

Ω FROM VELOCITIES IN VOIDS

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ABSTRACT

We propose a method for deriving a dynamical lower bound on Ω from diverging flows in low-density regions, based on the fact that large outflows are not expected in a low- Ω universe. The velocities are assumed to be induced by gravity from small initial fluctuations, but *no* assumptions are made regarding galaxy *biasing*, the exact statistical nature of the fluctuations or Λ . The derivatives of a diverging velocity field infer a nonlinear approximation to the mass density, which is an overestimate when the true value of Ω is assumed. This inferred density can become ridiculously negative when the assumed Ω is too low, thus bounding Ω . Observed radial peculiar velocities of galaxies allow the *POTENT* procedure to recover the required velocity field, Gaussian smoothed at 1200 km s⁻¹. The density and the associated errors are then inferred for different values of Ω , searching for a void which shows negative inferred density with high confidence. A preliminary application to data in a southern void indicates that $\Omega \leq 0.3$ can be ruled out at the 2.4 σ level. A detailed study of possible systematic errors is under way.

Subject headings: cosmology: theory — dark matter — galaxies: clustering — galaxies: formation — gravitation — large-scale structure of universe

1. INTRODUCTION

The observed large-scale peculiar velocities can place dynamical constraints on the cosmological density parameter Ω in several different ways. If one is willing to assume an explicit “*biasing*” relation between galaxy and mass density (see Dekel & Rees 1987), one can estimate Ω by comparing the velocity data to a uniform galaxy redshift survey. Using linear gravitational instability (GI) theory, and assuming a linear biasing relation with a factor b between the smoothed density fluctuation fields of galaxies and mass, one can measure the ratio $\Omega^{0.6}/b$ (e.g., Dekel et al. 1993). The degeneracy between Ω and b could, in principle, be removed by nonlinear analysis, but the interpretation is always confused by possible nonlinearities in the biasing relation.

Alternatively, one can determine Ω from the velocity field alone, if one is willing to assume a specific probability distribution function (PDF) for the initial fluctuations, e.g., Gaussian. This can be done by tracing the observed velocity field back in time, and comparing the recovered initial PDF, which depends on Ω , with the assumed initial PDF (Nusser & Dekel 1993). It can also be done by comparing the measured PDF of the velocity-divergence with the predictions for a system which evolved from Gaussian initial conditions under gravity with an assumed Ω (Bernardeau et al. 1994).

Here, we propose a method that relies *only* on the assumptions that the large-scale structure evolved under *gravity* from small-amplitude initial fluctuations, and that the galaxies are honest tracers of the underlying velocity field. The unique feature of this method is that it requires *no* assumptions about how the galaxies are distributed relative to the mass, or concerning the specific PDF of the initial fluctuations.

The intuitive idea underlying our method is that streaming motions arise from the “pull” of overdense regions and the “push” of underdense regions. In a low- Ω universe one would

not expect substantial deviations from the Hubble flow except for converging flows around very overdense regions, which only fill a small fraction of the volume. In particular, one can easily see that there is an Ω -dependent limit on the magnitude of the diverging flow around a void: even a completely empty void cannot induce substantial peculiar motions if the mean density around it is well below the critical density.

For a given diverging velocity field, and a given Ω , one can infer from the velocity derivatives, $\partial v/\partial x$, nonlinear approximations with small variance to the mass-density fluctuation field, $\delta \equiv \rho/\bar{\rho} - 1$ (Nusser et al. 1991, hereafter NDBB). One such approximation, δ_c , overestimates the true density in density wells ($\delta_c > \delta$) under very general conditions, as long as the assumed value of Ω is the true value. Analogously to the behavior of the linear approximation, $\delta_0 = -\Omega^{-0.6} \nabla \cdot \mathbf{v}$, the δ_c inferred from a given diverging velocity field decreases with decreasing Ω , and it may become smaller than -1 for assumed Ω values that are a severe underestimate. The fact that $\delta \geq -1$, because mass is never negative, implies a lower bound on Ω .

Given the observed radial peculiar velocities of galaxies, one can use the *POTENT* procedure to recover the three-dimensional velocity field, Gaussian-smoothed at ~ 1000 km s⁻¹, (Dekel, Bertschinger, & Faber 1990, hereafter DBF; Bertschinger et al. 1990, hereafter BDFDB). One can then derive the inferred mass-density field and the associated *error* field for different values of Ω . Focusing on the deepest density wells, the assumed Ω should be lowered until δ_c becomes significantly smaller than -1 , where the significance is evaluated based on the estimated errors. Such low values of Ω could be ruled out.

When trying to apply this test to the available data, severe difficulties are posed by the large random and systematic uncertainties due to distance errors and nonuniform, sparse sampling. In our effort to assign statistical significance to the result, we currently rely on the random error estimates of *POTENT* based on Monte Carlo noise simulations. The systematic errors have been partially corrected for in the *POTENT* procedure, but a more specific investigation of the biases affecting the smoothed velocity field in the deepest density wells is required for more reliable results.

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The method is outlined in § 2. The preliminary results from current data are briefly described in § 3. The analysis is discussed in § 4.

2. METHOD

The raw data consist of a large set of galaxies with measured redshifts z_i and redshift-independent estimated distances r_i of random Gaussian errors $\sigma_i \simeq (0.15-0.21)r_i$. The peculiar velocities are $u_i = z_i - r_i$. The distances are corrected for inhomogeneous Malmquist bias by heavy grouping (Willick et al. 1994) and by adopting a correction procedure which assumes that IRAS galaxies trace the underlying galaxy distribution (Dekel et al. 1994). In *POTENT*, the radial peculiar velocities are first smoothed with a Gaussian window of radius 1200 km s^{-1} onto a spherical grid, yielding a radial field $u(x)$. The smoothing suppresses the random uncertainties due to distance errors and sparse, nonuniform sampling. The adopted weighting scheme mimics equal-volume weighting in an attempt to minimize the associated systematic biases. *POTENT* then recovers the full velocity field by requiring a potential flow, $\mathbf{v} = -\nabla\phi$, as predicted by linear GI and maintained later via Kelvin's circulation theorem under appropriate smoothing (Bertschinger & Dekel 1989). The potential at each point x is computed by integrating $\phi(x) = -\int_0^x u(r') \cdot \partial_r \phi / \partial r' dr'$, and the transverse velocity components are derived by differentiating this potential.

Given the velocity field and Ω , one can approximate the corresponding mass-density field. In the linear approximation, $\delta_0 = -f(\Omega)^{-1} \nabla \cdot \mathbf{v}$, where $f(\Omega) = \dot{D}/D \approx \Omega^{0.6}$, and $D(t)$ is the fluctuation linear growth factor (Peebles 1980). The distances are measured in km s^{-1} , which eliminates any dependence on the Hubble constant. Although δ_0 is also an exact solution to the nonlinear Euler and Poisson equations for cosmological dust under the quasi-linear approximation of a universal time dependence for the displacements (Zel'dovich 1970), it always underestimates δ in the nonlinear regime (NDBB). Therefore, the linear approximation *cannot* be used to obtain the desired lower limit on Ω .

A better quasilinear approximation is the solution to the continuity equation assuming Zel'dovich displacements,

$$\delta_c(x) = \left\| I - \frac{1}{f(\Omega)} \frac{\partial \mathbf{v}}{\partial \mathbf{x}} \right\| - 1, \quad (1)$$

where I is the unit matrix, the derivatives are taken with respect to the Eulerian comoving position, and the double vertical bars denote the Jacobian determinant (NDBB). The desired inequality, $\delta_c \geq \delta$ at the negative tail, can be shown analytically to hold in a spherical negative top-hat model (NDBB, Fig. 1a). This inequality is *generic*, for any value of Ω in the range of interest, as long as the structure has grown according to the conventional wisdom. This is illustrated in Figure 1, which shows the mean and standard deviation of $\delta_c - \delta$ at a given true δ in N -body simulations of $\Omega = 1$ and 0.2 . These simulations started off as random Gaussian realizations of the CDM power spectrum in a box of $25,600 \text{ km s}^{-1}$, with 128^3 grid points and particles. They were evolved using a PM code until $b = 1$. The final density and velocity fields were smoothed with a Gaussian window of radius 500 km s^{-1} . The moments of $\delta_c - \delta$ are computed in bins of width 0.1 in δ . The figure shows that at the negative tail, δ_c is many standard deviations larger than δ (see Mancinelli et al. 1994 for more details). This behavior is not specific to the assumed fluctuation power-spectrum; a similar behavior is exhibited, for example,

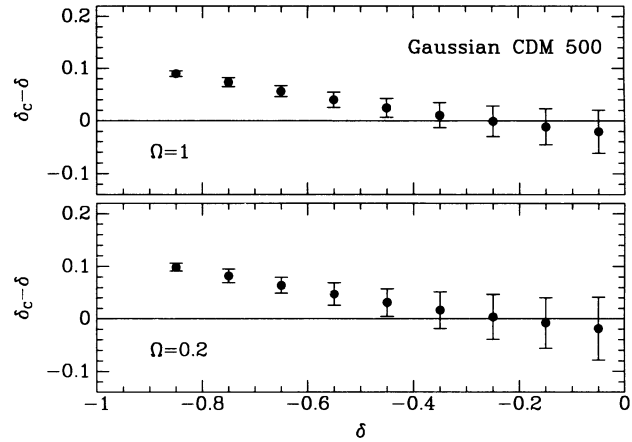


FIG. 1.—Approximation inferred from the velocity derivatives, δ_c , compared with the true density fluctuation, δ , in a Gaussian CDM N -body simulation. The fields are smoothed at 500 km s^{-1} . Only negative fluctuations are shown, focusing on the upward deviation of the mean at the negative tail, and the small standard deviation.

by HDM simulations (NDBB). The behavior of the mean is also similar when a larger smoothing is applied; for 1200 km s^{-1} smoothing the standard deviations get smaller by about a factor of 3, and the deepest void is of $\delta \approx -0.7$ only.

While the inequality $\delta_c \geq \delta$ is necessary for the method to work, the discriminatory power of the test could be improved by using an approximation which overestimates δ at the negative tail in the minimal possible way (Ganon & Dekel 1994).

The main source of uncertainty in the δ_c field such derived by *POTENT* is the random distance errors. To estimate this uncertainty we construct Monte Carlo samples in which the peculiar velocity of each galaxy is perturbed by a Gaussian random variable of dispersion σ_i . For each perturbed sample, we compute δ_c at each grid point. Its standard deviation over the Monte Carlo simulations, $\sigma(x)$, serves as our error estimate for the purpose of evaluating the significance of the limit on Ω .

The bias arising in the smoothing procedure due to sampling gradients remains severe in very poorly sampled regions despite our efforts to minimize it by appropriate weighting. We therefore exclude regions where the sampling is unacceptably poor, by considering only grid points where the distance to the fourth nearest neighboring object in the data set, R_4 , is smaller than a critical value, chosen here to be 1500 km s^{-1} . Such regions typically occur in the current data at distances beyond 6000 km s^{-1} , and in the galactic zone of avoidance.

3. PRELIMINARY RESULTS

As an illustration for the use of the proposed test, we apply it to the tentative data recently compiled by Willick et al. (1994), and *POTENT* analyzed by Dekel et al. (1994). It is based on ~ 3000 galaxies, mostly spread inside a sphere of radius 6000 km s^{-1} about the Local Group, and extending beyond 8000 km s^{-1} in selected regions.

The most promising “test case” provided by these data is a broad diverging region centered near the supergalactic plane at the vicinity of $(X, Y) = (-2500, -4000)$. This region can be roughly identified with the “Sculptor void” of galaxies next to the “Southern Wall” (da Costa et al. 1988, Fig. 7b; Kauffman & Fairall 1991; Rhee & Katgert 1991).

Figure 2 shows maps of δ_c in the relevant part of the supergalactic plane, for several different values of Ω . The void is

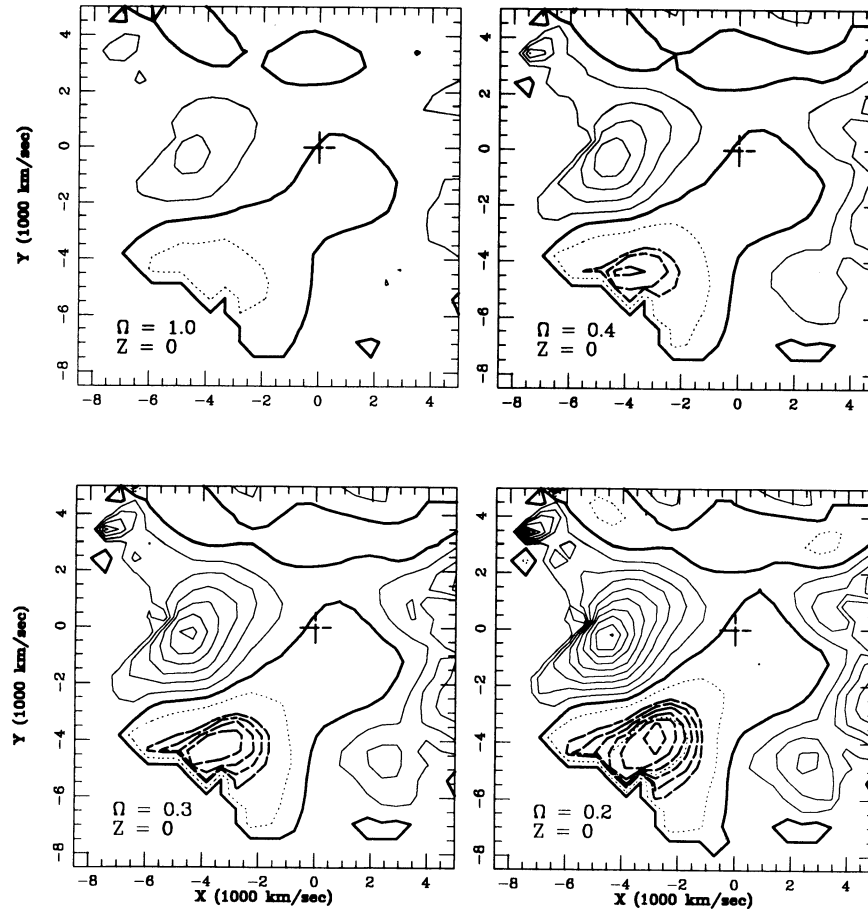


FIG. 2.—Maps of the density-fluctuation field inferred from the observed velocity derivatives, δ_c , in part of the supergalactic plane, for several different values assumed for Ω . The Sculptor void is confined by the Pavo part of the Great Attractor on the left and the Aquarius extension of the Perseus-Pisces supercluster on the right. The Local Group is marked by a “+”. Contour spacing is 0.5, with the mean, $\delta_c = 0$, marked by a heavy line, $\delta_c > 0$ marked by a solid line, and $\delta_c < 0$ marked by a dotted line. The heavy-dashed contours mark the downward deviation of δ_c from -1 in units of the standard deviation σ , starting from zero (which coincides with $\delta_c = -1$), and decreasing with spacing -0.5σ . The value $\Omega = 0.3$ is ruled out at the 2.4σ level.

confined by the Pavo part of the Great Attractor on the left, the Aquarius Wall extension of the Perseus-Pisces supercluster on the right, and the Cetus Wall from the bottom at a larger distance. Values of $\Omega \approx 1$ are perfectly consistent with the data, but δ_c becomes smaller than -1 in this void already for $\Omega = 0.6$. The confidence by which $\delta_c < -1$ at a given point is expressed in terms of the error σ there. The values $\Omega = 0.4, 0.3$, and 0.2 are ruled out at the $1.6, 2.4$, and 2.9σ levels, respectively.

4. DISCUSSION

We are proposing a dynamical method for determining a robust lower bound on Ω from observed diverging peculiar velocities. Our purpose is to present the basic method and to report the encouraging preliminary result of its application to data, in order to motivate further observations, tests and improvements.

Our preliminary result indicates that the method has the potential of ruling out low values of Ω . Based on the available data and the current bias-correction procedure of *POTENT*, the most pronounced diverging flow in our cosmological neighborhood indicates that $\Omega = 0.3$ or smaller can be ruled out at the greater than 2.4σ level or higher. Results of higher confidence should await a more careful study of the possible systematic errors.

Taking the current result at face value, it is consistent with other recent dynamical lower bounds to Ω . By comparing the density fields of *IRAS* galaxies and *POTENT* mass, Dekel et al. (1993) find $f(\Omega)/b_{IRAS} = 1.3^{+0.8}_{-0.6}$. A similar comparison between *POTENT* mass and optical galaxies yields $f(\Omega)/b_{optical} \approx 0.65 \pm 0.2$ (Hudson et al. 1994). By demanding Gaussian initial PDF (e.g., based on Nusser, Dekel, & Yahil 1994), Nusser & Dekel (1993) find that $\Omega \leq 0.3$ can be rejected at the 6σ level. A similarly high value for Ω is indicated by the skewness of $\mathbf{V} \cdot \mathbf{v}$, under the same assumption of Gaussian initial PDF (Bernardeau et al. 1994).

The following discussion focuses on certain features of the method:

The bound refers to the density parameter Ω and is insensitive to the cosmological constant Λ ; it is actually a bound on Peebles' $f(\Omega)$ which is hardly affected by Λ .

The method is *independent* of the biasing relation between galaxy and mass density fields; galaxies serve only as tracers of the gravitationally induced velocity field. It therefore bypasses the uncertainties of how galaxy formation may depend on the environment.

The main leverage which enables us to constrain Ω , either by the PDF methods or by the void method of the present letter, is the extreme sensitivity of the recovered density in low-density regions to the assumed Ω when it is small.

The method does not require any special topology for the isodensity contours associated with the diverging flow; it does not have to be an isolated spherical void. The *POTENT* procedure, which translates the data into a full velocity field, allows the computation of the key quantity δ_c wherever there are enough tracers.

The relation between the velocities and Ω assumes that the mass inhomogeneities evolved via GI from small initial fluctuations. The necessary properties of potential flow and the inequality $\delta_c > \delta$ have been tested to be valid for the smoothed fields in simulated gravitating systems with *Gaussian* initial fluctuations and “standard” power spectra (BD; NDBB). However, these properties are found to be valid under more general initial conditions; e.g., in explosion scenarios where the initial conditions are neither Gaussian nor of GI origin (Babul et al. 1994). We also know that these properties are valid in a spherical uniform void in which the velocity field is properly sampled interior to the caustic edge. Thus, the method does not rely explicitly on the specific statistical properties of the initial fluctuations. How peculiar and different from a random Gaussian field can the initial fluctuations be before the method possibly breaks down is yet to be investigated. One could probably invoke contrived, extremely non-Gaussian counter examples in a low- Ω universe, where the effective Ω in an extended diverging region is much larger (e.g., in an average-density region surrounded by several big clusters).

A disadvantage of the proposed method relative to the methods which use the whole PDF is that it makes use of only a limited portion of the available data—the bound is effectively based on the velocities near the deepest nearby void.

Two systematic effects deserve special attention. First, the sparse and nonuniform sampling. Although the method could be applied even if there were no correlation between the distribution of galaxies and the dark matter density, such correlation does exist, making it hard to trace the velocity field in the low-density regions. The smoothed velocity field at the center of a void could be biased by the interpolation from more populated regions surrounding it, introducing a “sampling-gradient bias” (SG, see DBF). The *POTENT* method tries to correct for this bias, and we exclude regions of extremely poor

sampling where the correction fails, but even in the best possible void one may be left with certain SG bias which should be accounted for in detail.

The second effect, the inhomogeneous Malmquist bias (IM), is the most fearsome enemy of all the dynamical estimates of Ω mentioned above. The random distance errors, combined with galaxy-density gradients along the line of sight, systematically enhance the inferred velocity gradients and the inferred density fluctuations. The IM bias is of particular relevance here in view of the possibly large log-density gradients near the void edges. This bias is always toward overestimating Ω . In the current *POTENT* analysis we correct for IM bias using a procedure which has been tested to work properly in general, but there is still room for improvement, with particular attention to the IM bias in voids.

These two systematic uncertainties are being carefully studied using simulations, with the aim to refine and calibrate the method (Ganon & Dekel 1994). But while this theoretical effort is going on, it would be of great importance to pursue a parallel observational effort to improve the sampling of peculiar velocities in low-density regions in order to reduce the errors. The void tentatively addressed above is an obvious place to start such an exploration, but it is certainly not the only void of possible relevance. For the method to be effective we need to find a void that is (a) bigger than the correlation length for its vicinity to represent the universal Ω , (b) deep enough for the lower bound to be strong, (c) nearby enough for the distance errors to be small, and (d) properly sampled to trace the velocity field in its vicinity.

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