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ON THE HYDROGEN LAYER THICKNESS IN PULSATING DA WHITE DWARFS

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ABSTRACT

We reexamine the question of the relationship between the onset of pulsational driving and the thickness of the outer hydrogen layer in ZZ Ceti stars. To this end, we solve the system of ordinary differential equations which describes the linear, nonadiabatic, nonradial pulsations of stellar models with the help of a new code based on the powerful Galerkin finite-element method of weighted residuals. We find that the theoretical blue edge temperature of the ZZ Ceti instability strip is insensitive to the hydrogen layer mass for $\log(M_H/M_*) \geq -12$. This is in line with mounting evidence that ZZ Ceti pulsators with relatively thick hydrogen layers do actually exist, but it is contrary to some previous calculations which have been widely accepted. We discuss the implications of our findings vis-à-vis current white dwarf spectral evolution models.

Subject headings: stars: interiors — stars: oscillations — white dwarfs

1. INTRODUCTION

In recent years, there has been an interesting and, sometimes, heated debate in the white dwarf community as to whether or not nonadiabatic pulsation calculations could be used to constrain the amount of hydrogen that floats on top of pulsating DA (ZZ Ceti) white dwarfs. On the one hand, Winget (1981), Winget et al. (1982), and Winget & Fontaine (1982) have found a correlation between the effective temperature at the blue edge of the theoretical ZZ Ceti instability strip and the mass of the outer hydrogen layer in a DA white dwarf. By matching their theoretical results with the blue edge temperature actually observed, they found that ZZ Ceti stars should have hydrogen layer masses in the range $-12 \lesssim \log q(\mathrm{H}) \equiv \log(M_H/M_*) \lesssim -8$ (actually extended to -7 in unpublished calculations of Winget & Fontaine 1982). Models with thicker hydrogen layers were also found to be unstable against g-mode pulsations, but only at effective temperatures too low to be consistent with the observations; such models were therefore considered not to represent real ZZ Ceti stars. These earlier results were confirmed as recently as 1989 in the reanalysis presented by Bradley, Winget, & Wood (1989), except for the upper limit of $\log q(\mathrm{H}) \simeq -7$, which returned to its former value $\log q(\mathrm{H}) \simeq -8$. Coupled with the suggestion that most, if not all, DA white dwarfs evolve to become ZZ Ceti pulsators (Fontaine et al. 1982; Greenstein 1982), the implication of the Winget et al. (1982) results is that DA white dwarfs, as a class, should have retained only "thin" hydrogen layers in the range $-12 \lesssim \log q(\mathrm{H}) \lesssim -8$ by the time they entered the ZZ Ceti instability strip.

On the other hand, Cox et al. (1987) have presented another set of nonadiabatic calculations from which they concluded that the onset of g-mode instability in DA white dwarfs is independent of the hydrogen layer mass. In that picture, nonadiabatic results provide no constraint on the thickness of the hydrogen layer. In particular, DA white dwarfs with "thick" layers corresponding to the largest amount of hydrogen allowed by standard evolution theory [$\log q(\mathrm{H}) \simeq -4$; see Iben & Tutukov 1984; Koestner & Schönberner 1986] should become ZZ Ceti pulsators. However, as we have pointed out before (Brassard et al. 1991; Fontaine et al. 1992), the results of Cox et al. (1987) are suspect.

Our own recent efforts in this field have led us to reexamine the question of the relationship between the onset of pulsational driving and the thickness of the outer hydrogen layer in ZZ Ceti stars. Using arguments based solely on *adiabatic* calculations (which have the distinct advantage over nonadiabatic computations of being much more reliable), we have uncovered evidence that at least some ZZ Ceti pulsators have relatively thick hydrogen layers. The most compelling case is that of G117-B15A for which we have recently inferred a hydrogen layer mass $\log q(\mathrm{H}) \simeq -5.9$ (Brassard et al. 1994). This result is based on a detailed seismological analysis of the observed properties of that star, including complete mode identification. Also, by comparing observed periods with theoretical values culled from our recent adiabatic survey (Brassard et al. 1992a, b, c), it has been possible to derive interesting constraints for two more objects. According to Fontaine et al. (1992), G226-29 has $\log q(\mathrm{H}) \simeq -4.4$ or -6.6 if its observed 109.3 s pulsation mode has a value $l = 1$ or 2, respectively. Likewise, Fontaine et al. (1993; see also Bergeron et al. 1993a) find that GD165 must have $\log q(\mathrm{H}) \simeq -3.7$ or -6.4 if its 120.4 s pulsation mode has a value $l = 1$ or 2, respectively. Note that the quoted values for $\log q(\mathrm{H})$ in these two stars are actually *lower* limits since they are based on the implicit assumption that the observed modes have a value of the radial order $k = 1$ (which corresponds to the shortest period for a sequence of g-modes belonging to a given value of the spherical harmonic index l).

The obvious conflict between these results based on adiabatic calculations and the previous nonadiabatic investigations of Winget and collaborators clearly suggests that the problem of the stability of ZZ Ceti star models should be examined anew. Based on our experience with various pulsation codes, we have reached the conclusion that progress in this area could be made only if more potent numerical tools than heretofore available were to become available. Accordingly, we have developed and thoroughly tested a nonadiabatic version of the powerful Galerkin finite-element nonradial pulsation code presented by Brassard et al. (1992c) in an adiabatic context. The main purpose of this *Letter* is to present the salient features of a stability study carried out with this new tool, with particular emphasis on the effects of the hydrogen layer mass on the theoretical blue edge of the ZZ Ceti instability strip.

2. COMPUTATIONS

Our new finite-element code efficiently solves the full sixth-order complex eigenvalue problem for nonradial, nonadiabatic stellar oscillations. We postpone a detailed description of the code to a future publication but point out that it shares many features with the adiabatic version described at length in Brassard et al. (1992c), to which the reader is referred. In particular, because the finite-element solver of Brassard et al. (1992c) was written in a quite general context, it was straightforward to apply it to the more complicated case of the system of six complex differential equations describing the nonadiabatic problem (as compared to the simpler system of four real differential equations in the adiabatic case). Unlike the homogeneous model in the adiabatic problem, there exists no standard benchmark for nonadiabatic, nonradial stellar pulsations, but extensive convergence tests were carried out by varying the number of shells in the equilibrium models, the number of finite elements, and the base (i.e., linear, quadratic, cubic) of the finite elements. As usual, we neglect the interaction between convection and pulsation and assume that the convective flux remains unperturbed. We fully recognize that this is a poor approximation (especially near the red edge where convection is fully developed in the envelopes of the models), but, to our knowledge, there is no reliable theory of convection/pulsation interaction currently available.

In the present computations, and to compare with the earlier results, we have considered the *same* equilibrium stellar models as those used by Winget and collaborators. These evolutionary models have been fully described by Tassoul, Fontaine, & Winget (1990). Typically, a model contains some 600 shells obtained by interpolation from the original mesh of 160–190 shells. The interpolation strategy was necessary to provide the resolution needed to define properly the eigenfunctions, particularly for the higher order modes. It is important to realize that, in our approach, *all* these ~ 600 shells have been used in the definition of the model in terms of *finite elements* (not to be confused with *shells*; see, e.g., Brassard et al. 1992c). By contrast, the earlier pulsation code relied on finite differences defined in terms of shells and met with convergence problems when the total number of shells considered exceeded ~ 400 (Winget 1981). Tests indicate that the stability of a mode (the sign of its growth rate) does not change when the evolutionary models of Tassoul et al. (1990) are defined in terms of 50 to 250 quadratic elements, although, of course, the accuracy increases with the number of elements. By varying the number of finite elements in this range, we find that the growth rate of a mode is derived to within a factor 2, while the period changes by $\sim 2\%$ at most. Nonadiabatic computations with 250 elements begin to be quite slow, and we have found that an acceptable compromise between speed and accuracy is to take 100 quadratic elements.

There exists some ambiguity in the definition of the theoretical blue edge because mode stability is function of the index l (Winget 1981; Dolez & Vauclair 1981; Dziembowski & Koester 1981). This dependence is weak, however, and, for the needs of the present *Letter*, we define the blue edge in terms of the stability of the $l = 1$ modes. This is possibly the best choice also as far as the *observed* blue edge is concerned. We have analyzed the stability of all g-modes with $l = 1$ in the 100–1000 s period window for models taken from the 0.6 $M_\odot$ H/He/C evolutionary sequences of Tassoul et al. (1990) which provide the most extensive coverage for the hydrogen layer mass, $-14 \leq \log q(\mathrm{H}) \leq -4$. Our results are summarized in Table 1, which assumes a format similar to that found in Winget et al. (1983). Column (1) refers to the sequence of evolutionary models in the notation of Tassoul et al. (1990). In this notation, the first digit gives the mass of the model in tenths of solar mass units. The next two digits refer to the mass of the outer helium layer [$-\log q(\mathrm{He})$]. Similarly, the fourth and fifth digits correspond to the mass of the hydrogen layer [$-\log q(\mathrm{H})$].

TABLE 1

PULSATION PROPERTIES OF ML1 EVOLUTIONARY MODELS FOR MODES WITH $L = 1$

			Short		Long		Maximum	
Sequence (1)	Model (2)	$T_{\rm eff}$ (3)	Π_k (s) (4)	τ_e (yr) (5)	Π_k (s) (6)	τ_e (yr) (7)	Π_k (s) (8)	τ_e (yr) (9)
60214L1	164	15138	...	...	...	...	...	...
	168	14276	207.9_1	3.7×10^5	730.1_{11}	2.4	683.6_{10}	2.2
60213L1	160	13432	...	...	...	...	...	...
	164	12725	222.4_1	1.1×10^6	663.3_9	8.2×10^1	611.9_8	2.5×10^1
60212L1	200	9980	...	...	...	...	...	...
	204	9512	264.5_1	1.2×10^6	785.9_9	3.8×10^1	713.5_8	6.7
60210L1	196	9999	...	...	...	...	...	...
	200	9527	264.2_1	9.7×10^5	993.9_{13}	2.5×10^{-1}	602.7_7	2.5×10^{-1}
60209L1	200	10395	...	...	...	...	...	...
	204	9851	259.4_1	4.6×10^5	867.5_{12}	2.5×10^{-1}	838.4_{11}	1.5×10^{-1}
60208L1	196	10099	...	...	...	...	...	...
	200	9580	263.2_1	1.8×10^5	980.7_{14}	1.6×10^{-1}	940.3_{13}	4.8×10^{-2}
60207L1	200	10388	...	...	...	...	...	...
	204	9865	257.7_1	1.8×10^4	831.9_{12}	1.5	738.7_{10}	3.6×10^{-1}
60206C1	208	10367	...	...	...	...	...	...
	212	9843	213.2_1	8.3×10^3	969.1_{15}	2.5×10^{-2}	922.4_{14}	2.3×10^{-2}
60204C1	196	9925	...	...	...	...	...	...
	200	9477	128.0_1	2.0×10^6	959.7_{17}	5.3×10^{-1}	917.4_{16}	1.6×10^{-1}

A letter follows, either C or L, depending on the actual set of radiative opacity data used in the calculations; C stands for the older Cox & Stewart (1970) tables and L for the new Los Alamos data (Huebner 1980). Finally, the last digit refers to the version of the mixing-length theory used: 1 for ML1, and 3 for ML3.

Columns (2) and (3) give, respectively, the number and effective temperature of a stellar model. The other entries give the periods and the *e*-folding times (the inverse of the imaginary part of the complex frequencies) for the shortest, the longest, and the most unstable excited modes belonging to the $l = 1$ sequence. If there are no entries in columns (4)–(9), the model is stable. For reasons of storage space, only one model out of four in the evolutionary sequence has been originally retained by Tassoul et al. (1990) for subsequent pulsation analysis. Hence, for each sequence, the first row corresponds to the last stable model that we could study, and the second row corresponds to the first unstable model near the blue edge.We note that the "resolution" in effective temperature used to locate the blue edge for each evolutionary sequence is somewhat coarse, ranging from ~ 200 K to ~ 800 K for the ML1 models.

In agreement with previous results, we find that the blue edge temperature depends strongly on the hydrogen layer mass for models with very thin layers, $\log q(\mathrm{H}) = -13$ and -14. In those instances, most of the driving is located near the base of the thin *helium* convection zone present just below the H/He transition zone. This is the well-known helium driving mechanism for ZZ Ceti stars discussed by Winget (1991), Dziembowski & Koester (1981), Dolez & Vauclair (1981), Winget et al. 1982), and Winget & Fontaine (1982). Since its theoretical blue edge temperature far exceeds the observed value, the model with the thinnest hydrogen layers [$\log q(\mathrm{H}) = -14$] can be dismissed, as was done by Winget et al. (1982).

Contrary to the previous results of Winget and collaborators, however, our stability analysis suggest that the theoretical blue edge temperature does *not* depend on the hydrogen layer mass in the range $-12 \leq \log q(\mathrm{H}) \leq -4$, corresponding to the regime where driving near the base of the *hydrogen* convection zone is responsible for the pulsational instabilities. Taking into account the finite resolution in effective temperature, the data presented in Table 1 are consistent with a *constant* blue edge temperature of 9895 ± 30 K for all the 0.6 $M_\odot$, ML1 models with $\log q(\mathrm{H}) \geq -12$. Moreover, looking at the *e*-folding times, there is no trend that would suggest that driving should diminish with increasing hydrogen layer mass. Table 2 shows some additional results which also suggest a constant blue edge temperature of $\sim 12{,}880 \pm 30$ K (comparable to the observed value according to Wesemael et al. 1991) for 0.6 $M_\odot$, ML3 models.

3. IMPLICATIONS

The new nonadiabatic calculations presented here suggest that the blue edge temperature of the ZZ Ceti instability strip is insensitive to the mass of the outer hydrogen layer. This casts serious doubts on the reliability of the method used in the past to constrain this quantity in the range $-12 \leq \log q(\mathrm{H}) \leq -8$ by comparing the observed blue edge with the theoretical blue edge temperature (Winget 1981; Winget et al. 1982; Winget & Fontaine 1982; Bradley et al. 1989). This last result has been interpreted by many (including ourselves) as compelling evidence in favor of thin hydrogen layers in cool DA white dwarfs in general. We can offer no definitive explanation for the observed dependence of the theoretical blue edge temperature on the hydrogen layer thickness in those previous calculations. However, given that we have used the *same* equilibrium stellar models as those used in the previous surveys, given that we have full confidence in our new nonadiabatic finite-element code for nonradial pulsations, and given that we are getting a different answer, we risk the suggestion that numerical inadequacies may have plagued past nonadiabatic calculations. On this account, our experience with the two types of nonadiabatic codes have convinced us that our new version is much more robust and less vulnerable to numerical noise than previously available tools. In addition, the present nonadiabatic results reconcile stability studies with growing evidence (based, we recall, on simpler and more solid *adiabatic* arguments) that ZZ Ceti pulsators with thick hydrogen layers do actually exist. As pointed out above, the cases of G117-B15A (Brassard et al. 1994), G226-29 (Fontaine et al. 1992), and GD 165 (Fontaine et al. 1993) are particularly noteworthy in this regard.

At the suggestion of the referee, we also point out that, while our results agree qualitatively with those of Cox et al. (1987), they should in no way be taken as support for the validity of their models. The referee also drew our attention to a paper by Bradley & Winget (1994) which suggests that sensitivity to zoning may have been responsible for the correlation discussed previously by Winget and collaborators between the blue edge temperature and the mass of the outer hydrogen layer.

The results of our stability analysis, when coupled to recent developments in related areas, also have profound implications on a model of the spectral evolution of white dwarfs which two of us have proposed in the past (Fontaine & Wesemael 1987)

TABLE 2

Pulsation Properties of ML3 Evolutionary Models for Modes with $L = 1$

			Short		Long		Maximum	
Sequence (1)	Model (2)	$T_{\rm eff}$ (3)	Π_k (s) (4)	τ_e (yr) (5)	Π_k (s) (6)	τ_e (yr) (7)	Π_k (s) (8)	τ_e (yr) (9)
60210L3	176	12957	...	...	...	...	...	...
	180	12848	221.1_1	5.8×10^7	956.1_{15}	2.4×10^{-2}	956.1_{15}	2.4×10^{-2}
60209L3	184	12949	...	...	...	...	...	...
	188	12732	219.6_1	5.1×10^5	865.9_{14}	1.2	822.6_{13}	6.4×10^{-2}
60208L3	170	12910	...	...	...	...	...	...
	174	12669	222.8_1	1.8×10^5	908.4_{15}	2.9×10^{-1}	874.5_{14}	3.8×10^{-2}
60207L3	180	12923	...	...	...	...	...	...
	184	12699	221.4_1	3.8×10^4	979.8_{17}	1.3×10^{-1}	891.2_{15}	3.3×10^{-2}
60206L3	180	13026	...	...	...	...	...	...
	184	12850	200.1_1	6.8×10^3	975.4_{18}	8.7×10^{-3}	928.3_{17}	6.2×10^{-3}

and which suggests that DA white dwarfs, *as a class*, should have relatively thin hydrogen layers. One of the cornerstones of this model was the constraint on the hydrogen layer mass derived from previous nonadiabatic pulsation studies. As we have demonstrated in this *Letter*, our results call into serious question the reality of these constraints. At the same time, other lines of evidence for thin layers in DA stars have been increasingly challenged. For instance, Vennes et al. (1988; see also Koester 1989a, b) have suggested that the bulk of the EUV/soft X-ray observations of hot DA white dwarfs could be accounted for in terms of stratified atmospheres with very thin layers of hydrogen floating on top of these stars. However, the results of Vennes et al. (1989), Vennes, Thejll, & Shipman (1991), and Vennes & Fontaine (1992) have revealed that an alternate model (radiative levitation of a complex of heavy elements with low individual abundances) may be more appropriate in some and, perhaps, most cases. The spectroscopic observations coming out of the *EUVE* satellite appear to confirm this suggestion (Vennes & Dupuis 1993). Moreover, a recent analysis of the DAO white dwarf Feige 55 (Bergeron et al. 1993b), as well as other similar objects (once believed to represent the best cases for atmospheric stratification because of the spectral visibility of helium; see, e.g., Vennes et al. 1988; MacDonald & Vennes 1991), indicates that the stratified atmosphere hypothesis and, consequently, the existence of thin hydrogen layers must be rejected for the majority of DAO stars (although an alternate explanation for the presence of helium remains to be found). Finally, the intriguing DAB stars which have been suggested as possibly being objects in a transitory mixing state between a DA (with a very thin hydrogen layer) and a DB (Liebert, Fontaine, & Wesemael 1987) may turn out to be much more mundane objects. Indeed, while the case for stratification appears to be still viable for the prototype of the class, GD 323 (Koester, Liebert, & Saffer 1994), there exists strong evidence that at least two DAB objects (out of the four known) are simply binary stars (Wesemael et al. 1994). Thus, although there is no question that spectral evolution *does occur* in white dwarfs (the most obvious manifestation of which is the existence of the so-called DB gap), it is clear from these considerations that current models calling on thin hydrogen layers in a majority of DA stars will require substantial revisions.

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