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CLUMPING AND MASS LOSS IN HOT STAR WINDS

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ABSTRACT

We construct a simple model for the continuum free-free or subordinate line emission from Wolf-Rayet star winds, in which full-scale, compressible, supersonic turbulence is assumed to prevail. Based on observed properties of the turbulent eddies with extrapolation by analogy with Giant Molecular Clouds, such clumpy winds could lead to a *decrease in the estimate of the mass-loss rate by a factor* $\gtrsim 3$ *compared to homogeneous winds of the same free-free or subordinate line flux*. Clumping may be important in *all* hot stars with winds.

In the particular case of the key eclipsing WR+O binary V444 Cygni, the observed difference between the higher values of $\dot{M}$ traditionally found from the radio free-free emission or the recombination flux of He I 1.08 μm assuming homogeneity, and the lower $\dot{M}$ values from dynamical arguments or polarimetry is found to be a factor $\gtrsim 3$, in agreement with the above turbulence-based factor. The former methods depend on the square of the density and hence are sensitive to clumping effects, while the latter methods are impervious to density fluctuations and are considered more reliable.

Subject headings: stars: early-type — stars: individual (V444 Cygni) — stars: mass loss — stars: Wolf-Rayet

1. INTRODUCTION

It is becoming clear that a parameter almost as basic among stars as the mass, is the mass-loss rate ($\dot{M}$). This is especially true for massive, hence luminous stars, where values of $\dot{M}$ lead to significant cumulative losses during the lifetime of the star. The epitome of such stars are probably the Wolf-Rayet (W-R) stars with $\dot{M}$ believed to lie in the range 10^{-5}–10^{-4} $M_\odot$ yr^{-1}, depending mainly on the W-R subclass, but also on the emission-line strength (e.g., Smith & Maeder 1990: their Fig. 2 and text, p. 74; but see Willis 1991). With a typical W-R lifetime of $\sim 5 \times 10^5$ yr (Maeder 1991), the total mass loss during the He-burning W-R phase alone (typically ~10% of the total lifetime) can be well over half the total initial mass. (For the Sun, the total mass lost during its main-sequence lifetime of 9×10^9 yr via the solar wind will be a mere ~0.01% of its initial mass.) Uncertainties of "only" a factor of 2 can have serious impact on massive star models (e.g., Maeder 1991) and hence on our interpretation of the related observations.

Because of their high $\dot{M}$'s, the winds of W-R stars can in practice be measured in more diverse ways than stars with lower rates (e.g., O stars have a mass-loss rate which is $\gtrsim 10$ times lower). This is especially true for the eclipsing system V444 Cygni (WN5+O6), a kind of Rosetta Stone for calibrating different techniques for determining $\dot{M}$. Although V444 Cygni is a binary, there is no reason to expect the WN5 component to behave differently from if it was single, in view of its high-speed, dense wind.

As shown recently by St.-Louis et al. (1993, hereafter S93) there is a serious discrepancy in the literature for estimates of $\dot{M}$ for the WN5 star in V444 Cygni. On the one hand, relatively low values are found (1) from dynamical arguments relating to the rate of period increase (Underhill, Grieve, & Louth 1990 found 0.4×10^{-5} $M_\odot$ yr^{-1} and Khaliullin, Khaliullina, & Cherepashchuk 1984, 1.0×10^{-5} $M_\odot$ yr^{-1}; S93's new data for the time of secondary minimum supports a value intermediate between these two) and (2) from the amplitude of linear polarization modulation (0.6×10^{-5} $M_\odot$ yr^{-1} from the double-wave orbital modulation and 0.75×10^{-5} $M_\odot$ yr^{-1} from the polarization eclipse at phase 0.5 when the O star eclipses the W-R star: S93). Note that the dynamical value is not very sensitive to the mode of mass loss (Khaliullin 1974). These two methods (dynamical and polarization) are mutually consistent, giving a mean of $\sim(0.7 \pm 0.1) \times 10^{-5}$ $M_\odot$ yr^{-1}.

On the other hand, significantly higher values of $\dot{M}$ for V444 Cygni are obtained from radio free-free (*f-f*) emission fluxes ($\sim 2.4 \times 10^{-5}$ $M_\odot$ yr^{-1} from Bieging, Abbott, & Churchwell 1982, corrected for the new lower v_∞ and lower ionization in the radio emitting region from Prinja, Barlow, & Howarth 1990; and 2–5 $\times 10^{-5}$ $M_\odot$ yr^{-1}—with preference for the former—from modeling the recombination flux of the He I 1.08 μm line by Howarth & Schmutz 1992). Errors of the radio data are typically ±0.2 dex.

Which value is correct: 0.7 or $\gtrsim 2 \times 10^{-5}$ $M_\odot$ yr^{-1}? (See S93 for a discussion of the effects on the different $\dot{M}$'s due to the adopted stellar parameters, such as v_∞, ionization structure, distance, magnitude difference ....) One potential source of deviation not fully explored so far could be related to clumpiness of the wind. A simple clump model has been proposed previously on several occasions, but generally rejected as a viable explanation for anomalies in IR/radio $\dot{M}$'s of O star winds (e.g., Barlow & Cohen 1977; Abbott, Telesco, & Wolff 1984). Clumping has also been modeled in W-R winds by Hillier (1991) using arbitrary, inhomogeneous shells. Here, we wish to model clumps in W-R winds based on viable physical grounds.

Indirect evidence for clumps in the W-R wind of V444 Cygni was suggested from the broad secondary eclipse in the IR (Cherepashchuk, Eaton, & Khaliullin 1984). The clumpiness of W-R winds is well documented observationally for a large sample of W-R stars of different subclass by Robert (1992);

Moffat et al. (1993); Robert, Moffat, & Drissen (1994); and Robert & Moffat (1994). If, as suspected by these last authors, the clumps are due to density fluctuations as a result of turbulence (especially in the outer wind, where radio and IR *f-f* emission and He I 1.08 μm line emission arise) then it is almost certain that the radio, IR, and He I 1.08 μm values give overestimates of $\dot{M}$. The reason is clear: *f-f* emission and subordinate line recombination emission are sensitive mainly to the *square* of the density (e.g., Osterbrock 1989). Thus, for a given outflow rate, clumpy structured winds will emit more *f-f* and subordinate line emission than smooth flow winds. The dynamical estimate of $\dot{M}$ from the rate of period change of V444 Cygni (unique among early-type binaries!) is independent of clumping, as is the polarization technique (electron scattering is proportional to the electron density n_e, not n_e^2); these two techniques (especially the former) are expected to be more accurate. Also, there is a trend that other W-R stars in binaries reveal $\dot{M}$'s from polarization that are below those based on radio *f-f* emission (St.-Louis et al. 1988). The same appears to apply to $\dot{M}$'s based on light-curve analysis of non–core-eclipsing WR+O binaries, in which n_e-dependent electron scattering also applies (Lamontagne et al. 1994).

In this paper, we derive a simple model to determine to what degree clumping can account for the observed difference in $\dot{M}$'s for V444 Cygni. We assume that the clumps are the consequence of supersonic turbulence in a compressible medium (Henriksen 1991) based on studies of the variable structures superposed on the wind profiles (Robert 1992; Moffat et al. 1993; Robert & Moffat 1994).

2. A SIMPLE CLUMPY WIND MODEL

Rather than attempt to derive an absolute value for $\dot{M}$, we prefer to construct a differential model, which should be less sensitive to basic, poorly known wind parameters. Thus, we compare the emission (*f-f* or subordinate line recombination emission, proportional to the square of the density) in a clumpy wind, with an extrapolated power-law spectrum based on the largest clumps (Robert & Moffat 1993), to a homogeneous, classical wind without clumps. In both cases (see Fig. 1), we assume that the radio and IR emission arises in regions of the wind between radii R_1 and R_2 that are relatively far from the central star (Willis 1983) and that hence, the density drops off relatively slowly in this region. (Even if it does not, however, it will have little consequence in our highly differential model.) Therefore we take the density (ρ) and the temperature (T) to be approximately constant in the region where the emission arises.

Thus, for the *homogeneous* wind (filling factor $f = 1$) we have the total volume ($V_{\rm tot}$), mass ($M_{\rm tot}$) and flux ($F_{\rm tot}$) for the *f-f* or subordinate line emission flux:

$$V_{\rm tot} = \tfrac{4}{3}\pi(R_2^3 - R_1^3)\,, \tag{1}$$

$$M_{\rm tot} = \rho V_{\rm tot}\,, \tag{2}$$

$$F_{\rm tot} = k\rho^2 V_{\rm tot}\,, \tag{3}$$

where k is a constant (we assume a constant temperature in the emitting region).

For the clumpy wind, with filling factor $0 < f \leq 1$, we assume that the emission arises in the same volume as above, and obeys the basic scaling law for supersonic turbulence in a compressible medium (Henriksen 1991), like that seen in giant molecular clouds (GMCs; e.g., Wolfire, Hollenbach, & Tielens 1992), which appear to have similar mass spectra as the W-R star wind clumps (Robert & Moffat 1993). Thus, for the *i*th clump:

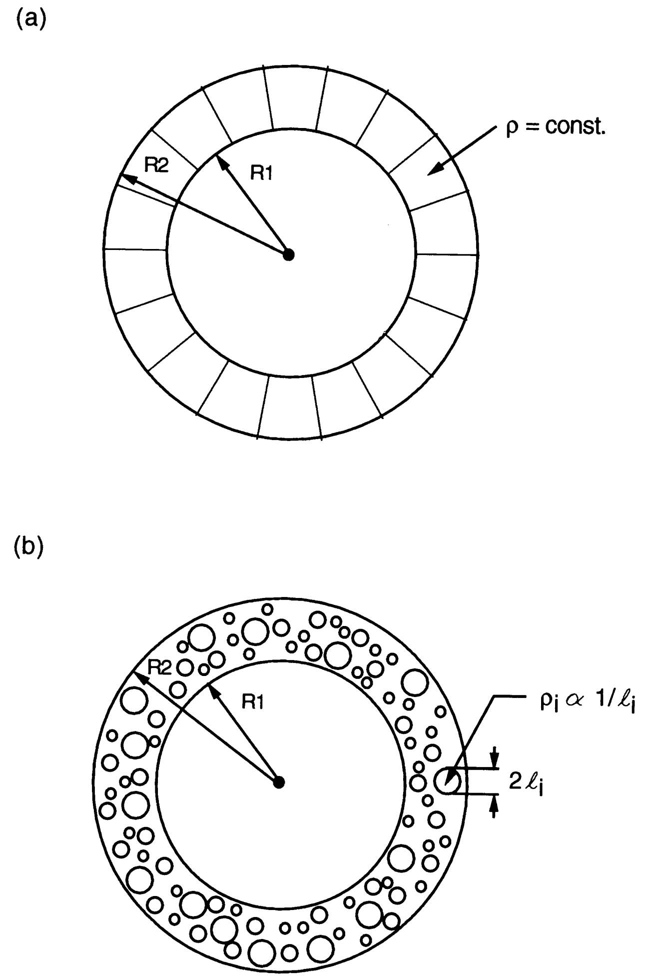

FIG. 1.—Cross section of wind from which continuum *f-f* emission or subordinate recombination line emission arises (radio and IR) for (*a*) a homogeneous wind and (*b*) a clumpy wind.

$$\rho_i = \frac{c_1}{l_i}\,, \tag{4}$$

where l_i is the radius ("length") of the *i*th clump. We confine the constant c_1 by taking $\rho_i = \rho$ in the limiting hypothetical case where the whole wind between R_1 and R_2 is effectively one big cell with the same density as above for a homogeneous wind, i.e.:

$$\rho = \frac{c_1}{l_0}\,, \tag{5}$$

where

$$\tfrac{4}{3}\pi l_0^3 = V_{\rm tot}\,. \tag{6}$$

Thus

$$c_1 = \rho\left(\frac{3}{4\pi}\,V_{\rm tot}\right)^{1/3}. \tag{7}$$

We furthermore adopt a flux spectrum as found by Robert & Moffat (1994) on the average for emission-line subpeaks in different W-R stars:

$$dN = N(F_i)dF_i , \quad (8)$$

where

$$N(F_i) = c_2 F_i^{\alpha} , \quad (9)$$

$$\alpha \simeq -2.5 , \quad (10)$$

and, by analogy with the homogeneous wind,

$$F_i = k\rho_i^2 V_i , \quad (11)$$

where

$$V_i = \tfrac{4}{3}\pi l_i^3 . \quad (12)$$

Hence for the *clumpy* wind:

$$V'_{\rm tot} = \int_{F_{\rm min}}^{F_{\rm max}} V_i N(F_i)dF_i = \frac{c_2 F_{\rm max}^{4+\alpha}}{(4\pi/3)^2 k^3 c_1^6 (4+\alpha)} (1 - \gamma^{4+\alpha}) , \quad (13)$$

where γ is the ratio of minimum to maximum flux of the clumps:

$$\gamma = \frac{F_{\rm min}}{F_{\rm max}} . \quad (14)$$

We also have for the clumpy wind:

$$M'_{\rm tot} = \int_{F_{\rm min}}^{F_{\rm max}} M_i N(F_i)dF_i = \frac{c_2 F_{\rm max}^{3+\alpha}}{(4\pi/3) k^2 c_1^3 (3+\alpha)} (1 - \gamma^{3+\alpha}) , \quad (15)$$

$$F'_{\rm tot} = \int_{F_{\rm min}}^{F_{\rm max}} F_i N(F_i)dF_i = \frac{c_2 F_{\rm max}^{2+\alpha}}{(2+\alpha)} (1 - \gamma^{2+\alpha}) . \quad (16)$$

By imposing:

$$V'_{\rm tot} = fV_{\rm tot} , \quad (17)$$

$$M'_{\rm tot} = mM_{\rm tot} , \quad (18)$$

$$F'_{\rm tot} = F_{\rm tot} , \quad (19)$$

we attempt to find m, the ratio of inhomogeneous to homogeneous mass in the shell for constant (observed) flux. Note that for a constant expansion velocity in the shell for each kind of wind, m also represents the ratio of mass-loss rates (inhomogeneous to homogeneous).

After substitution of equations (7), (13), (15), and (16) in equations (17), (18), and (19), we find:

$$c_2 F_{\rm max}^{4+\alpha}(1 - \gamma^{4+\alpha}) = k^3\rho^6(4+\alpha)fV_{\rm tot}^3 , \quad (20)$$

$$c_2 F_{\rm max}^{3+\alpha}(1 - \gamma^{3+\alpha}) = k^2\rho^4(3+\alpha)mV_{\rm tot}^2 , \quad (21)$$

$$c_2 F_{\rm max}^{2+\alpha}(1 - \gamma^{2+\alpha}) = k\rho^2(2+\alpha)V_{\rm tot} . \quad (22)$$

Our goal is to solve for m; to do this we solve equations (20) and (22) for $V_{\rm tot}$ and c_2. Substitution of these in equation (21) leads to

$$m = \frac{(2+\alpha)^{1/2}(4+\alpha)^{1/2}}{(3+\alpha)} \times \frac{(1 - \gamma^{3+\alpha})}{(1 - \gamma^{2+\alpha})^{1/2}(1 - \gamma^{4+\alpha})^{1/2}} f^{1/2} . \quad (23)$$

Note that m is independent of $V_{\rm tot}$, $F_{\rm max}$, ρ, k, and c_2, due to the normalization process.

Thus, for $\alpha = -2.5$, close to what is observed (Robert & Moffat 1994), and $f = 1$ (corresponding to an upper limit for m), we get

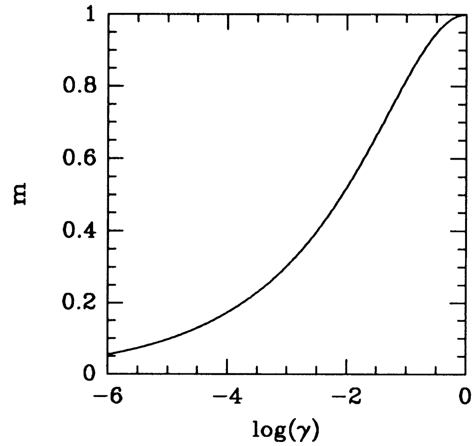

FIG. 2.—Mass (or mass-loss rate) ratio of clumpy shell to homogeneous shell for a filling factor $f = 1$, vs. the logarithm of the ratio of maximum to minimum flux of clumps: $\gamma = F_{\rm min}/F_{\rm max}$.

$$m = \left[3\gamma^{0.5}\left(\frac{1 - \gamma^{0.5}}{1 - \gamma^{1.5}}\right)\right]^{0.5} . \quad (24)$$

This yields

$$\lim_{\gamma \to 1} (m) = 1 , \quad (25)$$

as expected for no clumps, and

$$\lim_{\gamma \ll 1} (m) = \sqrt{3}\gamma^{0.25} . \quad (26)$$

3. DISCUSSION

In Figure 2, we show a plot of m as a function of log (γ) (for $f = 1$, i.e., with the reasonable assumption that there are no real density voids). This shows that correction factors (to multiply the homogeneous mass-loss rate to get the true rate) are $1/m = 1.2$, 2, 3, and 6 for $\gamma = 10^{-1}$, 10^{-2}, 10^{-3}, and 10^{-4}, respectively. Taken at face value, the *observed* range for W-R clumps over which $N(F)$ obeys a power law of mean index $\alpha \simeq -2.5$, is only Δ log (γ) $\lesssim 1.0$ (Robert & Moffat 1994). However, this is certainly a lower limit imposed by instrumental noise and the analysis technique for the inhomogeneities. A more realistic value can perhaps be estimated from more complete observation of cloudlets in GMCs, whose mass spectrum $N(M) \propto M^{-1.5}$, on average, with a spread in power index from ~ -1 to -2, depending on the age (Williams & Blitz 1993) is close to that for observed clumps in W-R winds: $N(M) \propto M^{(\alpha-1)/2} \propto M^{-1.75}$. [This follows from: $F \propto \rho^2 V$—i.e., neglecting temperature fluctuations, $\rho \propto l^{-1}$, $V \propto l^3$, $M \propto \rho V$, $N(F)dF = N(M)dM$, and $N(F) \propto F^{\alpha}$.] Sizes of GMC cloudlets vary over factors of at least 10^3 (Wolfire et al. 1992; Falgarone, Phillips, & Walker 1991), implying values of $\gamma = F_{\rm min}/F_{\rm max} = l_{\rm min}/l_{\rm max} \lesssim 10^{-3}$ or log (γ) $\lesssim -3$. If such a limit is also valid in W-R winds, this yields a factor $1/m \gtrsim 3$ to correct for the assumption of homogeneous winds.

4. CONCLUSION

If the clumps in W-R winds behave like cloudlets in GMCs as a result in both cases of supersonic turbulence in a compressible gas, then corrections of $\dot{M}$'s for inhomogeneous winds (by a factor $\gtrsim 3$) are compatible with observational estimates for the wind in V444 Cygni (which shows a difference of a factor $\gtrsim 3$), for which reliable $\dot{M}$'s independent of clumping versus those that are sensitive to clumping, are available.

If applicable to all W-R stars and indeed possibly all hot star winds, in view of the generally ubiquitous turbulence-induced variability of spectral lines from the winds of W-R (Robert 1992; Moffat et al. 1993; Robert & Moffat 1994) and O stars (Prinja & Howarth 1986, 1988), this means that *present estimates of most $\dot{M}$'s for hot stars based on smooth winds may have to be decreased by similar factors*. This would have an important impact on evolutionary models, where a change of $\dot{M}$ by even a factor of 2 produces remarkable changes in evolutionary tracks (Maeder 1991). Lower $\dot{M}$'s will also significantly diminish, or even eliminate, the so-called "momentum problem" in (even spherically symmetric) W-R winds, where the ratio of mechanical to photon momentum $\eta = \dot{M}v_{\infty}/(L/c)$, lies in the range $\eta \simeq 1$–30 (Willis 1991). If a correction factor $\gtrsim 3$ is necessary to decrease the wind flux due to inhomogeneities, revised values will be $\eta \lesssim 0.3$–10. Such values of η can probably be produced mainly in a strongly ionization-stratified model of W-R winds (Lucy & Abbott 1993).

However, better spectroscopic data of variable emission lines in W-R stars (simultaneous higher signal-to-noise ratio, spectral resolution and time resolution—a formidable challenge) will be necessary to derive more complete mass spectra of clumps and thus verify the correction obtained.

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