1994ApJ...424L..21N

THE ASTROPHYSICAL JOURNAL, 424:L21–L23, 1994 March 20

FINITE SOURCE SIZES AND THE INFORMATION CONTENT OF MACHO-TYPE LENS SEARCH LIGHT CURVES

ROBERT J. NEMIROFF[1,2] AND W. A. D. T. WICKRAMASINGHE[3]

Received 1993 November 1; accepted 1993 December 22

ABSTRACT

If the dark halo matter is primarily composed of MACHOs toward the lower end of the possible detection range ($<10^{-3}\ M_\odot$) a fraction of the lens detection events should involve the lens crossing directly in front of the disk of the background star. Previously, Nemiroff has shown that each crossing would create an inflection point in the light curve of the MACHO event. Such inflection points would allow a measure of the time it took for the lens to cross the stellar disk. Given an independent estimate of the stellar radius by other methods, one could then obtain a more accurate estimate of the velocity of the lens. This velocity could then, in turn, be used to obtain a more accurate estimate of the mass range for the MACHO or disk star doing the lensing.

Subject headings: dark matter — Galaxy: halo — gravitational lensing — stars: low-mass, brown dwarfs

1. INTRODUCTION

Recently Alcock et al. (1993, the MACHO collaboration, where MACHO stands for Massive Compact Halo Object), Aubourg et al. (1993, the Experience de Recherche d'Objets Sombres or EROS project), and Udalski et al. (1993, the Optical Gravitational Lensing Experiment or OGLE) have all reported seeing light curves of stars indicative of a fainter star passing in front of and gravitationally magnifying the light from a background star: a gravitational lens event. The probability of seeing such an event was predicted originally by Paczyński (1986) and by Griest (1991), while an estimate fully including relative lens and source motion was given by Nemiroff (1991). Assuming that the nature of these events is correctly identified, these events are measuring the mass and density of stars in our disk and dark-matter halo objects in the Galactic halo.

Paramount to the success of these efforts is the ability to turn light curves into useful information about the mass and density of the lenses. In this *Letter* we use the fact that a reasonable fraction of strong magnification lens events involving low-mass MACHOs ($<10^{-3}\ M_\odot$) would involve the lens crossing the finite stellar disk of the background star (Witt & Mao 1994). In general, the smaller the mass of the MACHO, the more of them are needed to explain the rotation curve of our Galaxy, and the more likely one will cross directly in front of a background stellar disk. Also, the smaller the mass of the lens, the smaller the angular size of its Einstein ring relative to the angular size of a background stellar disk, and the more likely that large magnitude lensing events will involve a disk crossing. For these reasons low-mass MACHOs are considered to be the most probable lenses for disk crossing events.

Lensing effects on a finite-sized source were discussed previously by Nemiroff (1987), by Schneider & Wagoner (1987) in the context of analyzing gravitational lensing effects of distant supernovae, and more generally in the book on gravitational lenses by Schneider, Ehlers, & Falco (1992). Gould (1992) discussed the logistics of detecting objects as low as $10^{-9}\ M_\odot$ and gave a description of the shape of a light curve for a lens crossing a stellar disk.

Clearly a gravitational lensing light curve will become more complex when finite source sizes are included in the lensing scenario, as shown in Nemiroff (1987). Although this may be thought of as unfortunate, since it makes understanding the light curves more complicated, the added information available in the light curve will be shown to be useful. In the next section the information content in point and finite source size MACHO lens events is discussed with a goal of using this extra information to better deconvolve the mass and relative velocity of the lens. The last section gives a summary and some discussion.

2. THE INFORMATION CONTENT OF POINT-LENS LIGHT CURVES

If a lens passes in front of a background source which is considered to be a point, one parameter completely describes the shape of the light curve: the angular impact parameter between the lens and the source (B). A second parameter acts like a multiplier in the duration of the light curve: the relative angular speed of the lens across the field containing the source which we will designate V. Yet a third parameter must be used to locate the time of light curve maximum, but we will assume that this zero-point temporal orientation is unambiguous here.

Both B and V are only determined from the light curve in terms of the projected angular Einstein ring size of the lens. More precisely, the measurable angular impact parameter

$$B = \frac{\beta}{E}, \qquad (1)$$

where β is the angular impact parameter between the lens and the source (the closest angular approach of the lens from the source) and E is the angular size of the Einstein ring of the lens, and is given by (Liebes 1964; Refsdal 1964)

$$E = \sqrt{\frac{2R_S(D_{OS} - D_{OL})}{D_{OL}D_{OS}}}, \qquad (2)$$

where D_{OL} is the distance between the observer and the lens, D_{OS} is the distance between the observer and the source, and R_S

---

[1] George Mason University, CSI, Fairfax, VA 22030.

[2] NASA/Goddard Space Flight Center, Code 668.1, Greenbelt, MD 20771.

[3] University of Pennsylvania, Philadelphia, PA 19104.

is the Schwarzschild radius of the lens (related to mass by $R_S = 3\ \mathrm{km}(M/M_\odot)$). B is related to the maximum magnification of lensing A by

$$B = \left(2\sqrt{\frac{A^2}{A^2-1}} - 2\right)^{1/2} , \tag{3}$$

such that for large A (small B), they are simply related by $B \approx 1/A$. Note that equation (3) holds even if B is interpreted as any relative projected distance of the lens from the source, not only the minimum distance (impact parameter) as used here.

The measurable relative angular velocity parameter between the lens and the source that can be determined from the light curve is

$$V = v/E , \tag{4}$$

where v is the actual angular velocity of lens relative to the source in the lens plane.

The most information a point source light curve can hope to provide is, through a well-determined shape, accurate determinations of B, V, and the time of maximum light. Once they are determined, one must assume a lens distance D_{OL}, a source distance D_{OS}, and a projected relative angular lens velocity v in order to solve for the mass of the lens. (If the source distance is much greater than the lens distance then the source distance is not important.) Relative angular lens velocities are particularly unconstrained as there is usually no indication what type of orbit the lens or source is on, so that its velocity may be uncertain by an order of magnitude. This uncertainty translates directly to uncertainty in the mass of the lens.

More information is discernible from the light curve of a source that has a finite angular size. Specifically, a parameter involving the size of the source is recoverable. Generally, finite size sources are only important if the angular size of the source is comparable with B. If B is much larger, light curves of point and finite sources will be practically indistinguishable.

If the MACHO crosses the disk of the source star, Nemiroff (1987) has found that lens crossing is closely matched in time with an inflection point of the light curve. This is reproduced here as Figure 1. This effect is also discernible from Figure 11.1 of Schneider et al. (1992). Therefore source sizes are relatively easy to discern from the lens deconvolution analysis of the light curve. Information is not lost from the previous point-source case: both B and V values that would have been measured in the absence of finite source size can be determined by fits to the light curve far from the peak, where the finite source is not important. But now, however, the crossing time of the lens in front of the source can be determined as

$$T \sim \frac{(R_*/D_{\mathrm{OS}})}{(v/D_{\mathrm{OL}})} , \tag{5}$$

where T is the measured time between lens crossings of the source disk, as determined by noting the time of inflection points, and R_* is the physical radius of the star. T is directly measured from the light curve, and R_* can be estimated independently (by noting the stellar type) for the source star. This allows one to independently estimate v, the projected angular velocity between the lens and the source. Possibly more important, one can then use equation (4) to compute E, the angular Einstein ring size, and then compute R_S and hence a more accurate mass of the lens through equation (2).

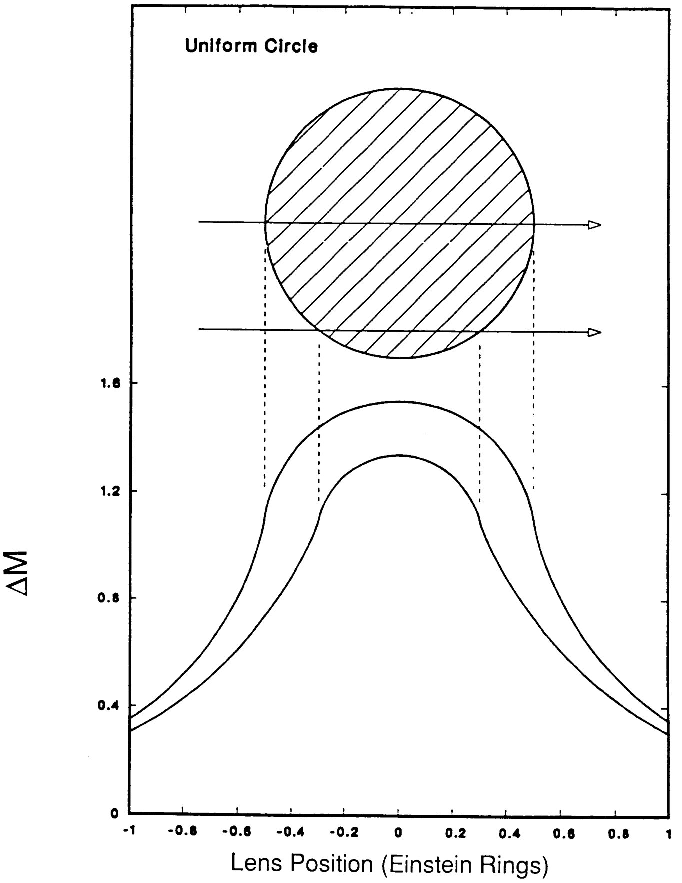

FIG. 1.—Magnification of a uniform circular disk by a point lens. Two independent lens crossings of the disk are shown, each marked by a long arrow. Position of each lens on the x-axis is listed in units of E, the angular Einstein ring size of the lens, with the zero point defined as the center of the source. Magnification of the source by each lens is listed in magnitudes as ΔM. Note that the times each lens crosses the disk boundaries are closely matched by inflection points on the light curve.

For a source star with $R_* = 30\ R_\odot$ at a distance of the LMC (taken to be 55 kpc), for a lens at a distance of 10 kpc, and for a relative lens-source velocity at the lens of 250 km s^{-1}, T is on the order of 4 hr. Note that this corresponds to the distance in time between the inflection points in Figure 1, and it does not depend on the mass of the lens. To detect inflection points one must then sample the light curve on a timescale significantly shorter than T, which is significantly less than the daily rate of most of the currently running MACHOs with the exception of the EROS CCD program (Aubourg et al. 1993), which has a sampling rate of 22 minutes. The crossing time is also significantly greater than the typical time for all the MACHO searches.

3. DISCUSSION AND SUMMARY

Currently only EROS's CCD program has the time resolution necessary to see a situation where a MACHO crosses in front of a stellar disk. We consider it somewhat unlikely that the present search techniques of the MACHO collaboration or OGLE will encounter such a situation. This is because the time between repeated images of the same stellar field are long compared to the duration of events that stars with masses less than $10^{-3}\ M_\odot$ are likely to create. Also, the MACHO collaboration and OGLE do not have the time resolution needed to accurately discern inflection points near the peaks of light curves. However, if their search techniques are augmented with a search on shorter time intervals, say, repeating some observations on timescales of hours instead of days, finite source size effects may become important (Gould 1992; Aubourg et al. 1993; Witt & Mao 1994). For expected values of V, one would expect this to encompass the mass scale between about 10^{-9} and 10^{-3} $M_{\odot}$.

A lens search of source stars in M31 (Crotts 1992) would be sensitive to smaller mass lenses, and hence finite source sizes might be more prominent and equally valuable in deconvolving lens and source parameters.

Nemiroff (1991) showed that with continuous monitoring, the smaller the mass of the lens that dominates the Galactic halo, the more frequently lens events would occur. This is expected primarily because there are more lenses needed at lower masses to populate the Galactic halo. These events would be expected to have increasingly shorter duration, since smaller mass lenses have smaller Einstein rings but equal spatial velocities, and so would cross their Einstein rings in a shorter time. The duration of the crossing time of the lens in front of the stellar disk, T, however, would be independent of the mass of the lens, since the radius of the source and the relative velocity of the lens are both independent of this mass.

Modulation effects on microlensing light curves caused by rotation of the Earth would have an effect only if the effective transverse lens velocity is not large compared to the Earth's spin speed, which is about 0.5 km s^{-1} at the surface. Modulation effects by the Earth's orbital motion (Gould 1992) would only significantly distort a part of a light curve which takes on the order of months, which is significantly longer than current estimates for T. Rapid binary motion of the lens or source might significantly change the light curve (Nemiroff 1991; Griest 1991; Gould 1992).

Photometric errors would necessarily create ambiguity in the time of the inflection points on a microlensing light curve. This ambiguity would propagate into T and so into the estimated mass of the lens. Even in the case of significant errors, however, it might be quite clear that a disk crossing event is being detected, because the center of the light curve could be significantly different than expected with a point source (Witt & Mao 1994).

We describe the advantage of probing the finite source size regime here because we feel that it is likely such capability is achievable in the next few years, and it is desirable to call attention to this phenomenon as a worthy goal. If, for example, a possible indication of the onset of a lensing event was taken as a "trigger," more frequent follow-up measurements might be made capable of better exploring the center regions of light curves, where finite source size effects could be dominant.

It is assumed that the source appears circular on the sky and that it has uniform surface brightness across its face. These do not appear unreasonable assumptions for a normal star, especially in the light of the inaccuracy of the other quantities known. It is possible that bumps in the light curves between clearly delineated inflection points could give evidence for starspots or other nonuniformities in the source star's appearance.

In sum, if high-magnification lens events could be measured to high time precision by MACHO-type search programs, one would expect to measure lens events where the lens passes directly across the background star's face. Such an event would create discernible inflection points in the lensing light curve that would, given an independent estimate of the radius of the star, yield an independent estimate of the velocity of the lens. This lens velocity is additional information that is useful in determining a more accurate mass of the microlens.

We thank Bohdan Paczyński and Shude Mao for helpful discussions, and Jerry Bonnell and an anonymous referee for comments and for carefully reading the manuscript. W. A. D. T. W. acknowledges the support of a Zaccheus Daniel Fellowship. This work was supported by a grant from NASA.

REFERENCES

Alcock, C. A., et al. 1993, Nature, 365, 621 (MACHO collaboration)
Aubourg, E., et al. 1993, Nature, 365, 623 (EROS project)
Crotts, A. P. S. 1992, ApJ, 399, L43
Gould, A. 1992, ApJ, 392, 442
Griest, K. 1991, ApJ, 366, 412
Liebes, S., Jr. 1964, Phys. Rev., 133, B835
Nemiroff, R. J. 1987, Ph.D. thesis, Univ. Pennsylvania
———. 1991, A&A, 247, 73
Paczyński, B. 1986, ApJ, 304, 1
Refsdal, S. 1964, MNRAS, 128, 295
Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses (Berlin: Springer), 313
Schneider, P., & Wagoner, R. V. 1987, ApJ, 314, 154
Udalski, A., Szymanski, M., Kaluzny, J., Kubiak, M., Krzeminski, W., Mateo, M., Preston, G. W., & Paczyński, B. 1993, Acta Astron., 43, 289 (OGLE)
Witt, J., & Mao, S. 1994, ApJ, submitted