

## CAN THE DARK MATTER BE $10^6 M_{\odot}$ OBJECTS?

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### ABSTRACT

If the dark matter in galactic halos is made up of compact, macroscopic objects (MOs), such as black holes with  $M_{\text{MO}} \gg M_{\text{stars}}$ , gravitational scattering will lead to kinematic heating of the stars. Observational constraints on the amount of heating in the disk of the Milky Way put upper limits on  $M_{\text{MO}} \lesssim 10^{6.3} M_{\odot}$ . We find limits that are three orders of magnitude more stringent by examining the heating limits in low-mass stellar systems, where higher densities of dark matter and lower relative velocities would destroy stellar disks or disperse the stars in less than a billion years.

Limits on  $M_{\text{MO}}$  are derived from two nearby dwarf galaxies, dominated by an extended dark matter halo: the presence of a flat stellar disk in the dwarf spiral galaxy DDO 154 is shown to imply  $M_{\text{MO}} \lesssim 7 \times 10^5 M_{\odot}$ , comparable to the limits derived for the Galactic disk. However, the structure and kinematics of the Local Group member GR 8 yield a limit of  $M_{\text{MO}} \lesssim 6 \times 10^3 M_{\odot}$ .

We also examine the possibility that the local disk heating is done by compact clusters of brown dwarfs rather than black holes. Such clusters could dissolve in the higher density halos of small galaxies. While theoretical arguments have been presented for such clusters, they should have been detected in the *IRAS* point source catalog.

If the properties of the dark matter are universal, these results preclude the dominance of dark matter constituents in the cosmologically interesting mass range  $\sim 10^6 M_{\odot}$  and limit them to  $M_{\text{MO}} \lesssim 10^{3.7} M_{\odot}$ . These results also rule out massive compact halo objects as significant contributors to the kinematic heating of the Galactic disks.

*Subject headings:* dark matter — galaxies: kinematics and dynamics

### 1. INTRODUCTION

The self-gravity of many astrophysical systems is clearly dominated by dark matter. Starting from Zwicky's original claim of dark matter in clusters of galaxies, evidence for dark matter has been found on both larger scales and on smaller, galactic scales (see, e.g., Trimble 1987 and Ashman 1992 for reviews).

While the existence of dark matter has been clearly established, its nature is still unknown. Candidates currently under discussion range from postulated elementary particles to compact objects with masses vastly greater than stellar masses—a mass range of  $10^{70}$ ! Compelling cases have been made for candidates throughout this mass range. Our current effort is directed at constraining this vast range of possible masses.

Here we examine the case for “macroscopic” dark matter candidates, ranging from compact objects with substellar masses to objects with masses in excess of  $10^6 M_{\odot}$ , possibly massive black holes. Small objects and black holes are particularly attractive candidates, because they have a low luminosity per unit mass and are thus “dark.” The existence and mass scale of such compact massive objects can be constrained principally by two observable effects: gravitational lensing and gravitational relaxation.

Efforts to detect these effects for the most part focused on the halo of the Milky Way:

A search for gravitational lensing amplification of background stars from compact halo objects in the mass range from  $10^{-6}$  to  $10^2 M_{\odot}$  is currently being attempted by various groups (see e.g., Alcock et al. 1993). An extension to larger masses has been considered by Gould (1992).

Black holes with masses of  $2 \times 10^6 M_{\odot}$  would provide a source for the gravitational heating needed to explain the increase in velocity dispersion with age of the stellar disk population (Lacey & Ostriker 1985). Insisting that they do not heat the disk too quickly places an upper limit on the typical mass of compact halo objects of  $10^{6.3} M_{\odot}$ .

A variation on the latter scenario has been proposed by Carr & Lacey (1987), who suggested compact clusters of dark objects instead of single black holes. They advocated such clusters because they can dissolve before they collect at the Galactic center due to dynamical friction (see also Hut & Rees 1993) and because gas accretion events during the passage of such objects through a molecular cloud in the Milky Way are less luminous in X-rays. As far as gravitational relaxation is concerned, tightly bound clusters of macroscopic dark matter are indistinguishable from point masses, and all results derived below apply equally to both cases.

At present there are no firm dynamical limits on compact objects with  $M_{\text{MO}} \lesssim 10^{6.3} M_{\odot}$  comprising the ubiquitous dark matter. However, mass scales near this upper limit are particularly interesting, since they correspond to the Jeans mass just after the epoch of recombination and thus may collapse to

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great overdensities at high redshifts (e.g., Dicke & Peebles 1968; White & Rees 1978; Ashman & Carr 1988).

In this *Letter* we show that progress can be made in limiting  $M_{\text{MO}}$  by considering smaller and less massive stellar systems than the Milky Way that nevertheless contain a dominant dark matter halo. There, relaxation due to two-body encounters between the dark compact objects and the stars is stronger for two reasons: first, for a given  $M_{\text{MO}}$  less massive galaxies contain a smaller number of such massive objects and hence have a “grainier” potential well. Second, small systems have lower velocity dispersions making the energy exchange in two-body encounters more efficient.

The arguments presented here hold under two assumptions: (1) the properties of the dark matter are universal and (2) a characteristic mass scale,  $M_{\text{MO}}$ , can be assigned to the dark matter constituents. The kinetic “heating rate” due to objects of a given mass is proportional to their (individual) mass and their contribution to the mass density (eq. [1]). If there is a range of masses, say, with number density  $N \propto M^\gamma$  for  $M_{\text{light}} < M_{\text{MO}} < M_{\text{heavy}}$ , the light objects will dominate the heating if  $\gamma < -3$ , the heavy ones otherwise.

## 2. RELAXATIONAL HEATING

### 2.1. Basics

In this section we will review a few relations describing the statistical effects of two-body encounters in stellar dynamical systems. For a detailed discussion we refer to Chandrasekhar (1960), Lacey & Ostriker (1985), and Spitzer (1987). The basic assumptions made for the present work are that  $M_{\text{MO}} \gg M_{\text{stars}} \sim M_\odot$  and that the dark matter is assumed to dominate the mass density and hence the stars can be viewed as test particles. The net effect of these statistical encounters will result in an approach to kinetic equipartition, increasing the rms velocity of the observable stars by up to  $\sqrt{M_{\text{MO}}/M_{\text{stars}}}$ .

We also assume that the dark matter velocity distribution is approximately Maxwellian and has a dispersion substantially larger than that of the stars. Then the expected one-dimensional energy increase (per unit mass) of the stars can be found as (see, e.g., Spitzer 1987, eq. [2–60]):

$$\langle \dot{E}_S \rangle = \sqrt{32\pi} \ln \Lambda G^2 M_{\text{MO}} \rho_{\text{DM}} \sigma_{\text{DM}}^{-1} \quad (1)$$

where  $\ln \Lambda$  is the Coulomb logarithm,  $M_{\text{MO}}$  is the mass of the and  $\sigma_{\text{DM}}$  is the one-dimensional velocity dispersion of the dark matter. For relaxation of a system of  $N$  equal mass particles the Coulomb logarithm is  $\ln \Lambda \approx \ln N \approx \ln(M_{\text{tot}}/M_{\text{MO}})$  (e.g., Binney & Tremaine 1987), where  $M_{\text{tot}}$  is the total mass of dark matter in the volume under consideration. As detailed in § 3, we will deal with systems of  $M_{\text{tot}} \sim 10^7\text{--}10^8 M_\odot$  and will be interested in objects with  $M_{\text{MO}} \sim 10^4\text{--}10^5 M_\odot$ , implying  $\ln \Lambda \approx 7$ . This value is quite similar to the value of 10 used by Lacey & Ostriker (1985).

A limit on  $M_{\text{MO}}$  can then be obtained by demanding that:

$$\int \langle \dot{E}_S \rangle dt < \sigma_S^2/2, \quad (2)$$

where  $\sigma_S^2$  is the velocity distribution of the stars and the integral extends over the lifetime of the stars.

### 2.2. Halo Model

We assume that the observable stars reside within the core of a halo with a mass profile:

$$\rho(r) = \frac{\rho_{\text{DM}}}{[1 + (r/a)^2]^2} \quad (3)$$

(e.g., Richstone & Tremaine 1986). This model has core radius of  $R_{\text{DM}} = 0.77a$ , a total mass of  $M_{\text{tot}} = \pi^2 \rho_{\text{DM}} a^3$ , and a central velocity dispersion of

$$\sigma_{\text{DM}}^2 = 1.6G\rho_{\text{DM}}R_{\text{DM}}^2. \quad (4)$$

For a given luminosity  $L$  of the whole system, the central velocity dispersion scales as

$$\sigma_{\text{DM}} \propto \rho_{\text{DM}}^{5/6} \cdot (M/L)^{1/3}. \quad (5)$$

The observational input needed in equations (1) and (2) consists of (1) the kinetic energy of the tracer stars,  $\sigma_S$ , and (2) the central mass density of the halo,  $\rho_{\text{DM}}$ , or a lower limit on it. Test particles (stars, gas) with an isothermal distribution function of dispersion  $\sigma_S$  residing in the dark matter core will have a Gaussian radial profile with width  $H$ . The central mass density of the halo is then given by

$$\rho_{\text{DM}} = \frac{3\sigma_S^2}{4\pi GH^2}. \quad (6)$$

The velocity dispersion of the halo constituents,  $\sigma_{\text{DM}}$ , or an upper limits on them. According to equations (4) and (5) this is best phrased in terms of the total  $M/L$  of the system.

Rewriting equation (1) in terms of the observable quantities, the increase in velocity of the kinematic tracers (e.g., stars) after time  $\Delta t$  is

$$\Delta\sigma_S^2 = (28 \text{ km s}^{-1})^2 \left( \frac{\rho_{\text{DM}}}{5 \times 10^{-24} \text{ g cm}^{-3}} \right)^{5/6} \left( \frac{M_{\text{MO}}}{10^5 M_\odot} \right) \times \left( \frac{M/L}{130 M_\odot/L_\odot} \right)^{-1/3} \left( \frac{L}{2.5 \times 10^5/L_\odot} \right)^{1/3} \left( \frac{\Delta t}{10^9 \text{ yr}} \right). \quad (7)$$

The particular value for parameterizing  $M/L$  is taken from Lake (1991). The  $M/L$  should, however, be viewed as an essentially free parameter within an order of magnitude. It is worth noting how these limits depend on the adopted distance to the galaxy. Equation (7) implies that the derived limit  $M_{\text{MO}}$  scales as  $D^2$  through the observables  $\rho_{\text{DM}}$  and  $L$ . Here we focus on the heating of stars by MOs, even though relaxation may also affect the dark matter itself. However, in contrast to the heating (eqs. [1] and [2]), the relaxation time increases as  $\sigma_{\text{DM}}^3$  (or  $R_{\text{DM}}^3$ ) for fixed  $\rho_{\text{DM}}$ . Therefore, the relaxation time depends very sensitively on quantities that are not directly observable, and this timescale may exceed the lifetime of the stars in the galaxies discussed below. In general, the heating of stars in an extended halo will provide a limit on  $M_{\text{MO}}$  which is stronger by at least  $(\sigma_S/\sigma_{\text{DM}})^2$ .

## 3. SMALL STELLAR SYSTEMS DOMINATED BY DARK MATTER

### 3.1. The Dwarf Spiral Galaxy DDO 154

Carignan & Beaulieu (1989, hereafter CB89) and Lake, Schommer, & van Gorkom (1990) have drawn attention to the dynamics of a class of small gas-rich spiral galaxies (e.g., DDO 154, DDO 170) whose H I kinematics indicate the presence of large amounts of dark matter. At an adopted distance of 4 Mpc, DDO 154 has a luminosity of  $L_B \sim 5 \times 10^7 L_\odot$  and  $\rho_{\text{DM}} \sim 0.009 M_\odot \text{ pc}^{-3}$ ; however, it must be noted that the distance to DDO 154 is uncertain by a factor of  $\sim 2$ .

Since stellar disks are kinematically “coldest” in the  $z$  direction, the most drastic effect of relaxational heating will be the increase in the velocity dispersion perpendicular to the plane. The stellar surface brightness in DDO 154 is too low for kinematic measurements. However, an increase of the velocity dispersion will lead to an increased vertical scale height of the

stellar distribution. We can use the intrinsic flattening of the stellar distribution to put upper limits on their  $z$  velocity dispersion. From fitting the H I rotation curve CB89 derive an inclination of the system of  $63^\circ$  is derived, while the axis ratio of the light at two disk scale lengths is 0.57. This implies an intrinsic thickness of  $\sim H_z/R_{\text{exp}} = 0.3$ , where  $H_z$  denotes the vertical scale height and  $R_{\text{exp}}$  is the radial scale length. The vertical velocity dispersion at a radial scale length is given by  $\sigma_S \sim v_{\text{circ}} \times H_z/R_{\text{exp}}$ , if the self-gravity of the disk is neglected. This yields  $\sigma_S \sim 17 \text{ km s}^{-1}$ , for a circular velocity of  $\sim 50 \text{ km s}^{-1}$ .

The age of the stellar disk may be obtained by its azimuthal smoothness.  $I(0.8 \mu\text{m})$  band images done by one of us (H.-W. R) show that  $\Delta I/I < 0.1$  along an elliptical annulus. Consequently, the age of the stellar disk must be at least of order five orbital periods to allow for the azimuthal smoothing of the disk. At 2 kpc the orbital time is  $\sim 3 \times 10^8 \text{ yr}$ , and so we adopt  $\Delta t \gtrsim 1.5 \text{ Gyr}$ .

Using the above parameters and equation (7), the present flatness of the observed stellar disk implies for the mass of the individual dark matter constituents:

$$M_{\text{MO}} \lesssim 7 \times 10^5 M_\odot \left( \frac{\rho_{\text{DM}}}{0.009 M_\odot \text{ pc}^{-3}} \right)^{-5/6} \times \left( \frac{1.5 \times 10^9 \text{ yr}}{\Delta t} \right) \times \left( \frac{\Delta \sigma_z}{17 \text{ km s}^{-1}} \right)^2. \quad (8)$$

Here we have used here a parameterization different from equation (7), because  $\sigma_S$  is inferred indirectly. Note that the dependence of  $M_{\text{MO}}$  on the adopted distance enters only linearly in this estimate if the lower limit on the disk age is derived from kinematic considerations. Similar limits can be derived for DDO 170 (Lake et al. 1990).

Even though DDO 154 is a much lower mass object than the Milky Way, the limits on  $M_{\text{MO}}$  are not much more stringent than the ones derived from the Galactic disk. This is because we cannot place stringent limits on both  $\sigma_S$  and on the age of the disk population.

### 3.2. The Gas-rich Dwarf Galaxy GR 8

By far the best system in the local group to derive limits on  $M_{\text{MO}}$  is GR 8. Its distance is estimated as 1.1 Mpc (de Vaucouleurs & Moss 1983) with an uncertainty of  $\sim 50\%$  (see also Hoessel & Danielson 1983, who advocate a distance of 1.4 Mpc). For a distance of 1.1 Mpc it has a luminosity of  $L_B \sim 2.5 \times 10^6 L_\odot$ . It is the most extreme, gas-rich dwarf mapped in H I (Carignan, Beaulieu, & Freeman 1990). In the inner parts ( $R < 250 \text{ pc}$ ) the H I gas rotates at a speed of  $6\text{--}8 \text{ km s}^{-1}$ , becoming increasingly pressure supported with  $\sigma_{\text{HI}} \sim 10 \text{ km s}^{-1}$  throughout (the rotation drops below  $3 \text{ km s}^{-1}$  at 500 pc). Its radial surface density profile can be fitted by a Gaussian with  $H = 294 \text{ pc}$ . The stars have an exponential light profile with  $R_{\text{exp}}(B) = 76 \text{ pc}$ .

Although the velocities and dispersions measured for the H I gas are small ( $\lesssim 10 \text{ km s}^{-1}$ , comparable to the turbulent velocities in Galactic H I), they can nevertheless be used for dynamical mass estimates. In our Galaxy, H I clouds need not be in virial equilibrium because they can be confined by the hot phase of the ISM, which in turn is confined by the deep potential well of the Galactic halo. Since, GR 8 is much too small a galaxy to retain a hot, gaseous halo, the H I it cannot be pressure confined and must hence be held together by gravity.

Carignan et al. (1990) used the Jeans equation for radial equilibrium to account for the radial surface density profile of

the H I as well as for its rotation and dispersion and inferred a mass profile. The dynamical mass is found to exceed greatly the observed stellar and H I mass, indicating the presence of an extended dark halo. Using equation (6), one finds for the central dark matter density  $\rho_{\text{DM}} \sim 5 \times 10^{-24} \text{ g cm}^{-3} \sim 0.07 M_\odot \text{ pc}^{-3}$  (Carignan et al. 1990). The kinetic energy of the stars can be estimated through  $\sigma_S = \sigma_{\text{HI}} \times [R_{0.5}(T)/R_{0.5}(\text{H I})] = 3.7 \text{ km s}^{-1}$ , where  $R_{0.5}$  denotes the half-light, or half-mass radius of the stars and the neutral gas.

Requiring that  $\Delta \sigma^2 < \sigma_S^2$ , equation (7) yields

$$M_{\text{MO}} \lesssim 6 \times 10^3 M_\odot \left( \frac{10^9 \text{ yr}}{\Delta t} \right) \left( \frac{M/L}{130 M_\odot/L_\odot} \right)^{1/3} \quad (9)$$

as the limit on the masses of the halo constituents.

Theoretical arguments and observational constraints can be used to place a lower limit on the age of the stellar population in GR 8. Lake (1991) has shown that the redshift of the expected turn-around epoch for a system of GR 8's central density and of  $M/L \sim 130$  is  $z_{\text{turn}} > 7$ . Even at the extreme assumptions about  $M/L$ , one expects  $z_{\text{turn}}(\text{GR 8}) \gtrsim z_{\text{turn}}(\text{MW})$ . At the epoch of turnaround the gas in GR 8 had a higher density and could cool more efficiently than the Milky Way's gas. As a result, the galaxy could collapse and cool to its current radius much earlier than the Milky Way. Star formation should occur promptly if the galaxy obeys the empirical star formation laws observed for larger spirals (Kennicutt 1989). We note that Skillman (1993) finds that these empirical laws are supported by the star formation in the extreme dwarf galaxy IC 1613. Hence, it is most likely that star formation in GR 8 started at least as early as in the Milky Way disk.

This scenario is supported empirically by two observations: in their color-magnitude diagram of GR 8 Hoessel & Danielson (1983) find evidence for the presence of carbon stars: a group of red stars with  $V = 22.0 \text{ mag}$  and  $B - V = 2.0$ , just as expected for carbon stars at GR 8's distance. Since carbon stars are evolved stages of low-mass progenitors, they must be old. Second, spectral population synthesis by Hunter & Gallagher (1985) show that at  $5000 \text{ \AA}$  40% of the light comes from O/B stars and 60% arises from G/K/M stars. They argue on this basis that even though GR 8 has had a very recent (and ongoing) episode of star formation, it also contains a very significant old stellar population.

In this light it is most likely that  $\Delta t > 1 \text{ Gyr}$ , and consequently  $M_{\text{MO}} \lesssim 6 \times 10^3 M_\odot$ . This limit does not change much if we allow for the uncertainties in the other parameters: even if we considered extreme values for  $M/L$ , say 1000, and a distance as large as 2 Mpc, the derived limits are still two orders of magnitude below what is required for the local heating of the Galactic disk.

### 3.3. Do "Dark Clusters" Provide a Loophole?

Rather than black holes, Carr & Lacey (1987) proposed clusters having a total mass  $10^{6.3} M_\odot$  and typical size  $\sim 1 \text{ pc}$ , consisting of stellar or substellar objects, such as brown dwarfs. Such clusters could circumvent our previous limits if they dissolved in these higher density dwarf galaxies. For the particular values chosen by Carr & Lacey, clusters will dissolve by mutual collisions in a Hubble time if the galactic density is  $\sim 1 M_\odot \text{ pc}^{-3}$  (see Hogan & Rees 1988). If such clusters were larger by factors of a few, they could get disrupted in GR 8 ( $\rho \sim 0.1 M_\odot \text{ pc}^{-3}$ ), yet still survive at  $R_0$  in the Galaxy. By fine-tuning the cluster's density, one could conceive a scenario whereby the clusters survive in the Milky Way's halo and dissolve in the high-density halos of dwarf galaxies.



Remarkably, we find that such clusters of brown dwarfs have been ruled out by exploration of the *IRAS* point source catalog. Beichman et al. (1990) have examined all sources that have faint optical counterparts and found no brown dwarfs. If the dark matter in the solar neighborhood were brown dwarfs with  $\rho \sim 0.1 M_{\odot} \text{ pc}^{-3}$  (Oort 1932, 1960; Bahcall 1984), they find that, for a variety of brown dwarf mass functions, there should have been between 0.12 and 2.4 objects in their sample. If we instead adopt a dark matter density appropriate for the halo,  $10^{-2} M_{\odot} \text{ pc}^{-3}$  (Caldwell & Ostriker 1981; Kuijken & Gilmore 1989), the expected number falls by one order of magnitude.

If we put the brown dwarfs into clusters, the number of objects detected in a magnitude limited sample increases by  $N_{\text{cl}}^{1/2}$ , where  $N_{\text{cl}}$  is the number of brown dwarfs in a cluster. If the clusters had a mass of  $10^{6.3} M_{\odot}$ , then there would be as many as  $10^8$  objects in each cluster and the number that would appear in the *IRAS* point source catalog would increase by  $10^4$ . Hence, we would have expected that the *IRAS* catalog would have had between  $10^2$  and  $10^4$  brown dwarf clusters if they were indeed responsible for the disk heating. Clusters of planetary mass objects,  $M < 2M_{\text{Jupiter}}$ , are still permitted by the *IRAS* data.

Nonetheless, we are led to conclude that clusters of brown dwarfs are not a viable option for the dark matter in the Milky Way. The only permissible configurations are clusters with  $M_{\text{cl}} = 10^{6.3} M_{\odot}$  (suitable for the local disk heating) made of planets (to avoid *IRAS* detection) with a radius of  $\sim 5$  pc (to dissolve in GR 8 but not at  $R_0$  in the Milky Way).

#### 4. CONCLUSIONS

The arguments presented above considerably strengthen the case that the constituents of dark matter in galactic halos, if

they are macroscopic, must not have masses of more than a few  $1000 M_{\odot}$ ; these kinematic limits are three orders of magnitudes lower than previous limits derived from the heating of the Milky Way's disk. In particular, the cosmologically interesting mass range for massive, dark halo constituents,  $M_{\text{MO}} \sim 10^6 M_{\odot}$ , is inconsistent with the dynamical state of the nearby dwarf galaxy GR 8.

It was possible to improve these limits because the body of information on the dynamics of low-mass stellar systems, dominated by dark matter, has improved dramatically over the last 5 years. Seemingly promising candidates, the dwarf spiral galaxies DDO 154 and DDO 170 yielded only  $M_{\text{MO}} \lesssim 7 \times 10^5 M_{\odot}$ , comparable to the limits from the Milky Way disk. However, the observations of Carignan et al. (1990) of the structure of the H I gas and the stars in GR 8 imply  $M_{\text{MO}} < 6 \times 10^3 M_{\odot}$ . Their data preclude the possibility that the entire dark matter in GR 8 is provided by a central object, but indicate that it exists in an extended distribution.

The mass range of  $10^{-6} M_{\odot} < M_{\text{MO}} < 10^4$  of potential massive halo objects will be probed by microlensing experiments in the next few years (Paczynski 1986; Griest 1991; Gould 1992). The presence of all possible dark matter constituents with  $M_{\text{MO}} > 10^{-6} M_{\odot}$  can therefore be tested.

These results also exclude massive compact halo objects as significant contributors to the kinematic heating of the local Galactic disk. Finally, we found that the *IRAS* point source catalog completely rules out large clusters of brown dwarfs.

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