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GRAVITATIONAL RADIATION FROM PAIRS OF REALISTIC, NONACCRETING COMPACT STARS

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ABSTRACT

The dependence is described of the gravitational-radiation luminosity on the changing internal characteristics of the members of a binary system which are assumed to be realistic compact stars. The new, general relativistic post-Newtonian contribution to the gravitational-radiation luminosity is due to the evolution of the nonaccreting members of the stellar pair, and it is nonvanishing even in the case of equal inertial masses of the members. For nonaccreting members of equal inertial masses in circular relative motion, the rate of emission of gravitational radiation due to the change of the members' internal characteristics is the same for both kinds of change, namely deceleration or acceleration of the axial rotations, or/and increase or decrease of the radii. In this case the value of the emission rate although generally small, can be comparable to the corresponding purely orbital one of the same post-Newtonian order. Also the dependence of the gravitational-wave phase change on the changing internal characteristics is examined and is compared with that due to simply orbital, Newtonian, or post-Newtonian characteristics. Finally, the underlying theoretical framework permits an analogous investigation for different inertial masses and, in principle, also for mass-accreting binaries.

Subject headings: binaries: close — radiation mechanisms: nonthermal — relativity — stars: evolution — stars: interiors

1. INTRODUCTION

The recent revival of interest in the detection of gravitational waves from astronomical sources, stimulated by the improvements achieved in the antenna technology, has as a consequence the need for as precise as possible calculation of the energy emitted in the form of gravitational waves from various astronomical sources.

One of the most promising sources for the detection of the gravitational waves with a likely detection rate of $(1\text{–}10^4)\ \mathrm{yr}^{-1}$ is the coalescence of the members of neutron-star binaries (Schutz 1989a, b, 1993a, b). Due to the lack of general-relativistic exact solutions for such systems, their study has been based on the use of post-Newtonian (pN) approximation (PNA) schemes.

If the binary's members are well separated, even the lowest-order, Newtonian treatment of the problem gives reliable, although crude, results regarding the rate of evolution of the system and of the energy emitted in the form of gravitational radiation. Since the more accurate treatment needs the consideration of pN methods of various orders, the inclusion of these terms becomes more important as the distance of the two bodies reduces and the system becomes more relativistic. However, close to the coalescence the pN approximation scheme seems to break down, and pN terms of all orders should be included.

In this last phase becomes important to know the answer to the following question: "*Since in all the previous post-Newtonian calculations mainly orbital characteristics have been taken into account, what will be the contribution resulting from the changing internal characteristics of both bodies, i.e., changes in their masses, radii, and periods of axial rotation?*"

Furthermore, as is known, for the detection of the gravitational waves, using the matched filtering techniques (Thorne 1987), it is of special importance to know the exact form of the waveform. This is necessary since small phase changes produced due either to inaccuracies in the chosen theory for gravitation (Kokkotas, Krolak, & Tsegas 1994) (i.e., omission of higher pN orbital effects) or to tidal effects (Kokkotas & Schäfer 1993) influence the accuracy of the calculation of the physical parameters of the system as well as its distance and position. For example, the detection of a signal produced by a binary system orbiting according to the pN theory, using a filter prepared on the basis of the Newtonian theory, leads to a decrease in their correlation of $\approx 30\%\text{–}40\%$ (Kokkotas et al. 1994). This implies that weaker signals will never be detected, and also that we will not be able to receive signals from the distances that we have expected based on Newtonian-theory calculations. For this reason, it is of great importance to know as completely as possible the influence of all possible effects on the generation of gravitational waves.

The present work aims to fill partially such a gap in the study of binary systems and to examine whether the changes of the internal characteristics can contribute significantly to the output of gravitational radiation. Here we shall be mainly interested in the change of the self-energies of the binary's members. The case of changing masses, due either to evolutionary effects or accretion effects will be examined independently. For a first general exposition of the problem considered here see Spyrou (1993). Also for the importance of rotating collapsed stellar cores to the emission of gravitational radiation see Mönchmeyer et al. (1991).

The paper is organized as follows: In § 2, the quadrupole-radiation formulae are described and the gravitational-radiation tensors are evaluated, to pN accuracy, in the case of a binary star composed of two realistic compact stars in relative motion. In § 3 the Newtonian total self-energies of the two members are evaluated, and the expressions for their first three time derivatives are given, assuming that the bodies are homogeneous, spherically symmetric compact stars (neutron stars or/and white dwarfs), axially and uniformly rotating in a rigid-body manner, endowed with a dipolar magnetic field, and whose interior is composed of a degenerate Fermi-Dirac gas (of neutrons or electrons, respectively). In § 4 the previous results, specialized to the case of circular relative motion, are applied in the evaluation of the gravitational-radiation luminosity of the stellar pair to pN accuracy, and its dependence on the changing internal characteristics of the two stars is given explicitly. In § 5 the numerical results are given and discussed, and possible future work is outlined. For reasons of clear exposition, the proof of some formulae, either basic or auxiliary, have been properly collected in three Appendices at the end. In Appendix A the detailed proof of the gravitational-radiation tensors is outlined, and a limited comparison and contrast of the relevant underlying theoretical results is attempted. In Appendix B the conditions are described under which the binary's relative position and velocity three vectors can be given a form similar to their corresponding Newtonian ones. Finally, in Appendix C the three first time derivatives of the binary's reduced inertial mass are evaluated in terms of the derivatives of the member's rest masses and self-energies.

2. QUADRUPOLE—RADIATION FORMULAE

For a nearly periodic, weakly gravitating and slowly moving source, the *near zone* or *radiation reaction* quadrupole formula (QF)

$$\left\langle -\frac{dE_0}{dt}\right\rangle = \frac{G}{5c^5}\langle \dddot{Q}_{\alpha\beta}\dddot{Q}_{\alpha\beta}\rangle , \tag{1}$$

where a dot denotes the total time derivative, and angle brackets denote average over time intervals large compared to some characteristic timescale of the source relates the rate of loss of the Newtonian total energy E_0 with the rate of emission of gravitational radiation described (Peters & Mathews 1963) in terms of the "quadrupole" mass-distribution tensor" $Q_{\alpha\beta}$ (α, $\beta = 1, 2, 3$). The above QF differs from the *far-field* or *wave-zone* QF which describes the source's gravitational-radiation *absolute luminosity* (Landau & Lifshitz 1975)

$$L = \frac{G}{5c^2}\dddot{Q}_{\alpha\beta}\dddot{Q}_{\alpha\beta} \tag{2}$$

determining, in the wave zone or at the future null infinity, the dependence of the radiation losses on the source's motions and internal structure.

Despite some open problems related to their proof (Persides 1991), the above formulae have been successsfully used in the description, *to lowest possible order*, of the emission of gravitational radiation from and its consequences on the motions of the gravitating source. This means that these formulae are given consistently up to and including terms of order ϵ^5/c^5, where c is the velocity of the light in vacuum and ϵ^2 is an upper bound of the values of the source's internal characteristics (see, e.g., Chandrasekhar 1965). Similarly for the description to the same order, the Newtonian Keplerian motions of the pair suffice to be used. Especially the QFs were applied (Weisberg & Taylor 1984; Damour 1987; Thorne 1987; Damour & Schäfer 1988, 1991; Taylor & Weisberg 1989; Damour et al. 1990; Damour & Taylor 1991) in the case of the well-known binary pulsar PSR 1913+16 (Hulse & Taylor 1975), as well as in the case of the possible change of the radii and periods of axial rotation of compact stars due to their continuous evolution (Spyrou 1985, 1987; for the corresponding accretion-induced changes for binary-participating neutron stars and white dwarfs, see Spyrou 1988).

Although there are no pN generalizations of equation (1), many authors (Epstein & Wagoner 1975; Thorne 1980; Damour & Schäfer 1988, 1991; Blanchet, Damour, & Schäfer 1990; Lincoln & Will 1990; Junker & Schäfer 1992) managed to provide generalizations of equation (2). From the various such generalizations we shall consider here the one due to Epstein & Wagoner (1975) referring to a bounded perfect-fluid gravitating source, namely,

$$\begin{aligned} L = \frac{G}{5c^5}\Bigg\langle N^{(0)}_{\alpha\beta}N^{(0)}_{\alpha\beta} + \frac{1}{21c^2}\,[&11N^{(1)}_{\alpha\beta\gamma}N^{(1)}_{\alpha\beta\gamma} - 6N^{(1)}_{\alpha\beta\beta}N^{(1)}_{\alpha\gamma\gamma} \\ &- 6N^{(1)}_{\alpha\beta\gamma}N^{(1)}_{\alpha\gamma\beta} + 22N^{(0)}_{\alpha\beta}N^{(2)}_{\alpha\beta\gamma\gamma} - 24N^{(0)}_{\alpha\beta}N^{(2)}_{\alpha\gamma\gamma\beta}] \\ &+ O\left[\left(\frac{M}{D}\right)^2 u^6\left(\frac{\epsilon}{c}\right)^4\right]\Bigg\rangle , \end{aligned} \tag{3}$$

where the spatial tensors $N^{(i)}_{\alpha\beta\gamma\ldots}$ denote time derivatives of various multipole moments of the source. To this pN order the rate of the orbital energy emitted in the form of gravitational radiation from a binary system consisting of two equal-mass neutron stars in circular relative orbit has been evaluated in Spyrou & Papadopoulos (1985), along with the rates of change of the orbital period and of the member's mutual distance (For further details on this see also Blanchet & Schäfer [1989] and the Appendix A at the end).

The basic assumption here, as in Spyrou & Papadopoulos (1985), for an appropriate energy-balance equation is that the time derivative of the source's total energy, E, up to the first PNA is equal to the corresponding gravitational-radiation luminosity given by equation (3); i.e.,

$$\langle -\dot{E}\rangle = L , \tag{4}$$

meaning that, in principle, *all the changing internal characteristics of the source are taken into account in evaluating the rate of emission of gravitational radiation, namely as some of its possible sources.*

Application of equations (3) and (4) to a binary star presupposes the knowledge of the pair's dynamical behavior and orbital motion up to the first PNA. This knowledge has been established long ago in the more general case of a realistic many-body system (Spyrou 1977a, b; see also Spyrou 1978, 1981a, b, 1983), for which the various dynamical laws of the pair (equations of motion, intrinsic and global conserved quantities, the virial theorems, etc.) have been given the same functional form as of their analogs for the Einstein, Infeld, & Hoffman (1938) system (hereafter EIH) of the ideal gravitating point masses. In that context the mass of the idealized EIH point mass is given a very precise physical meaning; namely, it is identified with an extended and realistic (but not tidally interacting) body's *inertial mass* (Contopoulos & Spyrou 1976)

$$m = \bar{m} + \frac{\mathscr{E}}{c^2}, \tag{5}$$

where $\bar{m}$ is the body's *rest mass* (namely the rest mass of the total number of baryons) and $\mathscr{E}$ is its total Newtonian *self-energy*. Therefore, if the body's rest mass remains constant, the rate of change of the inertial mass

$$\dot{m} = \frac{\dot{\mathscr{E}}}{c^2} \tag{6}$$

is of pN order.

Next we turn to the evaluation of the spatial tensor $N^{(0)}_{\alpha\beta}$. As it will be seen, the evaluation of solely $N^{(0)}_{\alpha\beta}$ will suffice for our present purposes. Thus, first, as described in Appendix B at the end, we refer the motions of the binary's members to the pair's *center-of-inertial-mass* frame of reference, which is uniformly moving to pN accuracy. It is straightforward to verify that

$$N^{(0)}_{\alpha\beta} = \frac{d^3}{dt^3}\left[\!\left[\mu\left(a^\alpha a^\beta - \frac{1}{3}\delta^{\alpha\beta}a^2\right)\left\{1 + \frac{1}{2c^2} \times\left[\left(1 - \frac{3\mu}{M}\right)u^2 - \left(1 - \frac{2\mu}{M}\right)\frac{GM}{a}(1 + O_2)\right]\right\}\right]\!\right], \tag{7}$$

where a^α and u^α are the components of the relative position and velocity three vectors of the two bodies' centers of inertial mass, and μ, M are the reduced mass and total inertial mass of the system, respectively, and O_l denotes terms of order l in the ratio of the bodies' linear dimensions over the mutual distance of their centers of inertial mass. (For an outline of the proof see the Appendix A at the end.)

From equations (5)–(7) it becomes transparent that in the case of nonchanging rest masses

$$\dot{\bar{m}}_1 = 0, \quad \dot{\bar{m}}_2 = 0, \tag{8}$$

namely for binaries whose members do not loose or/and mutually exchange material, but simply evolve independently of each other (as in the usual meaning of the term *stellar evolution*), the changing inertial masses will contribute to L only through the members' changing (due to their evolution) self-energies. It is exactly this property that enables us to use the EIH equations of motion for point particles (namely, to neglect tidal effects between the pair's members) and at the same time to consider changing (inertial) masses of the EIH particles. Obviously such a possibility does not arise in the context of the Newtonian theory of gravity, where changing masses (due to mass loss or mass gain) is a notion complementary to that of tidal effects. Therefore, not considering changes of the rest masses permits the bodies to preserve, in some sense, their identity and integrity. Furthermore, as described also in the Appendix C at the end, this contribution to L will be of order ϵ^7/c^7, and it will manifest itself through the time derivatives of only the tensor $N^{(0)}_{\alpha\beta}$, while all the other tensors in equation (3) will contribute terms of even higher pN order.

3. SELF-ENERGIES AND THEIR TIME DERIVATIVES

Before turning to the evaluation of the contribution to the gravitational-radiation luminosity of the changing self energies, we evaluate the members' self-energies and their first three time derivatives. First, we recall that according to Spyrou (1985, 1988) for a homogeneous, spherically symmetric compact star of radius R and rest mass $\bar{m}$, axially and uniformly rotating in a rigid-body manner with period P, endowed with a dipolar magnetic field of induction B, and whose interior is composed of a degenerate gas described by the Fermi-Dirac statistics, the total Newtonian energy $\mathscr{E}$ is

$$\frac{\mathscr{E}}{\bar{m}c^2} = \frac{4\pi^2}{5c^2}\frac{R^2}{P^2} - \frac{3}{5}\frac{G\bar{m}}{c^2R} + A\frac{g(x)}{x^3} + \frac{B^2R^3}{6\bar{m}c^2}. \tag{9}$$

In equation (9) the function $g(x)$ is defined by equation (23) of Chandrasekhar (1939 chap. X), and the relativisticity parameter x of the degenerate gas obeys (Spyrou 1985)

$$xR = \beta\bar{m}^{1/3}. \tag{10}$$

Finally the constants A and β are

$$A = \begin{cases} \dfrac{\bar{m}_e}{8\mu_e\bar{m}_p} & \text{for the electron gas,} \\ \dfrac{1}{8} & \text{for the neutron gas;} \end{cases} \tag{11}$$

$$\left(\frac{32\pi^2c^3}{9h^3}\right)^{1/3}\beta = \begin{cases} (\mu_e\bar{m}_e^3\bar{m}_p)^{-1/2} & \text{for the electron gas,} \\ \bar{m}_n^{-4/3} & \text{for the neutron gas;} \end{cases} \tag{12}$$

where $\mu_e(\leq 2)$ is the mean molecular weight per electron of the electron gas, and $\bar{m}_e$, $\bar{m}_p$, and $\bar{m}_n$ are the rest masses of the electron, proton and neutron, respectively. In evaluating the derivative $\dot{\mathscr{E}}$ (and $\ddot{\mathscr{E}}$, $\dddot{\mathscr{E}}$) we shall use equation (10) to eliminate the dependence on the time derivatives of x. Furthermore, we shall assume that the Fermi-Dirac gas is nonrelativistic, namely,

$$x \ll 1, \tag{13}$$

and that each member's magnetic flux is conserved, namely,

$$BR^2 = \text{constant}. \tag{14}$$

Under the above conditions we find

$$\dot{\mathscr{E}} = A_{\bar{m}}\dot{\bar{m}} + A_P\dot{P} + A_R\dot{R}, \tag{15}$$

with

$$A_{\bar{m}} = \frac{4\pi^2}{5}\frac{R^2}{P^2} - \frac{6G}{5}\frac{\bar{m}}{R} + 4A\beta^2c^2\frac{\bar{m}^{2/3}}{R^2},$$
$$A_P = -\frac{8\pi^2}{5}\frac{\bar{m}R^2}{P^3},$$
$$A_R = \frac{8\pi^2}{5}\frac{\bar{m}R}{P^2} + \frac{3G}{5}\frac{\bar{m}^2}{R^2} - \frac{24}{5}A\beta^2c^2\frac{(\bar{m})^{5/3}}{R^3} - \frac{1}{6}B^2R^2. \tag{16}$$

Also

$$\ddot{\mathscr{E}} = A_{\bar{m}\bar{m}}(\dot{\bar{m}})^2 + A_{PP}(\dot{P})^2 + A_{RR}(\dot{R})^2 + A_{\bar{m}P}\dot{\bar{m}}\dot{P} + A_{\bar{m}R}\dot{\bar{m}}\dot{R} + A_{RP}\dot{R}\dot{P}, \tag{17}$$

with

$$A_{\bar{m}\bar{m}} = -\frac{6G}{5R} + \frac{8}{3}(A\beta^2c^2)\frac{(\bar{m})^{-1/3}}{R^2},$$

$$A_{PP} = \frac{24\pi^2}{5}\frac{\bar{m}R^2}{P^4},$$

$$A_{RR} = \frac{8\pi^2}{5}\frac{\bar{m}}{P^2} - \frac{6G}{5}\frac{\bar{m}^2}{R^3} + \frac{72}{5}(A\beta^2c^2)\frac{(\bar{m})^{-5/3}}{R^4} + \frac{1}{3}B^2R,$$

$$A_{\bar{m}P} = -\frac{16\pi^2}{5}\frac{R^2}{P^3},$$

$$A_{\bar{m}R} = \frac{16\pi^2}{5}\frac{R^2}{P^2} + 12G\frac{\bar{m}}{R^2} - 16(A\beta^2c^2)\frac{(\bar{m})^{2/3}}{R^3},$$

$$A_{\dot{R}P} = -\frac{32\pi^2}{5}\bar{m}R. \quad (18)$$

Finally, although $\dddot{\mathscr{E}}$ will not be used, we shall give its expression retaining, for simplicity reasons, only first time derivatives of $\bar{m}$, P, and R. The physical justification for omitting these higher order derivatives is expressed mathematically as

$$\tau_{\dot{x}} \gg \tau_x,$$
$$\tau_{\ddot{x}} \gg \tau_y \quad (x \neq y),$$
$$\tau_x\tau_{\ddot{x}} \gg \tau_y^2 \quad (x \neq y),$$
$$\tau_x\tau_{\ddot{x}} \gg \tau_y\tau_z \quad (x \neq y, \quad x \neq z), \quad (19)$$

where x, y, and z are any of the parameters $\bar{m}$, P, and R, and by definition

$$\tau_x = \frac{x}{\dot{x}}, \quad \tau_{\dot{x}} = \frac{\dot{x}}{\ddot{x}}, \quad \tau_{\ddot{x}} = \frac{\ddot{x}}{\dddot{x}} \quad (20)$$

(similarly for y, z) are the characteristic time scales of the change x, $\dot{x}$, and $\ddot{x}$. Thus according to the first of equations (19) the timescale of the change of, e.g., P is much larger than that of $\dot{P}$, etc. Thus we find

$$\dddot{\mathscr{E}} = A_{\bar{m}PR}\dot{\bar{m}}\dot{P}\dot{R}, \quad (21)$$

with

$$A_{\bar{m}PR} = -\frac{64\pi^2}{5}\frac{R}{P^3}. \quad (22)$$

Next we turn to the evaluation of L.

4. THE GRAVITATIONAL-RADIATION LUMINOSITY FOR CIRCULARLY MOVING BINARIES

Since circularization of the orbit is expected during the late stages of the binary, especially when the members are close to coalesence, here we shall restrict ourselves to the case of circular relative orbit of radius a (for which in the Newtonian limit $u^2 = 2GM/a$, $a^\alpha u^\alpha = 0$). In this case equation (7) is written

$$N^{(0)}_{\alpha\beta} = \frac{d^3}{dt^3}\left[\mu\left(1 + \frac{E_0}{Mc^2}\right)S_{\alpha\beta}\right], \quad (23)$$

where

$$E_0 = -\frac{G\mu M}{2a}$$

is the Newtonian orbital energy, and, by definition,

$$S_{\alpha\beta} = a^\alpha a^\beta - \tfrac{1}{3}\delta^{\alpha\beta}a^2 \quad (24)$$

contains all the explicitly orbital terms. Therefore

$$\frac{d}{dt}\left[\mu\left(1 + \frac{E_0}{Mc^2}\right)\right] = \dot{\mu} + \text{terms of order } \frac{\epsilon^4}{c^4}, \quad (25)$$

and moreover

$$N^{(0)}_{\alpha\beta} = \mu\left(1 + \frac{E_0}{Mc^2}\right)\dddot{S}_{\alpha\beta} + 3\dot{\mu}\ddot{S}_{\alpha\beta} + 3\ddot{\mu}\dot{S}_{\alpha\beta} + 3\mu\dddot{S}_{\alpha\beta}. \quad (26)$$

In view of equation (26), equation (3) becomes

$$L = L_{\rm orb} + L_{\rm int}, \quad (27)$$

where by definition

$$L_{orb} = \frac{G}{5c^5}\Big\langle \mu^2\left(1 + \frac{2E_0}{Mc^2}\right)\dddot{S}_{\alpha\beta}\dddot{S}_{\alpha\beta} + \frac{1}{21c^2}(11N^{(1)}_{\alpha\beta\gamma}N^{(1)}_{\alpha\beta\gamma} - 6N^{(1)}_{\alpha\beta\beta}N^{(1)}_{\alpha\gamma\gamma} - 6N^{(1)}_{\alpha\beta\gamma}N^{(1)}_{\alpha\gamma\beta} + 22N^{(0)}_{\alpha\beta}N^{(2)}_{\alpha\beta\gamma\gamma} - 24N^{(0)}_{\alpha\beta}N^{(0)}_{\alpha\gamma\gamma\beta}) + O\left[\left(\frac{M}{D}\right)^2u^6\left(\frac{\epsilon}{c}\right)^4\right]\Big\rangle, \quad (28)$$

and

$$L_{\rm int} = \frac{G}{5c^5}\langle 2\mu\dddot{S}_{\alpha\beta}(3\dot{\mu}\ddot{S}_{\alpha\beta} + 3\ddot{\mu}\dot{S}_{\alpha\beta} + \mu S_{\alpha\beta})\rangle. \quad (29)$$

It is obvious that all the spatial tensors in equation (28) contribute only explicitly orbital terms up to first pN accuracy. Hence $L_{\rm orb}$ is the gravitational-radiation luminosity due to simply the orbital motion of the test particles (Spyrou & Papadopoulos 1985; Damour et al. 1990; Lincoln & Will 1990; Junker & Schäfer 1992; see also the relevant discussion in Appendix A at the end). On the other hand, $L_{\rm int}$ is a new part of the pair's luminosity including the pair's changing self energies. Furthermore, *$L_{\rm int}$ is in general nonvanishing even if the binary's members have equal inertial masses*:

$$m_1 = m_2 = m, \quad (30)$$

in which case there are no tidal interactions between the members, and hence no change of their internal characteristics in the standard meaning of the term. In the latter case, additionally for nonchanging rest masses (see eq. [8]) and, finally, in view of equations (C2)–(C4) of the Appendix C at the end, all the time derivatives of μ are of post-Newtonian order

$$\dot{\mu} = \frac{1}{4c^2}(\dot{\mathscr{E}}_1 + \dot{\mathscr{E}}_2), \quad \ddot{\mu} = \frac{1}{4c^2}(\ddot{\mathscr{E}}_1 + \ddot{\mathscr{E}}_2),$$

$$\dddot{\mu} = \frac{1}{4c^2}(\dddot{\mathscr{E}}_1 + \dddot{\mathscr{E}}_2), \quad (31)$$

and hence $L_{\rm int}$ is of order ϵ^7/c^7. More specifically, using the Newtonian equations for the circular relative orbital motion, we verify that

$$\dddot{S}_{\alpha\beta}\ddot{S}_{\alpha\beta}\ , \quad S_{\alpha\beta}\ddot{S}_{\alpha\beta} = 0\ , \tag{32}$$

$$\dot{S}_{\alpha\beta}\dddot{S}_{\alpha\beta} = -\frac{8GM}{a}u^2\ , \tag{33}$$

so that

$$L_{\rm int} = \frac{6G}{5c^5}\langle \mu\ddot{\mu}\ddot{S}_{\alpha\beta}\ddot{S}_{\alpha\beta}\rangle = -\frac{24}{5}\left\langle \frac{G^3}{c^7}\frac{m^3}{a^2}(\ddot{\mathscr{E}}_1 + \ddot{\mathscr{E}}_2)\right\rangle\ . \tag{34}$$

It is rather unexpected that the changing internal characteristics enter $L_{\rm int}$ through the *second* time derivatives of the self-energies. Moreover, according to equation (17), the dependence (for constant $\bar{m}$) of $\ddot{\mathscr{E}}$ (and hence λ defined by eq. [35] below) on $\dot{R}$ and $\dot{P}$ is the same independently of the latters' sign.

Furthermore, denoting by $L_{\rm orb}^{(7)}$ the $1/c^7$ part of the total orbital luminosity, we readily verify that the ratio

$$\lambda := \frac{L_{\rm int}}{L_{\rm orb}^{(7)}} = \left\langle \frac{3}{17}\frac{a^4}{G^2m^3}(\ddot{\mathscr{E}}_1 + \ddot{\mathscr{E}}_2)\right\rangle \tag{35}$$

is a measure of the relative importance of the $1/c^7$ parts of the gravitational-radiation luminosity due to the changing internal characteristics of the binary's members and to its orbital motions.

Finally, before proceeding to the numerical applications in the next section, we wish to recall that the knowledge of the waveform is important for the detection of gravitational waves by applying the *matched filtering technique*. The fact that the waveform is changing with the time induces inaccuracies in the calculation of the rate of change of the orbital period and leads to the corruption of the waveform. Consequently, the changing internal characteristics could lead to an analogous effect on the period's change. In order to examine this effect, we use the generalized Kepler's third law for the binary's motion, correct to pN accuracy, studied in Blanchet & Schäfer (1989), keeping in mind that the frequency of the gravitational wave, f, is twice the binary's orbital frequency. We readily verify that the orbital period P_b and the semimajor axis a are related to pN accuracy through the above law, namely,

$$\frac{a^3}{P_b^2 M}\left[1 - \frac{GM}{2ac^2}\left(\frac{\mu}{M} - 18\right)\right] = \frac{G}{4\pi^2}\ . \tag{36}$$

Direct consequence of equation (36) is the relation

$$\frac{\dot{P}_b}{P_b} = \frac{3}{2}\frac{\dot{a}}{a} - \frac{1}{2}\frac{\dot{M}}{M} = \left(\frac{\dot{P}_b}{P_b}\right)_{\rm PN} + \left(\frac{\dot{P}_b}{P_b}\right)_S\ . \tag{37}$$

In equation (37) we put

$$\left(\frac{\dot{P}_b}{P_b}\right)_{\rm PN} = \left(\frac{\dot{P}_b}{P_b}\right)_0\left[1 + \frac{GM}{6ac^2}\left(\frac{\mu}{M} - 18\right)\right]\ , \tag{38}$$

where

$$\left(\frac{\dot{P}_b}{P_b}\right)_0 = \frac{3}{2}\frac{\dot{a}}{a} \tag{39}$$

is the purely Newtonian orbital result for point masses and, for constant rest masses,

$$\left(\frac{\dot{P}_b}{P_b}\right)_S = -\frac{1}{c^2}\frac{\dot{\mathscr{E}}_1 + \dot{\mathscr{E}}_2}{2M} \tag{40}$$

includes the changing self-energies, and (for constant rest masses) is of pN order. Therefore the phase change due to the changing internal characteristics (eq. [40]) could be comparable to that due to (1) the Newtonian orbital parameters (eq. [39]) provided that

$$\left(\frac{\dot{P}_b}{P_b}\right)_0 = -\frac{1}{c^2}\frac{\dot{\mathscr{E}}_1 + \dot{\mathscr{E}}_2}{2M}\ , \tag{41}$$

and (2) to the post-Newtonian parameters (the explicitly $1/c^2$ part of eq. [38]) provided that

$$\left(\frac{\dot{P}_b}{P_b}\right)_0 \frac{GM}{6ac^2}\left(\frac{\mu}{M} - 18\right) = -\frac{1}{c^2}\frac{\dot{\mathscr{E}}_1 + \dot{\mathscr{E}}_2}{2M}\ . \tag{42}$$

5. NUMERICAL RESULTS AND DISCUSSION

Here first we show some numerical values of the relative-importance factor λ for a range of values of the distance of the binary's members, their radii and their masses, together with an expected range of possible values of $\dot{P}$ and $\dot{R}$ (recall that $\dot{m} = 0$).

We have chosen a binary system consisting of two typical millisecond neutron stars with equal masses $m_1 = m_2 = 1.4\ M_\odot$, radii $R_1 = R_2 = 10$ km and periods of axial rotation $P_1 = P_2 = 10^{-3}$ s. We list the values of the ratio λ (namely $L_{\rm int}/L_{\rm orb}^{(7)}$) for a range of values for the rate of period change from 10^{-17} to 10^{-5} s s^{-1} and a rate of radius change from 10^{-13} to 10^{-5} cm s^{-1} used in Spyrou (1985). The results are shown in Tables 1 and 2. From these results we conclude that the ratio λ is generally much smaller than unity. More specifically, when the stars are very close to each other, the gravitational-wave emission due to the orbital terms of the order ϵ^7/c^7 is very much larger than the energy emitted due to the change of the internal characteristics of the bodies. This, in

TABLE 1

RATIO λ AS A FUNCTION OF THE CONTRACTION RATE R

a (cm)	$\dot{R}$ (cm s^{-1})	$L_{\rm int}/L_{\rm orb}^{(7)}$
2×10^6	-10^{-5}	2.2×10^{-30}
2×10^6	-10^{-9}	6.6×10^{-37}
2×10^6	-10^{-13}	6.4×10^{-37}
10^7	-10^{-5}	1.4×10^{-27}
10^7	-10^{-9}	4.1×10^{-34}
10^7	-10^{-13}	4.0×10^{-34}
10^8	-10^{-5}	1.4×10^{-23}
10^8	-10^{-9}	4.1×10^{-30}
10^8	-10^{-13}	4.0×10^{-30}
2×10^{10}	-10^{-5}	2.2×10^{-21}
2×10^{10}	-10^{-9}	6.6×10^{-21}
2×10^{10}	-10^{-13}	6.4×10^{-21}
1.5×10^{12}	-10^{-5}	7.0×10^{-3}
1.5×10^{12}	-10^{-9}	2.1×10^{-9}
1.5×10^{12}	-10^{-13}	2.4×10^{-9}

NOTES.—Several values of the ratio λ of the internal-energy contribution $L_{\rm int}$ of order ϵ^7/c^7 to the orbital energy contribution $L_{\rm orb}^{(7)}$ of the same order are tabulated for various values of the separation distance a and the rate of decrease of the radii $\dot{R}_1 = \dot{R}_2 = \dot{R} < 0$. The system is composed of two neutron stars with equal masses $m_1 = m_2 = m = 1.4\ M_\odot$ and radii $R_1 = R_2 = R = 10^6$ cm, while their periods are $P_1 = P_2 = P = 10^{-3}$ s and the period change is $\dot{P}_1 = \dot{P} = \dot{P}_2 = 10^{-17}$ s s^{-1}. The range of values of $\dot{R} < 0$ is based on that used in Spyrou 1985.

TABLE 2

RATIO λ AS A FUNCTION OF THE PERIOD DECREASE RATE $\dot{P}$

a (cm)	$\dot{P}$ (s s^{-1})	$L_{int}/L_{orb}^{(7)}$
2×10^6	10^{-5}	6.4×10^{-13}
2×10^6	10^{-9}	6.4×10^{-21}
2×10^6	10^{-13}	6.4×10^{-29}
2×10^6	10^{-17}	6.4×10^{-37}
10^7	10^{-5}	4.0×10^{-10}
10^7	10^{-9}	4.0×10^{-18}
10^7	10^{-13}	4.0×10^{-24}
10^7	10^{-17}	4.0×10^{-34}
10^8	10^{-5}	4.0×10^{-6}
10^8	10^{-9}	4.0×10^{-14}
10^8	10^{-13}	4.0×10^{-20}
10^8	10^{-17}	4.0×10^{-30}
2×10^{10}	10^{-5}	$4.0 \times 10^{+2}$
2×10^{10}	10^{-9}	4.0×10^{-6}
2×10^{10}	10^{-13}	4.0×10^{-14}
2×10^{10}	10^{-17}	4.0×10^{-22}
1.5×10^{12}	10^{-5}	$2.0 \times 10^{+15}$
1.5×10^{12}	10^{-9}	$2.0 \times 10^{+7}$
1.5×10^{12}	10^{-13}	2.0×10^{-1}
1.5×10^{12}	10^{-17}	2.0×10^{-9}

NOTES.—Several values of the ratio λ of the internal-energy contribution L_{int} of order ϵ^7/c^7 to the orbital energy contribution $L_{orb}^{(7)}$ of the same order are tabulated for various values of the separation distance a and the change of the period, $\dot{P}$. The system is composed of two neutron stars with equal masses $m_1 = m_2 = m = 1.4\ M_\odot$ and radii $R_1 = R_2 = R = 10^6$ cm, while their periods are $P_1 = P = P_2 = 10^{-3}$ s and the change of radius $\dot{R} = -10^{-13}$ cm s^{-1}. The range of values of $\dot{P}$ is based on that used in Spyrou 1985.

a sense, is nonunexpected. Actually, when the two compact objects are very close to each other, the higher pN terms are comparable even to the Newtonian ones, since in this case the limits of the validity of the PNA theory have been almost reached. Thus the higher pN terms contribute a significant amount to the total energy emitted, which could be much larger than the energy emitted due to internal changes in the two stars. But, as we move to larger separations, since the energy emitted, as calculated by the QF, is proportional to a^{-5}, the ratio λ increases, and circumstances can be met under which, for appropriate values of $\dot{P}$ and $\dot{R}$, the ratio λ could be of the order of unity. However, although this result is very interesting from the theoretical point of view, the situation does not appear the same from the observational point of view as well. Actually the result is not important as far as gravitational-wave detectors of this generation are concerned. According to the numerical results, the ϵ^7/c^7 contribution to L of the changing internal characteristics balances the ϵ^7/c^7 orbital terms for values of the relative distance a in the range 10^{10}–10^{12} cm. Equivalently, through the Newtonian Kepler's third law (assuming $m_1 = m_2 = 1.4\ M_\odot$), this happens for gravitational-wave frequencies of at most a few tens of a mHz, which, however, is far out the present observational window ($\approx$10–1000 Hz). Therefore the importance of our proposed theoretical results to the observational ones is expected to be checked with the aid of gravitational-wave detectors of the next generation.

Analogous conclusions can be reached by considering the phase changes in the waveform defined by equations (37)–(40). For simplicity we shall assume that the only changing parameter of each star is its period of axial rotation $P(\dot{m} = 0, \dot{R}_1 = 0 = \dot{R}_2)$. Then, according to equation (41), for two identical members, the phase change $(\dot{P}_b/P_b)_S$ will be comparable to the purely orbital Newtonian phase change $(\dot{P}_b/P_b)_0$, provided that the gravitational-wave frequency satisfies

$$f = 4.36 \times 10^2(-\dot{P})^{3/8}\ \text{Hz}\ . \qquad (43)$$

Direct consequence of the last equation is that, for the assumed range of values of $\dot{P}$ in Tables 1 and 2, the corresponding range of the values of f is from a few tenths of a mHz to at most of a few hertz (For members with only radii changing we similarly obtain $f = 0.55(\dot{R})^{3/8}$ Hz with an upper bound of f of a few megahertz). We notice that, according to the equation (42), equating the self and orbital pN phase changes we obtain in a similar way $f = 1.98 \times 10^5\ (-\dot{P})^{3/5}$ Hz. In this case the range of values of f is from a few millionths of a hertz to a few killihertz. Therefore, for the currently accepted values of $\dot{P}$ for pulsars, the derived frequencies are out of the range of possible applicability of the matched filtering technique.

We shall conclude with some general remarks. To the extent of our knowledge, this paper is the first one in the literature dealing with the emission of gravitational radiation from a nonaccreting binary star, resulting from the evolutionary (in contrast to the accretion-induced binary star) changes of its members' internal characteristics. The use of the inertial mass (and the corresponding center-of-inertial-mass position and velocity three-vectors) for the description of each body permits the study of the emission of gravitational radiation due to the evolutionary change of their Newtonian total self-energies without the necessity of taking into account tidal effects (as appears to be the case in the Newtonian dynamical description of a two-point-masses binary star). Our theoretical framework is valid irrespective of the nature of the compact stars (white dwarfs or neutron stars). Also the theoretical framework takes into account all the internal characteristics of the members, and, hence, the physics of the members is considered in an as complete as possible and nontrivial way. It is important that the assumption for using among others a nonrelativistic rather than a relativistic, uniform Fermi-Dirac gas for describing the member's interior is only slightly changing the dependence of the results on the relativisticity parameter χ (see also § 4 of Spyrou 1988). Furthermore it is worth mentioning that, for binaries in circular relative motion, the newly proposed part of the gravitational-radiation luminosity depends on the second time derivatives of the member's self-energies. This is a rather unexpected result, because what one would have expected, based on the standard QFs, is the dependence on, additionally, the third time derivatives. As a consequence the luminosity appears quadratic in $\dot{P}$ and $\dot{R}$ (and $\dot{m}$), and is thus proved to be explicitly independent of the sign of $\dot{P}$ and $\dot{R}$, namely, the same for both a deceleration or acceleration of the axial rotation and, similarly, for both a contraction or expansion of each of the two compact stars. Finally, from an observational point of view, it is important that the frequency of the extra gravitational-radiation emission is far out of the present observational window, and that the induced phase change, although vanishing for the Newtonian Keplerian orbit, is nonvanishing for the post-Newtonian relative orbit. The relative importance of the purely orbital and the purely internal phase changes depends on the evolutionary stages of each of the pair's members and of the point-masses pair as a whole.

In spite of all the above advantages, we have to stress that our results are only approximate. Thus, our results have been based on the gravitational-radiation luminosity proposed in Epstein & Wagoner (1975), and hence they carry all its disadvantages, as the latter have been exposed in Blanchet & Schäfer (1989) and in Spyrou & Papadopoulos (1985). Furthermore, the compact stars have been treated as spherical bodies, a property that along with the assumption of secularly stationary members cannot be true when their relative distance is small. Also in deriving the gravitational-radiation luminosity we have assumed that the last (cross) term in equation (A14), being negligible compared to the first (purely orbital) one, has similarly negligible time derivatives. This surely deserves a further examination which will complete the present one on the importance of the evolutionary changing internal characteristics. Moreover, although our theoretical framework is valid independently of the nature of the compact members, our numerical results are valid only for pulsars, so that the cases of pulsar–white dwarf and white dwarf–white dwarf binaries could also present some at least theoretical interest. "Furthermore, the fact that the possibility for the newly proposed part of the gravitational-radiation emission to be currently detected is beyond present observational capabilities could also be considered a disadvantage." However, the further examination of the phase changes induced in the Newtonian and pN relative orbits could present some interest at least in the context of other techniques, beyond the matched filtering technique.

At the outcome, as far as future work is concerned, we have to stress that the case of nonspherical bodies should be examined, because then the first term on the right-hand side of equation (A9) (and its derivatives) is generally different from zero, and the role of the changing internal characteristics (due either to evolution or accretion) becomes important to the lowest (ϵ^5/c^5) approximation. Finally, and more important, the case of rest mass exchanging binaries ($\dot{m} \neq 0$) with the subsequent accretion-induced changes of internal characteristics should present additional interest, because then the full pN relative orbit's use become necessary, and tidal effects have to be taken into account. All these problems are currently under investigation in the context of a more general research program on gravitational radiation initiated in Spyrou (1985, 1987, 1988), Spyrou & Papadopoulos (1985), and Kokkotas & Schäfer (1993), and continued with the present article.

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APPENDIX A

PROOF OF EQUATION (7) IN THE TEXT

In this Appendix we outline the proof of the equation (7) in the text, namely,

$$N^{(0)}_{\alpha\beta} = \frac{d^3}{dt^3}\left[\mu\left(a^\alpha a^\beta - \frac{1}{3}\,\delta^{\alpha\beta}a^2\right)\left\{1 + \frac{1}{2c^2}\left[\left(1 - \frac{3\mu}{M}\right)u^2 - \left(1 - \frac{2\mu}{M}\right)\frac{GM}{a}(1 + O_2)\right]\right\}\right].$$

To this purpose we recall that according to equations (45)–(48) of Epstein & Wagoner (1975)

$$N^{(0)}_{\alpha\beta} = \frac{d^3}{dt^3}\, I_{\alpha\beta}\,, \tag{A1}$$

where, for a perfect-fluid source of three-dimensional volume V,

$$I_{\alpha\beta} = \int_V f(x^\alpha x^\beta - \tfrac{1}{3}\delta^{\alpha\beta}x^2)dV\,. \tag{A2}$$

In the case of a two-body system equation (A2), in an obvious notation, can be put in the form

$$I_{\alpha\beta} = \sum_{j=1}^{2} I^j_{\alpha\beta}\,, \tag{A3}$$

with

$$I^j_{\alpha\beta} = \int_{V_j} f_j(x^\alpha_j x^\beta_j - \tfrac{1}{3}\,\delta^{\alpha\beta}x^2_j)dV_j\,, \tag{A4}$$

where the subscript j denotes that the corresponding quantity is evaluated at the point x^α_j in the interior of the body j taking into account the existence of the other body.

Next we split x^α_j in the form

$$x^\alpha_j = a^\alpha_j + \xi^\alpha_j\,, \tag{A5}$$

$$v^\alpha_j = \dot{x}^\alpha_j = u^\alpha_j + \gamma^\alpha_j\,. \tag{A6}$$

In equations (A5) and (A6) a^α_j and

$$u^\alpha_j = \dot{a}^\alpha_j \tag{A7}$$

are, respectively, the components, in the system's center of inertial mass frame, of the position and velocity three vectors of the j-body's center of inertial mass, properly defined to post-Newtonian accuracy. Also, ξ_j^α and

$$\gamma_j^\alpha = \dot{\xi}_j^\alpha \tag{A8}$$

are, respectively, the components, in the j-body's center of inertial mass frame, of the position and velocity three vectors at the point x_j^α.

Then it is a matter of same algebra to show that

$$I_{\alpha\beta} = \left[\left[\int_{V_j} f_j\left(\xi_j^\alpha \xi_j^\beta - \frac{1}{3}\,\delta^{\alpha\beta}\xi_j^2\right)dV_j + m_j\left(a_j^\alpha a_j^\beta - \frac{1}{3}\,\delta^{\alpha\beta}a_j^2\right)\right] \times \left\{1 + \frac{1}{2c^2}\left[u_j^2 - G\sum_{k=j}\frac{m_k}{a_{jk}}(1+O_2)\right]\right\}$$
$$+ \frac{1}{c^2}\left(a_j^\beta u_j^\mu \delta^{\alpha\nu} + a_j^\alpha u_j^\mu \delta^{\beta\nu} - \frac{2}{3}\,a_j^\mu a_j^\nu \delta^{\alpha\beta}\right)\int_{V_j} f_j \xi_j^\nu \gamma_j^\mu \,\mu\, dV_j + \frac{1}{c^2}\,u_j^\mu \int_{V_j} f_j\left(\xi_j^\alpha \xi_j^\beta - \frac{1}{3}\,\delta^{\alpha\beta}\xi_j^2\right)\gamma_j^\mu\, dV_j\,, \tag{A9}$$

where f_j is the j-body's inertial-mass density and, as already defined, O_l will denote terms of order l in $L/D < 1$.

In the case of spherically symmetric bodies, the first and last terms in equations (A9) vanish. Moreover, we put

$$a^\alpha = a_1^\alpha - a_2^\alpha\,, \qquad u^\alpha = u_1^\alpha - u_2^\alpha \tag{A10}$$

for the relative position and velocity three vectors, which, as proved in Appendix B at the end, *obey the Newtonian-like formulae to* 1st *pN accuracy*

$$a_1^\alpha = \frac{\mu}{m_1}\,a^\alpha\,, \qquad a_2^\alpha = -\frac{\mu}{m_2}\,a^\alpha\,, \qquad u_1^\alpha = \frac{\mu}{m_1}\,u^\alpha\,, \qquad u_2^\alpha = -\frac{\mu}{m_2}\,u^\alpha\,. \tag{A11}$$

So equation (A9) is written as

$$I_{\alpha\beta}^j = \frac{\mu^2}{m_j}\left(a^\alpha a^\beta - \frac{1}{3}\,\delta^{\alpha\beta}\alpha^2\right)\left\{1 + \frac{1}{2c^2}\frac{\mu}{m_j}\left[\frac{\mu}{m_j}\,u^2 - \frac{GM}{a}(1+O_2)\right]\right\} + \frac{\mu^2}{c^2 m_j^2}\left(a^\alpha u^\mu \delta^{\beta\nu} + a^\beta u^\mu \delta^{\alpha\nu} - \frac{2}{3}\,a^\mu u^\nu \delta^{\alpha\beta}\right)l_j^{\nu\mu}\,, \tag{A12}$$

where

$$l_j^{\nu\mu} = \int_{V_j} f_j \xi_j^\alpha \gamma_j^\beta\, dV_j \tag{A13}$$

is the j-body's angular-momentum tensor, defined by equation (44) of Spyrou (1978). Then equation (A3) becomes

$$I_{\alpha\beta} = \mu\left(a^\alpha a^\beta - \frac{1}{3}\,\delta^{\alpha\beta}a^2\right) \times \left\{1 + \frac{1}{2c^2}\left[\left(1 - \frac{3\mu}{M}\right)u^2 - \left(1 - \frac{2\mu}{M}\right)\frac{GM}{a}(1+O_2)\right]\right\}$$
$$+ \frac{\mu^2}{c^2}\left(a^\alpha u^\mu \delta^{\beta\nu} + a^\beta u^\mu \delta^{\alpha\nu} - \frac{2}{3}\,a^\mu u^\nu \delta^{\alpha\beta}\right) \times \left(\frac{1}{m_1^2}\,l_1^{\mu\nu} + \frac{1}{m_2^2}\,l_2^{\mu\nu}\right), \tag{A14}$$

Equation (A14) generalizes equation (A5) in the Appendix of Spyrou & Papadopoulos (1985) in two ways, and also equation (A1) of Blanchet & Schäfer (1989). Thus, not only is it valid for circular relative orbital motion, but also it includes, in the form of its last term, the first corrections due to the interaction of internal and orbital characteristics. Moreover, we wish to stress, as it has been properly and correctly noted in Blanchet & Schäfer (1989), that it corrects the same equation by the inclusion of the term

$$1 + \frac{1}{c^2}\left(\frac{1}{2}\,u_j^2 - \frac{G}{2}\sum_{k\neq j}\frac{m_k}{|\boldsymbol{x}_j - \boldsymbol{a}_k|}\right)$$

in equation (A4) of Spyrou & Papadopoulos (1985) which has *not* been printed therein. One may notice that in the case of circular relative motion of two equal inertial masses, m, the omitted term equals $(1 - Gm/2c^2a)$, and as a consequence, equations (8) and (9) of Spyrou & Papadopoulos (1985) for $\langle \dot{E}/E\rangle$ and $(-\dot{P}_b/P_b)$, respectively, have to be multiplied by the term $(1 - Gm/4c^2a)^5$. This new term induces in the relative (with respect to the corresponding EIH result of Peters & Mathews 1963) post-Newtonian correction in $(-\dot{P}_b/P_b)$, as evaluated via equation (9) of Spyrou & Papadopoulos (1985), an extra term $-5Gm/4ac^2$. In the case of the binary pulsar PSR 1913−16, this extra term is $\sim -1.34 \times 10^{-6}$, and so the total post-Newtonian correction is -4.22×10^{-6}, namely, only $-\frac{1}{5}$ (and not -60, as noted in Blanchet & Schäfer 1989) of their value. Hence the two corrections practically differ only in their sign. It is of interest to notice that this remaining difference is surely attributed to the difference in the definition of the orbital period used in Spyrou & Papadopoulos (1985) on the one hand and Blanchet & Schäfer (1989) on the other hand. The latter is *definitely more appropriate*, because the former (for an elliptical orbit of semiaxes a and b) is $P_b = 2\pi ab/l_0$, where the orbital angular momentum (per unit reduced inertial mass μ) l_0 and the semiaxes a, b are defined to *post-Newtonian accuracy*, with the aid of the polar equation of the full relativistic orbit. Hence, due simply to initial assumptions, this P_b is something in between its Newtonian analogue and the more appropriate one of Blanchet & Schäfer (1989). The two periods differ by terms of pN order. In this sense, no "errors," at least in the standard meaning of the term, can be located in the definition of P_b in Spyrou & Papadopoulos (1985). In any case, one of us (N. S.) wishes to thank especially G. Schäfer for his private communication and some very useful discussions aiming to clear up the differences of the two definitions.

Next we turn to equation (A14). For a spherically symmetric body, axially rotating uniformly and rigidly, with angular velocity ω_j we find

$$l_j^{\alpha\beta} = \epsilon_{\beta\mu\nu}\,\omega_j^\mu \int_V f_j\,\xi_j^\alpha\,\xi_j^\nu\,dV_j = \tfrac{1}{3}\epsilon_{\beta\mu\alpha}\,\omega_j^\mu\,J_j\,, \tag{A15}$$

where $\epsilon_{\alpha\beta\gamma}$ is the completely antisymmetric pseudotensor of rank three (i.e., the Levi-Civita symbol),

$$J_j = \int_{V_j} f_j\,\xi_j^2\,dV_j = \tfrac{3}{5}m_j R_j^2\,, \tag{A16}$$

and so

$$\frac{1}{m_j^2}\,l_j^{\alpha\beta} = \epsilon_{\alpha\beta\mu}\,\omega_j^\mu\,\frac{1}{5}\,\frac{R_j^2}{m_j}\,, \tag{A17}$$

and moreover the last term of equation (A14) becomes

$$\mu^2\left(a^\alpha u^\mu\,\delta^{\beta\nu} + a^\beta u^\mu\,\delta^{\alpha\nu} - \frac{2}{3}\,a^\mu u^\nu\,\delta^{\alpha\beta}\right) \times \left(\frac{1}{m_1^2}\,l_1^{\mu\nu} + \frac{1}{m_2^2}\,l_2^{\mu\nu}\right) = \frac{\mu^2}{5}\,(\epsilon_{\mu\beta\rho}\,a^\alpha u^\mu + \epsilon_{\mu\alpha\rho}\,a^\beta u^\mu)\left(\omega_1^\rho\,\frac{R_1^2}{m_1^2} + \omega_2^\rho\,\frac{R_2^2}{m_2^2}\right). \tag{A18}$$

In view of equations (A3) and (A18) we now wish to point out that the last (or interaction) terms in the equation (A14), in orders to magnitude, are

$$\frac{c^{-2}\,DumLv}{c^{-2}mv^2\,D^2} \sim \frac{Lu}{Dv} \sim \frac{L}{D}\,\frac{(Gm/D)^{1/2}}{(Gm/L)^{1/2}} \sim \left(\frac{L}{D}\right)^{3/2} < 1 \tag{A19}$$

times smaller that the interaction terms hidden (through μ) in the first term of equation (A14). Also the last (interaction) terms in equation (A14), in orders of magnitude, are

$$\frac{c^{-2}\,DumLv}{m\,D^2u^2c^{-2}} \sim \frac{Lv}{Du} \sim \frac{L}{D}\,\frac{(Gm/L)^{1/2}}{(Gm/D)^{1/2}} \sim \left(\frac{L}{D}\right)^{1/2} < 1 \tag{A20}$$

times smaller than the pN purely orbital (second) terms. So we shall omit them obtaining for circular orbit ($u^2 = Gm/a$)

$$I_{\alpha\beta} = \mu\left(a^\alpha a^\beta - \frac{1}{3}\,\delta^{\alpha\beta}a^2\right)\left(1 - \frac{\mu}{Mc^2}\,\frac{GM}{2\alpha}\right) \tag{A21}$$

practically equation (7) in the text.

We notice that were we going to retain in the time differentiations the interaction terms, we *could not* use the pointlike equation of motions, and the problem becomes rather complicated, so that the perfect-fluid magnetohydrodynamical Euler's equations have to be used. However, we have to stress that although according to equations (19) and (20) the interaction terms are smaller than the first term on the right hand side of equation (A14), we have not checked out that this applies also for their time derivatives. We believe that this simplification will affect our results in a nonsignificant way.

APPENDIX B

THE CENTER OF INERTIAL MASS MOTION

In this Appendix we shall prove that the relative position and velocity three vectors a^α and u^α are expressed, in the terms of the inertial masses and the corresponding position and velocity three-vectors of the absolute motions, via the Newtonian-like formula (A11) which are valid to 1st PNA.

As it is known (Contopoulos & Spyrou 1976), the uniform, to PN accuracy, motion of the center of mass of an otherwise arbitrary perfect-fluid source is described by an equation of the form

$$P^\alpha = \frac{dS^\alpha}{dt}\,. \tag{B1}$$

In equation (B1) P^α and S^α are the components of the fluid source's total linear-momentum and dipole-moment three vectors, respectively. These vectors are defined by equation (1) and (18) of Contopoulos & Spyrou (1976), which, as it is readily verified with the aid of equations (A5)–(A7), (A10), and the Newtonian form of equation (A11), in the case of a two-body system reduce to

$$S^\alpha = m_1\,a_1^\alpha + m_2\,a_2^\alpha + \frac{1}{2c^2}\,\frac{\mu}{M}\,(m_2 - m_1)\left(u^2 - \frac{GM}{a}\right)a^\alpha + \frac{1}{c^2}\,\mu\left(\frac{l_1^{\alpha\beta}}{m_1} - \frac{l_2^{\alpha\beta}}{m_2}\right)u^\beta\,, \tag{B2}$$

$$P^\alpha = m_1\,u_1^\alpha + m_2\,u_2^\alpha + \frac{1}{2c^2}\,\frac{\mu}{M}\,(m_2 - m_1)\left(u^2 - \frac{GM}{\alpha}\right)u^\alpha + \frac{1}{c^2}\,\mu\left(\frac{V_1^{\alpha\beta}}{m_1} - \frac{V_2^{\alpha\beta}}{m_2}\right)u^\beta\,, \tag{B3}$$

where the j-body's virial-theorem tensor $V_j^{\alpha\beta}$ is defined by equation (46) of Spyrou (1978).

Now we observe that the explicitly pN terms on the right of equations (B2) and (B3) can be neglected compared to the remaining terms, if (1) the inertial masses, are equal and/or (2) the structures of the bodies are similar (apart, of course, from the vanishing of the virial-theorem tensors under conditions of secularly stationary equilibrium for each body not excluding tidal effects). Under such conditions equations (B2) and (B3) reduce to their Newtonian analogs. Hence the vanishing of both P^α and S^α, defining the center-of-inertial-mass frame of the fluid source consistently to pN accuracy, is equivalent to the Newtonian-like equations

$$m_1 a_1^\alpha + m_2 a_2^\alpha = 0 \,, \qquad m_1 u_1^\alpha + m_2 u_2^\alpha = 0 \,, \tag{B4}$$

direct consequence of which are equation (A11).

APPENDIX C

TIME DERIVATIVES OF THE TOTAL REDUCED INERTIAL MASS μ

By direct differentiation of

$$m_j = \bar{m}_j + \frac{\mathscr{E}_j}{c^2} \tag{C1}$$

and putting

$$\dot{\bar{m}}_1 = -\dot{\bar{m}}_2 = \dot{\bar{m}} \,, \tag{C2}$$

we readily verify that

$$\dot{\mu} = \frac{m_2 - m_1}{M}\,\dot{\bar{m}} + \frac{1}{c^2}\left[\left(\frac{m_2}{M}\right)^2 \dot{\mathscr{E}}_1 + \left(\frac{m_1}{M}\right)^2 \dot{\mathscr{E}}_2\right], \tag{C3}$$

$$\ddot{\mu} = \frac{m_2 - m_1}{M}\,\ddot{\bar{m}} - 2\,\frac{(\dot{\bar{m}})^2}{M} + \frac{4}{c^2}\,\frac{\dot{\bar{m}}}{M}\left[\left(\frac{m_1}{M}\right)^2 \dot{\mathscr{E}}_1 - \left(\frac{m_2}{M}\right)^2 \dot{\mathscr{E}}_2\right] + \frac{1}{c^2}\left[\left(\frac{m_2}{M}\right)^2 \ddot{\mathscr{E}}_1 + \left(\frac{m_1}{M}\right)^2 \ddot{\mathscr{E}}_2\right], \tag{C4}$$

$$\dddot{\mu} = \frac{m_2 - m_1}{M}\,\dddot{\bar{m}} - \frac{6\dot{\bar{m}}}{M}\,\dddot{\bar{m}} + \frac{6}{c^2}(\dot{\bar{m}})^2\,\frac{(\dot{\mathscr{E}}_1 + \dot{\mathscr{E}}_2)}{M_2} + \frac{6}{c^2}\,\frac{\dot{\bar{m}}}{M}\left(\frac{m_1}{M}\,\ddot{\mathscr{E}}_2 - \frac{m_2}{M}\,\ddot{\mathscr{E}}_1\right) + \frac{6}{c^2}\,\frac{\ddot{\bar{m}}}{M}\left(\frac{m_1}{M}\,\dot{\mathscr{E}}_2 - \frac{m_2}{M}\,\dot{\mathscr{E}}_1\right) + \frac{1}{c^2}\left[\left(\frac{m_2}{M}\right)^2 \ddot{\mathscr{E}}_1 + \left(\frac{m_1}{M}\right)^2 \ddot{\mathscr{E}}_2\right]. \tag{C5}$$

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