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RECYCLING PULSARS TO MILLISECOND PERIODS IN GENERAL RELATIVITY

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ABSTRACT

We use models of rapidly rotating neutron stars in general relativity to construct evolutionary tracks for the recycling of pulsars to millisecond periods. We give an exact treatment for a simple recycling scenario where accretion occurs from the inner edge of a Keplerian disk onto a bare neutron star. For this scenario we tabulate the shortest achievable period and corresponding accreted rest mass for 13 nuclear equations of state. For each equation of state, we determine whether a nonrotating 1.4 $M_\odot$ star can be spun up to millisecond periods without exceeding the maximum mass or rotation limits. We find that spin-up is possible for all of the equations of state, even those with a static maximum mass close to 1.4 $M_\odot$.

Subject headings: accretion, accretion disks — pulsars: general — relativity — stars: neutron

1. INTRODUCTION

It is currently believed that millisecond pulsars are recycled neutron stars that have been spun up by accretion in binary systems (Alpar et al. 1982; Bhattacharya & van den Heuvel 1991). Observations of spin period P and spin-down rate $\dot{P}$ imply that these systems have a combination of rapid spin and weak magnetic fields. A neutron star can achieve a rapid spin rate by accretion provided the magnetic field is dynamically negligible ($\lesssim 10^8$ G). In this case, the inner edge of the accretion disk can extend all the way to the star's surface. In equilibrium the star approaches corotation with the Keplerian disk, and so the smaller the inner disk radius the shorter the resulting period.

This recycling scenario raises a number of obvious questions. Is it possible to accrete sufficient angular momentum, with its associated mass, to achieve millisecond periods without exceeding the maximum allowed mass? If the answer to this question is yes, is there sufficient time for a neutron star to accrete this mass at a reasonable sub-Eddington accretion rate? Do our answers to these questions depend on the adopted nuclear equation of state? If the maximum allowed neutron star mass is low, can a typical nonrotating 1.4 $M_\odot$ neutron star achieve millisecond periods?

For example, simple Newtonian estimates suggest that the mass that must be accreted to spin a 1.4 $M_\odot$ neutron star to 1.5 ms is $\Delta M \sim 0.1$ $M_\odot$ (see, e.g., Phinney & Kulkarni 1990). It is therefore unclear whether a neutron star with a maximum mass of 1.5 $M_\odot$ (Brown & Bethe 1993) can achieve this period by recycling.

To answer these questions requires models of rapidly rotating neutron stars in full general relativity. Consider the simplest case of a nonrotating neutron star with no magnetic field accreting from a Keplerian disk in the equatorial plane. Suppose each accreted mass element originates from the inner edge of the disk and deposits all its angular momentum along with its rest mass onto the star. To follow the spin-up of the star, we need to calculate a sequence of rotating relativistic stellar models of increasing rest mass M_0 and angular momentum J. The sequence starts from the initial nonrotating spherical model and terminates either at the relativistic mass shedding limit, where the angular velocity of the star equals the Keplerian velocity at the surface, or at the onset of the relativistic radial instability to collapse. We also need the relativistic models to calculate the angular momentum per unit rest mass j_K associated with the inner edge of the accretion disk. Moreover, for some models the inner edge does not extend all the way to the stellar surface, but only to the radius of the innermost stable circular orbit r_{ms}.

We have recently constructed equilibrium sequences of uniformly rotating neutron stars in general relativity for 14 nuclear equations of state (Cook, Shapiro, & Teukolsky 1994, hereafter CST). We calculated a number of important physical parameters for these stars, including the maximum mass and maximum spin rate. We determined the stability of the configurations to radial collapse. We employed a numerical scheme particularly well suited to rapidly rotating configurations with large departures from spherical symmetry. It is now possible to use these models to address the recycling scenario. In this paper we focus on the simplest recycling scenario described above, because it can be treated exactly. These results will serve as a benchmark for more detailed treatments incorporating magnetic fields and other complications. We also briefly discuss the qualitative effects of some of these complications.

2. RESULTS

Our canonical sequence begins with a $M = 1.4$ $M_\odot$ nonrotating neutron star. We adopt this value for the mass because the neutron stars with known masses are all close to this value (Thorsett et al. 1993), consistent with theoretical expectations. Except for the millisecond pulsars, observed pulsar periods are dynamically unimportant for neutron stars of this mass. Accordingly we take our starting models to be nonrotating. We first determine whether there is a minimum stable circular orbit for test particles at a radius $r_{ms} > R$, following the prescription in CST. If there is, the inner edge of the accretion disk is at r_{ms}; otherwise it extends to the surface R of the star. We assume that each accreted rest mass increment δM_0 originates from the inner edge of the disk and deposits its rest mass and angular momentum onto the star. Thus the corresponding angular momentum increment is $\delta J = j_K \, \delta M_0$. We calculate j_K from equation (7) of CST, where it is denoted by L. We assume that the accretion proceeds in a quasi-stationary fashion, with

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TABLE 1

ACCRETION FROM KEPLERIAN DISK ONTO A $M_i = 1.4\ M_\odot$ NEUTRON STAR

EOS	$M_f/M_\odot$[a]	$M_0^i/M_\odot$[b]	$\Delta M_0/M_\odot$[c]	P_f[d]	$cJ_f/GM_\odot^2$[e]	T/W[f]	R_i[g]	R_f[h]	h_i[i]	h_f[j]	$M_{sm}/M_\odot$[k]	$M_{max}/M_\odot$[l]	P_{min}[m]	Fate[n]
A	1.77	1.57	0.428	0.604	2.10	0.120	9.59	12.8	2.82	···	1.66	1.95	0.472	MS
C	1.74	1.54	0.389	0.894	1.95	0.103	12.1	16.7	0.268	···	1.86	2.17	0.587	MS
D	1.76	1.56	0.405	0.730	2.01	0.111	10.7	14.6	1.70	···	1.65	1.95	0.571	MS
E	1.76	1.57	0.414	0.656	2.04	0.115	10.0	13.6	2.38	···	1.75	2.05	0.480	MS
F	1.52	1.59	0.172	0.715	0.827	0.0336	9.21	8.35	3.20	2.74	1.46	1.67	0.476	C
L	1.80	1.52	0.443	1.25	2.36	0.116	15.0	21.4	···	···	2.70	3.27	0.757	MS
M	1.74	1.50	0.367	1.49	1.98	0.0914	16.7	23.6	···	···	1.80	2.10	0.814	MS
N*	1.84	1.53	0.484	1.08	2.55	0.130	13.6	19.5	···	···	2.64	3.22	0.682	MS
KC	1.74	1.55	0.385	0.888	1.93	0.102	12.1	16.6	0.311	···	1.49	1.81	0.848	MS
AU	1.79	1.58	0.446	0.701	2.22	0.126	10.4	14.3	2.01	···	2.13	2.55	0.472	MS
UU	1.78	1.56	0.436	0.784	2.19	0.121	11.2	15.5	1.26	···	2.20	2.61	0.503	MS
UT	1.78	1.57	0.429	0.754	2.14	0.119	10.9	15.0	1.52	···	1.84	2.19	0.537	MS
FPS	1.76	1.56	0.416	0.747	2.07	0.114	10.9	14.9	1.56	···	1.80	2.12	0.533	MS

[a] Final total mass-energy.
[b] Initial rest mass.
[c] Accreted rest mass.
[d] Rotational period measured at infinity (ms).
[e] Total angular momentum.
[f] Rotational kinetic energy over gravitational energy.
[g] Initial circumferential radius (km).
[h] Final circumferential radius (km).
[i] Initial circumferential height of corotating marginally stable orbit (km).
[j] Final circumferential height of corotating marginally stable orbit (km).
[k] Maximum *static* total mass-energy for EOS.
[l] Maximum total mass-energy for EOS.
[m] Minimum rotational period measured at infinity for EOS (ms).
[n] MS: mass-shed limit; C: collapse.

the neutron star continuously adjusting to a new equilibrium state. Consequently we next construct a new stellar model with rest mass $M_0 + \delta M_0$ and angular momentum $J + \delta J$. We repeat the process, beginning with a redetermination of the inner edge of the accretion disk, and continue until the sequence terminates either at mass shedding or radial instability.

Table 1 summarizes the result of this calculation for 13 equations of state. All of these equations of state have maximum static masses greater than 1.44 $M_\odot$, the measured mass of PSR 1913+16 (Taylor 1992). All but equation of state KC were considered in CST. This equation of state incorporates kaon condensation and has a maximum nonrotating mass of only 1.5 $M_\odot$ (Brown 1992; Thorsson, Prakash, & Lattimer[2] 1993). We omit equations of state B and G of CST because they have a maximum mass less than 1.44 $M_\odot$. For each equation of state, we list the initial rest mass M_0^i corresponding to an $M = 1.4\ M_\odot$ spherical star, the initial radius R_i, and the height h_i above the surface of r_{ms}, if it exists. We also list the final mass M_f, the accreted rest mass ΔM_0, the period P_f, the angular momentum J_f, the equatorial radius R_f, and whether the process terminates in collapse or mass shedding. For comparison we list the maximum possible nonrotating and rotating masses M_{sm} and M_{max} for the given equation of state, and the minimum allowed period P_{min}. Note that in many cases r_{ms} disappears as the equatorial radius increases during accretion.

We see from Table 1 that the range of accreted rest mass is $\Delta M_0 \approx 0.2$–$0.5\ M_\odot$. The final periods are in the range 0.6–1.5 ms. The fastest known millisecond pulsar is PSR 1937+21 with a period of 1.558 ms. Thus *spin-up by this simple recycling scenario seems possible for all currently viable equations of state.* These equations of state span a large range of nuclear models and theory. Even the KC equation of state with its low maximum static mass can reach all observable millisecond periods by recycling.

In Table 2 we present results for the same scenario as Table 1, except we stop the spinup at 1.558 ms, the period of PSR 1937+21. Less mass is required in this case, and we find $\Delta M_0 \approx 0.1$–$0.3\ M_\odot$.

Selected evolutionary tracks for recycled pulsars are shown in Figure 1 for a representative equation of state (FPS). This figure is essentially Figure 1 of CST with the evolutionary tracks superposed. The dashed curve (a) shows the evolutionary track for the canonical spinup scenario summarized in Tables 1 and 2. The curve terminates at mass shedding, having crossed into the domain where the accretion disk reaches all the way to the stellar surface. Curve (b) starts from the same initial model, but assumes that only 50% of the test particle's angular momentum ends up in the star. This sequence terminates at radial instability at $M = 1.96\ M_\odot$ and period 0.66 ms, having accreted $\Delta M_0 = 0.71\ M_\odot$. Recycling in this case leads to a higher final spin rate despite the accretion of smaller angular momentum per unit mass; the price is a much larger mass deposition. The reason for the higher final spin rate is that a larger mass can support higher spin. Track (c) again begins at the same initial configuration except that the inner edge of the disk is held fixed at 24.8 km, twice the initial value of r_{ms}. The point of this track is to mimic the qualitative effect of a magnetic field strong enough to cause the inner edge of the disk to be at that larger radius. A factor of 2 change in the disk radius does not have much effect ($M_f = 1.70\ M_\odot$, $P_f = 0.77$ ms, $\Delta M_0 = 0.33\ M_\odot$). Curve (d) demonstrates that the canonical scenario can terminate in radial instability even when all

[2] The version we employ was constructed with $F(u) = u$, $a_3 m_3 = -134$ MeV, $K_0 = 190.7$ MeV and used a Maxwell construction to maintain positive compressibility.

TABLE 2

Accretion from Keplerian Disk to $P = 1.558$ Milliseconds

EOS	$M_f/M_\odot$ [a]	$M_0^i/M_\odot$ [b]	$\Delta M_0/M_\odot$ [c]	P_f [d]	$cJ_f/GM_\odot^2$ [e]	T/W [f]	R_i [g]	R_f [h]	h_i [i]	h_f [j]	$M_{sm}/M_\odot$ [k]	$M_{max}/M_\odot$ [l]	P_{min} [m]	Fate [n]
A	1.48	1.57	0.104	1.56	0.502	0.0145	9.59	9.69	2.82	2.07	1.66	1.95	0.472	MS
C	1.53	1.54	0.156	1.56	0.760	0.0275	12.1	12.6	0.268	···	1.86	2.17	0.587	MS
D	1.50	1.56	0.125	1.56	0.607	0.0197	10.7	11.0	1.70	0.892	1.65	1.95	0.571	MS
E	1.49	1.57	0.112	1.56	0.540	0.0164	10.0	10.2	2.38	1.57	1.75	2.05	0.480	MS
F	1.46	1.59	0.081	1.56	0.388	0.00923	9.21	8.80	3.20	2.90	1.46	1.67	0.476	C
L	1.65	1.52	0.285	1.56	1.46	0.0649	15.0	17.1	···	···	2.70	3.27	0.757	MS
M	1.70	1.50	0.330	1.56	1.76	0.0793	16.7	20.8	···	···	1.80	2.10	0.814	MS
N*	1.61	1.53	0.237	1.56	1.18	0.0503	13.6	14.9	···	···	2.64	3.22	0.682	MS
KC	1.53	1.55	0.153	1.56	0.746	0.0267	12.1	12.5	0.311	···	1.49	1.81	0.848	MS
AU	1.50	1.58	0.127	1.56	0.614	0.0202	10.4	10.7	2.01	1.13	2.13	2.55	0.472	MS
UU	1.52	1.56	0.143	1.56	0.696	0.0244	11.2	11.6	1.26	0.424	2.20	2.61	0.503	MS
UT	1.51	1.57	0.134	1.56	0.651	0.0220	10.9	11.2	1.52	0.696	1.84	2.19	0.537	MS
FPS	1.51	1.56	0.130	1.56	0.632	0.0210	10.9	11.1	1.56	0.756	1.80	2.12	0.533	MS

[a] Final total mass-energy.
[b] Initial rest mass.
[c] Accreted rest mass.
[d] Rotational period measured at infinity (ms).
[e] Total angular momentum.
[f] Rotational kinetic energy over gravitational energy.
[g] Initial circumferential radius (km).
[h] Final circumferential radius (km).
[i] Initial circumferential height of corotating marginally stable orbit (km).
[j] Final circumferential height of corotating marginally stable orbit (km).
[k] Maximum *static* total mass-energy for EOS.
[l] Maximum total mass-energy for EOS.
[m] Minimum rotational period measured at infinity for EOS (ms).
[n] MS: mass-shed limit; C: collapse.

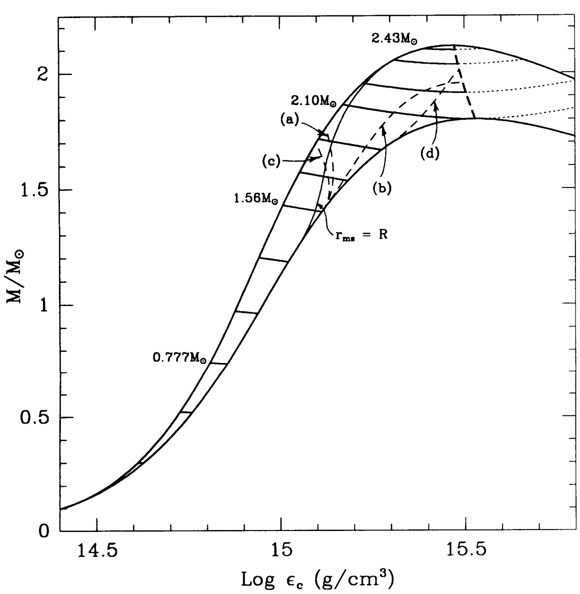

Fig. 1.—Selected evolutionary tracks for recycled pulsars for the FPS equation of state. Plotted is the total gravitational mass M vs. central energy density ϵ_c for equilibrium models. Constant rest mass sequences appear as nearly horizontal lines with selected sequences labeled by the value of the rest mass M_0. Stable portions are displayed as solid segments, unstable portions as dotted. The $M_0 = 1.56\ M_\odot$ sequence has a nonrotating member with $M = 1.4\ M_\odot$. The mass shed limit is the uppermost solid bounding curve while the static limit is the lower solid bounding curve. The vertical heavy dash line is the quasi-radial stability limit. To the right of the curving line models have $r_{ms} > R$, while to the left all corotating circular orbits are stable. The short dashed curves (a)–(d) are evolutionary tracks (see text).

the particle angular momentum is deposited onto the star, provided the initial mass is sufficiently large ($M > 1.7\ M_\odot$).

Figure 2 shows the same four evolutionary tracks on a plot of spin rate versus rest mass. The steepness of the tracks shows how little rest mass is required to achieve millisecond periods with the simple scenario.

3. DISCUSSION

Since it is now practical to construct rapidly rotating relativistic neutron stars of given rest mass and angular momen-

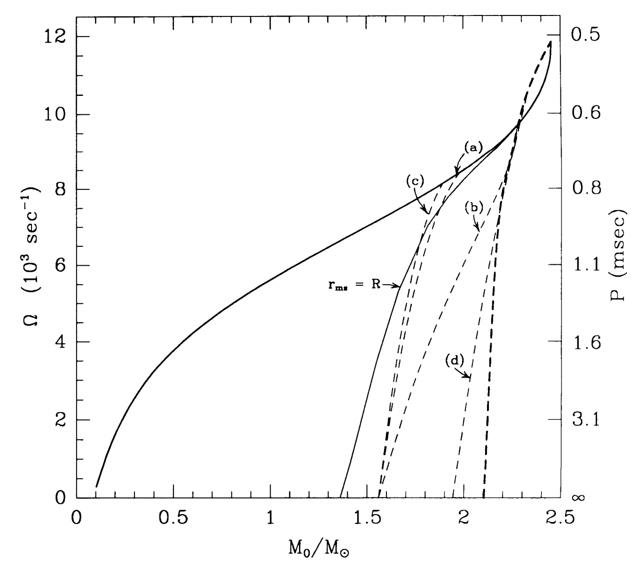

Fig. 2.—Spin rate vs. rest mass along evolutionary tracks for recycled pulsars for the FPS equation of state. Lines and labels are the same as for Fig. 1.

tum, we can track the spinup of a neutron star by recycling. The simplest scenario, for which the magnetic field is negligible ($\lesssim 10^8$ G), can be done exactly for a given equation of state and a given nonrotating initial star. We find that the accreted rest mass required to spin up a neutron star to millisecond periods is achievable without encountering instabilities or mass shed limits. This conclusion even applies to the KC equation of state, for which the maximum static mass is only 1.5 $M_\odot$. The reason why recycling works in this case is because in general relativity the maximum mass increases with rotation. In addition, not all of the accreted rest mass goes into increasing the total mass of the star because of gravitational binding energy.

The timescale to accrete the required rest mass, $\sim 0.1\ M_\odot$, at the Eddington limit, $\sim 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, is $\sim 10^7$ yr. This timescale is largely insensitive to the adopted nuclear equation of state. If other astrophysical considerations require a considerably shorter time scale, then the simple recycling scenario described here will have to be modified beyond the variations explored in this paper.

We have assumed in this paper that when the supply of accreting mass is unlimited and no radial instability is encountered, the recycling process terminates at mass shedding. It is possible that the process may extend beyond mass shedding, in which case the outermost layers of the neutron star would be forced to be in differential rotation. It is generally believed, however, that neutron star matter does not support appreciable differential rotation (Lindblom 1992). In addition, the high stability of millisecond pulsars as clocks argues against differential rotation.

It is also possible that the recycling process terminates earlier by encountering a nonaxisymmetric instability. No simple exact criterion for the onset of nonaxisymmetric instabilities in general relativity is known. However, in Newtonian physics the onset occurs when the ratio of rotational to magnitude of potential energy, T/W, gets large. For example, an incompressible MacLaurin spheroid is unstable to an $m = 2$ mode when $T/W > 0.14$. Several authors have analyzed the stability of compressible uniformly rotating polytropes in Newtonian theory. Usually only the $m = 2$ nonaxisymmetric mode is important (see Tassoul 1978 for a discussion and references), but it is only reached when the polytropic index $n \lesssim 0.808$. For rapidly rotating stars like millisecond pulsars, the modes with $m \lesssim 5$ are relevant, because they are the modes driven unstable by gravitational radiation (Ipser & Lindblom 1989, and references therein). Critical values of T/W for these modes have been tabulated by Managan (1985), Imamura, Friedman, & Durisen (1985), and Ipser & Lindblom (1990). In Tables 1 and 2 we list the relativistic T/W for the final members of the sequence. It is clear from Table 2 that periods as short as 1.56 ms are achieved with very modest values of T/W. This suggests that nonaxisymmetric instabilities are no obstacle to recycling.

Recently Miller & Lamb (1993) have argued that, when luminosities exceed $\sim 0.2 L_{\mathrm{edd}}$, radiation drag forces on accreting matter can transfer much of the matter angular momentum to the radiation field before the matter reaches the stellar surface. Our curve (b) in Figures 1 and 2 suggests that in this case recycling might still achieve high spin rates, but at the cost of higher accreted mass. If the loss of angular momentum is very efficient, however, recycling to high spin rates might be inhibited. Studies of radiation drag in a rotating relativistic gravitational field are required to address this issue.

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REFERENCES

Alpar, M. A., Cheng, A. F., Ruderman, M. A., & Shaham, J. 1982, Nature, 300, 728
Bhattacharya, D., & van den Heuvel, E. P. J. 1991, Phys. Rep., 201, 1
Brown, G. E. 1992, in Proc. 1st Symp. on Nucl. Phys. of the Universe, ed. M. Strayer (Singapore: World Scientific), in press
Brown, G. E., & Bethe, H. A. 1993, preprint
Cook, G. B., Shapiro, S. L., & Teukolsky, S. A. 1994, ApJ, in press (CST)
Imamura, J. N., Friedman, J. L., & Durisen, R. H. 1985, ApJ, 294, 474
Ipser, J. R., & Lindblom, L. 1989, Phys. Rev. Lett., 62, 2777
———. 1990, ApJ, 355, 226
Lindblom, L. 1992, in The Structure and Evolution of Neutron Stars, ed. D. Pines, R. Tamagaki, & S. Tsuruta (Reading: Addison-Wesley), 122
Managan, R. A. 1985, ApJ, 294, 463
Miller, M. C., & Lamb, F. K. 1993, ApJ, 413, L43
Phinney, E. S., & Kulkarni, S. R. 1990, Caltech Preprint GRP-221
Tassoul, J.-L. 1978, Theory of Rotating Stars (Princeton: Princeton Univ. Press)
Taylor, J. H. 1992, Phil. Trans. R. Soc. Lond. A, 341, 117
Thorsett, S. E., Arzoumanian, Z., McKinnon, M. M., & Taylor, J. H. 1993, ApJ, 405, L29
Thorsson, V., Prakash, M., & Lattimer, J. M. 1993, preprint