composition from Wolf-Rayet stars (Silberg & Tsao 1990). If this excess ^{22}Ne was generated in the Wolf-Rayet stars, it cannot be taken as representative of the initial composition of the F–K supergiants.

An adequate reservoir of Ne exists as ^{20}Ne. Prantzos, Coc, & Thibaud (1991) examined proton capture on ^{20}Ne to produce ^{21}Na and related reactions but, in order for the first dredge-up to result in Na enhancements similar to the observed values, the rate for ^{20}Ne(p, γ) had to be increased by a factor of 10 over the recommended value (Caughlan & Fowler 1988). Examination of the basis for the recommended rate led Prantzos et al. (1991) to conclude that "such an enhancement is unlikely."

These two apparently unsuccessful scenarios retain the standard prescription of stellar evolution but propose a modification to the Ne "reservoir" from which the Na enrichment is prepared. In an alternative picture, Denisenkov (1993) suggests that turbulent diffusive mixing in the radiative envelope of the main-sequence progenitors of the F–K supergiants leads to such large-scale conversion of ^{22}Ne to Na that the predicted surface Na abundances after the first dredge-up may match the observed Na abundances. The turbulent diffusive mixing is induced by the angular rotation so that the observed Na abundances are expected to show a star-to-star scatter arising from the variation of rotational velocity. Additional supergiants must be observed to confirm this expectation.

These three solutions seek to seed the atmospheres of the F–K supergiants with more Na than is predicted by the standard evolutionary calculations. As an alternative, one might seek to uncover systematic errors affecting the abundance analyses. The studies of Luck (1977) and Boyarchuk & Lyubimkov (1983) were classical local thermodynamic equilibrium (LTE) analyses. Even though both studies used subordinate lines of Na I, the reality of the Na excess, under the threat of departures from LTE in the low-density atmospheres of the supergiant stars, could be in some doubt. This objection has been answered, at least in part, by Boyarchuk et al. (1988a, b), whose probably adequate, non-LTE calculations indicated only small non-LTE effects in the observed Na I lines. Calculations by Drake (1991a) produced a similar answer, though with some equivocation.

Since the predicted Na synthesis occurs in H-burning layers, further clues to the puzzle might logically be found where there is evidence for strong CN processing. In this paper we look at the abundances of Na in the Weak G-band (WGB) giants. These relatively rare Population I giants were first noticed by Bidelman (1951). An analysis by Greenstein & Keenan (1958) of the prototype of the class, HD 18474, indicated it to be deficient in CH, the molecule largely responsible for Fraunhofer's G band, by a factor of 100. Narrow-band and spectroscopic abundance studies of several stars have now established that the atmospheres of the WGB stars are severely contaminated by CN-processed material (Hartoog, Persson, & Aaronson 1977; Cottrell & Norris 1978; Sneden et al. 1978; Day 1980).

Although the nucelosynthetic signatures of the WGB stars are easily read, the processes leading to the contamination of their atmospheres remain quite uncertain. Especially puzzling is the substantial presence of lithium in many WGB stars. Hartoog (1978) detected relatively "normal" Li abundances in 10 WGB giants, obtaining values ranging from an upper limit of log $\epsilon(\text{Li}) \leq 0.8$ to near the cosmic value of log $\epsilon(\text{Li}) \approx 3.0$, where log $\epsilon(x)$ is the abundance of x relative to H on the customary scale where log $\epsilon(\text{H}) = 12$. The additional sample observed by Lambert & Sawyer (1984) suggested that Li is, on average, overabundant relative to the abundance seen in normal G and K giants, and the maximum Li abundance is about equal to that expected for the main-sequence progenitors and exceeds by a factor of about 50 Li abundance expected and observed for giants that have undergone the first dredge-up. These Li abundances are a potent clue to the origins of the WGB stars and certainly present difficulties for otherwise attractive explanations invoking meridional circulation and turbulent diffusive mixing of the envelope through the high temperature sites of the CNO cycles at the convective core of a rapidly rotating main-sequence progenitor, since the preservation of the Li at the stellar surface is then most unlikely. Some compensatory Li production through operation of the ^{7}Be-transport mechanism (e.g., Cameron 1955; Cameron & Fowler 1971) or by spallation reactions at the stellar surface (e.g., Canal, Isern, & Sanahuju 1980) are not implausible. Lambert & Sawyer (1984) point out, however, that there is then no straightforward explanation for the observed cosmic Li abundance as the upper limit for Li in WGB stars. They speculate that large-scale diffusion within a (magnetic) protostar transported C inward to be converted to ^{13}C and ^{14}N, and Li outward to be largely preserved near the cooler surface layers. At the time of the first dredge-up, the red giant's convective envelope mixed the C-depleted deep layer with the Li-rich surface layers to create a WGB star enriched in CN-processed material but with substantial amounts of Li—perhaps even having its initial Li abundance. The WGB giants would then be descendants of one or more classes of the (magnetic) peculiar A stars.

The Na abundance could be both useful in resolving the WGB enigma and in untangling some of the outstanding puzzles in the Na enhancement scenarios. If the WGB giants are indeed descendants of Ap stars, then their masses must lie in the range $2 \leq M/M_\odot \leq 3$. Canonical stellar evolution models which include the Ne-Na cycle reactions offer predictions of the surface Na abundance following dredge-up.

Abundances of O determined for several WGBs (Cottrell & Norris 1978; Sneden et al. 1978; Day 1980) show no deficiencies which might suggest contamination due to ON-cycle products, providing an observational constraint on the maximum depth of penetration of dredge-up. Removing this uncertainty then reveals the potential value of the Na abundance. If significant Na enrichment beyond the levels allowed by classical models is observed and one rejects the notion of a drastically nonsolar ^{22}Ne abundance, then a reasonable conclusion must be that this is a result of transport of additional ^{22}Ne through the sites of CN processing. This result would support a mechanism such as large-scale diffusion in a main-sequence A star progenitor; the "normal" surface Li abundances rule out more disruptive turbulent diffusive mixing.

2. OBSERVATIONS

Stars were chosen from the WGB stars studied by Hartoog, Persson, & Aaronson (1977), for which they obtained J, H, and K colors. Spectra were obtained using the coudé spectrograph, equipped with a Tektronix 512 × 512 CCD detector, at the 2.1 m Struve telescope of the McDonald Observatory during 1991 April and May. A conventional grating blazed at 21° and with 1200 grooves mm^{-1} was used in first order for all observations. This configuration yielded a 2 pixel resolution of $\lambda/\Delta\lambda = 24{,}000$ near 6000 Å. We observed a spectral region centered near 6150 Å for each of our target stars and for the K0 III star Polux (β Gem) which we have adopted as a standard. Exposure times varied between 10 minutes and 1 hr for the WGB stars. This region was chosen primarily because it contains the Na I doublet lines of multiplet 5 at 6154.2 and 6160.7 Å that are not susceptible to large non-LTE effects in late-type giant stars (Drake 1991a) and which are both strong enough to be observed at our relatively modest spectral resolution and sufficiently weak so as to be insensitive to uncertainties in the microturbulence parameter. This region also features a number of relatively unblended calcium and iron lines suitable for analysis and is sufficiently uncrowded by spectral lines to enable accurate continuum placement. A tungsten filament lamp was observed following each stellar integration for flat-field purposes. The observations are summarized in Table 1. Raw spectra were reduced using standard IRAF tasks. The final extracted spectra had signal-to-noise ratios of between 100 and 200 in the continuum. Typical examples of these spectra are illustrated in Figure 1. In order to determine whether or not our spectra suffered contamination to any extent by scattered light in the spectrograph, we also observed Mars using the same spectrograph settings. While Mars is not a point source at the angular resolution of our spectrograph, it

TABLE 1

SPECTROSCOPIC OBSERVATIONS OF WEAK G-BAND GIANTS IN THIS STUDY

HD	HR	Exposure (s)	Date (day/month/year)
67728	...	1800	28.04.91
78146	...	3600	28.04.91
94956	...	3600	01.05.91
120170	...	3600	03.04.91
165462	6757	2700	28.04.91
165634	6766	2400	29.04.91
166208	6791	600	04.04.91
62509	2990[a]	200	03.04.91
112989	4924[b]	2400	01.01.91
Mars[c]	...	30	28.04.91

[a] The adopted standard star β Gem.

[b] 37 Com.

[c] Observed to monitor the quality of measured equivalent widths.

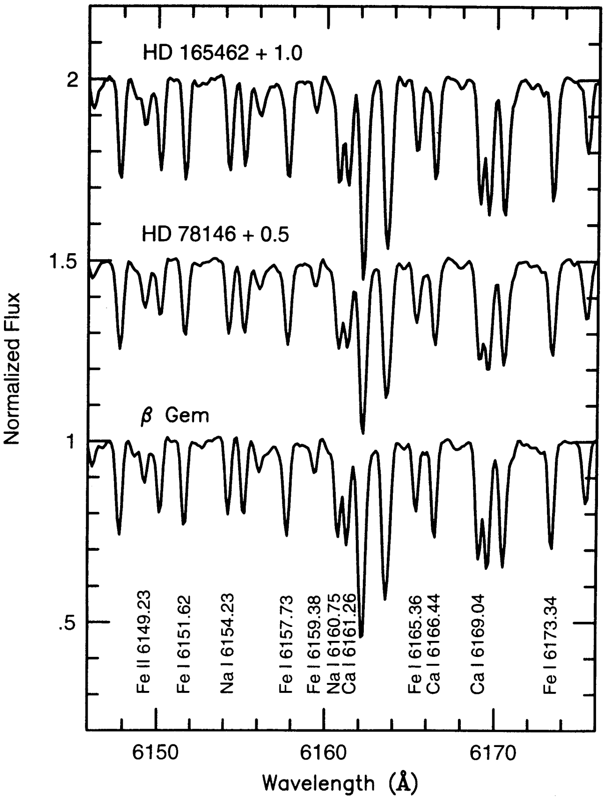

FIG. 1.—The 6160 Å region in β Gem and two stars in the sample of (weak G-band) WGB giants. The lines used in the analysis are labeled.

provides a reasonable approximation to the slit illumination characteristics of our stellar observations.

We have included an additional star, HD 112989 (37 Com), in the sample presented here. This star was observed as part of a program to look at Na abundances in a sample of normal G–K field giants (Drake & Lambert 1994). The observational and data reduction procedures adopted for HD 112989 were identical to those described above, except that a different diffraction grating with the same groove spacing but working in second order was employed. This configuration yielded a higher resolution of $\lambda/\Delta\lambda = 75{,}000$ at the cost of lower efficiency and wavelength coverage.

3. ANALYSIS

3.1. *Stellar Parameters*

The stellar parameters for our standard star, Pollux, have been determined spectroscopically from a fine analysis of high-quality spectra by Drake & Smith (1991), and we adopt those parameters in this study. Unfortunately, the wavelength coverage and resolution of the spectra obtained here did not permit a similar approach in the case of our WGB stars.

To estimate the effective temperatures, we used the $(J-K)$ colors of Hartoog et al. (1977). The temperature calibration of G and K giants is a problem that has still not been solved with universal satisfaction. The commonly used color-temperature calibrations of Ridgway et al. (1980) exhibit a rather disturbing scatter in $(J-K)$, and we eschewed these in favor of the synthetic photometry of Bell & Gustafsson (1989). The absolute $(J-K)$ calibration in the Johnson (1965) system of Bell & Gustafsson (1989) is probably too high; however, they obtained temperatures about 100–150 K lower using a modified version of the Infrared Flux Method (IRFM), devised by Blackwell & Shallis (1977) and subsequently modified (see Blackwell et al. 1986), which in turn are about 20–40 K hotter than the IRFM temperatures of Blackwell, Lynas-Gray, & Petford (1991). We prefer the latter scale, which gives excellent agreement with temperatures derived spectroscopically by Smith and coworkers, including the Drake & Smith (1991) temperature we have adopted for Pollox—see Smith (1992) for a discussion. The final temperatures are the Bell and Gustafsson $(J-K)$ values, corrected by -150 K. The $J\ H\ K$ photometry of Hartoog et al. (1977) was transformed to the Johnson (1965) system using the relations in Persson, Aaronson, & Frogel (1977). The spread in the Bell and Gustafsson $(J-K)$–$T_{\rm eff}$ relation caused by surface gravity and metallicity within the ranges $\log g = 2.25$ to 3.0 and $[M/H] = 0.0$ to -0.5, respectively, is about 0.02 mag, corresponding to about 30 K. Since this is about the same as the stated uncertainty in the $J\ H\ K$ photometry (0.03 mag), we did not take into account gravity or metallicity in our temperature calibration.

At the effective temperatures of our program stars the Blackwell et al. (1991) scale is only 50 K hotter than that of Ridgway et al. (1980). Since our analysis is entirely differential, and we have placed all of the stars on the same temperature scale, we emphasize that our fastidious choice of calibration is by no means critical.

For estimating surface gravities for field G–K giants, we adopted a similar approach to that of Lambert & Sawyer (1984), who assumed their sample of WGB stars were clump giants. This enables a reasonable, though somewhat probabilistic, constraint to be placed on stellar luminosity; we adopted $L/L_\odot = 1.65$ (Cannon 1970). Assuming a stellar mass of $M/M_\odot \approx 1.5$ then yields the gravity through the usual relation

$$\log g = -10.53 + 4 \log T_{\rm eff} - \log L/L_\odot \ .$$

The effective temperature for HD 112989 was derived slightly differently but has been placed on the same Blackwell et al. (1991) temperature scale. This star was included in the Li survey of Brown et al. (1989), who derived effective temperatures for each of the 644 G–K giants studied from photometric colors. This endeavor yielded a reasonably self-consistent calibration for a large number of field giants. We have compared the Brown et al. (1991) scale with that of Blackwell et al. (1991), whose star sample also included a number of G and K giants in common with the Brown et al. (1991) survey: on average the Blackwell et al. (1991) temperatures are hotter by $\sim$50 K. Since we prefer the Blackwell et al. (1991) scale, for HD 112989 we have therefore adopted a temperature that is hotter by 50 K than that derived by Brown et al. (1991). For consistency, we have adopted a surface gravity calculated using the relation above.

The final set of parameters for our program stars is listed in Table 2. For the purposes of calculating abundances, we initially adopted a metallicity of [M/H] = 0.0 for all stars.

3.2. *Abundances*

In order to minimize any possible systematic errors which might arise from a differential analysis of G–K giant stars with respect to the Sun, such departures from LTE, systematic errors in adopted parameters, and errors from using solar and stellar spectra of quite different quality, we have determined all abundances with respect to our adopted standard, Pollux. The abundance analysis of Pollux by Ruland et al. (1980; see also

TABLE 2

JHK Photometry Effective Temperatures and Surface Gravities for Weak G-Band Giants

HD	$E(B-V)$	K	$J-H$	$H-K$	T_{eff} (K)	log g (cm s^{-2})
67728	0.05	4.89	0.58	0.11	4790	2.5
78146	0.05	5.84	0.58	0.11	4790	2.5
94956	0.05	6.17	0.50	0.09	5115	2.6
120170	0.15	6.66	0.55	0.07	5265	2.7
165462	0.05	3.80	0.54	0.11	4915	2.5
165634	0.05	2.32	0.50	0.11	5040	2.6
62509[a]	...	...	...	...	4875	2.75
112989[a]	...	...	...	...	4625	2.3

Notes.—The *J H K* photometry for the weak G-band giants from Hartoog et al. 1977 together with our adopted effective temperatures and surface gravities.

[a] Refer to text for explanation of temperature and gravity determination.

Griffin 1976) using high-resolution photographic spectra revealed it to be a quite unremarkable star in that most elements appeared to have nearly solar abundances—precisely the quality which makes it a good standard for the present study.

We derived abundances for the unblended and unsaturated Ca and Fe lines within the 60 Å of our chosen wavelength interval as well as for the two Na I lines of multiplet 5. These lines are listed in Table 3. Some of the useful lines in our chosen wavelength region are not as clean as we would ideally prefer, but, since there is not a great number of useful lines available, we have retained them in the analysis. These lines were accorded lower weights by a factor of 2 in the final abundances. We have also given lower weights to the Fe I lines with low excitation potential (<3.5 eV) since these lines are known to be susceptible to larger non-LTE effects than lines of higher excitation (see § 3 below). Similarly, the best lines were accorded higher weights by the same factor. The "quality" category of each line is indicated in Table 3.

The Na I lines are more sensitive to temperature than the Fe I lines, owing to the lower ionization energy of Na I and the consequence that Na I is a more minor species than Fe I throughout the range of the parameters of our program stars. The ionization energy of Ca I (6.11 eV) is very similar to that of Na I (5.89 eV), and the Ca I lines serve as a useful "control," responding to uncertainties or errors in effective temperature in the same way as the lines of Na I. The Na/Ca and Na/Fe ratios are also quite insensitive to possible errors in the adopted surface gravities. Furthermore, it is unlikely that the Ca (or the Fe) abundance has been altered by nuclear reactions (presumably proton captures) in the layers where Na is manufactured from either ^{22}Ne or ^{20}Ne. In short, Ca is an ideal benchmark against which to measure a Na enrichment.

These atmospheres were generated for each star separately using the MARCS program. These atmospheres are similar to those described by Gustafsson et al. (1975) and published in grid form by Bell et al. (1976). Based on an analysis of strong Ca lines, Drake & Smith (1991) found that a MARCS model with parameters appropriate to Pollux provided a good representation of the atmospheric structure of this star. The MARCS models should also therefore, be appropriate to the sample of stars we consider here. There are, however, possible complications which might arise as a result of the WGB stars having low C abundances and hence weaker CO lines; we discuss these below.

Abundances were calculated using the LINFOR program (H. Holweger, M. Steffen, & W. Steenbock, Universität Kiel). In order to derive the final abundances, it is first necessary to adopt values for the microturbulence parameter. These were derived for each star by appealing to the Fe and Ca lines to yield abundances independent from line strength. The abundances relative to Pollux and the derived microturbulent velocities are listed in Table 4.

3.3. *Equivalent Widths*

The equivalent widths of the Na, Ca, and Fe lines were measured by fitting Gaussian profiles. The combined effects of rotational, turbulent, and thermal broadening in the spectra of our stars probably amount to 3–6 km s^{-1} (e.g., Gray 1992) which is considerably less than our spectral resolution. However, we have found through experience that the convoluted effects of stellar broadening and the 2.1 m coudé spectrograph instrumental profile can be reasonably approximated by a Gaussian function and that the fitting of Gaussian profiles is a quite reasonable approximation. Some of the lines used are rather blended in one wing. In these cases, we fitted Gaussian components to all of the lines in the blend: the wavelengths of the known components were fixed, and only the line strengths were allowed to vary.

TABLE 3

Lines Used and Their Equivalent Widths

λ	Species	χ	67728	78146	94956	120170	165462	165634	166208	112989	β Gem
6154.23	Na I	2.10	107	116	90	91	95	93	100	120	84
(6160.75)	Na I	2.10	119	116	98	108	121	115	112	133	113
(6161.30)	Ca I	2.52	126	134	99	102	110	104	108	130	98
<u>6166.44</u>	Ca I	2.52	129	120	106	100	111	106	105	133	108
(6169.04)	Ca I	2.52	135	142	128	120	135	122	135	...	144
(6149.23)	Fe II	3.89	72	75	66	56	70	65	71	60	57
(6151.62)	Fe I	2.18	124	115	101	89	...	98	104	128	96
6157.73	Fe I	4.07	135	123	108	90	107	105	102	133	113
6159.38	Fe I	4.61	48	44	27	20	31	30	41	43	35
<u>6165.54</u>	Fe I	4.14	90	78	76	66	83	74	75	93	77
(6173.33)	Fe I	2.22	158	149	120	116	129	123	125	...	121

Notes.—The lines used in the analysis and their equivalent widths in each of the program stars. Wavelengths in parentheses denote lines that were given lower weighting in the final abundances; lines given higher weighting were denoted by underlined wavelengths.

TABLE 4

ABUNDANCE RESULTS AND UNCERTAINTIES

STAR	ξ (km s^{-1})	ABUNDANCES[a] [Fe/H]	[Ca/H]	[Na/H]	[Ca/Fe]	[Na/Fe]	[Na/Ca]
HD 67728	2.00	0.12	0.04	0.15	−0.08	0.03	0.11
HD 78146	1.75	0.13	0.10	0.25	0.03	0.12	0.15
HD 94956	2.00	−0.02	−0.03	0.08	0.01	0.10	0.11
HD 112989/HR 4924[b]	2.0	−0.05	−0.10	0.12	−0.05	0.17	0.22
HD 120170	2.00	−0.07	0.05	0.22	0.13	0.29	0.17
HD 165462/HR 6757	1.75	0.00	0.00	0.15	0.00	0.15	0.15
HD 165634/HR 6766	1.60	−0.20	−0.07	0.08	0.13	0.28	0.15
HD 166208/HR 6791	1.50	0.12	0.10	0.28	−0.02	0.16	0.18
$\Delta T_{\rm eff} = -200$ K	...	−0.10	−0.20	−0.15	−0.10	−0.05	0.05
$\Delta \log g = +0.5$ cm s^{-2}	...	−0.05	0.05	0.02	0.10	0.07	−0.03
Δ[M/H] = −0.5	...	−0.02	0.01	0.02	0.03	0.04	0.01
$\Delta\xi = +0.5$ km s^{-1}	...	−0.25	−0.23	−0.13	0.02	0.12	0.10

NOTE.—Weak G-band giant abundance results and uncertainties arising from errors in the adopted stellar parameters (see text).

[a] Relative to Pollux.

[b] 37 Com.

The equivalent widths are listed in Table 3. In order to monitor their quality, we (1) compared equivalent widths for lines in the same spectral region measured from a spectrum of Mars with values measured from the Kurucz et al. (1984) solar atlas and (2) compared equivalent widths (see Table 3) measured from our spectrum of Pollux to those determined from very high quality Reticon spectra by Drake & Smith (1991). These latter were obtained using the McDonald Observatory 2.7 m coudé telescope and spectrograph with an echelle grating in a configuration which employed a grating monochromator to isolate the required spectral order.

Inspection of the different Pollux and solar equivalent widths suggests that our 2.1 m spectra yield values which are about 5% larger than those from the high-quality spectra. This is most probably a result of our inability to resolve weak spectral features that blend with the true line wings. Since such a systematic effect is small, we have made no attempt to correct for it.

3.4. *Uncertainties*

In order to estimate the likely influence of uncertainties in our adopted parameters on the abundance results, we have calculated the effects of changing, orthogonally, each of the stellar parameters $T_{\rm eff}$, log g, [M/H], and ξ, by 200 K, 0.5 dex, 0.5 dex, and 0.5 km s^{-1}, respectively. These effects are also dependent on the stellar parameters themselves, but, since our program stars do not differ widely in their parameters, they are represented to sufficient accuracy by the values calculated for Pollux. The abundance differences resulting from a change in a particular stellar parameter *centered on the parameters adopted for Pollux* are also listed in Table 4.

Since we have taken care to place all of the stars, including Pollux, on the same *absolute* temperature scale, the uncertainties in $T^*_{\rm eff} - T^{\rm Pollux}_{\rm eff}$ are likely to be dictated largely by the uncertainties in the photometry. The formal uncertainty in $T_{\rm eff}$, resulting from an uncertainty of 0.3 mag, is about 80 K: the total uncertainty in $T^*_{\rm eff} - T^{\rm Pollux}_{\rm eff}$ is therefore likely to be about 100 K. The absolute scale is possibly uncertain by a further 100 K, so the total uncertainty on any one temperature is probably as much as 200 K.

The surface gravities could be in error by more than 0.5 dex if our diagnoses of mass and evolutionary phase are significantly in error. Stars of mass $M/M_\odot \approx 2$ spend less than 10^8 yr on the first ascent of the red giant branch (RGB) and $1-2\times10^8$ years on the horizontal branch (HB) (e.g., Lambert & Ries 1981). Any low-mass field giant is therefore statistically more likely to be on the HB than the RGB.

The effects of errors in the metallicity adopted for the model calculations are negligible: lowering [M/H] of the model atmosphere by 0.5 dex results in a maximum change in the average abundances of 0.02 dex or so. The effects on the Na and Fe abundances are in the opposite sense, however, so that the net change in [Na/Fe] amounts to about 0.04 dex. Our derived Fe abundances, differing by a maximum of 0.13 dex from the [M/H] adopted, vindicate our choice of model parameters.

It is the uncertainty in the inconspicuous, but potent, microturbulence parameter that engenders the largest abundance uncertainty. The microturbulence for Pollux is well determined from a set of Ca I lines (Drake & Smith 1991); we estimate that our microturbulence values for the WGB stars have an accuracy of about 0.35 km s^{-1}. This uncertainty was based on the acceptable range of microturbulence values which yielded effectively no slope in the abundance-equivalent width plane. Such an uncertainty leads to an uncertainty in the Fe and Ca abundances of 0.15 dex or so and to about 0.1 dex in the Na abundance. The corresponding uncertainties in [Na/Fe], [Ca/Fe], and [Na/Ca] are 0.08, 0.01, and 0.07, respectively. Since, in one or two cases, the equivalent widths of some of the lines in our WGB stars are slightly larger than those of the corresponding lines in Pollux, for the lines in question the uncertainties due to microturbulence could be slightly larger than those quoted for Pollux by a couple of hundredths of a dex or so.

3.5. *Departures from LTE*

The major advantage of differential abundance analyses, between stars with very similar spectral types, is that systematic errors caused by inadequacies in the atmospheric modeling and radiative transfer schemes used tend to cancel one another. There is now a large body of both empirical and theoretical evidence which suggests that LTE is not an adequate approximation for the purposes of determining accurate element abundances in the atmospheres of G and K giants (e.g., Ruland et al. 1980; Brown, Tomkin, & Lambert 1983; Steenbock 1985; Drake & Smith 1991). The theoretical effects of departures from LTE on abundances derived from Fe and Ca lines in the spectrum of the K0 giant Pollux have been quantified by Steenbock (1985; see also Holweger 1988) and Drake (1991b), respectively. The non-LTE effect on the abundance derived from a given spectral line tends to be a rather slowly varying function of spectral type (e.g., Drake 1991b). Since our program stars range in effective temperature over less than 500 K, it is probably sufficient to ignore any residual non-LTE effects in a differential analysis with respect to Pollux. The detailed calculations for Fe carried out by Steenbock (1985) indicated that high-excitation Fe lines in the spectrum of Pollux were subject to non-LTE effects, δ log A, which amounted to $0.04 \leq [\mathrm{Fe/H}]_{\mathrm{NLTE}} - [\mathrm{Fe/H}]_{\mathrm{LTE}} \leq 0.1$, depending on line strength. Lines with equivalent widths of 100 mÅ or less generally had abundance corrections of δ log $A \sim 0.05$. We estimate that neglecting to account for departures from LTE in iron results in, at most, an error of 0.05 dex in our final Fe abundances. The calculations by Drake (1991b) and Drake & Smith (1991) indicate that errors in the Ca abundances should also be no larger than about 0.05 dex.

Departures from LTE in the Na spectra of G–K giants are being investigated in some detail and will form the subject of a future paper (Drake 1994). Our results for the K0 III star, Pollux, indicate that the two lines of multiplet 5 are subject to small negative non-LTE abundance corrections of δ log $A \sim 0.1$ dex, primarily as a result of a net radiative recombination overpopulating the $3p\ ^2P^{\circ}$ levels relative to LTE. Toward cooler effective temperatures this effect slowly diminishes to such an extent that δ log $A \sim 0$ at $T_{\mathrm{eff}} = 4000$ K. Throughout the range of parameters of our program stars, however, we have δ log $A \sim 0.1$: in the differential approach we have adopted residual non-LTE corrections that should be negligible.

3.6. *Model Atmosphere Temperature Structure*

Being severely depleted in C, the atmospheres of the WGB stars also have very weak CO bands (e.g., Hartoog et al. 1977). Lines due to the CO molecule are an important cooling mechanism in the outer layers of late-type stellar atmospheres (e.g., Gustafsson et al. 1975; Auman 1969) and therefore influence the temperature structures of these regions. It is possible then that the MARCS models we have adopted might not provide the appropriate temperature structure for the WGB stars. Moreover, the species with low ionization potential, such as Na I and Ca I, are more likely to be affected by any temperature structure differences in the outer atmospheric layers than are species with higher ionization potential, such as Fe I.

The MARCS models are line-blanketed atmospheres computed using opacity distribution functions. Unfortunately, using such models, it is not possible to investigate rigorously the effects on the model structure of differences in the relative abundances of elements important for the blanketing processes. However, using opacity sampling techniques, Drake, Plez, & Smith (1993) have recently investigated the effects of different C, N, and O compositions on Na, Ca, and Fe abundances derived for giants in the intermediate metallicity CN-bimodal globular clusters. While these stars are both significantly cooler and metal-deficient compared with those considered here, the Drake et al. (1993) results might at least provide some indication of the effects one might expect. Their "CN-weak" models are cooler in the outer layers than the "CN-strong" models, primarily because of CO: the latter models have lower C abundances by 0.5 dex, slightly lower O abundances (0.2 dex), and consequently weaker CO bands and less cooling through CO lines. However, the differences are small, amounting to at most 150 K in the regions higher than $\tau_{\mathrm{ross}} = -1$. In deeper layers there is no difference between the models. The effects on the abundances derived from Ca, Na, and Fe lines resulting from these differences in model structure were found to be largest for lines nearing saturation, but in no case were the effects larger than 0.1 dex. For weak lines, the effects were entirely negligible. Since our analysis is based on lines that are not saturated, and our estimated uncertainties are likely to be larger than 0.1 dex, we suggest that atmospheric structure effects related to the anomalous compositions of the WGB stars are unlikely to be important in our analysis.

4. DISCUSSION

Is Na enriched in the WGB giants? To provide an answer to this question we compare the abundances of Na, Ca, and Fe in the WGB giants and normal giants drawn from the sample selected by Lambert & Ries (1981) that we refer to as the LR sample. The latter stars were observed and analyzed by techniques identical to those discussed here; the LR sample will be discussed in detail elsewhere (Drake & Lambert 1994). This comparison of the two similar samples of giants surely minimizes systematic errors.

The Na and Fe abundances are compared in Figure 2*a*, and Ca and Fe abundances are compared in Figure 2*b*. The WGB giants appear to have a higher mean iron abundance than the

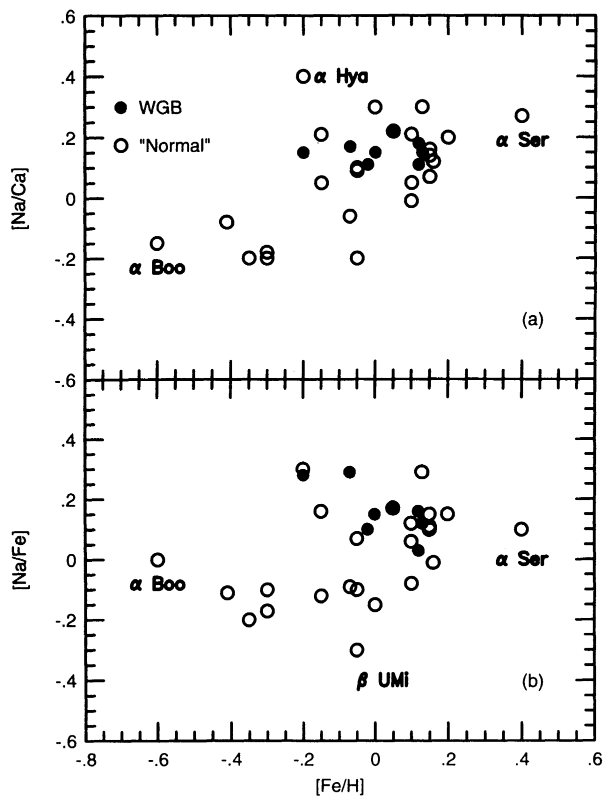

FIG. 2.—(*a*) The abundances [Na/Ca] and (*b*) [Na/Fe], relative to Pollux, as a function of [Fe/H] for the WGB giants and a sample of "normal" field giants (see text).

LR sample, but the maximum iron abundance is similar for the two samples. We presume that either selection effects currently militate against identification of metal-poor WGB stars or the mechanism producing these stars is less effective at metallicities less than [Fe/H] = −0.2. For stars in the two samples with [Fe/H] > −0.2 (relative to Pollux), we find the mean values given in Table 5. When the cutoff is put at [Fe/H] = −0.2, the metallicities of the two samples are identical to the mean value and the standard deviation. The mean temperatures of the samples are $T_{\rm eff} = 4930 \pm 220$ K for the WGB stars and 4760 ± 250 K for the LR sample. For the LR sample there is no significant trend in the abundances with effective temperature. Therefore, the slight difference in the mean $T_{\rm eff}$ of the samples seems unlikely to introduce a systematic difference in the abundance and the abundance ratios. Inspection of Table 5 seems to show that the WGB stars are slightly enhanced in Na relative to the LR giants of comparable metallicity: the mean [Na/Fe] with the standard error of the mean is 0.16 ± 0.03 for the WGBs but 0.04 ± 0.04 for the LR giants. The [Ca/Fe] ratios differ in the same sense but by a smaller amount: [Ca/Fe] = 0.01 ± 0.02 and -0.06 ± 0.03 for the WGB and LR samples, respectively. Since Na and Ca are rather similarly affected by errors related to the model atmospheres, and Na but not Ca is expected to be an associated product of ON cycling, the Na/Ca ratio may be a more sensitive indicator of ON-cycling products. The [Na/Ca] ratio shows only a slight enhancement in the WGB stars: [Na/Ca] = 0.16 ± 0.01 and 0.10 ± 0.03 for the WGB and LR samples, respectively. Evidently, the WGB stars do not share the Na enrichment of the F supergiants, where a factor of 5 (0.7 dex) has been reported.

The preceding discussion of the WGB stars is based on differential abundances with respect to Pollux. Pollux, as analyzed by Ruland et al. (1980), is perhaps very slightly metal-poor with [Fe/H] = -0.10 ± 0.12 relative to the Sun. Drake & Smith (1991) obtained [Fe/H] = -0.04 ± 0.05. If [Na/H] = [Ca/H] = [Fe/H] for Pollux, the [Na/Fe], [Ca/Fe], and then [Na/Ca] of the WGB and LR samples are essentially identical to the values found for local field F and G dwarfs (Edvardsson et al. 1993). This result is additional but weaker evidence that Na is not enriched in the WGB stars. It should be noted that Ruland et al. (1980) reported a Na abundance of [Na/H] = 0.03 ± 0.15 for Pollux relative to the Sun. For Ca, the reported result was [Ca/H] = -0.14 ± 0.12 (cf. -0.11 ± 0.1 found by Drake & Smith 1991), which places Pollux and therefore most of the WGB and LR stars with a [Ca/Fe] less than, and a [Na/Ca] greater than, the values reported for the field F and G dwarfs. In light of the uncertainties, and especially of the [Ca/H] for Pollux, the difference is probably not significant.

Standard calculations of the first dredge-up predict very small Na enhancements for low-mass red giants. Denisenkov (1989) predicts [Na/Fe] to increase by 0.04 for a 2.5 $M_\odot$, and even if the convective envelope were to extend to the center of the star, the predicted increase of [Na/Fe] is just 0.28 dex. For our stars with masses probably less than 2.5 $M_\odot$, the predicted increases are less than these values. (Dearborn 1992 predicts larger Na enrichments: [Na/Fe] ≃ 0.12 for the standard first dredge-up of a 2.5 $M_\odot$ star.) Denisenkov's predictions of a very small Na enrichment are fully compatible with our results for the normal giants in the LR sample: the [Na/Ca] results (=0.27, if [Na/H] = +0.03 and [Ca/H] = −0.14 are adopted for Pollux) might be taken as evidence of a Na enrichment in excess of standard predictions. Recall that the predictions are dependent on the adopted ^{22}Ne abundance. The possibly slight enhancement of Na in WGB stars could be attributed to greater mixing of the core during the main-sequence phase, a higher initial ^{22}Ne abundance, a deeper first dredge-up, or to mixing and nuclear processing in the red giant. This conjunction of a small and possibly null effect with a multitude of hypotheses is unfortunate.

As mentioned briefly in § 1, the presence of ON-cycling products should be revealed as directly by an O deficiency and compensating N overabundances as by a Na enrichment. Published O abundances based on the [O I] 6300 Å line confirm that WGB stars are not seriously contaminated by ON-cycling products—see Cottrell & Norris (1978), Sneden et al. (1978), and Day (1980). (These earlier studies did not discuss the Na abundance.) A renewed pursuit of the O abundances would be of interest, as O is a direct indicator of ON-cycling. Sodium enrichment is dependent on the ^{22}Ne abundance in the stellar interior which, if the WGB progenitors are subject to diffusion of elements, could be anomalously low (or high) so that the dredge-up of ON-cycling products need not necessarily result in a Na enrichment.

We note that the WGB stars with atmosphere clearly seriously contaminated with products of CN cycling are only very mildly, if at all, enriched in Na, an associated product of ON cycling. In conclusion, the differential analysis shows a possibly significant enrichment of the WGBs according to the [Na/Fe] ratio but a smaller enrichment in the [Na/Ca] ratio which is expected to be less sensitive to uncertainties related to the model atmospheres. The latter enrichment amounts to a difference in [Na/Ca] of a mere 0.06 dex when the standard errors of the mean ratios for the WGB and LR samples are 0.01 and 0.03. The severe carbon underabundances and the CN-cycle equilibrium ratios of ^{12}C/^{13}C possessed by the atmospheres of the WGB stars are clear evidence that the outer parts of these stars are severely contaminated in products of the H-burning CN-cycle reactions. Our report of the near absence of Na enrichment and published studies showing normal oxygen abundances demonstrate that WGB atmospheres are not contaminated with products of higher temperature ON-cycle reactions. If WGB stars are produced by extensive deep mixing (meridional currents?), these results show that the mixing did

TABLE 5

ABUNDANCES FOR WEAK G-BAND AND NORMAL GIANTS

Sample	Number of Stars	Mean Abundances[a]			
		[Fe/H]	[Na/Fe]	[Ca/Fe]	[Na/Ca]
Weak G-band	8	0.00 ± 0.11	0.16 ± 0.08	0.01 ± 0.07	0.16 ± 0.03
Normal giants	23	0.00 ± 0.12	0.04 ± 0.14	-0.06 ± 0.11[b]	0.10 ± 0.13

[a] All [X] are with respect to Pollux. The standard deviation is given.
[b] If β UMi is omitted, [Ca/Fe] = -0.04 ± 0.07 for the sample of 22 stars.

not extend into layers where the ON cycle ran. These results are not inconsistent with Lambert & Sawyer's (1984) speculation that carbon diffused inward prior to or during the main-sequence phase where it was destroyed by the CN cycle. Oxygen and sodium could similarly diffuse inwards, but their abundances would be restored to normal values when the star, as a red giant, develops a deep convection envelope

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