

## THE He II LYMAN-ALPHA OPACITY OF THE UNIVERSE

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## ABSTRACT

We discuss the recent *Hubble Space Telescope* FOC observations by Jakobsen et al. of a sharp drop in the spectrum of the high redshift ( $z = 3.29$ ) quasar Q0302–003 at the rest-frame wavelength of 304 Å, the Ly $\alpha$  line of He II. We assess two possible explanations for this flux reduction: line blanketing produced by He II in the Ly $\alpha$  forest, and the He II Gunn-Peterson effect from a diffuse intergalactic medium (IGM). The observed He II trough can be produced entirely by line blanketing only if the absorbing clouds are velocity broadened and/or their distribution extends to very low column densities. A significant contribution from absorption in a diffuse IGM is likely. We show that a steep ionizing UV background is required to explain the reported He II absorption and the simultaneous absence of H I. Such a soft metagalactic flux could arise from star-forming galaxies, decaying dark matter, or QSOs, after including the effect of the continuum opacity from intervening absorption systems on the integrated spectrum. We describe an alternative interpretation, that the He II reionization of the universe is not completed until  $z \sim 3$ , well after the H II region network has fully percolated at  $z \sim 5$ , and find that it too requires a steep ionizing spectrum for the individual sources. Photoionization from QSOs detected in optical surveys may satisfy the H I and He II limits if the Ly $\alpha$  forest contains a significant fraction of the baryons in the universe. If the ionizing background remains soft shortward of the H II edge, metal-line absorbers at high redshift must be ionized locally.

*Subject headings:* cosmology: observations — diffuse radiation — intergalactic medium  
— quasars: absorption lines

## 1. INTRODUCTION

The existence of a diffuse component of the IGM is predicted as a product of primordial nucleosynthesis. Its detection would serve both as a test of the standard cosmological model and as a probe of conditions at high redshift during the epoch of galaxy formation, in particular, the shape and possible origin of the metagalactic UV background. Despite considerable effort both theoretically and observationally, a uniformly distributed H I component of the IGM has to date evaded detection. The flux decrement observed on the blue side of the Ly $\alpha$  emission line of neutral hydrogen in the spectra of high-redshift QSOs appears entirely consistent with the blanketing by discrete absorption lines along the line of sight (e.g., Steidel & Sargent 1987; Giallongo, Cristiani, & Trevese 1992): we know now that the hydrogen component of a diffuse IGM must have been highly ionized by  $z \sim 5$  (Schneider, Schmidt, & Gunn 1989). Very recently, a large decrement at 304 Å (rest frame) has been reported by Jakobsen et al. (1994). Their *HST* Faint Object Camera objective prism spectrum of the high-redshift ( $z = 3.29$ ) quasar Q0302–003 shows a sharp drop, corresponding to  $\tau_{\text{Ly}\alpha}^{\text{He II}} \simeq 3.2_{-1.1}^{+1.1}$ , in what may be the first detection of He II Ly $\alpha$  absorption in the diffuse IGM. In this *Letter* we interpret the recently discovered He II trough in terms of the known structure of the IGM as determined by H I measurements, and discuss its implications for the spectrum of the ionizing background at high redshifts.

## 2. LINE BLANKETING

At wavelengths shortward of the rest-frame Ly $\alpha$  line of an atom of H I or H II, the source's continuum intensity is attenu-

ated by the combined blanketing effect of many absorption lines arising from intervening discrete systems. The average transmission over all lines of sight may be expressed as  $\langle e^{-\tau} \rangle \equiv e^{-\tau_{\text{eff}}}$ , with the effective optical depth due to Poisson-distributed absorbers given by

$$\tau_{\text{eff}}(z) = \frac{1+z}{\lambda_{\alpha}} \int_{W_{\text{min}}}^{W_{\text{max}}} \frac{\partial^2 N}{\partial W \partial z} W dW \quad (1)$$

(Paresce, McKee, & Bowyer 1980), where  $\partial^2 N / \partial W \partial z$  is the rest equivalent width distribution. The He II effective optical depth may be derived from equation (1) using a curve of growth analysis for a given H I equivalent width distribution and a distribution of Doppler parameters  $b$  for H I and He II. For a gas in photoionization equilibrium with an ionizing background of specific intensity  $J(\nu)$ , a helium to hydrogen cosmic abundance ratio of 8%, and for nearly fully ionized hydrogen and helium, the ratio of He II to H I number density is  $n_{\text{He II}}/n_{\text{H I}} \sim 1.8S_L$ , where we define the “softness” parameter  $S_L$  as the ratio of the incident flux  $J_{912}$  at the hydrogen Lyman edge to the flux  $J_{228}$  at the doubly ionized helium Lyman edge.<sup>1</sup> For the distribution of H I rest equivalent widths in the

<sup>1</sup> We note that, while He III starts to recombine for  $J_{228,-24} \lesssim n_{\text{H,-4}}$ , hydrogen is mostly ionized for  $J_{912,-24} \gtrsim 0.1n_{\text{H,-4}}$ , where the background intensity is measured in units of  $10^{-24}$  ergs  $\text{cm}^{-2}$   $\text{s}^{-1}$   $\text{Hz}^{-1}$   $\text{sr}^{-1}$ , and  $n_{\text{H,-4}}$  is the hydrogen density in units of  $10^{-4}$   $\text{cm}^{-3}$ .

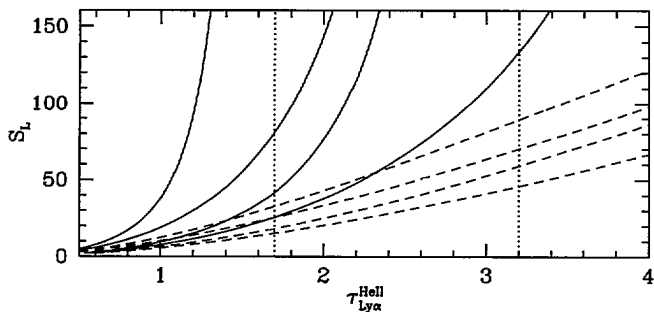


FIG. 1.—The softness parameter  $S_L$  as a function of He II Ly $\alpha$  optical depth for  $b^{\text{HI}} = 35 \text{ km s}^{-1}$ . The solid curves represent the effective opacity from intervening absorbers using eq. (1) of the text. From left to right, the two curves are for  $\xi = 0.5$  with  $W_{\text{min}}^{\text{HI}} = 0.01$  and  $0.0025 \text{ \AA}$ , respectively, and the remaining two are for  $\xi = 1$  with  $W_{\text{min}}^{\text{HI}} = 0.01$  and  $0.0025 \text{ \AA}$ . The He II effective opacity from clouds with  $W^{\text{HI}} > 0.01$  scales, to 10% accuracy, like  $(b^{\text{HI}})^{0.7} S_L^{0.25}$ , over the range  $30 < b^{\text{HI}} < 50 \text{ km s}^{-1}$  and  $25 < S_L < 100$ . The dashed curves show the corresponding total optical depths including a contribution from the He II Gunn-Peterson effect, eq. (3), with  $\tau_{\text{GP}}^{\text{HeII}} = 0.05$ . The two vertical dotted lines show the measurement of Jakobsen et al. (1994) and their 90% lower bound. For thermally broadened absorbers, the measured total opacity requires a soft spectrum,  $S_L > 30$ .

$$\frac{\partial^2 N}{\partial W^{\text{HI}} \partial z} = \begin{cases} 40.7 \exp(-W^{\text{HI}}/W_*) (1+z)^{2.46} & (0.2 < W^{\text{HI}} < 2 \text{ \AA}) ; \\ 11.4 (W^{\text{HI}}/W_*)^{-1.5} (1+z)^{2.46} & (W_{\text{min}}^{\text{HI}} < W^{\text{HI}} < 0.2 \text{ \AA}) , \end{cases} \quad (2)$$

where  $W_* = 0.3 \text{ \AA}$  (Murdoch et al. 1986). From equations (1) and (2) with  $W_{\text{min}}^{\text{HI}} = 0.01 \text{ \AA}$ , we obtain  $\tau_{\text{eff}}^{\text{HI}} \approx 0.55[(1+z)/4.3]^{3.46}$ , consistent with the recent determinations of Press, Rybicki, & Schneider (1993). The distribution of Doppler widths is fairly uncertain. We shall assume, for simplicity, that the Doppler parameter is the same for all clouds, and adopt the median value of  $b^{\text{HI}} = 35 \text{ km s}^{-1}$  measured by Rauch et al. (1992). It is unknown whether the broad range in measured Doppler widths is a result of bulk velocities within the clouds or contamination by line blending. We therefore parameterize the He II Doppler width as  $b^{\text{HeII}} = \xi b^{\text{HI}}$ , where  $\xi = 0.5$  for thermal broadening, and  $\xi = 1$  for velocity broadening.

In Figure 1, we show our estimate of the required softness parameter  $S_L$  as a function of  $\tau_{\text{eff}}^{\text{HeII}}$  at  $z = 3.3$ , for different values of  $W_{\text{min}}^{\text{HI}}$  and  $\xi$ .<sup>2</sup> As noted by Jakobsen et al. (1994), in the presence of a steep ionizing spectrum the He II lines lie predominantly on the flat part of the curve of growth, and the He II blanketing opacity rises rapidly for limiting H I equivalent widths lower than  $0.01 \text{ \AA}$ , while the H I effective opacity has nearly converged. In the case of thermal broadening and  $W_{\text{min}}^{\text{HI}} = 0.01 \text{ \AA}$ , we find that  $\tau_{\text{eff}}^{\text{HeII}}$  approaches  $\sim 1.3$  asymptotically for very soft spectra ( $S_L > 200$ ). Extrapolating the cloud distribution down to  $W_{\text{min}}^{\text{HI}} = 0.0025 \text{ \AA}$ , corresponding to an H I line center opacity of 0.01, does, however, result in  $\tau_{\text{eff}}^{\text{HeII}} > 1.7$  for  $S_L > 80$ , matching the 90% lower bound on the measured decrement. Note that the effective opacity increases

<sup>2</sup> Miralda-Escudé (1993) approximated the effective opacity associated with the He II Ly $\alpha$  forest blanketing as the product of equivalent width and number of lines near a line-center optical depth of unity, to derive a rough relation between the H I and He II effective opacities. Because the He II optical depth converges only for  $W_{\text{min}}^{\text{HI}} \ll 0.01 \text{ \AA}$ , we find this approximation is not suitable for our purposes.

if the lines are velocity broadened instead. With  $W_{\text{min}}^{\text{HI}} = 0.01 \text{ \AA}$ ,  $\tau_{\text{eff}}^{\text{HeII}} > 1.7$  is now reached for  $S_L > 40$ , while  $S_L > 25$  is required for  $W_{\text{min}}^{\text{HI}} = 0.0025 \text{ \AA}$ . Thus the 90% lower bound on the observed optical depth may be matched entirely by line blanketing from intervening absorbers only if velocity-broadened clouds provide a substantial contribution to the effective opacity and/or, as pointed out by Jakobsen et al. (1994), there exists a population of very low column density systems. In both cases, the softness parameter must be large,  $S_L \gtrsim 40$ . We remark that there is growing evidence from high-resolution studies of the Ly $\alpha$  forest for a much steeper equivalent width distribution than given by equation (2) (Kulkarni et al. 1994), an effect which may substantially reduce the estimated line-blanketing optical depth.

### 3. He II GUNN-PETERSON TROUGH

A very soft ionizing background would likely result in a significant contribution to the observed He II decrement from a diffuse IGM component. In the presence of uniform material of density  $n(z)$  along the path to a distant object, any resonance line with wavelength  $\lambda$  will produce a Gunn-Peterson (1965) absorption trough with optical depth  $\tau_{\text{GP}}(z) = (\pi e^2/m_e c) H(z)^{-1} \lambda f n(z)$ , where  $z$  is the redshift at which the observed radiation passes through the resonance,  $H(z)$  is the Hubble constant,  $f$  is the oscillator strength of the transition, and all other symbols have their usual meaning. The strongest limit on the amount of intergalactic neutral hydrogen at  $z \sim 3$  is provided by a direct estimate of the quasar Q2126–258 flux in regions of the spectrum where lines are absent (Giallongo et al. 1992). The derived 90% upper bound is  $\tau_{\text{GP}}^{\text{HI}} < 0.05$ . The He II Gunn-Peterson optical depth is related to the background intensity through the condition of photoionization equilibrium, which yields

$$\tau_{\text{GP}}^{\text{HeII}} \sim 25 \left( \frac{1+z}{4.3} \right)^{4.5} \Omega_D^2 h_{50}^3 S_L J_{912,-21}^{-1} \sim 0.45 S_L \tau_{\text{GP}}^{\text{HI}} \quad (3)$$

(Meiksin & Madau 1993), for an Einstein–de Sitter universe with  $H_0 = 50 h_{50} \text{ km s}^{-1} \text{ Mpc}^{-1}$ , and a contribution of the diffuse component of the IGM to the critical density of  $\Omega_D$ . A soft ionizing spectrum with  $S_L \gtrsim 40$  could produce a He II Gunn-Peterson optical depth  $\tau_{\text{GP}}^{\text{HeII}} \sim 1$ , and still be consistent with the lack of any detectable  $\tau_{\text{GP}}^{\text{HI}}$ . With an estimated baryon density from nucleosynthesis  $\Omega_D h_{50}^2 \sim 0.05$  (Walker et al. 1991),  $\tau_{\text{GP}}^{\text{HeII}}$  would actually be of order unity even for  $J_{912,-21}$  as high as 3, the  $1 \sigma$  upper limit for proximity effect estimates (Bajtlik, Duncan, & Ostriker 1988; Lu, Wolfe, & Turnshek 1991). We conclude that the decrement observed by Jakobsen et al. might partly be due to a diffuse IGM, unless the metagalactic flux at the hydrogen Lyman edge is much larger than  $10^{-21} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \text{ sr}^{-1}$ , and/or most of the baryons have been removed from the IGM, possibly into the Ly $\alpha$  forest.

#### 3.1. QSO-dominated UV Background

Although the He II data seem to require a very soft UV radiation field, the mean spectrum of the individual sources dominating the metagalactic flux need not be that steep. The reason is that the universe at high redshift is optically thick to ionizing photons: discrete clouds absorb radiation from the background through the ionization of H and He. The attenuated spectral energy distribution develops characteristic steps, being much harder than the local emissivity between the H I and He II Lyman limits, where two large breaks occur in the

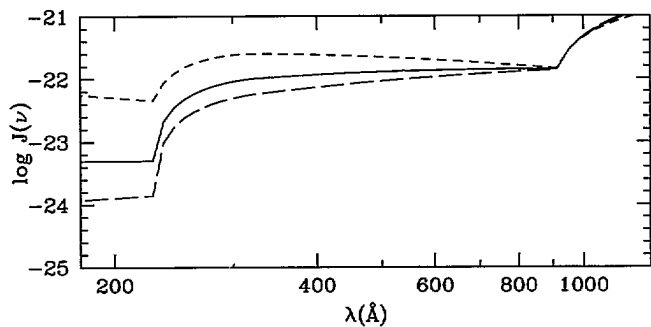


FIG. 2.—Spectrum of the background radiation field at  $z = 3.3$  produced by the observed QSOs in a  $q_0 = 0.5$  universe. A power-law has been adopted for the intrinsic quasar spectral energy distribution shortward of  $912 \text{ \AA}$ , with  $\alpha = 0.7$  (short-dashed line),  $1.5$  (solid line), and  $2$  (long-dashed line). The photoelectric opacity associated with a diffuse component of the IGM has been neglected. The steps result from the continuum absorption of H I and He II in intervening Ly $\alpha$  clouds and Lyman limit systems.

diffuse flux (Miralda-Escudé & Ostriker 1990; Madau 1991, 1992). This is illustrated in Figure 2, where we show such a QSO-dominated background at  $z = 3.3$ . We follow the procedure described in Madau (1992), normalizing to the luminosity function of Boyle (1991), and adopt a power-law  $F_\nu \propto \nu^{-\alpha}$  for the intrinsic quasar spectral energy distribution. We display three cases, with  $\alpha = 0.7, 1.5,$  and  $2$  shortward of rest frame  $912 \text{ \AA}$ . For the resulting ionizing metagalactic flux, we find  $S_L \sim 3, 30,$  and  $100$ , respectively. With  $S_L \sim 50$ , both the H I (upper) and He II (lower) limits may be explained by QSO photoionization for  $\Omega_D h_{50}^{3/2} \sim 0.03 J_{912, -21}^{1/2}$ . Values as high as  $\tau_{\text{GP}}^{\text{He II}} \gtrsim 5$ , however, would require intrinsic QSO spectra that are unacceptably soft. Very soft spectra could arise if additional sources such as star-forming galaxies (Bechtold et al. 1987; Miralda-Escudé & Ostriker 1990; Madau 1991), and/or decaying dark matter particles (e.g., Sciamma 1988) provide a substantial contribution to the UV background at the H I Lyman edge.

### 3.2. Delayed He II Reionization

An alternative interpretation is that the reionization of He II in the IGM was not yet completed by  $z = 3.3$ , and that remnant He II is responsible for the decrement observed in Q0302–003. In photoionization models of the intergalactic medium, sources of UV radiation turn on at high redshift and create isolated H II regions which expand and overlap by

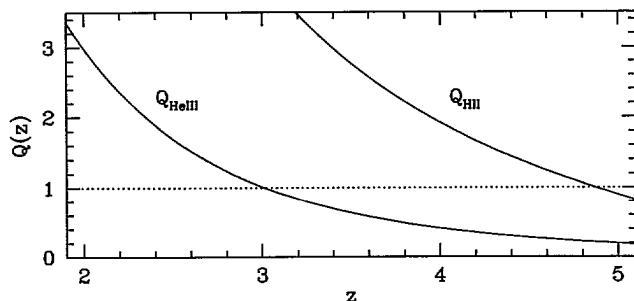


FIG. 3.—The evolution of the porosity parameter of fully ionized hydrogen,  $Q_{\text{H II}}$ , and helium,  $Q_{\text{He III}}$ , as a function of redshift  $z$  for a  $q_0 = 0.5$  universe ionized by QSOs turning on at  $z = 6$ . The total IGM density is taken to be  $\Omega_{\text{IGM}} h_{50}^2 = 0.04$ , with half of its baryons in the Ly $\alpha$  forest. The QSO intrinsic spectrum is taken to vary as  $\nu^{-1.9}$  shortward of the hydrogen Lyman edge. While the ionization of hydrogen is completed by  $z \lesssim 5$ , helium is not fully ionized until  $z = 3$ .

$z \sim 5$ . Equation (3) is only valid after the H II and He III region networks have fully percolated. Depending on the spectrum of the sources, however, the ionization of He II may be delayed until much later than for H I.

The evolution of an expanding cosmological H II region generated by a point source of ionizing radiation is governed by the equation  $3r_I^2(dr_I/dt - Hr_I) = (r_S^3 - r_I^3)/t_{\text{rec}}$  (Shapiro 1986), where  $r_I$  is the proper radius of the I-front,  $r_S$  is the Strömberg radius of the source, and  $t_{\text{rec}}$  is the hydrogen recombination timescale. Similar expressions apply for He II and He III regions. Most photons travel freely in the ionized bubble and are absorbed in a transition layer. An approximate solution to the general problem of the ionization of H and He may be provided by considering the innermost region, where H II and He III are the most abundant ions and the incident radiation is the unattenuated source flux  $F(\nu)$ . At  $228 \text{ \AA}$ , the ratio between the He II and H I optical depths is  $\sim 23 F_{912}/F_{228}$ . If the source spectrum is steep, the H I opacity is negligible compared to He II. In the He III zone, the He I abundance is also negligible, as  $n_{\text{He II}}/n_{\text{He I}} \sim 200 F_{504, -24} n_{\text{H}}^{-1} n_{\text{H}}^{-4}$ , where  $F_{504}$  is the radiation intensity at the He I edge. Since neutral hydrogen and neutral helium do not absorb a significant fraction of  $\nu > 54.4 \text{ eV}$  photons, the problem of He II ionization is decoupled from that of the other ionizations.

When  $r_I \lesssim 0.5 r_S$ , recombinations may be neglected, and the ratio between the H II and He III bubble volumes is  $r_I^3(\text{H II})/r_I^3(\text{He III}) \simeq (n_{\text{He II}}/n_{\text{He I}})S(\nu > 13.6 \text{ eV})/S(\nu > 54.4 \text{ eV})$ . Therefore, as argued by Miralda-Escudé & Rees (1993), the He III region will trail behind the H II region if the number  $S$  of ionizing photons emitted per second from the central source above  $4 \text{ Ryd}$  is less than 8% of the photons above  $1 \text{ Ryd}$ . Figure 3 shows the porosity parameters  $Q_{\text{H II}}$  and  $Q_{\text{He III}}$ , the products of the spatial number density of the radiating sources and the average volume of an ionized bubble, as a function of redshift for a case in which the transition from a singly to a doubly ionized helium universe takes place as late as  $z \sim 3$ . A QSO photoionization model from Meiksin & Madau (1993) is assumed, in which quasars turn on at  $z = 6$ , with a constant comoving QSO density between  $z = 3$  and  $6$ , and the IGM is taken to be clumpy. Recombinations in the medium increase the separation between the inner He III and the outer H II bubbles, as doubly ionized helium recombines much faster than hydrogen. Although the hydrogen in the IGM is completely reionized by  $z \sim 5$ , the breakthrough epoch when  $Q_{\text{H II}} = 1$  and all radiation sources can see each other in the hydrogen Lyman continuum, He II reionization is still incomplete for  $z \gtrsim 3$ . In this scenario, Ly $\alpha$  resonant scattering along the line of sight from He II still present at  $z \gtrsim 3$  between the expanding He III bubbles will produce a Gunn-Peterson absorption trough shortward of  $304 \text{ \AA}$  in the rest frame of a background source, with characteristic optical depth  $\tau_{\text{GP}}^{\text{He II}}(z = 3.3) \sim 3 \times 10^4 \Omega_D h_{50}$ . The probability of intercepting a sufficient number of patches of nonfully ionized intergalactic helium to black-out a region  $\sim 10^4 \text{ km s}^{-1}$  wide is significant (Meiksin & Madau 1993).

## 4. CONCLUSIONS

We have shown that the 90 percent lower bound of  $\tau_{\text{Ly}\alpha}^{\text{He II}} > 1.7$  measured by Jakobsen et al. (1994) may result from unresolved line blanketing, with a likely contribution from absorption in a diffuse IGM. A very soft spectrum for the UV background is required. We may derive a lower limit to the softness parameter  $S_L$  by assuming that line blanketing and the He II Gunn-Peterson effect are acting jointly. Combining the upper bound on H I set by Giallongo et al. (1992) with the

He II result, we obtain  $1.7 < \tau_{\text{Ly}\alpha}^{\text{He II}} < \tau_{\text{eff}}^{\text{He II}} + 0.02S_L$ . We show the constraint on  $S_L$  in Figure 1. For  $W_{\text{min}}^{\text{H I}} = 0.01 \text{ \AA}$ , we find  $S_L > 20(35)$  if the lines are velocity (thermal) broadened, while for  $W_{\text{min}}^{\text{H I}} = 0.0025 \text{ \AA}$ , we find  $S_L > 15(25)$ . These values require an absorption contribution from a diffuse IGM component of at least  $\tau_{\text{GP}}^{\text{He II}} \sim 0.5$ . Because He II lines are saturated, the He II Gunn-Peterson optical depth can be comparable to the effective blanketing opacity, unlike the case for H I.

Photoionization from QSOs alone cannot produce a value of  $\tau_{\text{GP}}^{\text{He II}} \geq 5$ , together with the simultaneous absence of H I diffuse absorption. We note that, for  $\tau_{\text{GP}}^{\text{He II}} > 10$ , the photoelectric opacity at the He II Lyman edge associated with the diffuse IGM would be significant, and it could result in most of the helium being singly ionized. It is possible that the reionization of He II in the universe was, in fact, completed only as late as  $z \sim 3$ . In this case, the flux of Q0302-003 shortward of He II Ly $\alpha$  could be completely absorbed by He II patches not yet incorporated into He III regions. The observed trough would result from the combined equivalent width of the patches.

While blanketing by velocity-broadened clouds alone may match the He II absorption limit for  $W_{\text{min}}^{\text{H I}} > 0.01 \text{ \AA}$ , thermal broadening requires the distribution to extend to much lower

equivalent width systems. In this case, the distinction between discrete Ly $\alpha$  clouds and the smooth IGM may no longer be meaningful. Spherical systems of average radius  $R_a \sim 10 h_{50}^{-1} \text{ kpc}$  (Smette et al. 1992) and  $W^{\text{H I}} = 0.0025 \text{ \AA}$  would have column densities  $N_{\text{H I}} \simeq 5 \times 10^{11} \text{ cm}^{-2}$ , total hydrogen gas densities  $n_a \simeq 10^{-5} (R_a/10 \text{ kpc})^{-1/2} J_{912, -21}^{1/2} \text{ cm}^{-3}$ , and baryonic masses  $M_a \simeq 10^6 (R_a/10 \text{ kpc})^{5/2} J_{912, -21}^{1/2} M_{\odot}$ . At  $z = 3.3$ , the filling factor of these absorbers would be appreciable:  $f_a \sim 0.2 (R_a/10 \text{ kpc})$ . Such diffuse clouds would be indistinguishable from a smoothly distributed IGM, the expected gas density of which is  $n_D(z = 3.3) \simeq 8 \times 10^{-6} (\Omega_D h_{50}^2 / 0.05) \text{ cm}^{-3}$ .

We finally note that a background spectrum which remains steep shortward of the He II edge would conflict with the observations by Reimers et al. (1992) of high-ionization stages of various metals (C IV, N V, and O VI), in the Lyman limit systems along the line of sight to the quasar HS 1700+6416. Vogel & Reimers (1993) argue that the observed ion column densities require a relatively hard spectrum, with  $S_L < 5$ . The indication is that the metals are ionized by local sources.

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