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SUPPLEMENTARY PARAMETERS IN THE PARAMETERIZED POST-KEPLERIAN FORMALISM

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ABSTRACT

New additional terms in relativistic time transformation contributing to the pulsar timing formula are discussed. Their influence on the separately measurable parameters in the framework of the parameterized post-Keplerian formalism constructed by Damour & Taylor (1992) is estimated.

Subject headings: gravitation — pulsars: individual (PSR 1913+16) — relativity — stars: neutron

1. INTRODUCTION

The physically important problem of testing of alternative theories of gravity in the strong gravitational field regime can be resolved by means of highly precise observations of binary pulsars in relativistic orbits (Taylor 1992). Parameterized post-Keplerian (PPK) formalism was developed by Damour & Deruelle (1986) and Damour & Taylor (1992) as a phenomenological approach to the binary pulsar tests. In its present form the formalism can be applied to measure five Keplerian and eight post-Keplerian (relativistic) parameters which can be directly (or separately) observed using pulsar timing data. Moreover, there are 11 other parameters contained in pulse structure data which can be used to give supplemental tests of alternative gravity theories. At the same time, as Damour & Deruelle (1986) have pointed out, there exist four additional but not separately measurable post-Keplerian parameters. They can be completely absorbed in the first relativistic approximation by suitable redefinitions of the separately measurable parameters. In other words, the fitting estimates of the separately measurable parameters are shifted from their physically meaningful values. Therefore, one finds some specific theoretical restrictions on the possibility of testing alternative theories of gravity. To overcome these obstacles and to make progress in getting the physically reasonable test of a gravitational theory in the strong field regime, one needs to know the explicit relationships between separately measurable and not separately measurable parameters. Then, in some favorable cases, the observational information about numerical values of relevant, not separately measurable parameters allows one to remove their "systematical error" influence on the directly measurable parameters and to refine the test. An excellent example of the application of such an approach has been recently demonstrated by Damour & Taylor (1991) while investigating the influence of the Galactic acceleration on the relativistic parameter, $\dot{P}_b$, describing the damping of orbital period of binary pulsar PSR B1913+16 due to the emission of gravitational waves (Damour 1983; Grishchuk & Kopeikin 1983). It is worthwhile to underline that Damour & Duruelle (1986) had not pointed out all of the possible not separately measurable parameters which are absorbed in the procedure of data processing. Additional parameters exist which are directly related to the relativistic part of time transformations. The aim of this *Letter* is to give explicit expressisions for the new parameters and estimate their physical significance.

2. TIME TRANSFORMATIONS

Physically consistent and mathematically justified derivation of a timing formula requires construction of six coordinate systems, one of which is global and the other five of which are local. They are as follows: (1) Galactic coordinate system covering all the Galaxy; (2) barycentric coordinate system of the solar system; (3) barycentric coordinate system of binary pulsar system; (4) geocentric coordinate system; (5) pulsar proper coordinate system; and (6) topocentric coordinate system of observer.

The Galactic coordinate system is global whereas all others are local. Therefore, it is impossible to use the barycentric coordinate system of the solar system to describe orbital motion of bodies in the binary system and propagation of radio pulses from the pulsar to observer as is usually done.

Construction of the coordinate systems under consideration should be based upon the relevant viable relativistic theory of gravitation. The case of application of the general relativity theory for solving this problem has been elaborated recently in the works of Kopeikin (1988), Brumberg & Kopeikin (1989, 1990), Klioner & Voinov (1993), and from a slightly different point of view by Damour, Soffel, & Xu (1991, 1992). The main idea of these works is the solution of the Einstein equations in global and local coordinate charts along with the following matching of both solutions. In this way, one can generalize an idea of the Fermi normal coordinate system (Misner, Thorne, & Wheeler, 1973) for the case of a self-gravitating body (or bodies) and write explicitly spacetime relativistic transformations between global and local coordinate systems constructed in the spacetime around the self-gravitating bodies.

We have used this procedure to reconsider the derivation of the timing formula for binary pulsars (Kopeikin 1992). The global Galactic coordinate system has been used as an intermediate reference frame to obtain an unambiguous solution of the equation of radio signal propagation through gravitational fields of binary and the solar systems. Subsequent application of relativistic spacetime transformations in the solution has allowed (as had been expected) for the cancellation of all essential terms depending on the absolute velocities of the binary and solar systems with respect to the center of the Galaxy. Nevertheless, we have found that in the solution under consideration, there are two new terms arising directly from the relativistic part of the time transformations. These terms can be obviously interpreted as the Lorentz part of the relativistic transformation between coordinate times of the corresponding reference frames.

More specifically, the relation between coordinate time t of the solar barycentric system and t_b of the binary pulsar barycentric system has the form:

$$t_b = \left[t - \frac{1}{c^2} (\boldsymbol{V} \cdot \boldsymbol{r}_p) \right] \Big/ \sqrt{1 - \frac{V^2}{c^2}} , \tag{1}$$

where $\boldsymbol{V}$ is the relative velocity of the barycenter of the binary pulsar with respect to that of the solar system, $\boldsymbol{r}_p$ is the radius vector from the barycenter of the binary system to the pulsar, and c is the speed of light.

The relativistic time transformation between t_b and pulsar (coordinate) time, T, is

$$T = t_b - c^{-2}[B(t_b) + (\boldsymbol{v}_p \cdot \boldsymbol{X})] + O(c^{-4}) , \tag{2}$$

where $\boldsymbol{v}_p$ is the velocity of the pulsar with respect to the barycenter of the binary system, and $\boldsymbol{X}$ is the radius vector pointing from the origin of the pulsar coordinate system toward the point of the pulse's radio emission ($\boldsymbol{X} = -\boldsymbol{K}X$, where $\boldsymbol{K}$ is the unit vector along the line of sight). The function, $B(t_b)$, is responsible for the quadratic Doppler and gravitational shifts (Blandford and Teukolsky 1976) and can be found from the equation:

$$\frac{dB}{dt_b} = \frac{1}{2} v_p^2 + \frac{Gm_c}{r} , \tag{3}$$

where the universal gravity constant is denoted by G, m_c is the pulsar's companion mass, and r is the relative distance between the pulsar and its companion.

The terms linear in velocity in equations (1) and (2) were never taken into consideration in pulsar timing data processing. Altogether they are expected to contribute to the set of the not separately measurable parameters in the PPK formalism. To proceed further and clarify this point, let us introduce a triad of the unit vectors ($\boldsymbol{I}_0$, $\boldsymbol{J}_0$, $\boldsymbol{K}_0$) attached to the barycenter of the binary system in the same manner as described in (Damour & Taylor 1992, Fig. 1). The vector $\boldsymbol{K}_0$ is pointing from the solar system barycenter toward the binary system one, and vectors $\boldsymbol{I}_0$, $\boldsymbol{J}_0$ lie in the plane of the sky with $\boldsymbol{I}_0$ directed to the east, and $\boldsymbol{J}_0$ to the north celestial pole. Two other sets of unit vectors are suitable, namely ($\boldsymbol{I}, \boldsymbol{J}, \boldsymbol{K}$) and ($\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$). They are related to ($\boldsymbol{I}_0$, $\boldsymbol{J}_0$, $\boldsymbol{K}_0$) by means of the following transformations (Damour & Deruelle 1986):

$$\begin{aligned} \boldsymbol{I} &= \cos \Omega \boldsymbol{I}_0 + \sin \Omega_{\boldsymbol{J}_0} , & \boldsymbol{i} &= \boldsymbol{I} , \\ \boldsymbol{J} &= -\sin \Omega \boldsymbol{I}_0 + \cos \Omega \boldsymbol{J}_0 , & \boldsymbol{j} &= \cos i\boldsymbol{J} + \sin i\boldsymbol{K} , \\ \boldsymbol{K} &= \boldsymbol{K}_0 , & \boldsymbol{k} &= -\sin i\boldsymbol{J} + \cos i\boldsymbol{K} . \end{aligned} \tag{4}$$

In the above transformations the angles Ω ($0 \leq \Omega < 2\pi$) and i ($0 \leq i < \pi$) designate, respectively, the longitude of the ascending node of the pulsar's orbit and its inclination to the plane of the sky. The radius vector of the osculating pulsar's orbit is represented as

$$\boldsymbol{r}_p = a_p(1 - e \sin u)\{\boldsymbol{i} \cos [\omega + A_e(u)] + \boldsymbol{j} \sin [\omega + A_e(u)]\} . \tag{5}$$

The pulsar's orbital velocity with respect to the binary system's barycenter has the form

$$\boldsymbol{v}_p = na_p(1 - e^2)^{-1/2}[\![\{-\sin [\omega + A_e(u)] + e \sin \omega\}\boldsymbol{i} + \{\cos [\omega + A_e(u)] + e \cos \omega\}\boldsymbol{j}]\!] , \tag{6}$$

where a_p is the semimajor axis of the pulsar's orbit, n is the mean orbital frequency, e is the orbital eccentricity, the angle ω is the longitude of periastron, u is the eccentric anomaly, and $A_e(u)$ is the true anomaly related to u by Kepler's equation.

The relative velocity of the binary pulsar with respect to the solar system is expressed as follows:

$$\boldsymbol{V} = d(\mu_\alpha \boldsymbol{I}_0 + \mu_\delta \boldsymbol{J}_0) + v_R \boldsymbol{K}_0 , \tag{7}$$

where d is the (varying) distance between the binary and solar systems, v_R is the relative radial velocity ($v_R = \dot{d}$), and μ_α and μ_δ are components of the proper motion of the pulsar in the sky. Using the relationships just given in this section, it is not difficult to generalize timing formula.

3. TIMING FORMULA

Let us designate $\Delta_L = c^{-2}(\boldsymbol{V} \cdot \boldsymbol{r}_p)$, and $\Delta_X = c^{-2}(\boldsymbol{v}_p \cdot \boldsymbol{X})$. Then an improved timing formula reads:

$$t - t_0 = DT + \Delta_R + \Delta_E + \Delta_S + \Delta_A + \Delta_B + \Delta_L + \Delta_X , \tag{8}$$

where $D = (1 + v_R/c)\sqrt{1 - v_R^2/c^2}$ is the (varying) Doppler factor; Δ_R, Δ_E, Δ_S, and Δ_A are the well-known propagation delays in the binary system due to "Roemer," "Einstein," "Shapiro," and "aberration" effects, respectively. Their analytical expressions may be easily found, for example, in the paper of Damour & Taylor (1992). Δ_B is the propagation delay in the binary system caused by the relativistic effect of gravitational deflection of the pulsar's beam in the gravitational field of its companion. This delay is important only for those binary pulsars whose orbits are visible nearly edge-on. Detailed discussion of the effect and explicit expression for the function Δ_B will be published elsewhere (Doroshenko & Kopeikin 1994). Explicit expressions for the "Lorentzian" delays Δ_L, Δ_X under discussion are given by

$$\Delta_L = x\{\mathscr{E}[\sin \omega(\cos u - e) + (1 - e^2)^{1/2} \cos \omega \sin u] + \mathscr{F}[\cos \omega(\cos u - e) - (1 - e^2)^{1/2} \sin \omega \sin u]\} , \tag{9}$$

and

$$\Delta_X = \delta_X\{\cos [\omega + A_e(u)] + e \cos \omega\} , \tag{10}$$

where $x = a_p \sin i/c$, and the new timing parameters $\mathscr{E}$, $\mathscr{F}$, and δ_X are defined as

$$\mathscr{E} = \frac{1}{c} [v_R + d \cot i(-\mu_\alpha \sin \Omega + \mu_\delta \cos \Omega)] , \tag{11}$$

$$\mathscr{F} = \frac{d}{c} \csc i(\mu_\alpha \cos \Omega + \mu_\delta \sin \Omega) , \tag{12}$$

and

$$\delta_X = -\frac{X}{c} nx(1 - e^2)^{-1/2} . \tag{13}$$

It is worthwhile to recall again that the quantity X in formula (13) is the radial distance from the pulsar's center of mass to the point of the pulse's emission.

Doppler shift factor D and relativistic delays Δ_A, Δ_L, and Δ_X are not directly observable. The factor D redefines rotational frequency ν_p of the pulsar and its orbital period, P_b, as well as time derivatives of these quantities (Doroshenko & Kopeikin 1990; Damour & Taylor 1992). Functions Δ_A, Δ_L, and Δ_X are absorbed into the classical "Roemer" delay Δ_R by suitable redefinition of the five Keplerian parameters of the binary pulsar orbit. Let us note that the statement of Damour & Taylor (1992) that the Doppler factor D influences the observational value of parameter x (see eq. [2.4b] from that paper) is not quite correct. It is the new parameter $\mathscr{E}$ which gives the contribution (instead of D):

$$x^{\rm obs} = (1 + \epsilon_A + \mathscr{E})x^{\rm intrinsic}\,, \tag{14}$$

where $\epsilon_A = \mathscr{A}/x$, and $\mathscr{A}$ is one of two aberration parameters (Damour & Deruelle 1986) characterizing amplitude of the aberration delay Δ_A. Parameters $\mathscr{F}$ and δ_X redefine, respectively, the longitude of periastron and the initial epoch T_0:

$$\omega^{\rm obs} = \omega^{\rm intrinsic} + \mathscr{F}\,, \tag{15}$$

and

$$T_0^{\rm obs} = T_0^{\rm intrinsic} - \epsilon_B n^{-1}(1-e^2)^{1/2} + \frac{X}{c}\,, \tag{16}$$

where $\epsilon_B = \mathscr{B}/x$, and $\mathscr{B}$ is the second aberration parameter (Damour & Deruelle 1986). Values of the other Keplerian parameters are perturbed only by means of ϵ_A and ϵ_B as well as the Doppler factor D (Damour & Taylor 1992).

Time variations of the parameters are given by the following expressions:

$$\left(\frac{\dot{x}}{x}\right)^{\rm obs} = \left(\frac{\dot{x}}{x}\right)^{\rm intrinsic} + \cot i\,\frac{di}{dt} + \frac{d\epsilon_A}{dt} + \frac{d\mathscr{E}}{dt}\,, \tag{17}$$

$$\dot{\omega}^{\rm obs} = \dot{\omega}^{\rm intrinsic} + \frac{d\mathscr{F}}{dt}\,. \tag{18}$$

Here the first term in the right-hand side of equation (17) is linked to the gravitational-wave damping of the orbital motion (Damour 1983; Grishchuk & Kopeikin 1983), and $\dot{\omega}^{\rm intrinsic}$ in equation (18) is the relativistic apsidal motion of the orbit (Damour & Shäfer 1985; Kopeikin & Potapov 1994). Since the measurement of the binary pulsar periastron advance, $\dot{\omega}^{\rm obs}$, serves as a tool in the determination of neutron star masses and plays a crucial role in high-precision tests of alternative theories of gravitation, it is quite important to know the contribution of $d\mathscr{F}/dt$ to the $\dot{\omega}^{\rm obs}$. Knowledge of $d\mathscr{E}/dt$ is necessary to provide a test of the spin-orbit relativistic interaction causing temporal changing of ϵ_A (Damour & Taylor 1992).

4. DISCUSSION

To estimate contributions of $d\mathscr{E}/dt$ and $d\mathscr{F}/dt$ to the variations of the parameters x and ω, let us show the explicit formulas for these time derivatives:

$$\frac{d\mathscr{E}}{dt} = \frac{1}{c}\left[\dot{v}_R + \left(v_R \cot i - d \csc^2 i\,\frac{di}{dt}\right) \times (-\mu_\alpha \sin\Omega + \mu_\delta \cos\Omega)\right] - \mathscr{F}\cos i\,\frac{d\Omega}{dt}\,, \tag{19}$$

$$\frac{d\mathscr{F}}{dt} = \frac{\csc i}{c}\left[\left(v_R - d\cot i\,\frac{di}{dt}\right)(\mu_\alpha \cos\Omega + \mu_\delta \sin\Omega) + d(-\mu_\alpha \sin\Omega + \mu_\delta \cos\Omega)\frac{d\Omega}{dt}\right]\,, \tag{20}$$

where we have omitted terms depending on the proper motion accelerations $\dot{\mu}_\alpha$ and $\dot{\mu}_\delta$ since they are usually negligibly small.

Relevant quantities in the right-hand side of equations (19) and (20) have the following numerical values in the case of PSR B1913+16 (Taylor & Weisberg 1989; Damour & Taylor 1991): $d = 8.3 \pm 1.4$ kpc, $\mu_\alpha = -3.27 \pm 0.35$ mas yr^{-1}, $\mu_\delta = +1.04 \pm 0.42$ mas yr^{-1}, $v_R = 22.75$ km s^{-1}, $\dot{v}_R/c \simeq 0.6 \times 10^{-18}$ s^{-1}, and $\sin i = 0.734$ ($\cos i = 0.679$). Orbital inclination, i, and longitude of the ascending node, Ω, are changes due to the existence of the relativistic spin-orbit coupling (Brumberg 1991; Damour & Taylor 1992)

$$\frac{di}{dt} = a_R^{-3}(1-e^2)^{-3/2}[S_p\sigma_p \sin\lambda\cos\eta + \sigma_c(\boldsymbol{S}_c \cdot \boldsymbol{i})]\,, \tag{21}$$

$$\frac{d\Omega}{dt} = a_R^{-3}(1-e^2)^{-3/2} \times [S_p\sigma_p(\cos\lambda + \cot i \sin\lambda\sin\eta) + \sigma_c(\boldsymbol{S}_c \cdot \boldsymbol{j})]\,, \tag{22}$$

where $S_p = |\boldsymbol{S}_p|$ and $\boldsymbol{S}_c$ are the spin vectors of the pulsar and its companion, respectively; σ_p and σ_c are the relativistic parameters characterizing the spin-orbit part of the Lagrangian describing the orbital dynamics of the binary system (Damour & Taylor 1992); and λ and η are polar angles of the pulsar's spin vector. Our calculations show the maximal amplitude of variation of di/dt and $d\Omega/dt$ is about 1×10^{-13} rad s^{-1} for PSR B1913+16 (within the framework of general relativity) where we have taken the pulsar's radius to be 30 km. Using this estimate and observational parameters of PSR B1913+16, we conclude that the possible maximal contribution of time variations of the parameters $\mathscr{E}$ and $\mathscr{F}$ into the observational values of $\dot{\omega}^{\rm obs}$ and $(\dot{x}/x)^{\rm obs}$ cannot be more than 0.3 mas yr^{-1} and 2×10^{-16} rad s^{-1}, respectively. It is completely beyond the present observational accuracy for $\dot{\omega}^{\rm obs}$ and $(\dot{x}/x)^{\rm obs}$. Nevertheless the contribution of time derivatives of the parameters $\mathscr{E}$ and $\mathscr{F}$ may happen to be important for those pulsars whose radial accelerations and proper motions have relatively large values, while $\sin i$ is close to zero.

As for the Keplerian parameter, $T_0^{\rm obs}$, the contribution of the post-Keplerian parameter δ_X (in the form of X/c in eq. [16]) can lead, in principle, to a small difference in two values of $T_0^{\rm obs}$ having been measured in two different electromagnetic frequencies. This may happen because the emission at different frequencies can arise at different altitudes (Taylor & Stinebring 1986).

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