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THE ORBITAL EVOLUTION OF HIGHLY ECCENTRIC BINARIES

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ABSTRACT

The tidal evolution of close binaries in the limit of $e \to 1$ is studied in this work. We use Hut equations to obtain the time derivatives and timescales for the evolution of the eccentricity, semimajor axis, and stellar rotation rate in the high-eccentricity binaries.

We find that in some of the highly eccentric binaries the tidal shear changes near periastron on a timescale shorter than the convective timescale, so that the turbulent viscosity could be reduced. We consider three different recently proposed approaches to viscosity reduction and show that for all three theories the tidal evolution for highly eccentric binaries is quite different from that encountered in low-eccentricity systems. In particular, the semimajor axis decreases on a timescale much shorter than the eccentricity, and the periastron distance stays constant in time.

We suggest to test the different approaches to viscosity reduction by comparing the age of any known highly eccentric binary with its tidal timescales. The proposed test is applied to Gl 586A, a nearby binary recently found to have an extremely high eccentricity. The test indicates that this binary may indeed be used to reject the approach which assumes a nonreduced viscosity.

Subject headings: binaries: close — stars: rotation — stars: individual (Gliese 586A) — stars: interiors

1. INTRODUCTION

The orbital elements of the nearby double-line spectroscopic binary Gl 586A were determined recently by Duquennoy et al. (1992). The most remarkable feature of this system is its extremely high orbital eccentricity of 0.9752 ± 0.0003. Duquennoy et al. pointed out that despite the long orbital period of 890 days, the very high eccentricity implies quite a small periastron distance, of $\sim 10\ R_\odot$. Duquennoy et al. noted also that the tidal interaction is expected to decrease the orbital period and eccentricity, implying even higher *initial* eccentricity. They argued that this suggestion would have resulted in periastron separation too small to accommodate the two stars in the pre–main-sequence phase, when the stellar radii were larger.

The problematic evolutionary history of Gl 586A, as noted by Duquennoy et al., is typical of extremely high eccentric spectroscopic binaries. With the expectation that more systems of this kind will be discovered in the near future, we study in this work tidal evolution of binaries in the limit of $e \to 1$. We obtain the time derivatives and timescales for the evolution of the eccentricity, semimajor axis, and stellar rotation rate for highly eccentric binaries. The timescales are functions of the coupling between the shear (velocity gradient) in the stellar envelopes, induced by the tidal interaction between the two components, and the turbulent viscosity.

In highly eccentric binaries the tidal interaction varies on a timescale which is a small fraction of the orbital period, and thus can be in some cases comparable to or shorter than the stellar convective timescale. In such cases, the turbulent viscosity could be reduced (Zahn 1989) by an amount which is still not clear. We consider here three approaches to this reduction.

The first original approach ignores the reduction of the turbulent viscosity (e.g., Zahn 1977). The second assumes a reduction linear in the ratio between the timescale for variation of the tidal interaction and the convective timescale (Zahn 1989, 1992). The third approach (Goldman & Mazeh 1991) argues for a reduction which is quadratic in the above ratio.

In systems with highly eccentric orbits, the only relevant reduction of the tidal efficiency is the one at periastron. This is so because the tidal interaction is effectively confined to the periastron passage, due to the strong dependence of the tidal forces on the separation between the two stars. We take advantage of this simplifying feature and derive tidal timescales for these systems for the three different approaches.

The orbital periods of the highly eccentric binaries are typically long. Nevertheless, we show that the high eccentricity could yield tidal evolution timescales which are shorter than the age of the system. This fact suggests that some of these systems can be used to test the different prescriptions for the reduction of the turbulent viscosity. We show that Gl 586A can serve as one of these test cases.

We also show that for the three theories the tidal evolution of the eccentricity and the semimajor axis in the limit of $e \to 1$ are such, that the periastron distance stays constant in time. This surprising result resolves the apparent difficulty raised by Duquennoy et al. regarding the periastron separation of Gl 586A at its formation epoch.

2. HUT EQUATIONS IN THE LIMIT OF $e \to 1$

The tidal evolution of a binary system originates in the interaction between its two components, which induces a shear (velocity gradient) in the stellar envelopes. The coupling of the shear with the turbulent viscosity in the convective envelopes of late-type stars causes the induced tidal bulges to lag (precede) the line connecting the stars centers. The resulting torques act to synchronize and circularize the orbit (Zahn 1966, 1977; Alexander 1973; Lecar, Wheeler, & McKee 1976; Hut 1981).

To study the tidal evolution of binaries with extremely high eccentricity, we employ the time derivative of the orbital elements obtained by Hut (1981), which are valid for any value of eccentricity. Hut equations are based on the weak friction approximation (Darwin 1879; Alexander 1973), which assumes that the tidal dissipation causes a *constant* time lag between the varying tidal force and the resulting stellar deformation. The magnitude of the time lag is proportional to the viscosity in the stellar envelope. Thus, the assumption of constant time lag translates into an assumption of a constant viscosity with respect to the orbital phase. In cases where the tidal torque and the induced tidal shear vary on a timescale which is comparable to or shorter than the convective timescale, the effective viscosity could be reduced (Zahn 1989). For an eccentric orbit, the tidal shear and the timescale on which it changes are different for different orbital phases. Therefore, the reduced viscosity varies with the orbital phase, so that the tidal time lag is no longer constant, and the analysis of Hut (1981) may need some modifications.

There are, however, two limits in which Hut equations are valid even for a reduced viscosity. The first is the limit of $e \to 0$, where the timescale for variations of the tidal interaction is constant along the orbit. The other limit is that of $e \to 1$, for which the tidal interaction is effective *only* in a small time interval around periastron. This is so because of the steep dependence of the tidal torque on the instantaneous orbital separation r. Therefore, the only relevant values of the reduced viscosity are those in the narrow time interval around periastron. In the limit of $e \to 1$ one can use a representative value for the reduced viscosity in this time interval.

Hut assumed tidal dissipation only in one of the binary members, which we will denote as the primary, and treated the other star, denoted the secondary, as a mass point. He also assumed an alignment of the primary rotation angular momentum and the orbital angular momentum. Taking the limit of $e \to 1$ and replacing $(1 - e^2)$ by $2(1 - e)$, we obtain from Hut (1981) that the tidal time derivative of the semimajor axis a, the eccentricity e and the stellar rotation angular velocity Ω, are given by

$$\dot{a} = -2 \frac{k}{T} q(1 + q)\left(\frac{R}{a}\right)^8 (1 - e)^{-15/2}\left(1 - 0.79 \frac{\Omega}{\Omega_{ps}}\right)a , \quad (1)$$

$$\dot{e} = -2 \frac{k}{T} q(1 + q)\left(\frac{R}{a}\right)^8 (1 - e)^{-13/2}\left(1 - 0.79 \frac{\Omega}{\Omega_{ps}}\right)e , \quad (2)$$

$$\dot{\Omega} = -0.58 \frac{k}{T} q^2 r_g^{-2}\left(\frac{R}{a}\right)^6 (1 - e)^{-9/2}(\Omega - \Omega_{ps}) , \quad (3)$$

where k is the apsidal motion constant of the primary (e.g., Kopal 1978) and T is a characteristic timescale which is inversely proportional to the primary turbulent viscosity. The mass ratio between the secondary and the primary is denoted by q, R is the primary radius and r_g is its gyration radius:

$$r_g^2 = \frac{I}{MR^2} , \quad (4)$$

where I is the primary moment of inertia, and M is its mass.

Hut (1981) defines Ω_{ps}—the pseudosynchronization stellar rotation angular velocity, as the value of Ω for which $\dot{\Omega}$ vanishes. He obtained an expression for Ω_{ps} as function of the eccentricity. For small values of e, it equals the mean orbital angular velocity ω. In the limit of $e \to 1$, of interest here,

$$\Omega_{ps} = 0.825\omega_p = 1.167(1 - e)^{-3/2}\omega , \quad (5)$$

where ω_p is the orbital angular velocity at periastron. The value of Ω_{ps} reflects the fact that the tidal interaction is most effective near periastron, so that the stellar rotation is synchronized not with the mean orbital angular velocity but with a value determined near periastron.

3. TIDAL EVOLUTION TIMESCALES FOR $e \to 1$

For general values of P, e, and Ω, one can define timescales for tidal evolution of the semimajor axis and eccentricity by

$$T_a(P, e, \Omega) = -\frac{a}{\dot{a}} , \quad (6)$$

$$T_e(P, e, \Omega) = -\frac{e}{\dot{e}} . \quad (7)$$

Another interesting timescale is the one needed for reaching pseudosynchronization:

$$T_{ps}(P, e, \Omega) = -\frac{\Omega - \Omega_{ps}}{\dot{\Omega} - \dot{\Omega}_{ps}} . \quad (8)$$

This timescale is relevant only in systems for which pseudosynchronization could indeed be reached, namely, systems where the variation timescale of Ω is much shorter than that of Ω_{ps} itself.

We wish to express the three timescales in terms of $T_c(P)$—the timescale for circularization in nearly circular binaries. Using the definition of $T_c(P)$ as

$$T_c(P) = T_e(P, e = 0, \Omega = \omega) ,$$

Hut equations yield

$$T_c(P) = \frac{2}{21} \frac{T}{k} \frac{1}{q(1 + q)}\left(\frac{a}{R}\right)^8 . \quad (9)$$

Therefore, for a nonreduced viscosity, $T_c(P)$ is proportional to $(P/P_0)^{16/3}$ (Zahn 1977; Mathieu & Mazeh 1988) and can be expressed by

$$T_c(P) = T_0(P/P_0)^{16/3} , \quad (10)$$

where P_0 and T_0 are arbitrary fiducial values.

In case that the tidal shear varies on a timescale τ_s which is comparable to or shorter than the convective timescale, the turbulent viscosity ν_t is reduced to a fraction η of the nonreduced viscosity $\nu_{t,0}$:

$$\nu_t = \eta\nu_{t,0} . \quad (11)$$

If we set the fiducial value of P_0 as equal to the convective timescale, then

$$\eta = 1 \quad \text{for } \tau_s \geq P_0 , \quad (12)$$

$$\eta = \left(\frac{\tau_s}{P_0}\right)^n \quad \text{for } \tau_s < P_0 \quad (13)$$

where $n = 0$, 1, 2 for the three theories of viscosity reduction. The timescales for tidal evolution are inversely proportional to the viscosity and thus inversely proportional to η.

Substituting equations (9) and (10) in equations (1)–(3) and (6)–(8) yields

$$T_a = \eta^{-1}T_0\left(\frac{P}{P_0}\right)^{16/3}\frac{5.25}{1-0.79(\Omega/\Omega_{\rm ps})}(1-e)^{15/2}\,, \quad (14)$$

$$T_e = \eta^{-1}T_0\left(\frac{P}{P_0}\right)^{16/3}\frac{5.25}{1-0.79(\Omega/\Omega_{\rm ps})}(1-e)^{13/2}\,, \quad (15)$$

$$T_{\rm ps} = \eta^{-1}T_0\left(\frac{P}{P_0}\right)^{16/3}1.1\,\frac{1+q}{q}\left(\frac{r_g^2}{0.06}\right)\left(\frac{R}{a}\right)^2(1-e)^{9/2}\,, \quad (16)$$

where r_g^2 is normalized to its solar value (Allen 1978).

The reduction in the turbulent viscosity is determined by the timescale for the variation of the tidal shear near periastron, τ_s. There are two natural timescales at periastron: ω_p^{-1}, and Ω^{-1}. In Appendix A we study the time dependence of the tidal shear near periastron, for given Ω in the limit of $e \to 1$, and adopt as a representative expression

$$\tau_s = \frac{2}{\omega_p}\left[1 + 0.3\left(\frac{\Omega}{\Omega_{\rm ps}}\right)^2\right]^{-1/2}\,, \quad (17)$$

so that in the limit of $e \to 1$

$$\frac{\tau_s}{P_0} = 0.225(1-e)^{3/2}\,\frac{P}{P_0}\left[1 + 0.3\left(\frac{\Omega}{\Omega_{\rm ps}}\right)^2\right]^{-1/2}\,. \quad (18)$$

Equations (14)–(16) indicate that the three tidal timescales are not constant. In fact, they could vary on timescales shorter than themselves. Therefore, the time dependence of e, a, and Ω is not necessarily exponential, and could be more complex. Such time dependence is indeed exhibited by the solutions of the tidal equations, presented in the next section.

We are interested in binaries consisting of late type stars with masses around 1 $M_\odot$. For these stars we take $P_0 = 20$ days, a value that appears in the theories for reduced viscosity as the boundary between the regimes of reduced and non-reduced viscosity (Goldman & Mazeh 1992). Using the value of 18.7 days for the transition period of the Galactic halo stars (Latham et al. 1992), and $T_c(P) = 1.5 \times 10^{10}$ yr (the Galactic halo age) we obtain $T_0 = 2 \times 10^{10}$ yr.

When tidal dissipation takes place in the envelopes of both stars, the rates T_a^{-1} and T_e^{-1} are the sum of the rates due to the dissipation in each of the stars. However, the rate of the evolution of the stellar rotation depends on dissipation only in one star. Therefore, for two equal stars, the ratio of $T_{\rm ps}$ to T_a (or to T_e) is larger by a factor of 2 compared to the case where the dissipation takes place only in the primary. This factor has to be taken into account whenever equations (14)–(16) are applied to a specific system.

4. EVOLUTION EQUATIONS FOR $a(t)$, $e(t)$, AND $\Omega(t)$ IN THE LIMIT $e \to 1$

From equations (14) and (15) follows that in highly eccentric binaries, the timescales for tidal evolution of the eccentricity and the semimajor axis are related by the simple expression

$$T_a = (1-e)T_e\,, \quad (19)$$

independently of the values of Ω, ω, T_0, and η. It implies that $T_a \ll T_e$, contrary to synchronized nearly circular binaries, where the opposite is true. Inserting the definitions of T_a and T_e (eqs. [6] and [7]) into equation (19) results in

$$\frac{\dot a}{a} = \frac{\dot e}{1-e}\,. \quad (20)$$

Equation (20), which was obtained also by Lecar et al. (1976) from an analysis similar to but less general than that of Hut (1981), is very interesting. It implies that, as long as $e \to 1$, the periastron separation r_p does not change in time:

$$r_p = a(1-e) = \text{constant}\,, \quad (21)$$

in spite of the evolution of a and e. In Appendix B we comment on the physical interpretation of this result. As an illustration, we note that for the observed eccentricity of Gl 586A, equation (20) is satisfied to a precision of one part in 10^3.

The constancy of the periastron distance during tidal evolution, as long as the eccentricity stays close to unity, together with angular momentum conservation, implies that ω_p is constant during the evolution. Thus, from equation (5) it follows that $\Omega_{\rm ps}$ is constant too. If during this phase of evolution Ω is much smaller than $\Omega_{\rm ps}$ or does not change in time, then τ_s and consequently the viscosity reduction factor η are also constant. The constancy of these quantities enables us to obtain simple equations for the time evolution of $a(t)$, $e(t)$, and $\Omega(t)$ in the limit of $e \to 1$.

Inserting equations (6) and (8) in equations (14) and (16), respectively, yields

$$-\frac{\dot a}{a} = \frac{1}{T_a} = \eta A(1-e)^{-15/2}a^{-8}\left(1 - 0.79\,\frac{\Omega}{\Omega_{\rm ps}}\right)\,, \quad (22)$$

$$-\frac{\dot\Omega}{(\Omega-\Omega_{\rm ps})} = \frac{1}{T_{\rm ps}} = \eta B(1-e)^{-13/2}a^{-8}\,, \quad (23)$$

where A and B are constant in time. Using equation (21) again, the last two equations yield

$$-\frac{\dot a}{a} = \eta C a^{-1/2}\left(1 - 0.79\,\frac{\Omega}{\Omega_{\rm ps}}\right)\,, \quad (24)$$

$$-\frac{\dot\Omega}{(\Omega-\Omega_{\rm ps})} = \eta D a^{-3/2}\,, \quad (25)$$

where C and D are also constant in time.

To solve these equations we introduce the dimensionless variables

$$x = \frac{a}{a_0}\,, \quad (26)$$

$$y = \frac{\Omega}{\Omega_{\rm ps}}\,, \quad (27)$$

where a_0 is the value of a at $t = 0$. Similarly, we denote by y_0 the value of y at $t = 0$. The constants C and D can then be expressed in terms of $T_{a,0}$ and $T_{\rm ps,0}$, the two timescales at $t = 0$, and in terms of a_0, and y_0. Doing so, and using equations (13) and (18) to express η, yields

$$\dot x = -\frac{1}{T_{a,0}(1-0.79y_0)}\left(\frac{1+0.3y_0^2}{1+0.3y^2}\right)^{n/2}x^{1/2}(1-0.79y)\,, \quad (28)$$

$$\dot y = -\frac{1}{T_{\rm ps,0}}\left(\frac{1+0.3y_0^2}{1+0.3y^2}\right)^{n/2}x^{-3/2}(y-1)\,. \quad (29)$$

Equations (28) and (29) yield the relation:

$$1 - \frac{1}{x} = \frac{T_{\rm ps,0}}{T_{a,0}(1-0.79y_0)}\left[0.79(y_0-y) + 0.21\ln\left(\frac{1-y}{1-y_0}\right)\right]. \quad (30)$$

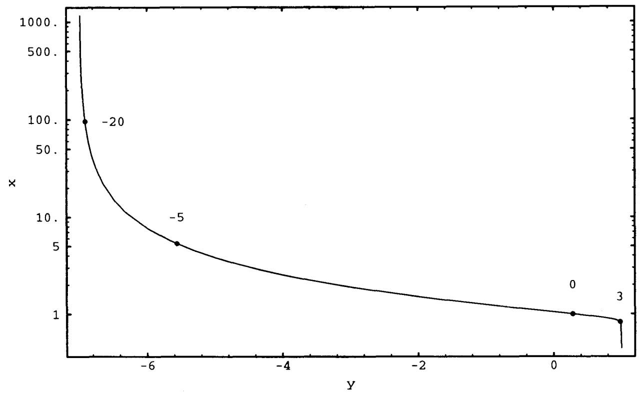

FIG. 1.—The semimajor axis as function of the stellar rotation rate. The dots indicate time in units of $T_{ps,0}$, for nonreduced viscosity.

This relation is *independent* of the value of η or the absolute values of $T_{ps,0}$ and $T_{a,0}$. It depends on two parameters: the ratio between $T_{ps,0}$ and $T_{a,0}$ and on y_0—the value of y at $x = 1$. For given values of these two parameters it is possible to solve numerically equations (28) and (29) for each of the theories and obtain x and y as functions of time, measured in units of $T_{ps,0}$. The translation between this dimensionless time and the real time is through the absolute value of $T_{ps,0}$, which is different for the different theories for viscosity reduction.

Figure 1 displays the variable x, which measures the semimajor axis in units of its value at $t = 0$, as a function of y—the stellar rotation in units of Ω_{ps}. The values of the parameters $T_{ps,0}/T_{a,0}$ and y_0 were chosen in Figure 1 as 0.125 and 0.28, respectively, to present the case of Gl 586A, as discussed in the next section. The dots indicate some representative values of time in units of $T_{ps,0}$, for nonreduced viscosity theory, for which $n = 0$. For $n = 1$ and $n = 2$ the same dots will correspond to earlier times.

The graph includes three main parts. The left part exhibits a steep decrease, representing an early phase of the system, where $T_a \ll T_{ps}$ and $x \gg 1$. At that phase of the evolution the semimajor axis decreases almost without any change in the stellar rotation. However, as a decreases, equation (21) implies that $(1 - e)$ increases, so that T_{ps} eventually becomes shorter than T_a. The system enters, therefore, another phase of the evolution, where Ω varies faster than a. This phase is presented in the graph by the middle part with the moderate slope. At the last phase of the evolution Ω is very close to Ω_{ps}, which implies that y is very close to unity, and therefore only a keeps decreasing. This stage of the evolution is presented by the right steep part of the graph.

5. PROPOSED TEST

We expect a binary system with an age substantially longer than T_{ps} to be pseudosynchronized at the present epoch. The evolutionary timescale depends, as we have shown, on the specific viscosity reduction approach. This suggests an observational test for these theories. A theory can be rejected if we find several systems with rotational rate substantially different from their Ω_{ps}, and for which the theory predicts evolutionary timescales shorter than the systems ages.

Moreover, even pseudosynchronized systems with highly eccentric orbits or systems with unknown rotational period, can help us to reject a viscosity reduction approach. This can be done if we find systems with T_a substantially shorter than their lifetimes. A theory can be rejected if it implies astrophysically unrealistic *initial* parameters, like a very large semimajor axis, and/or an eccentricity which is different than unity only by a very small fraction.

Obviously, highly eccentric systems can help us to distinguish between the different approaches to viscosity reduction, only if the timescale for variation of the induced shear is shorter than the convective timescale. The different theories will predict different timescales only for such systems. It can be shown that if the timescales T_a or T_{ps} for nonreduced viscosity are significantly shorter than $\sim 10^9$ yr, the relation between the period and the eccentricity is such that it guarantees that $\tau_s < P_0$. Thus, the theories that allow for a reduction of the turbulent viscosity will predict substantially longer timescales. A comparison of the different timescales and the age of the system could therefore help in assessing which of the three approaches is in better agreement with the observations.

6. APPLICATION TO Gl 586A

We apply now the evolution equations and the proposed test to Gl 586A. Combining the measured orbital elements with the observed stellar spectral types allowed Duquennoy et al. (1992) to estimate the orbital inclination $i \sim 60°$. From this value we estimate masses of 0.75 $M_\odot$ and 0.5 $M_\odot$, and corresponding stellar radii (Allen 1973) of 0.85 $R_\odot$ and 0.65 $R_\odot$, for the two components.

The circularization timescale, $T_c(P)$, depends on the masses of the stars and their metallicities (Mathieu & Mazeh 1988). The value we have used for T_0 was calibrated for a binary consisting of two 1 $M_\odot$ stars. From Mathieu & Mazeh (1988) follows that for the two components of Gl 586A, $T_c(P)$ is shorter by a factor of ~ 1.5 than that of a binary of two 1 $M_\odot$ stars. In view of the other uncertainties in the theory, we disregard this factor; keeping it would only strengthen our conclusions. As explained in § 3, we take T_{ps} for Gl 586A to be larger by a factor of 2 than the value computed for the case when tidal dissipation takes place in one star only. We adopt a value that is the mean of the values corresponding to the two stars.

An observational upper limit of 3 km s^{-1} on the projected rotational velocities of the stars was obtained by Duquennoy et al. (1992). For $i \sim 60°$ and the stellar radii estimated above this implies for the two stars

$$\frac{\Omega_1}{\Omega_{ps}} \lesssim 0.25 \; ; \quad \frac{\Omega_2}{\Omega_{ps}} \lesssim 0.32 \, . \tag{31}$$

In what follows a mean value of

$$\frac{\Omega}{\Omega_{ps}} \lesssim 0.28 \tag{32}$$

is adopted.

Substituting the values of e and P of Gl 586A in equations (13) and (18) yields

$$\eta = (0.04)^n \, . \tag{33}$$

The timescales for tidal evolution, at the present time, are therefore

$$T_{e,0} \lesssim (25)^n \times 3 \times 10^9 \text{ yr} \, , \tag{34}$$

$$T_{a,0} \lesssim (25)^n \times 8 \times 10^7 \text{ yr} \, , \tag{35}$$

$$T_{ps,0} \sim (25)^n \times 10^7 \text{ yr} \, , \tag{36}$$

where $r_p = 10.5 \, R_\odot$ was used.

Let us assume for Gl 586A an age which is typical to its environment, say 5×10^9 yr. Support for this assumption comes from the lack of photometric or spectroscopic indications for stellar activity which characterize young stars (Duquennoy et al. 1992).

We solved numerically equations (28) and (29) for the parameters of Gl 586A: $T_{ps,0}/T_{a,0} = 0.125$ and $\Omega(t = 0)/\Omega_{ps} = 0.28$, which is the observational upper limit. We obtained the semimajor axis and the stellar rotation rate as functions of time measured in units of $T_{ps,0}$. The solution is presented in Figure 1, where time ticks in units of $T_{ps,0}$ are shown for the case of nonreduced viscosity, $n = 0$, and the present time is denoted by $t = 0$.

We first address the theory of nonreduced viscosity, corresponding to $n = 0$. For this theory, $T_{ps,0}$ is $\sim 2 \times 10^{-3}$ of the system age. The numerical solution of equations (28) and (29) for the present parameters of Gl 586A implies that had we waited a time equal to $2T_{ps,0}$, Ω would be $\sim 0.93\Omega_{ps}$, namely, almost pseudosynchronized. Therefore, if the system age was just larger by a fraction of $\sim 4 \times 10^{-3}$ of its present age, it would have been pseudosynchronized. Thus, the chance to observe the system in its present state is quite small, $\sim 4 \times 10^{-3}$. Assuming that the stars are pseudosynchronized but not aligned (so the observed velocities are smaller than $V \sin i$) does not alleviate the difficulty, since the timescale for alignment roughly equals that for pseudosynchronization.

Another serious difficulty follows from the required initial parameters of the system. The adopted age corresponds to a formation epoch at $t = -500T_{ps,0}$. For this value, the numerical solution of equations (28) and (29) implies an initial eccentricity which differed from 1 by merely $\sim 4 \times 10^{-7}$. Such an extreme initial eccentricity is indeed quite unlikely according to observed eccentricity distributions (e.g., Boffin, Cerf, & Paulus 1993). Moreover, the initial semimajor axis was $\sim 7 \times 10^4$ times larger than the present one: ~ 0.7 pc. Most of the time the orbital separation was twice this value, and thus the system had no chance to survive against the tidal disruption in the Galaxy (e.g., Close, Richer, & Crabtree 1990; Wasserman & Weinberg 1991). The initial rotation velocity had to be ~ 86 km s^{-1} in a direction opposite to the orbital rotation. These extreme initial conditions, and the very short $T_{ps,0}$ compared to the system age, point quite strongly to the conclusion that the turbulent viscosity has to be reduced.

For a linear reduction ($n = 1$), $T_{ps,0}$ is a factor of ~ 20 shorter than the age of the system. Therefore the chance to observe it in a nonpseudosynchronized rotation is ~ 0.1. The initial semimajor axis had to be a factor of ~ 14 larger than the present one, corresponding to a value of 27 AU and an initial eccentricity differing from 1 by $\sim 1.8 \times 10^{-3}$. The initial stellar rotation rate was in this case ~ 80 km s^{-1} in a direction opposite to the orbital rotation. These parameters are quite unlikely, but not as extreme as in the case of nonreduced viscosity. Therefore, this system cannot conclusively prove the theory nonviable, and more similar binaries are required to establish this conclusion.

For quadratic reduction ($n = 2$) $T_{ps,0}$ is a factor of ~ 1.25 longer than the system age. Therefore, there is no problem in Gl 586A not being pseudosynchronized. The initial conditions of the system were very similar to the present ones. The eccentricity was ~ 0.978, the semimajor axis was larger than the present one by $\sim 15\%$, and the rotation velocity was ~ 6 km s^{-1} in an opposite direction to the orbital rotation. These parameters are very reasonable for the formation epoch of the system.

7. DISCUSSION

This work studied the tidal evolution of highly eccentric binaries. We have shown that in some of these binaries the tidal interaction varies on a timescale which is comparable to or shorter than the stellar convective timescale. In such cases, the turbulent viscosity could be reduced, and we considered here three different approaches to this reduction. The analysis was done by utilizing the fact that the tidal interaction is effectively confined to a short time interval around periastron passage. We have shown that for all three theories tidal evolution for highly eccentric binaries is quite different from that encountered in low eccentricity systems. In particular,the semimajor axis decreases on a timescale much shorter than the eccentricity, and the periastron distance stays constant in time.

One should be aware that all three theories considered here assume an evolution due to the interaction between the induced tidal shear and the turbulent viscosity in the convective envelopes of late type stars. A completely different theory is that of Tassoul (1987, 1988), which attributes the tidal evolution to large-scale transient meridional currents induced by the tidal distortion of the stellar axial symmetry. Tassoul's theory was not considered in this paper, since the published timescales referred only to circular binaries and not to highly eccentric systems considered here.

We pointed out that the highly eccentric binaries can serve as laboratories for testing the theories for viscosity reduction. An unpseudosynchronized system would strongly constrain any theory which predicts for this system an evolutionary timescale significantly shorter than the system age. Moreover, we found that if the timescale for the evolution of the semimajor axis is much shorter than the system age, that system was created with extreme initial parameters. For example, for nonreduced viscosity Gl 586A had to be formed with a semimajor axis of the order of 1 pc. This value would have made the system susceptible to tidal disruption in the Galaxy.

The theories for viscosity reduction have been tested recently against samples of short-period binaries, by studying the dependence of the circularization timescale on the orbital period in the limit of $e \rightarrow 0$. This is done by deriving a transition orbital period which separates circular from eccentric binaries in coeval samples of different ages (Mazeh et al. 1990). However, the data cannot yet point to the correct theory because of the fuzziness of the transition between the circular and eccentric binaries (Mathieu et al. 1992).

The advantage of the new proposed test is that it requires only a small number of systems with age estimation. The example of Gl 586A shows that the theory with nonreduced viscosity can be proven nonviable on the basis of even one system. For the linear reduction approach, the predicted initial conditions of Gl 586A are not as extreme as for nonreduced viscosity. Therefore, a number of similar systems are required to distinguish between the linear and the quadratic reductions.

The proposed test relies partly on the assumption that pseudosynchronization will be achieved within a few corresponding timescales. This requires that the other effects on the stellar rotation rate be negligible compared to the tidal interaction. This assumption breaks down in magnetic active stars with strong stellar winds, which could result in an efficient magnetic braking. Mass transfer between the two components of the binary system can also delay the pseudosynchronization. Unless one of these effects is strong enough, we expect old enough binaries to be pseudosynchronized.

Hall & Henry (1990) studied the pseudosynchronization of a sample of 22 binaries with moderate eccentricity and known rotational periods. They found only three unpseudo-synchronized systems, all of which contain chromospherically active stars, suggesting strong magnetic braking at the present epoch. Moreover, all three binaries contain a giant star that almost fills its Roche lobe, so that mass transfer had been also possible. In contrast, all binaries in the Hall & Henry sample of confirmed pseudosynchronization involve small dwarfs well within their Roche lobes. The two stars in Gl 586A are far from filling their instantaneous Roche lobes, even at periastron, and the observed spectral types of the stars imply quite weak stellar winds.

So far, the only known high-eccentricity binary with relatively short period is Gl 586A. Hopefully, the ongoing large radial-velocity surveys for spectroscopic binaries (e.g., Griffin 1992; Latham 1992; Mayor et al. 1992) will yield in the near future more interesting eccentric systems, from which we will be able to learn some more astrophysics about tidal circularization and synchronization.

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APPENDIX A

TIDAL SHEAR TIMESCALES

The reduction in the turbulent viscosity depends on τ_s—the timescale for variation of the tidal shear near periastron. General expressions for the tidal shear, at different locations in the stellar envelope and for any orbital phase, are given in Alexander (1973). In order to arrive at an estimate for the above timescale, we consider here for simplicity only the shear in the equatorial plane of the star (which is assumed to coincide with the orbital plane) and in the envelope regions that are on the line connecting the centers of the two stars. In this case the shear, at a true anomaly θ and separation r, is given by

$$S(t) \propto \frac{d}{dt}\left[r^{-3}\sin(2\theta - 2\Omega t)\right], \tag{A1}$$

where $\theta = t = 0$ denote the periastron.

We note in passing that equation (A1) explicitly shows that the average of $S(t)$ over a long enough time vanishes by definition. Over one period the average is only a fraction of $\sim 0.1 \times (1-e)^{3/2}$ of $S(t=0)$. A zero average is a necessary condition for the shear reduction.

Expressing r in terms of θ

$$r = a\frac{1-e^2}{1+e\cos\theta}, \tag{A2}$$

and evaluating the differentiation in equation (A1) we obtain $S(t)$ in terms of the shear at periastron for the case of a nonrotating star $S(t=0, \Omega=0)$:

$$\frac{S(t)}{S(t=0,\Omega=0)} = -\frac{3e(1+e\cos\theta)^2}{2(1+e)^3\omega_p}\sin\theta\sin(2\theta-2\Omega t)\dot{\theta} + \frac{(1+e\cos\theta)^3}{(1+e)^3\omega_p}\cos(2\theta-2\Omega t)(\dot{\theta}-\Omega). \tag{A3}$$

Since $\dot{S}(t=0) = 0$, a possible estimate for τ_s is

$$\tau_s = 2\pi\left(\frac{S}{\ddot{S}}\right)^{1/2}_{t=0}, \tag{A4}$$

which is the period of a cosine function matched to $S(t)$ at periastron. We used this estimate, as well as plots of $S(t)$ for different values of Ω and found that for absolute value of Ω much smaller than ω_p

$$\tau_s \sim \frac{2}{\omega_p}, \tag{A5}$$

while for absolute value of Ω much larger than ω_p

$$\tau_s \sim \frac{3}{\Omega}. \tag{A6}$$

The overall behavior can be represented by an approximate interpolating formula

$$\tau_s = \frac{2}{\omega_p}\left[1+\left(\frac{\Omega}{1.5\omega_p}\right)^2\right]^{-1/2} = \frac{2}{\omega_p}\left[1+0.3\left(\frac{\Omega}{\Omega_{ps}}\right)^2\right]^{-1/2}. \tag{A7}$$

This result is not surprising, as there are two natural timescales for the shear variation. One is ω_p^{-1} and the other Ω^{-1}. Our result is a combination of these two timescales.

APPENDIX B

CONSTANCY OF PERIASTRON DISTANCE DURING TIDAL EVOLUTION FOR $e \to 1$

The constancy of the periastron distance in the limit $e \to 1$ can be derived from relatively simple physical considerations. The tidal force comprises of nondissipative and dissipative parts. The nondissipative force is independent of the viscosity, and it cannot change the orbital energy and angular momentum. Its only effect is to induce a rotation of the periastron.

The dissipative force, which is due to the lag caused by the viscosity, is the one that can change the orbital energy and angular momentum. Clearly, only the dissipative part is relevant for the tidal evolution discussed in this work. The result of the two parts of the tidal force is a rotation of the periastron combined with evolution of the period and eccentricity.

The dissipative force includes a tangential and a radial component, the latter being proportional to $\dot{r}$ (Hut 1981). For highly eccentric binaries the tidal interaction is effective essentially only at periastron. There, the radial component of the tidal dissipative force vanishes and the tidal dissipative force is thus purely tangential.

At periastron, this tangential force imparts to each star a differential orbital velocity which is perpendicular to the radius vector, without changing the latter. Therefore, in the next revolution, it will be also the periastron. The actual location in space of the periastron after one period has been slightly rotated, as a result of the nondissipative force, but the periastron distance is the same, as stated in equation (21). This explains why equation (21) was obtained also by Lecar et al. (1976), who assumed a purely tangential tidal dissipative force.

More formally, for a dissipation confined to a time interval δt and a phase interval $\delta\theta$ centered at periastron, the changes in the orbital energy and in the orbital angular momentum are given by

$$\delta E_{\rm orb} = N\,\delta\theta = N\omega_p\,\delta t\,, \qquad \text{(B1)}$$

$$\delta h = N\,\delta t\,, \qquad \text{(B2)}$$

implying

$$\delta E_{\rm orb} = \omega_p\,\delta h\,, \qquad \text{(B3)}$$

where N denotes the tidal torque at periastron,

$$E_{\rm orb} = -G\,\frac{M_1 M_2}{2a} \qquad \text{(B4)}$$

is the orbital energy, and

$$h = G^{1/2}\,\frac{M_1 M_2}{(M_1 + M_2)^{1/2}}\,a^{1/2}(1 - e^2)^{1/2} \qquad \text{(B5)}$$

is the orbital angular momentum.

Using equations (B4) and (B5) to express $\delta E_{\rm orb}$ and δh in terms of δa and δe and substituting in equation (B3) yields equation (21).

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