

COSMOLOGICAL DIPOLES IN DEPTH

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ABSTRACT

We review and discuss the problem of the convergence of the peculiar acceleration (proportional to the “dipole”) felt by the Local Group (LG) of galaxies, as inferred by the motion with respect to the cosmic microwave background (CMB) frame. In the light of the conflicting results that appeared in the literature, the basic issue we discuss here is at what depth such convergence is achieved.

By matching the dipole growth with radial distance obtained from galaxy samples, which map better the closer regions, to that obtained from samples of galaxy clusters, which map better the far regions, we estimate that almost 1/2 of the LG peculiar velocity with respect to the CMB, v_{LG} , is due to mass fluctuations located at a distance greater than $\sim 60 h^{-1}$ Mpc ($\lesssim 30\%$ of v_{LG} is due to fluctuations at depths greater than $100 h^{-1}$ Mpc). This result is at variance with earlier claims which suggested that *all* of this velocity is generated within such inner sphere. Our results make a prediction for the final level of dipole anisotropy (scaled to *IRAS* galaxies) on the scale of $\sim 180 h^{-1}$ Mpc which should be found by future, fainter all-sky galaxy samples: the asymptotic expected value of the peculiar velocity should be $v_l^\infty \simeq (4/b_{cl})(\Omega_0^{0.6}/b_{lp}) 1100 \pm 150 \text{ km s}^{-1}$. This result favors an open universe, unless *IRAS* galaxies are biased with respect to the matter distribution (i.e., $b_{lp} \sim 1.8$). The growth with depth of the dipole favors models with a coherence length larger than standard CDM.

Subject headings: cosmology: theory — galaxies: distances and redshifts — large-scale structure of universe

1. INTRODUCTION

In the past few years a considerable effort and a large number of papers have been devoted to the study of the distribution of matter around the Local Group (LG), in order to fully explain within the gravitational instability picture the peculiar velocity of the LG itself with respect to the cosmic microwave background frame (CMB). The available data have steadfastly grown since the first results of CfA1 (Davis & Huchra 1982), limited to the $\hat{z}$ component in Galactic coordinates, and a strong impulse has been given by galaxy samples derived from the *IRAS* database. A key point for cosmological conclusions and inferences on the value of Ω_0 is the depth within which the peculiar acceleration responsible for the peculiar velocity of the LG is generated (convergence depth). The first claims of a “local” (on a cosmological scale) convergence depth were somewhat counterargued through theoretical warnings by Vittorio & Juszkiewicz (1987). Since the first study of Yahil et al. (1986), most of the many papers which appeared in the literature on this subject studied two-dimensional samples, and we mention here just a few, since we will discuss below only the three-dimensional results. From a technical point of view, noteworthy papers are the following: for the two-dimensional error analysis Villumsen & Strauss (1987), Fabbri & Natali (1990); for extrapolation through spherical harmonics Lahav (1987), Plionis (1989), and Scharf et al. (1992). The last paper identifies as *Puppis* a nearby mass at low Galactic latitude which was a needed player in the game (cf. the displaced Pe-Pi of Shaya, Tully, & Pierce 1992).

In § 2 we briefly review some of the dipole results published on three-dimensional data and we present our results obtained through matching the data from clusters and galaxies samples; in § 3 we discuss the sensitivity needed in all-sky samples of galaxies in order to detect (if present) the deep dipole anisotropy we observed in the cluster distribution; in § 4 we show how the predictions from different shapes of the cosmic fluctuation spectra compare with our results on the dipole behaviour with depth; finally, in § 5 we give a brief summary and a discussion of the main topics of the paper.

2. WHERE IS CONVERGENCE REACHED?

To answer with confidence this question using galaxy samples, we would like to have at our disposal an all-sky galaxy sample with measured redshifts which is volume limited up to at least $200 h^{-1}$ Mpc, roughly corresponding to a magnitude limited sample with $b_J \sim 17$ (cf. discussion in § 3.2). However, the presently available data from galaxy samples reach at best a maximum depth of $\sim 100 h^{-1}$ Mpc, so that it is still very useful to consider other cosmic tracers, such as galaxy clusters, to investigate what happens beyond such distance. Clusters, in turn, suffer of having a relatively low space density, so that they fail to trace satisfactorily our nearby surroundings. Therefore, at the present time we face a situation which calls for an attempt to match the two complementary pieces of information from galaxies and clusters. We recall the present situation in the next subsection, discuss some aspects of it in the following one, and finally present the matching in the last subsection.

2.1. A Panoramic of Three-Dimensional Results

As mentioned in the introduction, while there have been several determinations of surface dipoles by different groups, and sometimes with conflicting results, the study of the detailed

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growth in depth of the acceleration on the LG has been limited up to now to the fewer cases in which redshift information is available. Indeed, all-sky three-dimensional results have been published so far on: (i) *IRAS* galaxies from a flux-limited sample (Yahil 1988, hereafter Y88; Strauss & Davis 1988, hereafter SD88; and subsequent papers); (ii) optical galaxies from a combination of diameter-limited samples (Lynden-Bell, Lahav, & Burstein 1989, hereafter LLB89); (iii) *IRAS* galaxies from a sparse sample (Rowan-Robinson et al. 1990, hereafter RR90); (iv) Abell clusters (Scaramella et al. 1991a, hereafter Paper I; Plionis & Valdarnini 1991, hereafter PV91; Scaramella et al. 1991c; hereafter Paper II); (v) X-ray selected clusters (Lahav, Kaiser, & Hoffman 1990a); (vi) X-ray selected AGNs (Miyaji & Boldt 1990); (vii) a nearby sample of galaxies (Shaya et al. 1992).

Each of these samples has given results which are strongly dependent on the kind of objects examined, the adopted weighting schemes, and on the various incompleteness of the samples themselves. Some characteristics and related problems for these samples are, respectively: (i) a selection from a uniform database up to low Galactic latitudes, tracers which under-represent the densest regions, an apparent limiting flux which gives a maximum effective depth of $\sim 80 h^{-1}$ Mpc in tracing structures; (ii) the most populated catalog, tracers which sample effectively all environments, depth information which is inferred (and, therefore, diluted) through a diameter function, a combination of different databases, a maximum effective depth of $\sim 80\text{--}100 h^{-1}$ Mpc; (iii) same as in (i), but with fewer redshifts, for a lower limiting flux which gives a better number density at intermediate distances, a maximum effective depth of $\gtrsim 100 h^{-1}$ Mpc; (iv) tracers which represent only the densest spots, which are rather sparse, a combination of different databases, and a maximum effective depth of $\sim 300 h^{-1}$ Mpc; (v) a quite sparse subset of (iv); (vi) a very sparse sample with a maximum effective depth of $\sim 40\text{--}50 h^{-1}$ Mpc and a rather uncertain relationship with the galaxy distribution; (vii) a local sample with intrinsic distance indicators, which gives indications up to a maximum depth of $\sim 30 h^{-1}$ Mpc. In most of the above studies, additional uncertainty on the results is given by the different weighting schemes which can be adopted within the given sample, and by the different methods for taking into account missing data in the Galactic plane. We will not discuss any further samples (v) and (vi), since both of them give too dilute information with respect to that contained in (iv) and (i) or (ii), respectively.

2.2. Intrinsic Dipole Behavior with Depth

To fix the notation, we define the monopole, $\mathcal{M}(R)$, and the dipole, $\mathcal{D}(R)$, to be the following quantities:

$$\mathcal{M}(R) \equiv \sum_{i=1}^{N(r \leq R)} \frac{w_i}{r_i^2}; \quad \mathcal{D}(R) \equiv \sum_{i=1}^{N(r \leq R)} \frac{w_i \hat{r}_i}{r_i^2}. \quad (1)$$

The quantity w_i is the weight associated to the i th object, and the ratio

$$\mathcal{R}(R) \equiv \frac{\mathcal{D}(R)}{\mathcal{M}(R)} \quad (2)$$

gives the *observed* fractional anisotropy of the distribution of objects within the distance R (notice that the present dipole definition differs by a factor of 3 from that used by other authors and is such to give 100% anisotropy in the presence of

a single object). The continuous counterpart of equation (1) is given by

$$\mathcal{M}(R) = \int_{V(r \leq R)} d^3r n(r) w(r) \frac{1}{r^2} \\ \mathcal{D}(R) = \int_{V(r \leq R)} d^3r n(r) w(r) \frac{\hat{r}}{r^2}, \quad (3)$$

where $n(r)$ is the number density field.

In linear theory, the *full* peculiar velocity of the LG, taken to be at the origin of the coordinate system, is given by (Peebles 1980)

$$\mathbf{v}_p^\infty = H_0 \Omega_0^{0.6} \frac{1}{4\pi} \int d^3x \delta(\mathbf{x}) \frac{\hat{\mathbf{x}}}{x^2}, \quad (4)$$

where $\Omega_0 = \rho_b/\rho_c$ is the present density in terms of the critical value, $\delta(\mathbf{x}) \equiv [\rho(\mathbf{x}) - \rho_b]/\rho_b$ is the fractional matter density perturbation, and $\mathbf{v}_p^\infty$ denotes the asymptotic value of the peculiar velocity, given by integration over the whole space (actually, over a volume large enough to assure the convergence of the integral in eq. [4], but much smaller than the horizon volume since the simple formula above does not contain relativistic effects). The above equation is especially important since, in principle, it can yield the value of Ω_0 , because both the modulus, $\sim 620 \text{ km s}^{-1}$, and the direction of $\mathbf{v}_p^\infty$ are known from the observed dipole of the cosmic microwave background (CMB) radiation, while the integral can be evaluated from astronomical data. Equation (4) has often been rewritten and estimated as

$$\mathbf{v}_p^\infty = \frac{H_0 \kappa}{\langle n \rangle 4\pi} \sum_{i=1}^N w_i \frac{\hat{r}_i}{r_i^2} \equiv \kappa \mathcal{B}, \quad (5)$$

where $\kappa \equiv \Omega_0^{0.6}/b$, and $\langle n \rangle$ is the average number density of objects in the sample. This form makes the assumption that the set of objects under study traces to a given degree the unknown mass distribution. More precisely, it is assumed that $\langle n \rangle \langle m \rangle = \rho_b$, and that there is a constant scaling between number density fluctuations and matter density fluctuations, i.e., a linear bias such that $[n(\mathbf{x}) - \langle n \rangle]/\langle n \rangle = b \delta(\mathbf{x})$.

As noted first by Vittorio & Juszkiewicz (1987), there is a potential problem in equation (5), which can seriously affect some of the conclusions drawn by its use in the literature. This problem arises when the effective depth of the data at hand, R_{eff} , is smaller than the intrinsic convergence depth, R_{conv} , which can be operationally defined such as $\mathbf{v}_p(R \geq R_{\text{conv}}) \cong \mathbf{v}_p^\infty$ (unprimed velocities refer to the true, intrinsic ones). When this is the case, it may happen that the measured $\mathcal{B}$ is different from $\mathcal{B}^{(\text{true})}$. Now, if it unfortunately happens that there is a fair directional agreement between $\mathbf{v}_p^\infty$ and the measured $\mathcal{B}$, one can be erroneously led to conclude that convergence has been reached and so to derive the value $\kappa' = |\mathbf{v}_p^\infty|/|\mathcal{B}|$. Since $|\mathcal{B}| \neq |\mathcal{B}^{(\text{true})}|$, this conclusion then has the possible following consequences: (i) an over/underestimate of the true value of κ [we think that some of the previous literature (SD88; Y88; LLB89; RR90) gave an overestimate], which then leads to an over/underestimate of the value of Ω_0 for a given b , and to an underestimate of the length R_{conv} . This in turn has important consequences in testing cosmological models which have a different expected degree of spatial coherence in cosmic bulk flows (Vittorio & Juszkiewicz 1987; Juszkiewicz et al. 1991; Lahav et al. 1991).

Therefore, it is always necessary, for any sample under analysis, to *prove* that convergency has been reached, in order to derive meaningful results from this method. Obviously, in an asymptotically homogeneous and isotropic universe, one expects to find a convergence, not because of the inverse distance square weight (one finds again Olbers' paradox), but because of the symmetry in the distribution of objects, when spatial fluctuations are small compared to the width of the distant spherical shells. In fact, it is easy to see that, when $n(r) \rightarrow \langle n \rangle$ for $R > R_{\text{conv}}$, one has that the peculiar velocity due to distant shells becomes negligible by the symmetry of the angular integral: $\Delta v_p \simeq \langle n \rangle \int_{R > R_{\text{conv}}} dr r^2 \int_{4\pi} d\omega, \hat{r} \simeq 0$. Notice that, by the same means, one expects the monopole from equation (3) to grow linearly with distances for $R > R_{\text{conv}}$, $\mathcal{M}(R) \rightarrow 4\pi \langle n \rangle R$. This trend has been verified by us in our analysis of the clusters dipole (see Figs. 1 of Paper II and Scaramella et al. 1990b). It may also be of interest to notice that, if the clusters had followed a pure fractal distribution of dimension D , $n(r) \sim r^{-D}$, as advocated by some authors (e.g., Coleman & Pietronero 1992), one would have found $\mathcal{M}(R) \rightarrow \text{constant } R^{1-D}$, which is in contrast with the observed linear behavior for the monopole.

For the analysis of the peculiar velocity it is more direct to consider $\mathcal{R}(R)$, by rewriting equation (5) as

$$\frac{v_p(R)}{v_H(R)} = \kappa \mathcal{R}(R), \quad (6)$$

where $v_H(R) = H_0 R$ is the Hubble velocity at depth R . Moreover, since for different tracers one expects different bias values and, therefore, different values for κ , in order to make direct comparisons among different tracers it is better to leave aside the factor κ and to study the quantity

$$\psi(R) \equiv v_H(R) \mathcal{R}(R). \quad (7)$$

We present in Figure 1 results which appeared in the literature for $\psi \equiv [\psi(R)]$ obtained from different tracers. In the left panel we show results from Abell clusters (from Papers I and II; a similar analysis by PV91 gives similar results up to $\sim 200 h^{-1}$ Mpc, then their sample suffers of the aforementioned depth incompleteness). The $\psi(R)$, plotted here from direct computation of $\mathcal{R}(R)$, is the average between the values obtained by weighting the clusters with the inverse of the angular selec-

tion function, and with or without mass weighting (see Paper II). Notice that this sample is volume limited up to $\sim 300 h^{-1}$ Mpc, and therefore can probe depths which presently are not reached by other samples. Another good property of this sample is that the selection function ϕ is always greater than 0.5, to be compared with values $\phi \ll 1$ which are typically used in the analysis of magnitude limited samples of galaxies (see further discussion in the next section). The main problems for this sample are the fact that clusters are relatively rare objects, so that they fail to represent satisfactorily our surroundings, the lack of an objective procedure in selecting the original samples (Abell and ACO catalogs), and the fact that clusters are strongly correlated and therefore highly biased with respect to the matter distribution. The convergence shown at $\lesssim 180 h^{-1}$ Mpc is real and it is not due to depth incompleteness, since the monopole is still linearly growing beyond that depth (see Paper II). The zero point, however, is subject to the uncertainty due to the sparseness of this sampling. In fact, the observed almost 100% relative anisotropy at $\sim 30\text{--}40 h^{-1}$ Mpc is obviously unrealistic (e.g., think of the absence of Virgo from the original sample) and it is simply due to the lack of these objects in our immediate surroundings (see the discussion following Paper I).

In the middle panel we show data for optical galaxies kindly supplied from O. Lahav, and which have been published in the analysis of LLB89. From the raw data of dipole and monopole we derive and plot three different curves for ψ . The bottom curve (*open hexagons*) is obtained from the unweighted data, the topmost one (*filled hexagons*) is obtained from the data with the correction of the "cloned" sky over the Galactic plane (see LLB89), and the middle one (*triangles*) is the average of the other two (see also PV91). We think that the last is the best one to consider since the cloning procedure yields an over-correction (the final anisotropy doubles because of such procedure), and the uncorrected data do not take into account the Galactic extinction and underestimate the final signal, which happens to be mainly due to features at low Galactic latitudes. The vertical dashed line denotes the maximum effective depth of this sample according to LLB89: beyond that distance the galaxy monopole $\mathcal{M}$ flattens and this causes the unrealistic steepening of the three curves. Since detailed redshift information is not available for this sample, distances were estimated through a diameter function (a recent dis-

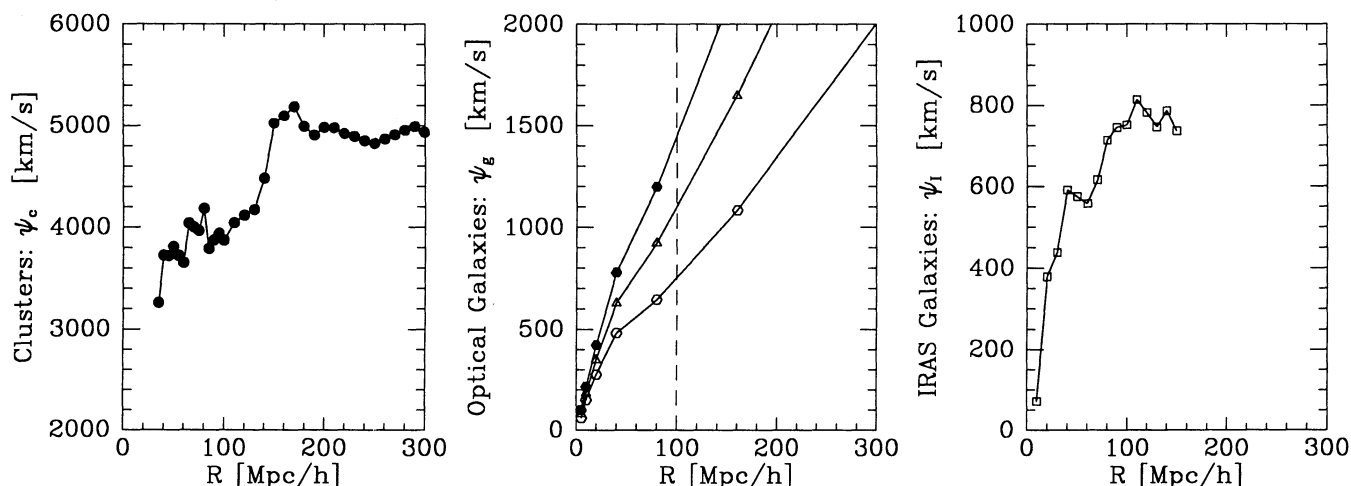


FIG. 1.— ψ -values as a function of depth for three different kinds of objects: clusters (left panel), optical galaxies (middle panel), and IRAS galaxies (right panel)

cussion on this point is given by Bardelli et al. 1991 and Hudson & Lynden-Bell 1991). The intrinsic uncertainties in this estimate, added to the logarithmic sampling in distance, causes these curves to be almost featureless, when confronted with those in the other panels. It must be recalled that from these data LLB89 concluded that convergence was essentially reached by the depth of $\sim 40 h^{-1}$ Mpc, although all three curves still increase significantly at deeper distances. Such a conclusion suggested a large value of κ (and of Ω_0), and implied that mass fluctuations beyond that depth ("Great Attractor" [GA] region included!) were small, since they did not influence the motion of the LG.

In the right panel we show the results from a study of a sparse sample of IRAS galaxies (RR90). In the literature it has appeared also a previous result from a complete sample of IRAS galaxies, but with a higher limiting flux (Y88, DS88). In this panel it is important to notice the temporary flattening or even decrease of the ψ curve beyond $40 h^{-1}$ Mpc. This decrease happens when one encompasses in the computation the gravitational pull of the Pe-Pi region, which is almost opposite and slightly farther from us than the GA region, which in turn is mostly responsible for the signal in the inner regions. A similar flat behavior is present at the same distances also in the cluster distribution, although to a lesser extent here, since for completeness purposes we cut the cluster sample at $|b| = 20^\circ$, therefore losing the Pi contribution (cf. Paper II). While Y88 and Strauss & Davis (1988) claimed convergence and stated that the LG peculiar velocity was essentially of local origin (within $\sim 40 h^{-1}$ Mpc, as subsequently advocated also by LLB89), RR90 pushed deeper ($\sim 100 h^{-1}$ Mpc) the convergence depth, supporting further our previous findings of a significant nonlocal contribution (Paper I). It must be noticed that also the RR90 sample becomes less and less believable beyond $100 h^{-1}$ Mpc, due to the sparseness of their sampling; beyond that distance there are fewer galaxies in their sample than there are rich clusters in the sample of Abell-ACO clusters we examined (see discussion below in § 3.2).

It is also interesting to note that the study of Shaya et al. (1992) maps well the nearby region up to $30 h^{-1}$ Mpc, and that these authors find two results of interest here. First, they find that the inner region does not contain enough anisotropy to bring in agreement the acceleration vector from the inner zone with that derived from the LG peculiar velocity, and that there is the need for more distant sources which provide part of the final dipole, implying a nonlocal dipole convergence. These main contributors have been identified through our maps of clusters (Zamorani et al. 1991), and with their inclusion the final result seems to be satisfactory. Second, the direction of the LG peculiar velocity due to structures within $30 h^{-1}$ Mpc is in good agreement with the values given by RR90. This agreement shows the stability of the results from the inner to the outer regions over a radial range up to at least $80\text{--}100 h^{-1}$ Mpc.

2.3. Matching

As we discussed earlier, if the different astronomical objects used in different analyses were all perfect tracers of the mass distribution, the derived ψ should be identical. Since this is not the case, we will try to maximize the information presently at our disposal by joining the cluster information with the one obtained from the other samples.

We will assume that the simple linear biasing (Kaiser 1984; Bardeen et al. 1986) of number density fluctuations holds

throughout and specifically that, if b_{Ap} and b_{Bp} are the biasing factors of objects of type A and B with respect to the matter (i.e., $\xi_{AA} \simeq b_{Ap}^2 \xi_{pp}$, $\xi_{BB} \simeq b_{Bp}^2 \xi_{pp}$), with $b_{Ap} > b_{Bp}$, the following relation holds: $b_{Ap} \sim b_{AB} b_{Bp}$, where b_{AB} is the biasing factor of objects of type A with respect to those of type B (cf. Lahav, Nemiroff, & Piran (1990b)).

In what follows we will consistently normalize our results to the bias of IRAS galaxies with respect to matter, b_{Ip} . We will adopt for the bias of clusters with respect to optical galaxies the value of $b_{cg} \simeq (r_{0,c}/r_{0,g})^{1/2} \simeq 3$, where $r_{0,c} \simeq 20 h^{-1}$ Mpc (Postman, Huchra, & Geller 1992) and $r_{0,g} \lesssim 6 h^{-1}$ Mpc (Davis & Peebles 1983) are their correlation lengths, respectively, and $\gamma \simeq 1.8$. On the basis of the same argument, we will also adopt for the biasing of optical galaxies with respect to the IRAS galaxies the value $b_{gI} \sim 1.3$ (obtained as the average of the value $b_{gI} \sim 1.4$ by Lahav et al. 1990b and the value $b_{gI} \sim 1.2$ by Babul & Postman 1990), so that $b_{cI} \approx 4$ follows. One must bear in mind, however, that all these b -values are still pretty uncertain, and that this uncertainty reflects in the following discussion.

As we mentioned above, the zero point of the cluster dipole is not well determined because of the lack of nearby clusters (Paper I). In the following we try two different solutions to this problem, which give similar results.

The first method consists in vectorially rescaling and normalizing the cluster $\psi(R)$ to that of RR90 at $R_* = 100 h^{-1}$ Mpc (both samples were corrected for incompleteness in the same way, the only difference being the limiting Galactic latitude, which differs by 10°). In formulas, where henceforth primed values are rescaled to IRAS galaxies, we obtain

$$\psi'_{c,k}(R) = b_{cI}^{-1} [\psi_{c,k}(R) - \psi_{c,k}(R_*)] - \psi_{I,k}(R_*), \quad (8)$$

where $k = 1, 2, 3$ denotes the Cartesian component. Since the deeper part of the cluster dipole is very well aligned with the inner part, the final direction of ψ'_c differs little from that of $\psi_I(R_*)$ ($\sim 30^\circ$ from the CMB dipole). We plot in the left panel of Figure 2 the RR90 ψ as squares, the rescaled optical average, $\psi'_g = \psi_g/b_{gI}$, as triangles, and ψ'_c as solid dots. The first thing to notice is that ψ'_g at its deepest point at $80 h^{-1}$ Mpc practically coincides with the IRAS level. Then, by looking to ψ'_c one sees that also in the inner region ($R < R_*$), where the procedure of equation (8) is not supposed to work at its best, there is a fair similarity in shapes and level between ψ'_c and ψ_I , with ψ'_c being slightly higher than ψ_I , as expected. On the other side, the behavior is different beyond $120 h^{-1}$ Mpc. While the cluster signal still rises up to $\lesssim 180 h^{-1}$ Mpc and then flattens off, the IRAS signal quickly dies. We think that this discrepancy is essentially due to the lack of adequate deep data in RR90 (see discussion in § 3.2 below). The rise seen in the cluster sample is largely due to the presence of the very large supercluster known as the "Distant Centaurus Supercluster" or the "Shapley Supercluster" to honor the astronomer who made pioneering studies of this sky area (we will follow the latter denomination). This supercluster has the peculiarity of being aligned with the GA region, causing their combined gravitational pull to add in the most effective way (Scaramella et al. 1989 and Paper I). Because of their sparse sampling at great depths [the selection function is $\phi(r) \approx 5 \times 10^{-4}$ at $r \sim 150 h^{-1}$ Mpc; see Fig. 3 below], the Shapley Supercluster does not show up significantly in the RR90 data, although it is obviously one of the largest overdensities around us: in fact this appears as very remarkable in rich optical clusters (Scaramella et al. 1989; Vettolani et al. 1990), in X-ray clusters (Lahav et al.

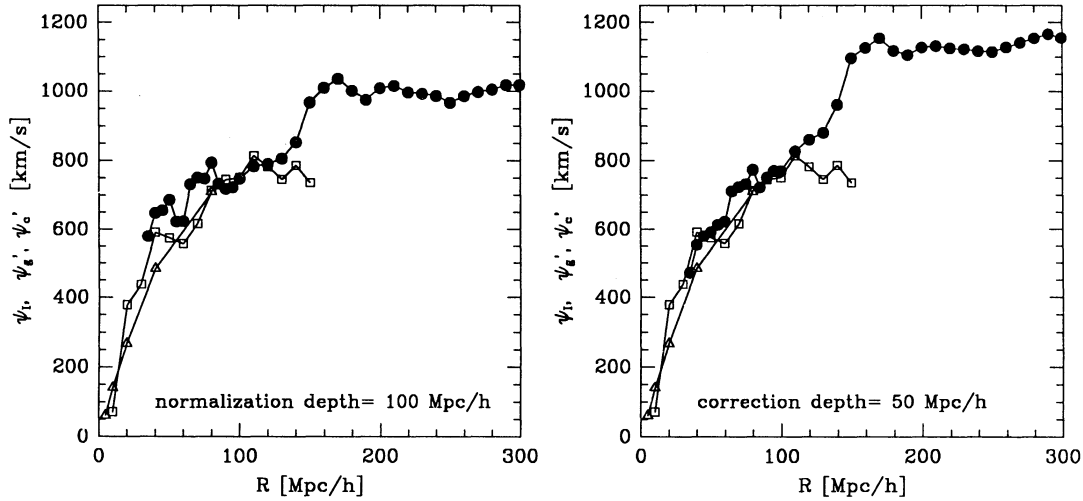


FIG. 2.—Same curves as in Fig. 1 (same symbols) after having been rescaled to each other for the assumed biasing values (see text). The zero-point of the cluster curve has been corrected according to eq. (8) in the left panel, and according to eq. (9) in the right panel.

1989), in optical galaxies (Raychaudhuri 1990), and also in *IRAS* galaxies from a different sample (Allen et al. 1990).

The second method we used to put clusters and galaxies' dipole on the same scale consists in the addition of a constant correction term to the cluster monopole in the inner region, in order to avoid starting with 100% anisotropy at $\sim 40 h^{-1}$ Mpc. We normalized this correction so to have the same rescaled anisotropy level as the RR90 data at a distance of $50 h^{-1}$ Mpc, and therefore we implemented the following substitution,

$$\mathcal{R}_c(R) = \frac{\mathcal{D}_c(R)}{\mathcal{M}_c(R)} \rightarrow \mathcal{R}_c^*(R) = \frac{\mathcal{D}_c(R)}{[\mathcal{M}_c(R) + \Delta\mathcal{M}_c]}, \quad (9)$$

where $\Delta\mathcal{M}_c$ is fixed by the requirement that $|\mathcal{R}_c^*(50 h^{-1} \text{ Mpc})| = b_{cl} |\mathcal{R}_l(50 h^{-1} \text{ Mpc})|$. The correction $\Delta\mathcal{M}_c$, which is $\sim 65\%$ of the $\mathcal{M}_c$ value at $50 h^{-1}$ Mpc, has a significant effect only in the inner zone, because of the rapid increase of $\mathcal{M}_c$ with distance, and it yields good results over the whole overlap distance range of cluster and *IRAS* data. This good agreement can be easily seen in the right panel of Figure 2, where we plot $\psi'_c = |\mathcal{R}_c^*|/b_{cl}$. Also in this case the final direction of the cluster anisotropy ($\sim 20^\circ$ from CMB dipole; see Paper II) is almost unaffected by the zero-point correction [here we did not use the extrapolation across the galactic belt through spherical harmonics as discussed by Lahav 1987, Plionis 1989, and Scharf et al. 1992; to do so would slightly increase the final anisotropy estimate (by $\sim 10\%$; see end of Paper II), since the final dipole direction is at low Galactic latitude; the effects of subtraction of “randoms” and of a lower cut in Galactic latitude for the cluster sample are shown in Fig. 1c of Paper II].

From this analysis we conclude that:

- (i) Given all the uncertainties, linear theory and the constant bias factors assumed from correlation functions seem to work well in describing the scaling in anisotropy amplitudes of different tracers;
- (ii) Even after the zero point corrections to the cluster dipole there is still evidence that there is non-negligible anisotropy signal up to $\lesssim 180 h^{-1}$ Mpc;
- (iii) By averaging the results of the two above corrections and taking into account the $\sim 10\%$ spread in the cluster dipole due to the different original weights, we make the prediction

that deep enough *IRAS* samples should find a final level of $\psi'_l \simeq (4/b_{cl}) 1100 \pm 150 \text{ km s}^{-1}$, which is roughly twice the value at $40 h^{-1}$ Mpc that was suggested to be the relevant one by Y88, SD88, and LLB89;

(iv) From ψ'_l and equation (6) we have that $\kappa_l \simeq 0.55$, which implies $\Omega_0 \sim 0.4$ if *IRAS* galaxies trace the mass, or $\kappa_g = \kappa_l/b_{gl} \gtrsim 0.4$ and $\Omega_0 \gtrsim 0.2$ if optical galaxies trace the mass. This result points to an open universe, unless both populations are strongly biased with respect to the matter distribution: $b_{gp} \simeq 2.4$ and $b_{lp} \simeq 1.8$ would be needed in order to have $\Omega_0 \simeq 1$. We want to stress, however, that uncertainties on the detailed numerical values are likely to be larger than the nominal $\sim 15\%$ on bias and $\sim 25\%$ on Ω_0 .

3. SIGNAL VERSUS SHOT NOISE UNCERTAINTIES AND FUTURE GALAXY SAMPLES

We now briefly discuss the significance of the signal of the deep anisotropy. Both for completeness and the benefit of the reader we recall in the first subsection material which has been presented elsewhere (Scaramella et al. 1993). In the following one we will discuss how faint an *IRAS* galaxy flux limited catalog has to go in order to detect at large distances a signal of the same level as that detected from clusters.

3.1. Significance of the Cluster Deep Signal

Let us now discuss in detail the shot noise aspect. It is convenient to rewrite equation (1) as $\mathcal{D} \equiv \sum_k \mathbf{d}_k$, and $\mathcal{M} \equiv \sum_k \mathbf{d}_k$. A major point to be considered here is the direction of the vectors. Indeed, as we have repeatedly stressed, a key point is the remarkable alignment of surrounding structures which causes a bootstrap effect and a not negligible large-scale anisotropy. It is useful to define three different volumes (A, B, C) in such a way that $\mathcal{D}_{\text{tot}} = \mathcal{D}_A + \mathcal{D}_B + \mathcal{D}_C$, where $\mathcal{D}_A \equiv \mathcal{D}(R \leq R_1)$, $\mathcal{D}_B \equiv \mathcal{D}(R_1 \leq r \leq R_2)$, and $\mathcal{D}_C \equiv \mathcal{D}(r \geq R_2) \simeq 0$, with $R_1 = 60 h^{-1}$ Mpc, and $R_2 = 180 h^{-1}$ Mpc. Notice that $\mathcal{D}_B$ is the signal from more than 220 clusters and only because it is almost parallel to $\mathcal{D}_A$ gives a contribution to $|\mathcal{D}_{\text{tot}}|$ almost equal to its modulus. Let us now compute the probability of the null hypothesis that $\mathcal{D}_B$ is drawn from a field realization which has no “true” signal beyond $60 h^{-1}$ Mpc, i.e. with $\langle \mathcal{D}_B \rangle = 0$, as advocated by Y88, SD88, and LLB89. We expect the shot noise vector, $\mathcal{D}_B^{\text{shot}}$, to have its three Cartesian com-

ponents, $\mathcal{D}_{B,i}^{\text{shot}}$, normally distributed with zero average and the same dispersion $\Sigma_{\mathcal{D}}$, with

$$3\Sigma_{\mathcal{D}}^2 \equiv \langle \mathcal{D}_B^{\text{shot}} \cdot \mathcal{D}_B^{\text{shot}} \rangle \Rightarrow \sum_k^{(R_1 \leq r_k \leq R_2)} \left(\frac{w_k}{r_k^2} \right)^2 = N_{c,B} \langle d^2 \rangle_B, \quad (10)$$

where averages are taken on all those observers who study the same intrinsic configuration of the cluster density field, $n_c(x)$, but observe different $N_c(x_i)$ Poisson “draws” from such field configuration (see Kaiser & Lahav 1988). Now, since we assumed as our null hypothesis $\langle \mathcal{D}_B \rangle = 0$, the shot noise contribution can be directed anywhere. For instance, it could be orthogonal to $\mathcal{D}_A$ or even antiparallel to it, so that $\mathcal{D}_{\text{tot}} < \mathcal{D}_A$. On average we expect a projected contribution along the direction of $\mathcal{D}_A$ of the order of $\pm \Sigma_{\mathcal{D}}$. What we have to compute is the probability that $\mathcal{D}_B^{\text{shot}}$ is such to have a modulus at least as large as that of the observed $\mathcal{D}_B$, and to have a direction within θ degrees from $\mathcal{D}_A$. Numerically we have $\Sigma_{\mathcal{D}} \simeq 5.1 \times 10^{-8} (\text{km s}^{-1})^{-2}$, $|\mathcal{D}_B| \simeq 1.6 \times 10^{-7} (\text{km s}^{-1})^{-2}$. If we normalize such that $\chi_3^2 \equiv |\mathcal{D}_B^{\text{shot}}|^2 / (3\Sigma_{\mathcal{D}}^2)$ we then have $P(\chi_3^2 \geq 3.3) \simeq 2 \times 10^{-2}$. Now, if we very generously take $\theta = 30^\circ$ (in reality it is $\theta \sim 20^\circ$), then the probability of $\mathcal{D}_B^{\text{shot}}$ being within θ degrees from $\mathcal{D}_A$ is given by the ratio of the cone solid angle to that of the whole sphere, that is $P' = [1 - \cos(\theta)]/2 \simeq 7 \times 10^{-2}$. The final result is therefore a very low probability for the null hypothesis: $P_T = P \cdot P' < 2 \times 10^{-3}$. This probability has also been verified by explicit computation on “random” samples which take into account the catalogs biases and differences (see Scaramella et al. 1990). We can then conclude that the observed $\mathcal{D}_B$ signal for clusters is quite unlikely to be spurious and due only to shot noise effects.

3.2. Sensitivity to the Catalog Limiting Flux

Another interesting question is what is the limiting magnitude that an all-sky galaxy catalog with redshifts should reach in order to confirm or disprove the deep anisotropy found in the cluster distribution. This means to test for the presence in the deep region, i.e., region B above, of a dipole with the same amplitude as that shown in Figure 2. In this region we expect an anisotropy level such to give $\lesssim v_p^2/2$. This means that we expect a value of $\mathcal{R}_{12} = \psi_{12}/[H_0(R_2 - R_1)] \approx 500/12,000 = 4 \times 10^{-2}$. A similar expected value of $\sim 4 \times 10^{-2}$ is obtained by considering the even deeper shell with $R_1 = 100$ and $R_2 = 180 h^{-1} \text{ Mpc}$.

In order to test this level of anisotropy it is required to beat down the shot noise effects in such a way that $v \equiv \mathcal{R}_{12}/\Delta\mathcal{R}_{12}^{\text{shot}} \sim$ a few. As a concrete example, let us consider IRAS galaxies, for which Yahil et al. (1991) and Strauss et al. (1991) derived the following radial incompleteness function:

$$\phi(r) = Ar^{-2\alpha} \left[1 + \left(\frac{r}{r_*} \right)^2 \right]^{-\beta}, \quad (11)$$

where $\alpha \sim 0.55$, and $\beta \sim 1.8$, are derived from the intrinsic luminosity function, and $r_* = r_*^0 \sim 45 h^{-1} \text{ Mpc}$ is a characteristic depth, which is a function of the limiting flux of the sample ($f_{\text{min}}^0 = 2 \text{ Jy}$) and of the adopted minimum threshold in intrinsic luminosity. The normalization is such that $\phi(r \leq 5 h^{-1} \text{ Mpc}) = 1$, and $n_1 \simeq 0.045 (h^{-1} \text{ Mpc})^{-3}$ is the number density, so that $n(r) = n_1 \phi(r)$. This selection function is usually used in order to weight the distant galaxy contributions, typically as $w(r) = 1/[\phi(r)]$, so to take into account also the unseen galaxies at distance r . Obviously one should be aware of using too small values of ϕ , because of the uncertainties induced by large correction factors. A sensible and generous rule of thumb could be

that of limiting the sample depth at R_{max} such that $\phi(R_{\text{max}}) = 0.1$ (cf. Davis & Huchra 1982 in their study of the first CfA sample). A still rather conservative option would be to roundly set $R_{\text{max}} = r_*$, so that one has the value $\phi_{\text{min}} \simeq \phi_1 \equiv 2.5 \times 10^{-2}$, i.e. the ratio of unseen galaxies to the tracer is at most $w = 40$. In the following we will consider also the rather low value $\phi_{\text{min}} = \phi_3 \equiv 4 \times 10^{-3}$ ($w \leq 2500$), which was used by Y88 and SD88 by their having $R_{\text{max}} = 80 h^{-1} \text{ Mpc}$ with $r_* = r_*^0$, and the intermediate value $\phi_{\text{min}} = \phi_2 \equiv 1 \cdot 10^{-2}$ ($w \leq 100$).

As an example, we show in Figure 3 the selection function $\phi(R)$ for the Y88 and SD88 sample (solid line), for the new fainter sample with $S_{\text{lim}} = 1.2 \text{ Jy}$ (Fisher et al. 1992; dashed line) and for the sparse sample of RR90 with $S_{\text{lim}} = 0.6 \text{ Jy}$ (dotted-dashed line). The next five curves (short dashed lines) have been obtained by lowering the limiting flux in half a magnitude steps starting from $S_{\text{lim}} = 1.2 \text{ Jy}$. The three horizontal lines show the three values of ϕ_{min} discussed above. It is immediate to see that, in order to be able to map the galaxy distribution to a depth of $180 h^{-1} \text{ Mpc}$, one would need a sample fainter by more than one magnitude with respect to the best current sample, even using the lowest value $\phi_{\text{min}} = \phi_3$. In this simple-minded approach (one, in fact, can use different weighting schemes, as well as sparse sampling strategies), we proceed as follows to derive more detailed information on this point. From equation (10) it is easy to see that the shot noise error on the quantity $\mathcal{R}$ is given by

$$(\Delta\mathcal{R}_{12}^{\text{shot}})^2 \equiv \langle \mathcal{R}_{12}^{\text{shot}} \times \mathcal{R}_{12}^{\text{shot}} \rangle = \frac{\sum_k (w_k/r_k^2)^2}{[\sum_k w_k/r_k^2]^2} = N_B^{-1} \frac{\langle d^2 \rangle_B}{\langle d \rangle_B^2}, \quad (12)$$

where N_B is the number of objects in the B region. We compute the expected number of galaxies in the B region for the three arbitrarily chosen values of ϕ_{min} above and for the same limiting fluxes or, equivalently, for the same values of r_* used for the curves in Figure 3. N_B can be written as

$$N_B(r_*) = \omega n_1 \int_{R_1}^{\tilde{R}(r_*)} dr r^2 \phi(r, r_*), \quad (13)$$

where $\tilde{R}(r_*) = \min(R_{\text{max}}, R_2)$ and ω is the sample solid angle. Similarly, we have

$$\begin{aligned} \langle d_B^2(r_*) \rangle &= \frac{\omega}{V_e(r_*)} \int_{R_1}^{\tilde{R}(r_*)} \frac{dr}{r^2} \phi(r, r_*) \\ \langle d_B(r_*) \rangle^2 &= \left[\frac{\omega}{V_e(r_*)} \int_{R_1}^{\tilde{R}(r_*)} dr \right]^2, \end{aligned} \quad (14)$$

where $V_e(r_*) = \omega \int_{R_1}^{\tilde{R}(r_*)} dr r^2 \phi(r, r_*)$ is the sample effective volume. In Figure 4 we show $\Delta\mathcal{R}_{12}^{\text{shot}}$ for the B zone. The solid curves show $\Delta\mathcal{R}_{12}^{\text{shot}}$ as a function of r_* for the three values of ϕ_{min} chosen above (left to right: ϕ_3, ϕ_2, ϕ_1). The horizontal lines denote the noise values required to have $v = 1, 2, 3$, while the vertical lines correspond to the same limiting fluxes as in Figure 3. The $\Delta\mathcal{R}_{12}^{\text{shot}}$ curves have a vertical asymptote at an r_* value corresponding to the limiting flux for which the ϕ_{min} cut makes no galaxies available in the examined volume for the given limiting flux. Then, when the limiting flux decreases, the number of objects increases and the uncertainty decreases. Beyond a given value of r_* , curves corresponding to two different ϕ_{min} collapse over each other. This happens when the limiting flux is low enough that the same number of galaxies in the examined volume is sampled with the two different ϕ_{min} , i.e., when for both values is $R_{\text{max}} > R_2$.

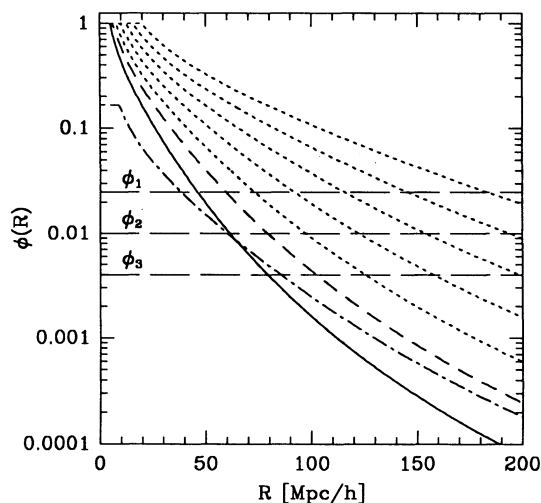


FIG. 3.—Radial selection function of flux-limited *IRAS* samples. (solid line) Y88 and SD88 sample; (dot-dashed line) sparse sample of RR90; (dashed line) sample described in Fisher et al. (1992); (dotted lines) samples with limiting flux decreased in half a magnitude steps. The three horizontal long-dashed lines denote the values of $\phi_{\min}$ discussed in the text.

From Figure 4 we see that the new, ongoing *IRAS* sample (long-dashed vertical line) has $\nu \sim 1.5$ for $\phi_{\min} = \phi_3$ and $\nu \sim 1$ for $\phi_{\min} = \phi_2$, so we expect that, differently from similar but more myopic samples, this new sample may find, although still at a low level of significance, some evidence for anisotropy in the far zone. However, since from our cluster analysis it is clear that most of the action in the volume B happens for distances greater than $100 h^{-1}$ Mpc, it is interesting to divide this volume in a first region with $R_1 = 60$ and $R_2 = 100 h^{-1}$ Mpc, which is examined in the left panel of Figure 5, and a second deeper region, with $R_1 = 100$ and $R_2 = 180 h^{-1}$ Mpc, shown in the right panel of the same figure. For $\phi_{\min} = \phi_2, \phi_3$, we see that the expected noise is equal or smaller than the expected signal (i.e., $\nu \gtrsim 1$) in the first region, so that our predicted

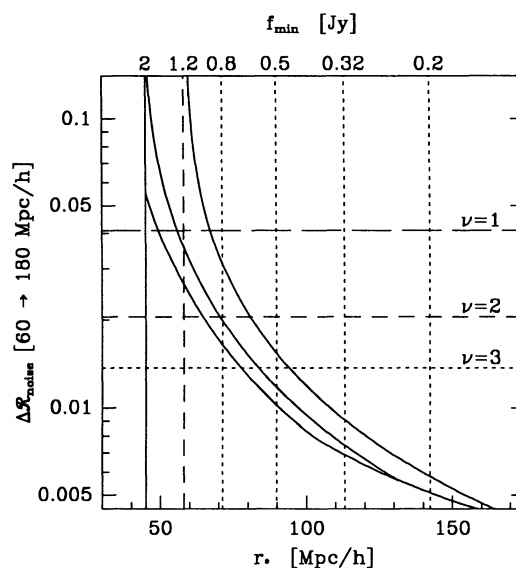


FIG. 4.—Shot noise uncertainty as a function of the depth of an *IRAS* galaxy sample for the far region beyond $60 h^{-1}$ Mpc. The expected signal ($\nu = 1$) is shown by the horizontal long-dashed line ($\nu = 2$ is given by the horizontal dashed line, $\nu = 3$ is given by the horizontal dotted line). The vertical lines indicate the depth of the samples shown in Fig. 3 (same symbols), while the three solid curves show the effect of the three different choices of the $\phi_{\min}$ values described in the text (left to right: ϕ_3, ϕ_2, ϕ_1).

anisotropy on these larger scales should be found by the new *IRAS* sample, as it was in fact found with the sparse sample used by RR90, whose r_* is intermediate between the solid and dashed vertical lines. On the contrary, in the deepest region the noise is still larger than the expected signal by more than a factor of 3. So, even for $\phi_{\min} = \phi_3$ a sample limited at least half a magnitude fainter than the 1.2 Jy limit would be needed in order to reduce the noise just below the expected signal. More conservative criteria, i.e., $\phi_{\min} > \phi_3$ and/or larger ν would of course require much fainter samples.

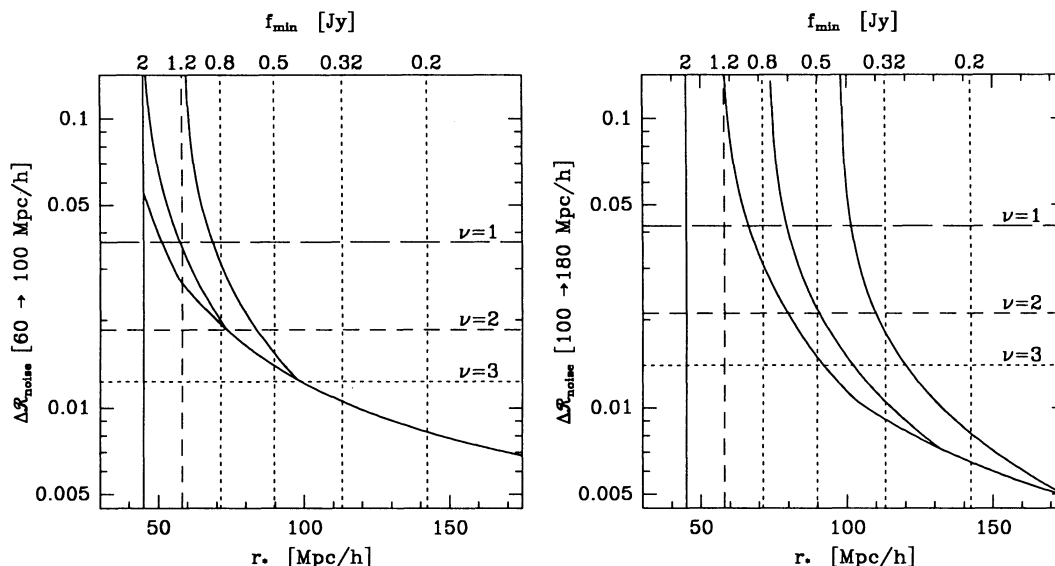


FIG. 5.—Same as Fig. 4, with the expected shot noise signal computed for the intermediate (left panel) and deep (right panel) regions, separately

4. DIPOLE IN DEPTH AND POWER SPECTRA

The LG motion with respect to the CMB frame is only a single experimental point in the problem of large-scale peculiar velocities; however, it is a point of particular interest because it is reasonably well determined and it can therefore constrain to a given extent also theoretical models for the large-scale distribution of matter. As a consequence, there have been several papers over the years on the comparison between dipole data and models, among which those by Vittorio & Juszkiewicz (1987), Kaiser & Lahav (1989), Juszkiewicz et al. (1990, hereafter JYW90), and Lahav et al. (1990a). The first of these papers pointed out that a considerable fraction of the acceleration might be missed by the first *IRAS* sample if one had a universe with significant large-scale power, the second paper made a detailed comparison of different data sets, and the last two papers gave further theoretical developments and confronted in detail cold dark matter (CDM) and isocurvature models with the galaxy data.

It is then interesting to check how some models fare in the light of the information we derived in Figure 2. Deferring a fuller discussion elsewhere, here we concentrate on a very simple comparison between theory and data, which is essentially a test of the *shape* of the power spectrum of density fluctuations. For the reader's sake we recall only the formulae of interest here, while the general formalism is described in JYW90 and Lahav et al. (1990a).

In the linear theory of large-scale velocities, a model is essentially characterized by the density fluctuation power spectrum, $P(k)$, by a normalization amplitude which also reflects into a biasing factor, and by the value of the cosmic density in terms of the critical one, Ω_0 . Also, the fluctuation field is assumed to be a Gaussian random field. Then, if one considers the ratio of the peculiar velocity (in terms of its rms value, v_a/σ_a) induced on the origin from fluctuations within a sphere of radius R_a to that from within a sphere with the same origin and larger radius R_b , one has that the distribution of this ratio, $\chi \equiv (v_a/\sigma_a)/(v_b/\sigma_b)$, is given by (JYW90, their eq. [10])

$$\mathcal{R}(\chi)d\chi = \frac{16(1 - \Gamma_{ab}^2)^{3/2}}{\pi} \frac{\chi^2(1 + \chi^2)d\chi}{[(1 + \chi^2)^2 - 4\Gamma_{ab}\chi^2]}, \quad (15)$$

where the correlation coefficient, Γ_{ab} , depends only upon the shape of the power spectrum and not on the normalization amplitude (JYW90, their eq. [11]), and is given by

$$\Gamma_{ab} = \frac{\int_0^\infty dk P(k)w(kR_a)w(kR_b)}{[\int_0^\infty dk P(k)w^2(kR_a) \int_0^\infty dk P(k)w^2(kR_b)]^{1/2}}, \quad (16)$$

where $w(x) = 1 - \sin(x)/x$ is a window function. For any given model, i.e., a given $P(k)$, it is then easy to derive the appropriate Γ_{ab} from equation [16] and the distribution $f(v_a/v_b) = \mathcal{R}(\chi)d\chi$, with $\chi' = \chi\sigma_a/\sigma_b$.

As an example, we examine now six simple spectra, which have quite different shapes, and study their predictions with respect to velocity ratios, without any concern about possible agreement with absolute values, which depend on Ω_0 and bias values. These models are two CDM models, three simple phenomenological spectra, and one "hybrid" cold plus hot spectrum. The two CDM models have parameter values $h\Omega_0 = 0.5, 0.2$, and correspondingly increasing power on large scales; the second value yields a spectrum which is in good agreement with a number of different astronomical observables (Efstathiou, Bond, & White 1991; Park 1991; Scaramella 1992; Vogeley et al. 1992; Bond et al. 1992; Bahcall & Cen 1992;

Taylor & Rowan-Robinson 1992; Fisher et al. 1993), although fares less well when confronted with the amplitude of large-scale bulk flows. Then we consider three different phenomenological "kinked" spectra (Scaramella 1992), described by a power law $P(k) \propto k^{n_1}$ on small scales up to a break at wavelength λ_k , where they suddenly change to the scale-invariant ($n_2 = 1$) slope. These two phenomenological spectra have $n_1 = -1.2, -1.4, -2$ with $\lambda_k \sim 290, 220, 150 h^{-1}$ Mpc, respectively. The n_1 -values are in turn motivated by: the observations of galaxy distribution on small scales [from the canonical slope of the two point correlation function, $\xi(r) \sim r^{-\gamma}$ and $n_1 = \gamma - 3$]; the distribution of radio (Peacock & Nicholson 1991; Peacock 1991) and *IRAS* galaxies (Saunders et al. 1992; Fisher et al. 1993); optical (Guzzo et al. 1991) and X-ray estimates (Henry et al. 1992). The break values were derived by the study of the deep part of the cluster dipole (Scaramella 1992). Another interesting physical model is the "hybrid" model "cold + hot" [CHDM]. A CHDM model with 2/3 of C and 1/3 of H confronts *COBE* observations well (Smoot et al. 1992; Wright et al. 1992). This is also in agreement with the earlier findings that the fraction of matter in the H component had to be at most 1/3 of the total in order not to delay too much the galaxy formation epoch (Achilli, Occhionero, & Scaramella 1985; Occhionero & Scaramella 1988). Indeed, the CHDM model has been reanalysed and confronted again well with more recent observations by Shaefer, Shafi, & Stecker (1989) and, after the *COBE* results, the CHDM model is recently considered with favour by linear (Taylor & Rowan-Robinson 1992; van Dalen & Shaefer 1992) and nonlinear (Davis, Summers, & Schlegel 1992; Klypin et al. 1993) models. In Figure 6 we show results of the ratio v_a/v_b for spheres of radius 40 and 80 h^{-1} Mpc in the upper panel, and 80 and 180 h^{-1} Mpc in the lower panel. The heavy long-dashed curves represent the observed values from Figure 2 assuming as minimal estimated error the uncertainty of the larger distance ($\sim 10\%$ and $\sim 15\%$, respectively). The CDM spectra are given by the dotted curves ($h\Omega_0 = 0.2$; *left dotted curve*); the CHDM is given by the dashed line, while the kinked spectra have solid lines. In the upper panel, the window function relative to the sphere of 80 h^{-1} Mpc radius is sensitive to the difference of effective slope on intermediate scales, so that different amount of power in the CDM models beyond 40 h^{-1} Mpc neatly separate the corresponding curves. Still in the upper panel, the $n_1 = -1.2, -1.4$ kinked models and the CHDM yield results very similar to the CDM $h\Omega_0 = 0.2$ model, while the best agreement is obtained for the $n = -2$ kinked model. All these models but the standard CDM have very similar behavior in the lower panel, reflecting their similar properties on the largest scales.

From Figure 6 it is clearly seen that spectra with a rather large coherence scale are preferred by the present data. Notice that the experimental value in the top panel appears fairly robust, since all three curves from Figure 2b yield the same result. The one in the lower panel, though, is due only to the "rescaled" cluster signal, so it might move toward higher values, if this rescaled signal were to be revised downward. Also, notice that we expect this experimental value to be on the left tail of the expected distribution (remember that we are considering here a single event), because of our location in an unusual alignment of large masses, as have repeatedly stressed in several occasions. It may be of interest to notice that in a recent work, Peacock (1992) predicts from theoretical considerations on power spectra that there should be more power on

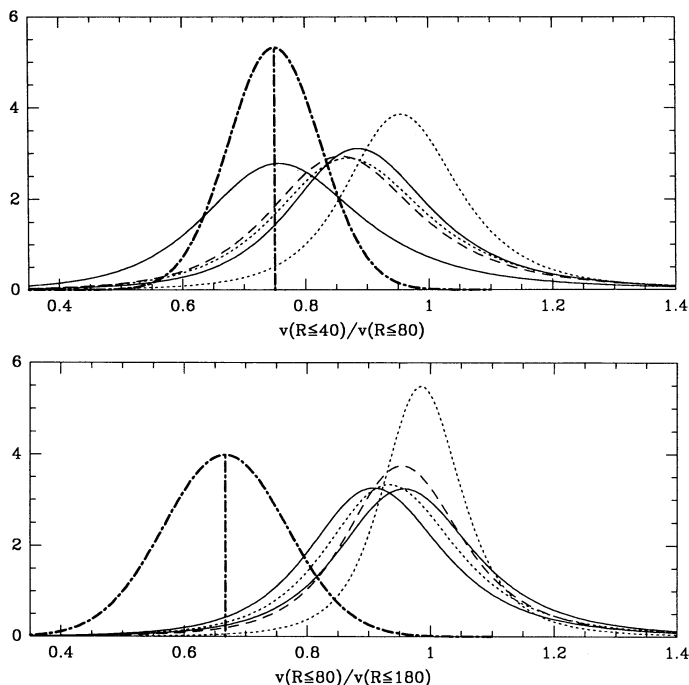


FIG. 6.—Comparison for data and models of probability density distributions of the ratio of peculiar velocities induced on the origin from nested spheres of different radii $R_a < R_b$. (Upper panel): $R_a = 40 h^{-1}$ Mpc, $R_b = 80 h^{-1}$ Mpc. (Lower panel): $R_a = 80 h^{-1}$ Mpc, $R_b = 180 h^{-1}$ Mpc. Dotted-dashed curves denote values obtained from Fig. 2; dotted curves denote CDM spectra ($h\Omega_0 = 0.2, 0.5$ from left to right); dotted-dashed curves denote CHDM; solid curves denote kinked models (upper panel: $n_1 = -2, -1.4, -1.2$ from left to right; lower panel: $n_1 = -2$ on the right).

large scales and therefore a deeper convergence than found up to now by galaxy samples. This “prediction,” although it makes no mention of them, is in remarkable good agreement with the experimental findings published earlier on this specific point (Scaramella et al. 1991a, b, c; PV91)].

5. SUMMARY

While on the one hand we feel that there is still a fair amount of uncertainty in the results and samples we examined here, on the other hand we think that previous claims of a “local” convergence of the LG peculiar acceleration, which contrasted our earlier results and relied on samples which were not deep enough, were not well founded. Indeed, these claims both assumed and implied a very smooth universe beyond the typical scale claimed for convergence, $\sim 40 h^{-1}$ Mpc, a concept hard to reconcile with the existence of known large superclusters (Postman, Geller, & Huchra 1988 found $\delta N/N \sim 10$ on $\sim 20 h^{-1}$ Mpc for the Cor-Bor supercluster) and voids (Strauss & Huchra 1988 found $\delta N/N \sim -3/4$ on $\sim 60 h^{-1}$ Mpc for the Bootes void), given the fact that gravity cannot be shielded. Moreover, in our opinion also “local” data gave early indications that part of the peculiar flow was due to larger scales, since the peculiar flow predicted by the *IRAS* sample yielded a bifurcation at the LG (Y88) in contrast with observations, and also the detailed agreement between the predicted peculiar velocities and the observed ones improved a great deal by changing the origin of the coordinates from the CMB rest frame to the LG frame (SD88), possibly implying more power originating from outside their local sample.

As we considered first, one important player in this game is the Shapley supercluster [SSC], because of both its richness

and its excellent alignment with the nearer mass concentration identified as the Great Attractor (GA). Given the fact that the presence of the SSC should not be ignored, how important a player is this particular supercluster? Its direction and hence its effectiveness because of the alignment with the GA is obviously undisputable, so the still open questions are related to quantitative data on its mass, its mean overdensity, and its size. Three years ago from the all-sky cluster distribution we had estimated (Scaramella et al. 1991a) that the contribution to the dipole from a deep volume beyond $100 h^{-1}$ Mpc was $\approx 30\%$, which can be interpreted as an upper bound to the contribution of the SSC. At the same time, Raychaudhury (1989) from a study of a galaxy catalogs in the SSC area estimated such contribution to be around $15\%–20\%$, a fraction not very different from our previous and present ($\approx 30\%$ from beyond $100 h^{-1}$ Mpc, $\approx 20\%$ from beyond $120 h^{-1}$ Mpc) estimates, since he possibly underestimated somewhat this contribution by using as reference zone an area very rich in galaxies of 17th magnitude (R. B. Tully 1992, private communication). Conversely, we may have overestimated the SSC influence because of a possible bias in the original ACO catalog: poor clusters in a rich environment may have been “promoted” to be rich by the global background correction.

It is therefore interesting to make the following simple calculation, starting from the model described in RR90, which is quite similar to the general picture we had exposed. In the latter paper the authors implemented from their sparse *IRAS* data a model of “extended” clusters (as we had argued on the basis of $\xi_{cg} \neq 0$), which yields very large masses for clusters [$\sim \mathcal{O}(10^{16}) M_\odot$], with values interestingly close to our independent estimates (Scaramella et al. 1989; Papers I and II). Albeit crude, this description seems to explain to a large extent the observed peculiar velocities. However, the RR90 model stops at a depth of $\sim 100 h^{-1}$ Mpc, and therefore the SSC is absent from it. Now, if we are conservative and, among the ~ 15 clusters of the SSC, we consider only the eight clusters which represent the core as defined by Vettolani et al. (1990), and if we express their total mass in terms of the Coma mass via rescaling of $(N_g/106)^{3/2}$ (Kaiser 1986), we get a value of $M_{\text{core}} \sim 11 \times M_{\text{Coma}}$, which is much larger than the background mass in the same volume.

Therefore, the peculiar velocity contribution from the SSC alone, which was omitted by the RR90 model, is roughly given by

$$\Delta v \approx \frac{M_{\text{core}}}{M_{\text{Coma}}} \frac{d_{\text{Coma}}^2}{d_{\text{core}}^2} \Delta v_{\text{Coma}} \approx 3 \Delta v_{\text{Coma}} \approx 180 \text{ km s}^{-1}, \quad (17)$$

where we used the value $\Delta v_{\text{Coma}} \sim 60 \text{ km s}^{-1}$ from Table 5 of RR90, $d_{\text{Coma}} \approx 70 h^{-1}$ Mpc, and $d_{\text{core}} \approx 140 h^{-1}$ Mpc. The value estimated through equation (17) is (again !?) $\sim 25\%$ of the final value derived by RR90, and therefore it cannot be neglected. Moreover, in the above estimate we did not include the other clusters outside the core region nor did we weight the number of clusters with the inverse of the cluster selection function, which would tend to further increase this estimate, since the SSC is located at low Galactic latitude. Therefore, this simple estimate also clearly shows the need for samples at least as deep as $\sim 200 h^{-1}$ Mpc in the study of masses which cause the LG peculiar motion. Also, the close alignment between different structures we pointed out (Scaramella et al. 1989) is likely to be responsible in part for the large coherence of the flow observed by Willick (1990) and Mathewson, Ford, &

Buchhorn (1992), as discussed by Shaya et al. (1992). A simple and very rough estimate would yield a possible bulk velocity of $\approx 300\text{--}400\text{ km s}^{-1}$ at the GA position due to the presence of the SSC: because of the presence of two aligned masses, the peculiar velocity pattern in the CMB frame should be that due to a sort of Roche lobe, still with the presence of a backinfall on the GA (Dressler & Faber 1991), but of smaller amplitude than the forward infall (i.e., the result of an “S” wave superposed to a forward bulk motion).

We have made quantitative predictions for the level of the anisotropy which should be found by deeper, all-sky galaxy samples. These predictions will be directly tested soon. A noteworthy issue is also the fair agreement obtained by the use of biasing values to rescale fluctuation levels of different tracers: indeed, when better and deeper samples are available (including cluster samples) the direct matching of dipoles will allow a clear test of biasing, complementing and improving upon results obtained by statistical means.

Note added in manuscript.—After this paper was submitted, two papers appeared in the literature which are relevant to the present discussion. Plionis, Coles, & Catelan (1993) derived again a dipole from the same data used by RR90 and compared this with the PV91 cluster data. In agreement with our above discussion, their main conclusion is that there is no compelling evidence of dipole convergence from the sample used by RR90, and that from the RR90 sample can only be derived the upper bound $\kappa_I \leq 0.6$, in agreement with the value

given at the end of § 2. More interesting is the result of Strauss et al. (1992), who present a long and detailed analysis of the new 1.2 Jy *IRAS* sample and which can be compared with our predictions. These authors still prefer a local convergence of their data at $\sim 40\text{ }h^{-1}\text{ Mpc}$, but they state that they cannot presently draw conclusions on the relevance of large scales and the issue of the convergence depth. Indeed, there are some very interesting features in their figures of the dipole behavior with depth (Figs. 3, 4, and 5). First, all their dipoles spread out and have a sharp rise at depths $\gtrsim 150\text{ }h^{-1}\text{ Mpc}$, an effect which is most probably due to shot noise and is in good agreement with our discussions of §§ 2.2 and 3.2, which call for deeper galaxy samples. Second, all of their the dipoles, however, agree in the range $\sim 130\text{--}150\text{ }h^{-1}\text{ Mpc}$, with values of $\psi \sim 1000\text{--}1200\text{ km s}^{-1}$, larger than the RR90 dipole, which flattens off at $\psi \sim 800\text{ km s}^{-1}$ beyond $100\text{ }h^{-1}\text{ Mpc}$. This level is in fair agreement with our values from our Figure 2, given the fact that the cluster value at $150\text{ }h^{-1}\text{ Mpc}$ is practically the same as the one at the convergence depth (the small “bump” between 150 and $180\text{ }h^{-1}\text{ Mpc}$ is due to the combined effects of the SSC on the one side and of the Horologium SC on the other side).

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